IEA Task 37 on System Engineering in Wind Energy
The Aerodynamic Only Optimization Case Study

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Abstract

This report outlines an aerodynamic only case study to understand how numerical design optimization could be applied in blade design. This report provides defines the optimization problems and outlines what information should be provided when researchers share their results.

Contents

1 Introduction

This report outlines an aerodynamic only case study to understand how numerical design optimization could be applied in blade design. The purpose of this case study is for multiple participants to test their optimization set-ups and share their results for comparison. This will help researchers and industry to understand the opportunities and challenges in applying numerical optimization in blade design.

This case study is not meant to perfectly represent industrial design practices. To achieve realistic designs, additional disciplines like structural dynamics, control theory and other disciplines need to be included. Such comprehensive optimization problems are expected to be quite complicated. Simple case studies like this one are meant to provide a simple baseline to help understand the results in these more complicated problems. Second, these simple case studies allow researchers to develop their system engineering capabilities gradually so more participants are able to perform the more complicated test cases later.

This report has three sections. The first (section 2), the optimization is based on an initial design that is described in this report and also given in a YAML file. The first section describes the coordinate systems and the various conventions used to describe this initial design. The YAML file represents a machine readable format so different groups have a common format to share data and run their models on these designs in an automated way. The format of the YAML file is described in this section. Finally, for the convenience of the reader the initial design information is also provided in this section.

The optimization problem is given in section 3. This describes the objective, the environmental conditions and the optimization constraints.

Finally section 4 explains how participants should report their results. It is desired that these participants share them in the same YAML format described in section 2. It is expected that there will be differences between the different results. To understand these differences participants are asked to fill out survey to describe their analysis packages.

2 The Initial Design

2.1 Terminology

The design description is given in terms of non-dimensional quantities common for wind turbine design. For clarification these terms are defined here. In equations, vector and matrix quantities are given in bold.

Tip Speed Ratio The ratio of the tip speed to the wind speed as defined by $\lambda = \frac{\omega R}{V}$ where ω is the rotational rate in radians per second, R is the radius of the rotor and V is the wind speed.

Relative Thickness The ratio of the airfoil thickness to the chord as defined by $r = \frac{t}{c}$ where t is the absolute thickness, c is the chord.

Coefficient of Lift The non-dimensional lift force as defined by $C_l = \frac{2F_l}{\rho c W^2}$ where f_l is the distributed lift force, ρ is the fluid density, c is the chord, W is the fluid velocity relative to the airfoil.

Coefficient of Drag The non-dimensional drag force as defined by $C_d = \frac{2F_d}{\rho c W^2}$ where f_d is the distributed drag force, ρ is the fluid density, c is the chord, W is the fluid velocity relative to the airfoil.

Coefficient of Moment The non-dimensional drag force as defined by $C_m = \frac{2m_z}{\rho c^2 W^2}$ where m_z is the distributed pitching moment, ρ is the fluid density, c is the chord, W is the fluid velocity relative to the airfoil.

2.2 The design description conventions

In the blade description, there is a coordinate system that is attached to the blade such that the positive $\boldsymbol{Z}$ direction points in the radial direction of the rotor plane, the positive $\boldsymbol{Y}$ direction point in the downwind direction and the positive $\boldsymbol{X}$ direction points in the direction of the blades rotation. This coordinate system rotates with rotor and rotor tilt, but does not rotate with deformation, sweep, pre-bend, coning or any other changes in blade shape. Since it rotates with the rotor, it is not a global coordinate system, so it will be referred to as the blade coordinate system. In this coordinate system, the blade rotates in the clock-wise direction when viewed from an up-wind location. This is shown in Figure 1 for the lower blade.

The distributed design data is described along a non-dimensional S parameters where 0 represents the root of the blade and 1 is the tip of the blade. The shape of the blade is given by a set of X_b, Y_b and Z_b coordinates for a set of S coordinates.

Along this curve, there are a set of local coordinate system for each cross sections. In this local coordinate system the $\boldsymbol{Z}'$ axis is tangent to the blade curve and the $\boldsymbol{X}'$ axis stays within the rotor plane described by the blade coordinates $\boldsymbol{Z}$ and $\boldsymbol{X}$ axis. This coordinate system is rotated with prebend, sweep, coning and other curves in the blade. For this problem, the initial blade shape and optimal blade shape are expected to remain straight so the $\boldsymbol{X}'$, $\boldsymbol{Y}'$ and $\boldsymbol{Z}'$ coordinate system is the same as the original $\boldsymbol{X}$, $\boldsymbol{Y}$ and $\boldsymbol{Z}$ system. This rotated coordinate system is defined here now because future optimization cases are expected to include pre-bend and possibly sweeps. Researchers should make their pre-processing tools to conform the curved coordinate system. The curved coordinate system can be related to the global coordinate system with equations (1a) through (1c).

$$\boldsymbol{Z}' = \frac{\frac{dX_b}{dS}\hat{\imath} + \frac{dY_b}{dS}\hat{\jmath} + \frac{dZ_b}{dS}\hat{k}}{\left\| \frac{dX_b}{dS}\hat{\imath} + \frac{dY_b}{dS}\hat{\jmath} + \frac{dZ_b}{dS}\hat{k} \right\|} \tag{1a}$$

$$\boldsymbol{X}' = \frac{\boldsymbol{Y} \times \boldsymbol{Z}'}{\|\boldsymbol{Y} \times \boldsymbol{Z}'\|} \tag{1b}$$

$$\boldsymbol{Y}' = \frac{\boldsymbol{Z}' \times \boldsymbol{X}'}{\|\boldsymbol{Z}' \times \boldsymbol{X}'\|} \tag{1c}$$

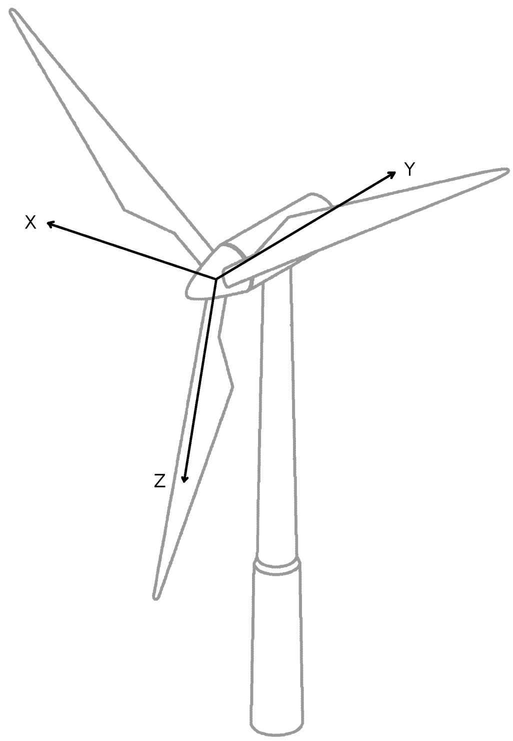

Figure 1: Coordinate System for the Lower Blade

The profile shapes are given in a normalized $\boldsymbol{x}-\boldsymbol{y}$ coordinate system where the leading edge is placed at the origin, and the trailing edge is placed at $(1,0)$. Note that the negative $\boldsymbol{x}$ direction corresponds to the positive $\boldsymbol{X}$ direction. The profile shapes for each airfoil are described by a set of coordinates that follow the profile shape. These points defines a two dimensional curve, that can be parameterized by a curve coordinate s where 0 is the beginning of the curve and 1 is the end of the curve.

The cross section shape for the blended profile is given by a linear blend of these airfoil shapes. This blended shape is given by a weight for each cross section. The shape is blended by re-mapping the points so the profile coordinates are given at a common set of s curve coordinates. Then the corresponding points are blended linearly according to the blending weights to obtain the final shape.

The final shape of the blade is obtained by first applying the normalized shift which moves the profile in the positive $\boldsymbol{X}'$ direction (or negative $\boldsymbol{x}$ direction). Initially the leading edge of the cross section points in the $\boldsymbol{X}'$ direction. Then the twist is applied, the twist is given in degrees and a positive twist rotates the profile about the $\boldsymbol{Z}'$ axis so the leading edge points more in the upwind direction. Then the profile shape is scaled according to the chord parameter to obtain the final shape. This is defined mathematically in terms the blade curve coordinate S and the profile curve coordinate s as shown in equation in equation (2). Where X_s, Y_s and Z_s denotes the position of a surface point in the blade coordinate system. The reference curve for the blade is denoted by X_b, Y_b and Z_b. The rotation matrix T' rotates vectors from the local coordinate system to the blade coordinate system. The twist is given by θ and the chord is given by c. The profile shape is given by x_p and y_p. The lateral shift is denoted by x_l.

$$\left\{\begin{array}{c} X_s(S,s) \\ Y_s(S,s) \\ Z_s(S,s) \end{array}\right\} = \left\{\begin{array}{c} X_b(S) \\ Y_b(S) \\ Z_b(S) \end{array}\right\} + c(S)\boldsymbol{T}' \begin{bmatrix} -\cos(\theta(S)) & -\sin(\theta(S)) & 0 \\ -\sin(\theta(S)) & \cos(\theta(S)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{\begin{array}{c} x_p(s) - x_l(S) \\ y_p(s) \\ 0 \end{array}\right\} \tag{2}$$

For aerodynamic models based on lift drag and moment coefficients, a set of coefficients are given for the angles of attack between $-180°$ and $180°$. These coefficients correspond to the most of the airfoils given for the profile shapes with the exception of the TC72 shape. Also, there is an additional set of coefficients for the FFA-W3-600 airfoil. The coefficients for the blended airfoils can be obtained by linearally blending the lift, drag and moment curves. A set of linear blending weights are given along the blade to describe how these coefficients should be blended. Since the set of airfoils for the shape and the coefficients are different, there are two sets of blending weights for each set.

2.3 The YAML file conventions

The YAML format is for encoding structured data with nested mappings, lists and values. Data is identified by keys, followed by the corresponding values. YAML allows for different data structures to be nested under these keys. For the sake of brevity, this document will not explain the details of the YAML syntax, the reader should refer to YAML tutorials for these details. We have defined a set of keys to correspond with the data given in this report. This section explains the structure of the YAML file we have chosen and how these keys are mapped.

The YAML format for this project has the following sections: 'input_format_version', 'planform', 'airfoils', 'control', 'environmental', 'optimization'. Each are described separately with an example of the YAML file given below. All the examples can be combined to create the complete YAML file used to communicate the design data here.

This is the first YAML file convention for communicating design data. It is expected that revisions will be made in the future. For the sake of maintaining backwards compatibility, the 'input_format_version' is used to indicate the format version used for a given file.

```
input_format_version: 0
```

The next section 'planform' is used to provide the design variables that describe the blade shape. This section is essentially a table of values. The first variable is 'SParam', this gives a list[1] of locations along the blades reference curve where design information is given. All other variables will have the same number of values. The variables 'Chord', 'Twist', 'X', 'Y', 'Z' and 'Lateral_Shift' are self explanatory and correspond to the variables described in section 2.2. The key 'Airfoil_Coefficient_Blending_Weights' gives the blending weights for the airfoil coefficients and 'Airfoil_Shape_Blending_Weights' the weights for the airfoil shape. In both of these keys is a nested collection of keys that correspond to airfoils given in the 'airfoils' section. Then under each of these keys the weights are given.

This is the section users will have to fill out and provide to describe their optimal designs. Only the 'Chord', 'Twist' and 'Blending_Weights' should be modified when entering your optimal design solution.

```
planform:
    hub_radius: 2.8
    SParam:
        - 0.00000
        - 0.03057
        ...
    Chord:
        - 5.380
        - 5.380
        ...
    Twist:
        - 14.500
        - 14.500
        ...
    X:
        - 0.0
        - 0.0
        ...
    Y:
        - 0.0
        - 0.0
        ...
    Z:
        - 0.000
        - 2.643
        ...
    Lateral_Shift:
        - 0.500
        - 0.500
        ...
    Airfoil_Coefficient_Blending_Weights:
```

[1] The '...' in the YAML file example indicates a continuation of data. The data has been truncated in the examples and replaced with this symbol throughout

```
        FFA_W3_600:
            - 0.0
            - 0.008
            ...
        Cylinder:
            - 1.0
            - 0.992
            ...
    Airfoil_Shape_Blending_Weights:
        TC_72:
            - 0.0
            - 0.012
            ...
        Cylinder:
            - 1.0
            - 0.988
            ...
        ...
```

The ‘airfoils’ key gives a collection of airfoils. Each airfoil is identified by a key, followed by properties identified by the nested keys. The property names are self explanatory. Each airfoil is not required to provide all the different attributes. Where airfoil coefficients are given, there is one key that lists the angles of attack where this information is given, then additional keys for the various coefficients. The number of values in these lists must be consistent.

```
airfoils:
    FFA_W3_600:
        Relative_Thickness: 0.600
        Angle_Of_Attack:
            - −180.000
            - −175.000
            ...
        Cl:
            - 0.000000
            - 0.173600
            ...
        Cd:
            - 0.000000
            - 0.009900
            ...
        Cm:
            - 0.000000
            - 0.021800
            ...
    TC_72:
        Relative_Thickness: 0.720
        Shape:
            - [0.99937, −0.02908]
            - [0.98757, −0.03411]
            ...
    Cylinder:
        Relative_Thickness: 1.000
        Shape:
            - [1.00000, 0.00000]
            - [0.99901, −0.03140]
            ...
        Angle_Of_Attack:
            - −180.000
            - −175.000
            ...
        Cl:
            - 0.000000
```

```yaml
            - 0.000000
            ...
        Cd:
            - 0.600000
            - 0.600000
            ...
        Cm:
            - 0.000000
            - 0.000000
            ...
    ...
```

The ‘control’ section given here only gives the minimum amount of information to solve this particular problem and may not be sufficient to provide controller information in general. The key names are self explanatory and correspond to variables described in section 3.

```yaml
control:
    Maximum_Mechanical_Power: 10638300
    Design_Tip_Speed_Ratio: 7.8
    Minimum_Pitch: 0.0
    Minimum_Rotational_Rate: 6.0
    Maximum_Rotational_Rate: 9.6
```

The ‘environment’ describes the reference wind resource that should be used in this optimization. The ‘Turbulence’ and ‘Shear’ keys are left empty to indicate that there is no shear or turbulence. The ‘Wind_Speed_Frequency’ gives the probability distribution for the wind speeds, in this problem it is Weibull distribution with a scale and shape parameter. These variables are explained in greater detail in section 3.

```yaml
environment:
    Air_Density: 1.225
    Turbulence:
    Shear:
    Wind_Speed_Frequency:
        Distribution: 'Weibull'
        Scale_Param: 8.0
        Shape_Param: 2.0
```

The ‘optimization’ section is used to specify optimization parameters specific to this problem. This may not be sufficient to describe general wind turbine optimization problems. Basically, this section gives ‘Objectives’, the ‘Design_Variables’ and ‘Constraints’. Within each of these keys is a second set of keys that give the names for the various variables. These are user defined names that describe the entry. These names are then followed by the attributes. These attributes are self explanatory. The attribute ‘Variable’ is given as a place-holder, it is meant to give the YAML key that corresponds to that objective, design variable or constraint.

```yaml
optimization:
    Objectives:
        AEP:
            Is_Minimization: False
            Variable: 'AEP'
    Design_Variables:
        Chord:
            Lower_Limit:
            Upper_Limit:
            Variable: 'Chord'
        Twist:
            Lower_Limit:
            Upper_Limit:
            Variable: 'Twist'
        Airfoil_Coefficients:
            Lower_Limit: 0.0
            Upper_Limit: 1.0
            Variable: 'Airfoil_Coefficient_Blending_Weights'
        Airfoil_Shape:
```

```
            Lower_Limit: 0.0
            Upper_Limit: 1.0
            Variable: 'Airfoil_Shape_Blending_Weights'
    Constraints:
        Rotor_Thrust:
            Lower_Limit:
            Upper_Limit: 1.14
            Equality:
            Variable: 'Normalized_Rotor_Thrust'
        Root_Flap_Wise_Bending_Moment:
            Lower_Limit:
            Upper_Limit: 1.11
            Equality:
            Variable: 'Normalized_Root_Flap_Wise_Bending_Moment'
        Absolute_Thickness:
            Lower_Limit:
                SParam:
                    - 0.00000
                    - 0.03057
                    ...
                Limit:
                    - 5.38
                    - 5.3630
                    ...
            Upper_Limit:
            Equality:
            Variable: 'Absolute_Thickness'
```

2.4 The initial design data

The initial design for this case study is based on the DTU 10MW Reference Wind Turbine Design (RWT). The initial RWT design was already aerodynamically optimized and provided the optimization little room to improve the design. To give optimization more room, the chord and twist distributions were modified so the initial design is sub-optimal and the optimization had room to make improvements. The data for this initial design is given in the following sections.

To simplify the optimization, the initial and optimal solution should not have any tilt or coning.

2.4.1 Planform data

The planform data is given in Table 1. The blending weights for the airfoil coefficients are given in Table 2. The blending weights for the profile shapes are given in Table 3. Some of the planform data is graphed in Figure 2 to 5. The relative thickness for these airfoils is required to calculate the absolute thickness and ensure geometric constraints are satisfied. These thicknesses are given in table 4.

Table 1: The Initial Planform Design

S	$\boldsymbol{X}$	$\boldsymbol{Y}$	$\boldsymbol{Z}$	Twist [deg]	Chord [m]	Lateral Shift [-]
0.00000	0.0	0.0	0.000	14.500	5.380	0.500
0.03057	0.0	0.0	2.643	14.500	5.380	0.500
0.06222	0.0	0.0	5.380	14.500	5.380	0.500
0.09487	0.0	0.0	8.203	14.445	5.474	0.494
0.12841	0.0	0.0	11.103	14.222	5.620	0.472
0.16274	0.0	0.0	14.071	13.727	5.727	0.440
0.19772	0.0	0.0	17.095	12.959	5.730	0.412
0.23321	0.0	0.0	20.164	12.078	5.630	0.391
0.26907	0.0	0.0	23.265	11.236	5.475	0.376
0.30515	0.0	0.0	26.384	10.476	5.325	0.365
0.34128	0.0	0.0	29.508	9.806	5.190	0.358
0.37731	0.0	0.0	32.623	9.206	5.044	0.353
0.41309	0.0	0.0	35.716	8.656	4.869	0.350
0.44846	0.0	0.0	38.773	8.102	4.676	0.350
0.48328	0.0	0.0	41.782	7.517	4.473	0.350
0.51741	0.0	0.0	44.732	6.916	4.263	0.350
0.55073	0.0	0.0	47.611	6.315	4.052	0.350
0.58312	0.0	0.0	50.410	5.727	3.844	0.350
0.61449	0.0	0.0	53.120	5.164	3.642	0.350
0.64476	0.0	0.0	55.734	4.635	3.449	0.350
0.67385	0.0	0.0	58.247	4.147	3.265	0.350
0.70172	0.0	0.0	60.653	3.701	3.093	0.350
0.72833	0.0	0.0	62.950	3.298	2.931	0.350
0.75364	0.0	0.0	65.135	2.935	2.780	0.350
0.77766	0.0	0.0	67.208	2.608	2.641	0.350
0.80038	0.0	0.0	69.167	2.314	2.512	0.350
0.82181	0.0	0.0	71.016	2.045	2.395	0.350
0.84197	0.0	0.0	72.755	1.797	2.286	0.350
0.86089	0.0	0.0	74.386	1.566	2.187	0.350
0.87861	0.0	0.0	75.913	1.350	2.097	0.350
0.89517	0.0	0.0	77.340	1.148	2.011	0.350
0.91061	0.0	0.0	78.671	0.959	1.925	0.350
0.92498	0.0	0.0	79.908	0.784	1.836	0.350
0.93833	0.0	0.0	81.059	0.625	1.741	0.350
0.95072	0.0	0.0	82.125	0.483	1.642	0.350
0.96220	0.0	0.0	83.113	0.358	1.532	0.350
0.97281	0.0	0.0	84.026	0.251	1.402	0.350
0.98262	0.0	0.0	84.870	0.157	1.294	0.350
0.99166	0.0	0.0	85.649	0.070	1.176	0.350
1.00000	0.0	0.0	86.366	0.000	1.035	0.350

Table 2: The blending weights for the airfoil coefficients

S	Cylinder	FFA-W3-600	FFA-W3-480	FFA-W3-360	FFA-W3-301	FFA-W3-241
0.00000	1.000					
0.03057	0.992	0.008				
0.06222	0.883	0.117				
0.09487	0.720	0.280				
0.12841	0.530	0.470				
0.16274	0.325	0.675				
0.19772	0.119	0.881				
0.23321		0.768	0.232			
0.26907		0.250	0.750			
0.30515			0.879	0.121		
0.34128			0.602	0.398		
0.37731			0.393	0.607		
0.41309			0.228	0.772		
0.44846			0.065	0.935		
0.48328				0.805	0.195	
0.51741				0.484	0.516	
0.55073				0.171	0.829	
0.58312					0.870	0.130
0.61449					0.545	0.455
0.64476					0.301	0.699
0.67385					0.146	0.854
0.70172					0.057	0.943
0.72833					0.014	0.986
0.75364						1.000
0.77766						1.000
0.80038						1.000
0.82181						1.000
0.84197						1.000
0.86089						1.000
0.87861						1.000
0.89517						1.000
0.91061						1.000
0.92498						1.000
0.93833						1.000
0.95072						1.000
0.96220						1.000
0.97281						1.000
0.98262						1.000
0.99166						1.000
1.00000						1.000

Table 3: The blending weights for the cross-section shape

S	Cylinder	TC72	FFA-W3-480	FFA-W3-360	FFA-W3-301	FFA-W3-241
0.00000	1.000					
0.03057	0.988	0.012				
0.06222	0.832	0.168				
0.09487	0.597	0.403				
0.12841	0.323	0.677				
0.16274	0.028	0.972				
0.19772		0.693	0.307			
0.23321		0.381	0.619			
0.26907		0.124	0.876			
0.30515			0.879	0.121		
0.34128			0.602	0.398		
0.37731			0.393	0.607		
0.41309			0.228	0.772		
0.44846			0.065	0.935		
0.48328				0.805	0.195	
0.51741				0.484	0.516	
0.55073				0.171	0.829	
0.58312					0.870	0.131
0.61449					0.545	0.455
0.64476					0.301	0.699
0.67385					0.146	0.854
0.70172					0.057	0.943
0.72833					0.014	0.986
0.75364						1.000
0.77766						1.000
0.80038						1.000
0.82181						1.000
0.84197						1.000
0.86089						1.000
0.87861						1.000
0.89517						1.000
0.91061						1.000
0.92498						1.000
0.93833						1.000
0.95072						1.000
0.96220						1.000
0.97281						1.000
0.98262						1.000
0.99166						1.000
1.00000						1.000

Table 4: The relative thickness of profiles shapes

Airfoil	Relative Thickness
FFA-W3-241	24.1%
FFA-W3-301	30.1%
FFA-W3-360	36.0%
FFA-W3-480	48.0%
FFA-W3-600	60.0%
TC72	72.0%
Cylinder	100.0%

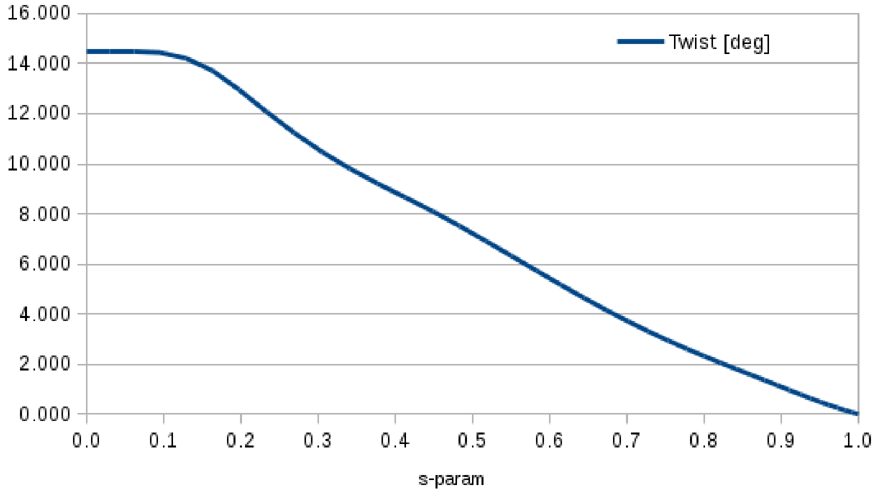

Figure 2: Initial Twist

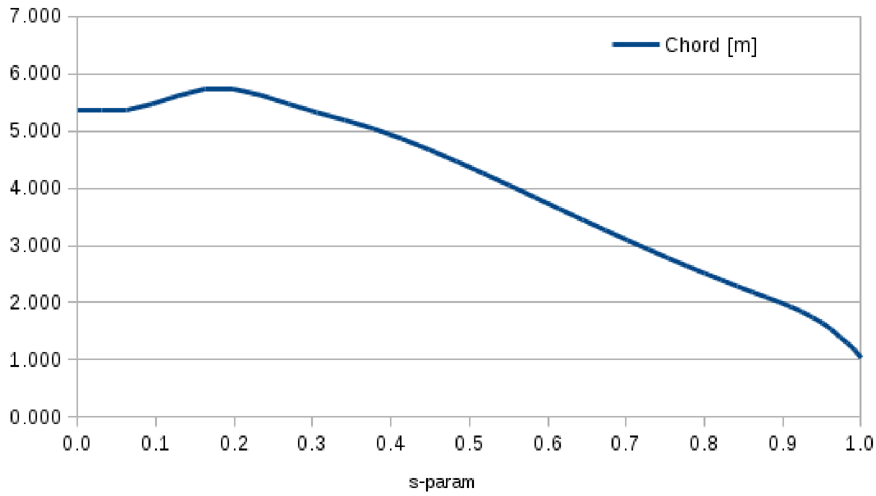

Figure 3: Initial Chord

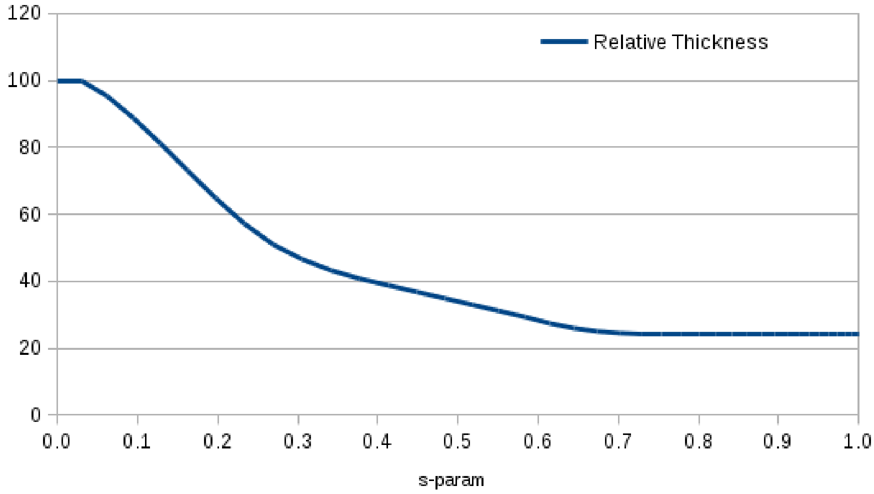

Figure 4: Initial Relative Thickness

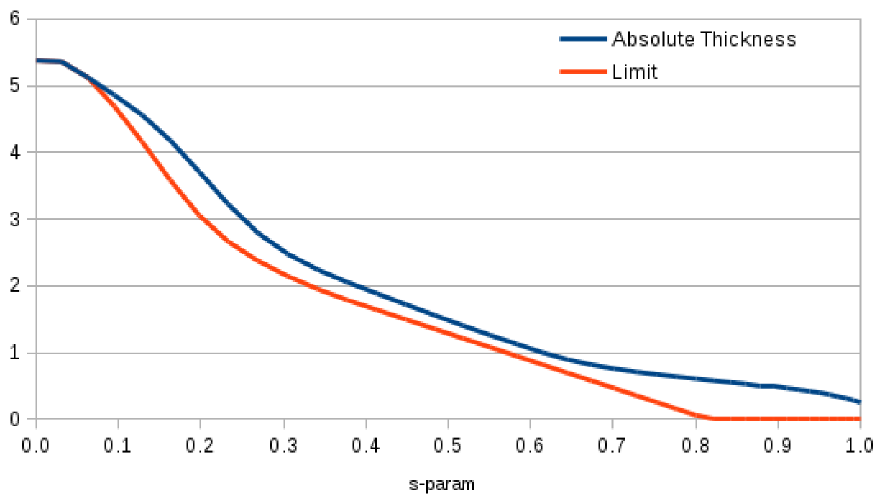

Figure 5: Initial Absolute Thickness

2.4.2 Airfoil coefficients

The initial design and the subsequent optimized design is based on the FFA-W3 airfoil family. The coefficients for these airfoils are given in Tables 5 through 8 and graphed in Figures 6 through 8. These airfoil coefficients were solved using Ellipsis 2D for Reynolds numbers of 10×10^6. The Reynolds number for the FFA-W3-241 airfoil data is 12×10^6. The turbulence intensity was 10% and a free transition model was used.

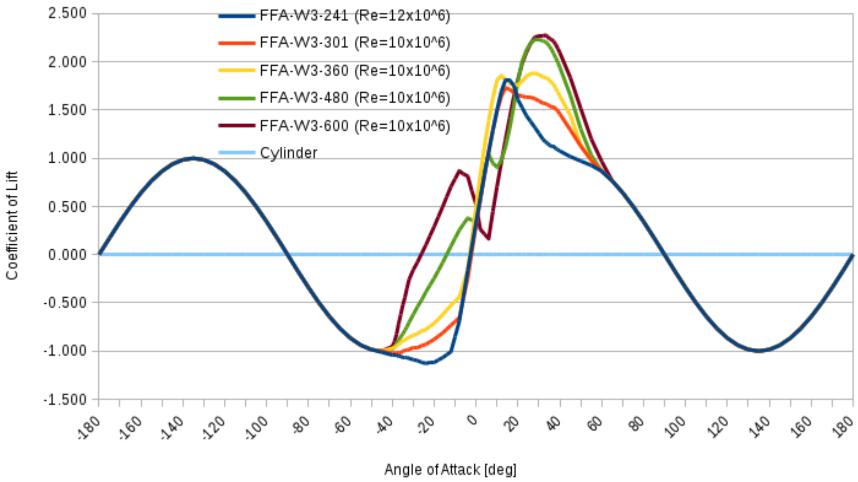

Figure 6: Coefficients of Lift

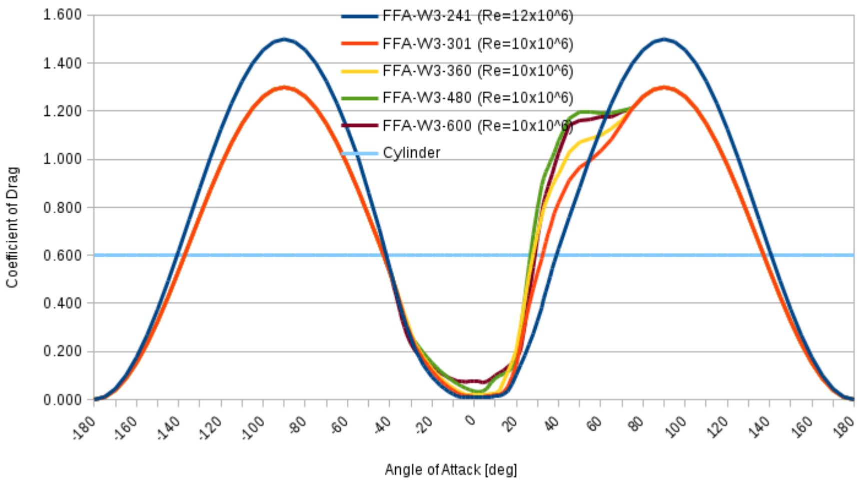

Figure 7: Coefficients of Drag

Table 5: Airfoil Coefficients for Thin Airfoils at Negative Angles of Attack

	FFA-W3-241			FFA-W3-301			FFA-W3-360		
AoA [deg]	C_l	C_d	C_m	C_l	C_d	C_m	C_l	C_d	C_m
-180	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-175	0.1736	0.0114	0.0218	0.1736	0.0099	0.0218	0.1736	0.0099	0.0218
-170	0.3420	0.0452	0.0434	0.3420	0.0392	0.0434	0.3420	0.0392	0.0434
-165	0.5000	0.1005	0.0647	0.5000	0.0871	0.0647	0.5000	0.0871	0.0647
-160	0.6428	0.1755	0.0855	0.6428	0.1521	0.0855	0.6428	0.1521	0.0855
-155	0.7660	0.2679	0.1057	0.7660	0.2322	0.1057	0.7660	0.2322	0.1057
-150	0.8660	0.3750	0.1250	0.8660	0.3250	0.1250	0.8660	0.3250	0.1250
-145	0.9397	0.4935	0.1434	0.9397	0.4277	0.1434	0.9397	0.4277	0.1434
-140	0.9848	0.6197	0.1607	0.9848	0.5371	0.1607	0.9848	0.5371	0.1607
-135	1.0000	0.7500	0.1768	1.0000	0.6500	0.1768	1.0000	0.6500	0.1768
-130	0.9848	0.8803	0.1915	0.9848	0.7629	0.1915	0.9848	0.7629	0.1915
-125	0.9397	1.0065	0.2048	0.9397	0.8723	0.2048	0.9397	0.8723	0.2048
-120	0.8660	1.1250	0.2165	0.8660	0.9750	0.2165	0.8660	0.9750	0.2165
-115	0.7660	1.2321	0.2266	0.7660	1.0678	0.2266	0.7660	1.0678	0.2266
-110	0.6428	1.3245	0.2349	0.6428	1.1479	0.2349	0.6428	1.1479	0.2349
-105	0.5000	1.3995	0.2415	0.5000	1.2129	0.2415	0.5000	1.2129	0.2415
-100	0.3420	1.4548	0.2462	0.3420	1.2608	0.2462	0.3420	1.2608	0.2462
-95	0.1736	1.4886	0.2490	0.1736	1.2901	0.2490	0.1736	1.2901	0.2490
-90	0.0000	1.5000	0.2500	0.0000	1.3000	0.2500	0.0000	1.3000	0.2500
-85	-0.1736	1.4886	0.2490	-0.1736	1.2901	0.2490	-0.1736	1.2901	0.2490
-80	-0.3420	1.4548	0.2462	-0.3420	1.2608	0.2462	-0.3420	1.2608	0.2462
-75	-0.5000	1.3995	0.2415	-0.5000	1.2129	0.2415	-0.5000	1.2129	0.2415
-70	-0.6428	1.3245	0.2349	-0.6428	1.1479	0.2349	-0.6428	1.1479	0.2349
-65	-0.7660	1.2321	0.2266	-0.7660	1.0678	0.2266	-0.7660	1.0678	0.2266
-60	-0.8660	1.1250	0.2165	-0.8660	0.9750	0.2165	-0.8660	0.9750	0.2165
-55	-0.9397	1.0065	0.2048	-0.9397	0.8723	0.2048	-0.9397	0.8723	0.2048
-50	-0.9848	0.8603	0.1915	-0.9848	0.7629	0.1915	-0.9848	0.7629	0.1915
-45	-1.0120	0.7120	0.1708	-1.0000	0.6500	0.1768	-1.0000	0.6500	0.1768
-40	-1.0376	0.5475	0.1416	-1.0216	0.5352	0.1416	-0.9716	0.5352	0.1416
-39	-1.0419	0.5165	0.1346	-1.0239	0.5066	0.1346	-0.9639	0.5136	0.1346
-38	-1.0462	0.4816	0.1276	-1.0162	0.4851	0.1276	-0.9462	0.4851	0.1276
-37	-1.0525	0.4487	0.1185	-1.0185	0.4565	0.1205	-0.9285	0.4635	0.1205
-36	-1.0568	0.4177	0.1095	-1.0108	0.4279	0.1135	-0.9108	0.4349	0.1135
-35	-1.0611	0.3848	0.1005	-1.0031	0.3994	0.0965	-0.8931	0.4064	0.0965
-34	-1.0654	0.3558	0.0894	-0.9954	0.3708	0.0794	-0.8854	0.3778	0.0794
-33	-1.0717	0.3289	0.0824	-0.9877	0.3353	0.0624	-0.8677	0.3523	0.0624
-32	-1.0765	0.3031	0.0678	-0.9835	0.3097	0.0515	-0.8564	0.3315	0.0447
-30	-1.0889	0.2560	0.0508	-0.9703	0.2663	0.0389	-0.8378	0.2777	0.0327
-28	-1.0993	0.2090	0.0337	-0.9672	0.2229	0.0263	-0.8191	0.2338	0.0208
-26	-1.1168	0.1756	0.0206	-0.9441	0.1941	0.0154	-0.7924	0.2045	0.0102
-24	-1.1282	0.1423	0.0075	-0.9310	0.1654	0.0045	-0.7756	0.1751	-0.0004
-22	-1.1215	0.1183	-0.0008	-0.9047	0.1417	-0.0039	-0.7441	0.1513	-0.0089
-20	-1.1148	0.0943	-0.0091	-0.8784	0.1181	-0.0123	-0.7126	0.1274	-0.0174
-18	-1.0919	0.0765	-0.0123	-0.8459	0.0986	-0.0175	-0.6678	0.1085	-0.0229
-16	-1.0691	0.0587	-0.0156	-0.8134	0.0792	-0.0227	-0.6231	0.0896	-0.0285
-14	-1.0379	0.0454	-0.0156	-0.7728	0.0643	-0.0235	-0.5742	0.0748	-0.0300
-12	-1.0067	0.0321	-0.0155	-0.7322	0.0495	-0.0244	-0.5252	0.0600	-0.0314
-10	-0.8479	0.0230	-0.0318	-0.6935	0.0381	-0.0227	-0.4827	0.0485	-0.0279
-8	-0.6892	0.0138	-0.0480	-0.6547	0.0267	-0.0210	-0.4402	0.0370	-0.0244
-6	-0.4278	0.0118	-0.0611	-0.4507	0.0204	-0.0389	-0.2983	0.0294	-0.0359
-4	-0.1665	0.0098	-0.0742	-0.2467	0.0140	-0.0569	-0.1564	0.0219	-0.0474
-2	0.0863	0.0095	-0.0811	0.0295	0.0129	-0.0717	0.1744	0.0203	-0.0782

Table 6: Airfoil Coefficients for Thin Airfoils at Positive Angles of Attack

	FFA-W3-241			FFA-W3-301			FFA-W3-360		
AoA [deg]	C_l	C_d	C_m	C_l	C_d	C_m	C_l	C_d	C_m
0	0.3391	0.0092	-0.0880	0.3056	0.0118	-0.0865	0.5053	0.0187	-0.1090
2	0.5867	0.0094	-0.0933	0.5670	0.0119	-0.0954	0.8241	0.0188	-0.1329
4	0.8301	0.0099	-0.0977	0.8199	0.0125	-0.1024	1.1209	0.0196	-0.1510
6	1.0656	0.0109	-0.1008	1.0614	0.0136	-0.1071	1.3897	0.0213	-0.16298
8	1.2914	0.0124	-0.1026	1.2874	0.0152	-0.1094	1.6254	0.0240	-0.16908
10	1.5012	0.0144	-0.1024	1.4840	0.0180	-0.10796	1.8109	0.0279	-0.16832
12	1.6886	0.0173	-0.0998	1.6388	0.0224	-0.10256	1.8589	0.0365	-0.15848
14	1.8103	0.0226	-0.0941	1.7327	0.0303	-0.09524	1.8159	0.0760	-0.15576
16	1.8139	0.0354	-0.0874	1.7142	0.0539	-0.09104	1.7786	0.1165	-0.16696
18	1.7545	0.0647	-0.0850	1.6828	0.0954	-0.09794	1.7560	0.1571	-0.18094
20	1.6071	0.1035	-0.0913	1.6567	0.1435	-0.1103	1.7630	0.2063	-0.19598
22	1.5257	0.1437	-0.1026	1.6444	0.2280	-0.12996	1.8002	0.3057	-0.21428
24	1.4428	0.1841	-0.1140	1.6329	0.3148	-0.15332	1.8495	0.4153	-0.23648
26	1.3826	0.2290	-0.1282	1.6333	0.3926	-0.1760	1.8775	0.5163	-0.25882
28	1.3218	0.2738	-0.1423	1.6175	0.4623	-0.19658	1.8828	0.6069	-0.27966
30	1.2583	0.3278	-0.1582	1.5975	0.5266	-0.2149	1.8689	0.6892	-0.2983
32	1.1944	0.3816	-0.1739	1.5708	0.5855	-0.2313	1.8439	0.7625	-0.31466
33	1.1734	0.4209	-0.1790	1.5758	0.6215	-0.2366	1.8349	0.7954	-0.3166
34	1.1568	0.4504	-0.1831	1.5571	0.6562	-0.2406	1.8297	0.8232	-0.3106
35	1.1379	0.4815	-0.1873	1.5475	0.6897	-0.2447	1.8136	0.8446	-0.3047
36	1.1248	0.5105	-0.1914	1.5268	0.7148	-0.2487	1.7854	0.8684	-0.2987
37	1.1177	0.5450	-0.1953	1.5259	0.7464	-0.2496	1.7669	0.8912	-0.2926
38	1.1024	0.5737	-0.1993	1.5139	0.7762	-0.2455	1.7269	0.9066	-0.2865
39	1.0891	0.5999	-0.2032	1.4918	0.7982	-0.2444	1.6864	0.9255	-0.2844
40	1.0755	0.6280	-0.2071	1.4596	0.8197	-0.2423	1.6458	0.9385	-0.2803
45	1.0175	0.7578	-0.2161	1.2951	0.9122	-0.2373	1.4473	1.0290	-0.2633
50	0.9716	0.8820	-0.2184	1.1256	0.9665	-0.23537	1.1356	1.0715	-0.25437
55	0.9268	1.0104	-0.2214	0.9808	0.9957	-0.23243	0.9808	1.0857	-0.24543
60	0.8660	1.1250	-0.2255	0.8660	1.0350	-0.2335	0.8660	1.1000	-0.2365
65	0.7660	1.2321	-0.2306	0.7660	1.0828	-0.2336	0.7660	1.1278	-0.2366
70	0.6428	1.3245	-0.2349	0.6428	1.1479	-0.2349	0.6428	1.1629	-0.2389
75	0.5000	1.3995	-0.2415	0.5000	1.2129	-0.2415	0.5000	1.2129	-0.2415
80	0.3420	1.4548	-0.2462	0.3420	1.2608	-0.2462	0.3420	1.2608	-0.2462
85	0.1736	1.4886	-0.2490	0.1736	1.2901	-0.2490	0.1736	1.2901	-0.2490
90	0.0000	1.5000	-0.2500	0.0000	1.3000	-0.2500	0.0000	1.3000	-0.2500
95	-0.1736	1.4886	-0.2490	-0.1736	1.2901	-0.2490	-0.1736	1.2901	-0.2490
100	-0.3420	1.4548	-0.2462	-0.3420	1.2608	-0.2462	-0.3420	1.2608	-0.2462
105	-0.5000	1.3995	-0.2415	-0.5000	1.2129	-0.2415	-0.5000	1.2129	-0.2415
110	-0.6428	1.3245	-0.2349	-0.6428	1.1479	-0.2349	-0.6428	1.1479	-0.2349
115	-0.7660	1.2321	-0.2266	-0.7660	1.0678	-0.2266	-0.7660	1.0678	-0.2266
120	-0.8660	1.1250	-0.2165	-0.8660	0.9750	-0.2165	-0.8660	0.9750	-0.2165
125	-0.9397	1.0065	-0.2048	-0.9397	0.8723	-0.2048	-0.9397	0.8723	-0.2048
130	-0.9848	0.8803	-0.1915	-0.9848	0.7629	-0.1915	-0.9848	0.7629	-0.1915
135	-1.0000	0.7500	-0.1768	-1.0000	0.6500	-0.1768	-1.0000	0.6500	-0.1768
140	-0.9848	0.6197	-0.1607	-0.9848	0.5371	-0.1607	-0.9848	0.5371	-0.1607
145	-0.9397	0.4935	-0.1434	-0.9397	0.4277	-0.1434	-0.9397	0.4277	-0.1434
150	-0.8660	0.3750	-0.1250	-0.8660	0.3250	-0.1250	-0.8660	0.3250	-0.1250
155	-0.7660	0.2679	-0.1057	-0.7660	0.2322	-0.1057	-0.7660	0.2322	-0.1057
160	-0.6428	0.1755	-0.0855	-0.6428	0.1521	-0.0855	-0.6428	0.1521	-0.0855
165	-0.5000	0.1005	-0.0647	-0.5000	0.0871	-0.0647	-0.5000	0.0871	-0.0647
170	-0.3420	0.0452	-0.0434	-0.3420	0.0392	-0.0434	-0.3420	0.0392	-0.0434
175	-0.1736	0.0114	-0.0218	-0.1736	0.0099	-0.0218	-0.1736	0.0099	-0.0218
180	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 7: Airfoil Coefficients for Thick Airfoils at Negative Angles of Attack

	FFA-W3-480			FFA-W3-600			Cylinder		
AoA [deg]	C_l	C_d	C_m	C_l	C_d	C_m	C_l	C_d	C_m
-180	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6000	0.0000
-175	0.1736	0.0099	0.0218	0.1736	0.0099	0.0218	0.0000	0.6000	0.0000
-170	0.3420	0.0392	0.0434	0.3420	0.0392	0.0434	0.0000	0.6000	0.0000
-165	0.5000	0.0871	0.0647	0.5000	0.0871	0.0647	0.0000	0.6000	0.0000
-160	0.6428	0.1521	0.0855	0.6428	0.1521	0.0855	0.0000	0.6000	0.0000
-155	0.7660	0.2322	0.1057	0.7660	0.2322	0.1057	0.0000	0.6000	0.0000
-150	0.8660	0.3250	0.1250	0.8660	0.3250	0.1250	0.0000	0.6000	0.0000
-145	0.9397	0.4277	0.1434	0.9397	0.4277	0.1434	0.0000	0.6000	0.0000
-140	0.9848	0.5371	0.1607	0.9848	0.5371	0.1607	0.0000	0.6000	0.0000
-135	1.0000	0.6500	0.1768	1.0000	0.6500	0.1768	0.0000	0.6000	0.0000
-130	0.9848	0.7629	0.1915	0.9848	0.7629	0.1915	0.0000	0.6000	0.0000
-125	0.9397	0.8723	0.2048	0.9397	0.8723	0.2048	0.0000	0.6000	0.0000
-120	0.8660	0.9750	0.2165	0.8660	0.9750	0.2165	0.0000	0.6000	0.0000
-115	0.7660	1.0678	0.2266	0.7660	1.0678	0.2266	0.0000	0.6000	0.0000
-110	0.6428	1.1479	0.2349	0.6428	1.1479	0.2349	0.0000	0.6000	0.0000
-105	0.5000	1.2129	0.2415	0.5000	1.2129	0.2415	0.0000	0.6000	0.0000
-100	0.3420	1.2608	0.2462	0.3420	1.2608	0.2462	0.0000	0.6000	0.0000
-95	0.1736	1.2901	0.2490	0.1736	1.2901	0.2490	0.0000	0.6000	0.0000
-90	0.0000	1.3000	0.2500	0.0000	1.3000	0.2500	0.0000	0.6000	0.0000
-85	-0.1736	1.2901	0.2490	-0.1736	1.2901	0.2490	0.0000	0.6000	0.0000
-80	-0.3420	1.2608	0.2462	-0.3420	1.2608	0.2462	0.0000	0.6000	0.0000
-75	-0.5000	1.2129	0.2415	-0.5000	1.2129	0.2415	0.0000	0.6000	0.0000
-70	-0.6428	1.1479	0.2349	-0.6428	1.1479	0.2349	0.0000	0.6000	0.0000
-65	-0.7660	1.0678	0.2266	-0.7660	1.0678	0.2266	0.0000	0.6000	0.0000
-60	-0.8660	0.9750	0.2165	-0.8660	0.9750	0.2165	0.0000	0.6000	0.0000
-55	-0.9397	0.8723	0.1978	-0.9397	0.8723	0.1978	0.0000	0.6000	0.0000
-50	-0.9848	0.7629	0.1775	-0.9848	0.7629	0.1775	0.0000	0.6000	0.0000
-45	-1.0000	0.6500	0.1558	-1.0000	0.6500	0.1488	0.0000	0.6000	0.0000
-40	-0.9816	0.5352	0.1246	-0.9516	0.5282	0.1176	0.0000	0.6000	0.0000
-39	-0.9539	0.5136	0.1106	-0.9039	0.5026	0.1076	0.0000	0.6000	0.0000
-38	-0.9262	0.4851	0.0966	-0.8262	0.4671	0.0906	0.0000	0.6000	0.0000
-37	-0.8885	0.4565	0.0825	-0.7185	0.4345	0.0805	0.0000	0.6000	0.0000
-36	-0.8508	0.4279	0.0655	-0.6208	0.3989	0.0665	0.0000	0.6000	0.0000
-35	-0.8231	0.3924	0.0585	-0.5231	0.3634	0.0595	0.0000	0.6000	0.0000
-34	-0.7854	0.3638	0.0484	-0.4454	0.3278	0.0524	0.0000	0.6000	0.0000
-33	-0.7477	0.3383	0.0384	-0.3477	0.3013	0.0454	0.0000	0.6000	0.0000
-32	-0.7011	0.3123	0.0305	-0.2571	0.2729	0.04699	0.0000	0.6000	0.0000
-30	-0.6208	0.2663	0.0200	-0.1574	0.2332	0.0606	0.0000	0.6000	0.0000
-28	-0.5406	0.2402	0.0094	-0.0776	0.2034	0.0742	0.0000	0.6000	0.0000
-26	-0.4694	0.2169	-0.0006	0.0115	0.1858	0.08749	0.0000	0.6000	0.0000
-24	-0.3881	0.1936	-0.0106	0.1007	0.1681	0.10079	0.0000	0.6000	0.0000
-22	-0.3161	0.1732	-0.0197	0.1962	0.1526	0.11329	0.0000	0.6000	0.0000
-20	-0.2442	0.1529	-0.0289	0.2918	0.1370	0.12579	0.0000	0.6000	0.0000
-18	-0.1641	0.1355	-0.0368	0.3932	0.1240	0.1370	0.0000	0.6000	0.0000
-16	-0.0841	0.1180	-0.0448	0.4947	0.1109	0.14822	0.0000	0.6000	0.0000
-14	0.0021	0.1035	-0.0510	0.6004	0.1009	0.15711	0.0000	0.6000	0.0000
-12	0.0883	0.0890	-0.0572	0.7061	0.0908	0.1660	0.0000	0.6000	0.0000
-10	0.1722	0.0773	-0.0604	0.7880	0.0836	0.16771	0.0000	0.6000	0.0000
-8	0.2561	0.0656	-0.0636	0.8700	0.0765	0.16942	0.0000	0.6000	0.0000
-6	0.3179	0.0563	-0.0605	0.8358	0.0748	-0.1470	0.0000	0.6000	0.0000
-4	0.3798	0.0470	-0.0574	0.8162	0.07311	-0.12588	0.0000	0.6000	0.0000
-2	0.3573	0.0405	-0.0351	0.6603	0.07553	-0.08764	0.0000	0.6000	0.0000

Table 8: Airfoil Coefficients for Thick Airfoils at Positive Angles of Attack

	FFA-W3-480			FFA-W3-600			Cylinder		
AoA [deg]	C_l	C_d	C_m	C_l	C_d	C_m	C_l	C_d	C_m
0	0.3348	0.0341	-0.0128	0.5199	0.07795	-0.05062	0.0000	0.6000	0.0000
2	0.5652	0.0316	-0.0494	0.2636	0.07795	-0.00078	0.0000	0.6000	0.0000
4	0.8769	0.0343	-0.0894	0.2146	0.07082	0.06962	0.0000	0.6000	0.0000
6	1.0425	0.0451	-0.11168	0.1656	0.07483	0.01974	0.0000	0.6000	0.0000
8	0.9487	0.0700	-0.12084	0.4475	0.08587	-0.02718	0.0000	0.6000	0.0000
10	0.9088	0.0886	-0.1376	0.7071	0.0999	-0.06712	0.0000	0.6000	0.0000
12	0.9761	0.0993	-0.15942	0.9540	0.11148	-0.10334	0.0000	0.6000	0.0000
14	1.1130	0.1070	-0.18234	1.1891	0.12072	-0.13694	0.0000	0.6000	0.0000
16	1.3065	0.1163	-0.20664	1.4183	0.13678	-0.16866	0.0000	0.6000	0.0000
18	1.5414	0.1317	-0.2315	1.6392	0.14998	-0.19886	0.0000	0.6000	0.0000
20	1.8049	0.1570	-0.25662	1.8216	0.16894	-0.22524	0.0000	0.6000	0.0000
22	2.0020	0.2757	-0.28058	1.9837	0.20657	-0.25094	0.0000	0.6000	0.0000
24	2.1216	0.4224	-0.30574	2.0985	0.30919	-0.27474	0.0000	0.6000	0.0000
26	2.1916	0.5610	-0.3294	2.1904	0.4344	-0.29658	0.0000	0.6000	0.0000
28	2.2291	0.6861	-0.35198	2.2541	0.5512	-0.31712	0.0000	0.6000	0.0000
30	2.2322	0.7958	-0.3710	2.2731	0.6732	-0.33352	0.0000	0.6000	0.0000
32	2.2188	0.8915	-0.38784	2.2740	0.7756	-0.34854	0.0000	0.6000	0.0000
33	2.2092	0.9229	-0.3845	2.2780	0.8209	-0.3466	0.0000	0.6000	0.0000
34	2.1927	0.9472	-0.3806	2.2658	0.8551	-0.3476	0.0000	0.6000	0.0000
35	2.1625	0.9664	-0.3747	2.2408	0.8862	-0.3487	0.0000	0.6000	0.0000
36	2.1313	0.9889	-0.3687	2.2331	0.9141	-0.3427	0.0000	0.6000	0.0000
37	2.0878	1.0073	-0.3626	2.2134	0.9390	-0.3366	0.0000	0.6000	0.0000
38	2.0435	1.0284	-0.3565	2.1820	0.9709	-0.3305	0.0000	0.6000	0.0000
39	1.9984	1.0521	-0.3504	2.1391	0.9996	-0.3244	0.0000	0.6000	0.0000
40	1.9418	1.0769	-0.3443	2.0951	1.0254	-0.3183	0.0000	0.6000	0.0000
45	1.6535	1.1697	-0.3233	1.8370	1.1419	-0.3003	0.0000	0.6000	0.0000
50	1.3016	1.1965	-0.30437	1.5156	1.1615	-0.29137	0.0000	0.6000	0.0000
55	1.0308	1.1957	-0.29543	1.2008	1.1657	-0.28243	0.0000	0.6000	0.0000
60	0.8660	1.1900	-0.2865	0.9660	1.1750	-0.2735	0.0000	0.6000	0.0000
65	0.7660	1.1928	-0.2766	0.7660	1.1778	-0.2636	0.0000	0.6000	0.0000
70	0.6428	1.2029	-0.2649	0.6428	1.1979	-0.2549	0.0000	0.6000	0.0000
75	0.5000	1.2129	-0.2515	0.5000	1.2129	-0.2515	0.0000	0.6000	0.0000
80	0.3420	1.2608	-0.2462	0.3420	1.2608	-0.2462	0.0000	0.6000	0.0000
85	0.1736	1.2901	-0.2490	0.1736	1.2901	-0.2490	0.0000	0.6000	0.0000
90	0.0000	1.3000	-0.2500	0.0000	1.3000	-0.2500	0.0000	0.6000	0.0000
95	-0.1736	1.2901	-0.2490	-0.1736	1.2901	-0.2490	0.0000	0.6000	0.0000
100	-0.3420	1.2608	-0.2462	-0.3420	1.2608	-0.2462	0.0000	0.6000	0.0000
105	-0.5000	1.2129	-0.2415	-0.5000	1.2129	-0.2415	0.0000	0.6000	0.0000
110	-0.6428	1.1479	-0.2349	-0.6428	1.1479	-0.2349	0.0000	0.6000	0.0000
115	-0.7660	1.0678	-0.2266	-0.7660	1.0678	-0.2266	0.0000	0.6000	0.0000
120	-0.8660	0.9750	-0.2165	-0.8660	0.9750	-0.2165	0.0000	0.6000	0.0000
125	-0.9397	0.8723	-0.2048	-0.9397	0.8723	-0.2048	0.0000	0.6000	0.0000
130	-0.9848	0.7629	-0.1915	-0.9848	0.7629	-0.1915	0.0000	0.6000	0.0000
135	-1.0000	0.6500	-0.1768	-1.0000	0.6500	-0.1768	0.0000	0.6000	0.0000
140	-0.9848	0.5371	-0.1607	-0.9848	0.5371	-0.1607	0.0000	0.6000	0.0000
145	-0.9397	0.4277	-0.1434	-0.9397	0.4277	-0.1434	0.0000	0.6000	0.0000
150	-0.8660	0.3250	-0.1250	-0.8660	0.3250	-0.1250	0.0000	0.6000	0.0000
155	-0.7660	0.2322	-0.1057	-0.7660	0.2322	-0.1057	0.0000	0.6000	0.0000
160	-0.6428	0.1521	-0.0855	-0.6428	0.1521	-0.0855	0.0000	0.6000	0.0000
165	-0.5000	0.0871	-0.0647	-0.5000	0.0871	-0.0647	0.0000	0.6000	0.0000
170	-0.3420	0.0392	-0.0434	-0.3420	0.0392	-0.0434	0.0000	0.6000	0.0000
175	-0.1736	0.0099	-0.0218	-0.1736	0.0099	-0.0218	0.0000	0.6000	0.0000
180	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6000	0.0000

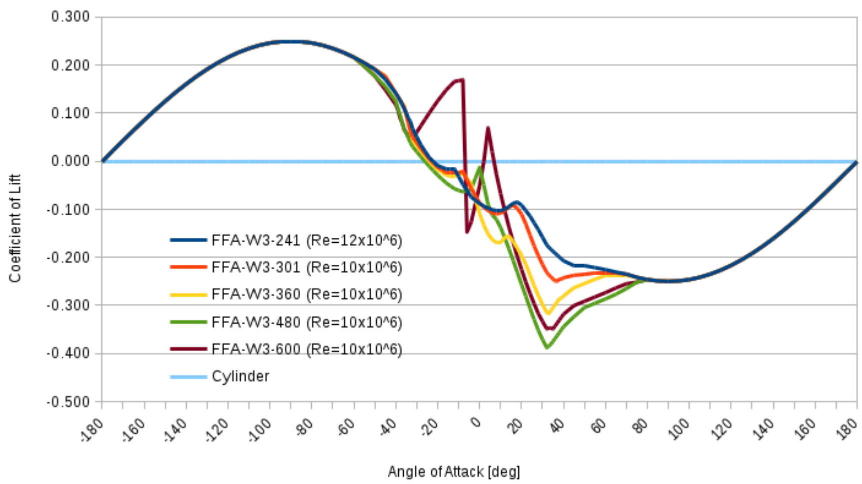

Figure 8: Coefficients of Moment

2.4.3 Airfoil shapes

The tabulated points that describe the airfoil shapes are given in Tables 9 through 15. The point ID and the s location along the curve are also given for each point. The Table for the cylinder is redundant since this is merely a circle, but is given for completeness. These shapes are also given graphically in Figures 9 through 14.

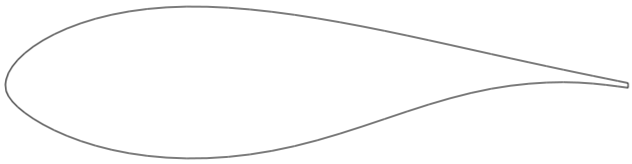

Figure 9: FFA-W3-241 Shape

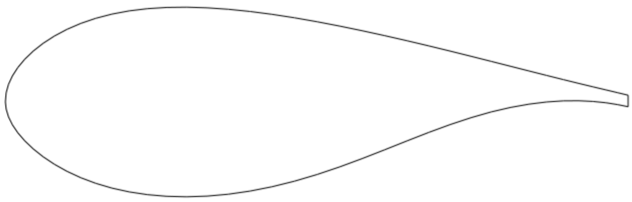

Figure 10: FFA-W3-301 Shape

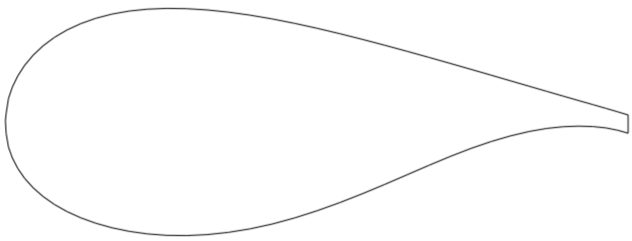

Figure 11: FFA-W3-360 Shape

Table 9: FFA-W3-241 Shape

ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$		ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$
1	0.00000	1.00000	-0.00360		41	0.50369	0.00130	0.00956
2	0.00794	0.98338	-0.00140		42	0.50897	0.00507	0.02005
3	0.01692	0.96457	0.00096		43	0.51468	0.01133	0.03033
4	0.02689	0.94365	0.00311		44	0.52116	0.02006	0.04087
5	0.03778	0.92072	0.00466		45	0.52841	0.03122	0.05135
6	0.04954	0.89589	0.00528		46	0.53648	0.04474	0.06170
7	0.06215	0.86928	0.00473		47	0.54536	0.06056	0.07176
8	0.07557	0.84102	0.00291		48	0.55506	0.07861	0.08139
9	0.08976	0.81124	-0.00030		49	0.56554	0.09880	0.09044
10	0.10469	0.78010	-0.00514		50	0.57678	0.12104	0.09873
11	0.12036	0.74772	-0.01183		51	0.58876	0.14522	0.10613
12	0.13669	0.71428	-0.02024		52	0.60144	0.17121	0.11247
13	0.15363	0.67994	-0.03015		53	0.61479	0.19891	0.11764
14	0.17107	0.64485	-0.04127		54	0.62877	0.22817	0.12153
15	0.18887	0.60918	-0.05308		55	0.64336	0.25885	0.12409
16	0.20687	0.57311	-0.06502		56	0.65851	0.29081	0.12526
17	0.22491	0.53681	-0.07651		57	0.67418	0.32389	0.12504
18	0.24285	0.50045	-0.08706		58	0.69033	0.35794	0.12350
19	0.26055	0.46422	-0.09623		59	0.70690	0.39279	0.12076
20	0.27795	0.42827	-0.10370		60	0.72380	0.42827	0.11696
21	0.29497	0.39279	-0.10938		61	0.74098	0.46422	0.11223
22	0.31158	0.35794	-0.11324		62	0.75834	0.50045	0.10670
23	0.32775	0.32389	-0.11531		63	0.77582	0.53681	0.10052
24	0.34342	0.29081	-0.11567		64	0.79331	0.57311	0.09382
25	0.35857	0.25885	-0.11441		65	0.81072	0.60918	0.08672
26	0.37317	0.22817	-0.11167		66	0.82798	0.64485	0.07935
27	0.38717	0.19891	-0.10754		67	0.84498	0.67994	0.07187
28	0.40054	0.17121	-0.10213		68	0.86163	0.71428	0.06441
29	0.41324	0.14522	-0.09560		69	0.87785	0.74772	0.05707
30	0.42523	0.12104	-0.08811		70	0.89355	0.78010	0.04996
31	0.43647	0.09880	-0.07985		71	0.90866	0.81124	0.04316
32	0.44691	0.07861	-0.07104		72	0.92309	0.84102	0.03673
33	0.45650	0.06056	-0.06188		73	0.93678	0.86928	0.03070
34	0.46519	0.04474	-0.05258		74	0.94966	0.89589	0.02510
35	0.47295	0.03122	-0.04334		75	0.96168	0.92072	0.01996
36	0.47976	0.02006	-0.03427		76	0.97276	0.94365	0.01529
37	0.48562	0.01133	-0.02550		77	0.98287	0.96457	0.01107
38	0.49063	0.00507	-0.01700		78	0.99197	0.98338	0.00727
39	0.49480	0.00130	-0.00903		79	1.00000	1.00000	0.00391
40	0.49987	0.00004	0.00160					

Table 10: FFA-W3-301 Shape

ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$		ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$
1	0.00000	1.00000	-0.00891		41	0.50771	0.00126	0.01326
2	0.00783	0.98338	-0.00558		42	0.51414	0.00503	0.02667
3	0.01662	0.96457	-0.00261		43	0.52095	0.01130	0.04001
4	0.02634	0.94364	-0.00051		44	0.52833	0.02003	0.05339
5	0.03694	0.92071	0.00058		45	0.53629	0.03118	0.06655
6	0.04841	0.89588	0.00040		46	0.54490	0.04470	0.07938
7	0.06072	0.86927	-0.00134		47	0.55415	0.06052	0.09167
8	0.07387	0.84101	-0.00479		48	0.56406	0.07858	0.10323
9	0.08783	0.81124	-0.01002		49	0.57459	0.09877	0.11385
10	0.10259	0.78009	-0.01715		50	0.58576	0.12101	0.12332
11	0.11813	0.74772	-0.02631		51	0.59754	0.14518	0.13146
12	0.13438	0.71428	-0.03722		52	0.60994	0.17118	0.13814
13	0.15125	0.67993	-0.04966		53	0.62294	0.19888	0.14325
14	0.16862	0.64484	-0.06318		54	0.63655	0.22813	0.14673
15	0.18632	0.60917	-0.07722		55	0.65075	0.25882	0.14866
16	0.20418	0.57310	-0.09116		56	0.66551	0.29078	0.14908
17	0.22203	0.53680	-0.10442		57	0.68080	0.32386	0.14807
18	0.23973	0.50044	-0.11652		58	0.69656	0.35791	0.14578
19	0.25716	0.46420	-0.12710		59	0.71274	0.39276	0.14230
20	0.27426	0.42824	-0.13591		60	0.72926	0.42824	0.13776
21	0.29095	0.39276	-0.14283		61	0.74605	0.46420	0.13229
22	0.30721	0.35791	-0.14780		62	0.76304	0.50044	0.12601
23	0.32300	0.32386	-0.15080		63	0.78013	0.53680	0.11909
24	0.33829	0.29078	-0.15188		64	0.79724	0.57310	0.11169
25	0.35305	0.25882	-0.15108		65	0.81428	0.60917	0.10391
26	0.36728	0.22813	-0.14847		66	0.83117	0.64484	0.09587
27	0.38093	0.19888	-0.14411		67	0.84782	0.67993	0.08766
28	0.39403	0.17118	-0.13810		68	0.86413	0.71428	0.07939
29	0.40653	0.14518	-0.13052		69	0.88004	0.74772	0.07116
30	0.41844	0.12101	-0.12155		70	0.89545	0.78009	0.06309
31	0.42974	0.09877	-0.11136		71	0.91028	0.81124	0.05528
32	0.44040	0.07858	-0.10018		72	0.92445	0.84101	0.04780
33	0.45039	0.06052	-0.08825		73	0.93791	0.86927	0.04074
34	0.45969	0.04470	-0.07582		74	0.95057	0.89588	0.03417
35	0.46825	0.03118	-0.06312		75	0.96237	0.92071	0.02812
36	0.47608	0.02003	-0.05035		76	0.97326	0.94364	0.02266
37	0.48320	0.01130	-0.03764		77	0.98319	0.96457	0.01775
38	0.48971	0.00503	-0.02502		78	0.99211	0.98338	0.01333
39	0.49573	0.00126	-0.01256		79	1.00000	1.00000	0.00937
40	0.50101	0.00000	-0.00119					

Table 11: FFA-W3-360 Shape

ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$		ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$
1	0.00000	1.00000	-0.01393		41	0.50753	0.00138	0.01755
2	0.00773	0.98339	-0.00928		42	0.51835	0.00515	0.04140
3	0.01633	0.96457	-0.00555		43	0.52693	0.01141	0.05950
4	0.02575	0.94365	-0.00342		44	0.53576	0.02014	0.07716
5	0.03603	0.92072	-0.00273		45	0.54482	0.03126	0.09405
6	0.04716	0.89589	-0.00361		46	0.55420	0.04481	0.11001
7	0.05914	0.86929	-0.00636		47	0.56389	0.06063	0.12474
8	0.07199	0.84103	-0.01120		48	0.57393	0.07868	0.13804
9	0.08569	0.81126	-0.01816		49	0.58441	0.09887	0.14982
10	0.10022	0.78011	-0.02713		50	0.59536	0.12111	0.15996
11	0.11553	0.74774	-0.03807		51	0.60684	0.14528	0.16844
12	0.13156	0.71431	-0.05083		52	0.61887	0.17128	0.17512
13	0.14821	0.67996	-0.06500		53	0.63146	0.19897	0.17994
14	0.16531	0.64487	-0.08000		54	0.64464	0.22823	0.18291
15	0.18269	0.60921	-0.09522		55	0.65840	0.25891	0.18414
16	0.20017	0.57314	-0.11009		56	0.67272	0.29086	0.18370
17	0.21761	0.53685	-0.12416		57	0.68757	0.32394	0.18171
18	0.23490	0.50049	-0.13705		58	0.70290	0.35799	0.17825
19	0.25192	0.46426	-0.14848		59	0.71866	0.39283	0.17347
20	0.26860	0.42831	-0.15816		60	0.73478	0.42831	0.16749
21	0.28486	0.39283	-0.16578		61	0.75120	0.46426	0.16048
22	0.30067	0.35799	-0.17134		62	0.76781	0.50049	0.15262
23	0.31601	0.32394	-0.17488		63	0.78454	0.53685	0.14408
24	0.33085	0.29086	-0.17643		64	0.80130	0.57314	0.13505
25	0.34517	0.25891	-0.17603		65	0.81800	0.60921	0.12568
26	0.35896	0.22823	-0.17375		66	0.83454	0.64487	0.11611
27	0.37219	0.19897	-0.16970		67	0.85085	0.67996	0.10647
28	0.38486	0.17128	-0.16402		68	0.86683	0.71431	0.09687
29	0.39695	0.14528	-0.15683		69	0.88240	0.74774	0.08740
30	0.40845	0.12111	-0.14818		70	0.89749	0.78011	0.07815
31	0.41937	0.09887	-0.13821		71	0.91201	0.81126	0.06920
32	0.42973	0.07868	-0.12695		72	0.92589	0.84103	0.06063
33	0.43958	0.06063	-0.11440		73	0.93907	0.86929	0.05251
34	0.44898	0.04481	-0.10064		74	0.95147	0.89589	0.04489
35	0.45796	0.03126	-0.08587		75	0.96303	0.92072	0.03783
36	0.46662	0.02014	-0.07006		76	0.97371	0.94365	0.03137
37	0.47509	0.01141	-0.05330		77	0.98345	0.96457	0.02543
38	0.48323	0.00515	-0.03624		78	0.99223	0.98339	0.01997
39	0.49308	0.00138	-0.01459		79	1.00000	1.00000	0.01503
40	0.50237	0.00012	0.00612					

Table 12: FFA-W3-480 Shape

ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$		ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$
1	0.00000	1.00000	-0.01857		41	0.51000	0.00138	0.02340
2	0.00752	0.98339	-0.01237		42	0.52358	0.00515	0.05520
3	0.01577	0.96457	-0.00740		43	0.53415	0.01141	0.07933
4	0.02472	0.94365	-0.00456		44	0.54479	0.02014	0.10288
5	0.03445	0.92072	-0.00364		45	0.55544	0.03126	0.12540
6	0.04499	0.89589	-0.00481		46	0.56614	0.04481	0.14668
7	0.05637	0.86929	-0.00848		47	0.57683	0.06063	0.16632
8	0.06866	0.84103	-0.01493		48	0.58756	0.07868	0.18405
9	0.08188	0.81126	-0.02421		49	0.59840	0.09887	0.19976
10	0.09603	0.78011	-0.03617		50	0.60944	0.12111	0.21328
11	0.11108	0.74774	-0.05076		51	0.62075	0.14528	0.22459
12	0.12698	0.71431	-0.06777		52	0.63240	0.17128	0.23349
13	0.14361	0.67996	-0.08667		53	0.64445	0.19897	0.23992
14	0.16073	0.64487	-0.10667		54	0.65697	0.22823	0.24388
15	0.17812	0.60921	-0.12696		55	0.67000	0.25891	0.24552
16	0.19557	0.57314	-0.14679		56	0.68354	0.29086	0.24493
17	0.21289	0.53685	-0.16555		57	0.69761	0.32394	0.24228
18	0.22994	0.50049	-0.18273		58	0.71218	0.35799	0.23767
19	0.24661	0.46426	-0.19797		59	0.72720	0.39283	0.23129
20	0.26280	0.42831	-0.21088		60	0.74261	0.42831	0.22332
21	0.27845	0.39283	-0.22104		61	0.75836	0.46426	0.21397
22	0.29355	0.35799	-0.22845		62	0.77435	0.50049	0.20349
23	0.30812	0.32394	-0.23317		63	0.79051	0.53685	0.19211
24	0.32218	0.29086	-0.23524		64	0.80672	0.57314	0.18007
25	0.33572	0.25891	-0.23471		65	0.82290	0.60921	0.16757
26	0.34880	0.22823	-0.23167		66	0.83896	0.64487	0.15481
27	0.36141	0.19897	-0.22627		67	0.85480	0.67996	0.14196
28	0.37358	0.17128	-0.21869		68	0.87034	0.71431	0.12916
29	0.38533	0.14528	-0.20911		69	0.88549	0.74774	0.11653
30	0.39668	0.12111	-0.19757		70	0.90018	0.78011	0.10420
31	0.40767	0.09887	-0.18428		71	0.91432	0.81126	0.09227
32	0.41834	0.07868	-0.16927		72	0.92784	0.84103	0.08084
33	0.42877	0.06063	-0.15253		73	0.94067	0.86929	0.07001
34	0.43904	0.04481	-0.13419		74	0.95274	0.89589	0.05985
35	0.44918	0.03126	-0.11449		75	0.96400	0.92072	0.05044
36	0.45928	0.02014	-0.09341		76	0.97439	0.94365	0.04183
37	0.46945	0.01141	-0.07107		77	0.98387	0.96457	0.03391
38	0.47945	0.00515	-0.04832		78	0.99242	0.98339	0.02663
39	0.49180	0.00138	-0.01945		79	1.00000	1.00000	0.02004
40	0.50352	0.00012	0.00816					

Table 13: TC-72 Shape, part 1

ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$	ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$	ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$
1	0.00000	0.99937	-0.02908	51	0.24835	0.44713	-0.35336	101	0.44384	0.02643	-0.13282
2	0.00491	0.98757	-0.03411	52	0.25292	0.43529	-0.35489	102	0.44693	0.02343	-0.12533
3	0.00986	0.97569	-0.03922	53	0.25746	0.42347	-0.35608	103	0.44998	0.02065	-0.11786
4	0.01487	0.96410	-0.04534	54	0.26199	0.41168	-0.35694	104	0.45300	0.01807	-0.11041
5	0.01992	0.95277	-0.05209	55	0.26649	0.39993	-0.35747	105	0.45599	0.01569	-0.10298
6	0.02500	0.94160	-0.05923	56	0.27097	0.38822	-0.35768	106	0.45894	0.01350	-0.09560
7	0.03009	0.93059	-0.06672	57	0.27543	0.37658	-0.35756	107	0.46185	0.01149	-0.08825
8	0.03521	0.91973	-0.07449	58	0.27986	0.36500	-0.35714	108	0.46473	0.00964	-0.08095
9	0.04033	0.90900	-0.08251	59	0.28427	0.35350	-0.35642	109	0.46758	0.00796	-0.07371
10	0.04543	0.89851	-0.09073	60	0.28866	0.34208	-0.35541	110	0.47039	0.00646	-0.06651
11	0.05048	0.88835	-0.09911	61	0.29302	0.33076	-0.35411	111	0.47317	0.00515	-0.05936
12	0.05553	0.87826	-0.10761	62	0.29736	0.31954	-0.35251	112	0.47592	0.00404	-0.05227
13	0.06058	0.86822	-0.11619	63	0.30167	0.30843	-0.35063	113	0.47864	0.00312	-0.04524
14	0.06564	0.85821	-0.12481	64	0.30596	0.29744	-0.34847	114	0.48132	0.00234	-0.03828
15	0.07070	0.84819	-0.13345	65	0.31023	0.28658	-0.34602	115	0.48397	0.00169	-0.03139
16	0.07577	0.83815	-0.14208	66	0.31446	0.27585	-0.34330	116	0.48658	0.00115	-0.02457
17	0.08084	0.82809	-0.15069	67	0.31867	0.26526	-0.34031	117	0.48917	0.00069	-0.01783
18	0.08591	0.81800	-0.15927	68	0.32286	0.25482	-0.33705	118	0.49172	0.00033	-0.01117
19	0.09098	0.80787	-0.16782	69	0.32702	0.24455	-0.33354	119	0.49425	0.00010	-0.00458
20	0.09605	0.79770	-0.17630	70	0.33115	0.23444	-0.32977	120	0.49674	0.00002	0.00193
21	0.10112	0.78747	-0.18472	71	0.33525	0.22450	-0.32576	121	0.49924	0.00010	0.00845
22	0.10619	0.77719	-0.19307	72	0.33932	0.21473	-0.32151	122	0.50176	0.00038	0.01504
23	0.11126	0.76684	-0.20131	73	0.34337	0.20516	-0.31703	123	0.50431	0.00078	0.02168
24	0.11632	0.75641	-0.20945	74	0.34739	0.19576	-0.31234	124	0.50689	0.00125	0.02842
25	0.12138	0.74590	-0.21747	75	0.35141	0.18649	-0.30743	125	0.50951	0.00179	0.03524
26	0.12643	0.73532	-0.22536	76	0.35539	0.17742	-0.30233	126	0.51216	0.00241	0.04214
27	0.13148	0.72464	-0.23310	77	0.35934	0.16858	-0.29702	127	0.51485	0.00316	0.04911
28	0.13652	0.71388	-0.24069	78	0.36325	0.15995	-0.29154	128	0.51756	0.00405	0.05615
29	0.14156	0.70302	-0.24812	79	0.36713	0.15154	-0.28588	129	0.52031	0.00511	0.06325
30	0.14658	0.69207	-0.25537	80	0.37098	0.14335	-0.28005	130	0.52309	0.00636	0.07041
31	0.15160	0.68104	-0.26244	81	0.37479	0.13540	-0.27406	131	0.52591	0.00782	0.07762
32	0.15661	0.66991	-0.26931	82	0.37857	0.12768	-0.26791	132	0.52876	0.00945	0.08488
33	0.16161	0.65868	-0.27599	83	0.38231	0.12019	-0.26161	133	0.53164	0.01125	0.09219
34	0.16659	0.64737	-0.28245	84	0.38602	0.11294	-0.25517	134	0.53455	0.01321	0.09955
35	0.17157	0.63596	-0.28870	85	0.38970	0.10593	-0.24861	135	0.53750	0.01533	0.10696
36	0.17654	0.62447	-0.29471	86	0.39334	0.09915	-0.24194	136	0.54049	0.01764	0.11441
37	0.18149	0.61289	-0.30050	87	0.39695	0.09261	-0.23514	137	0.54351	0.02013	0.12189
38	0.18643	0.60124	-0.30604	88	0.40052	0.08632	-0.22825	138	0.54656	0.02282	0.12940
39	0.19136	0.58950	-0.31134	89	0.40406	0.08027	-0.22125	139	0.54965	0.02571	0.13693
40	0.19628	0.57768	-0.31638	90	0.40757	0.07446	-0.21417	140	0.55277	0.02880	0.14448
41	0.20118	0.56579	-0.32116	91	0.41104	0.06890	-0.20700	141	0.55593	0.03210	0.15204
42	0.20608	0.55382	-0.32568	92	0.41448	0.06359	-0.19976	142	0.55912	0.03561	0.15960
43	0.21097	0.54178	-0.32992	93	0.41788	0.05852	-0.19246	143	0.56235	0.03934	0.16717
44	0.21576	0.52990	-0.33388	94	0.42125	0.05370	-0.18510	144	0.56561	0.04329	0.17472
45	0.22048	0.51813	-0.33755	95	0.42458	0.04912	-0.17770	145	0.56891	0.04747	0.18225
46	0.22518	0.50633	-0.34094	96	0.42788	0.04477	-0.17026	146	0.57224	0.05188	0.18976
47	0.22985	0.49451	-0.34404	97	0.43114	0.04066	-0.16279	147	0.57561	0.05653	0.19723
48	0.23451	0.48268	-0.34683	98	0.43437	0.03677	-0.15531	148	0.57901	0.06142	0.20466
49	0.23914	0.47083	-0.34932	99	0.43756	0.03310	-0.14782	149	0.58245	0.06656	0.21202
50	0.24376	0.45898	-0.35150	100	0.44072	0.02966	-0.14032	150	0.58592	0.07195	0.21932

Table 14: TC-72 Shape, part 2

ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$		ID	s	$\boldsymbol{x}$	$\boldsymbol{y}$
151	0.58943	0.07759	0.22655		201	0.80312	0.59151	0.33077
152	0.59297	0.08348	0.23369		202	0.80805	0.60368	0.32658
153	0.59655	0.08962	0.24073		203	0.81299	0.61582	0.32217
154	0.60016	0.09601	0.24767		204	0.81795	0.62791	0.31753
155	0.60381	0.10267	0.25448		205	0.82292	0.63995	0.31267
156	0.60748	0.10957	0.26116		206	0.82790	0.65195	0.30759
157	0.61119	0.11672	0.26771		207	0.83290	0.66388	0.30229
158	0.61494	0.12411	0.27411		208	0.83791	0.67576	0.29678
159	0.61871	0.13174	0.28035		209	0.84294	0.68757	0.29105
160	0.62252	0.13962	0.28643		210	0.84797	0.69930	0.28513
161	0.62636	0.14775	0.29232		211	0.85301	0.71097	0.27900
162	0.63023	0.15610	0.29802		212	0.85807	0.72255	0.27267
163	0.63413	0.16468	0.30351		213	0.86313	0.73406	0.26614
164	0.63806	0.17348	0.30880		214	0.86820	0.74547	0.25942
165	0.64203	0.18250	0.31388		215	0.87328	0.75679	0.25251
166	0.64602	0.19173	0.31873		216	0.87836	0.76802	0.24541
167	0.65004	0.20117	0.32334		217	0.88345	0.77914	0.23813
168	0.65408	0.21079	0.32773		218	0.88854	0.79016	0.23067
169	0.65816	0.22060	0.33187		219	0.89364	0.80107	0.22303
170	0.66226	0.23059	0.33578		220	0.89874	0.81187	0.21523
171	0.66640	0.24074	0.33945		221	0.90384	0.82255	0.20726
172	0.67056	0.25105	0.34287		222	0.90894	0.83312	0.19912
173	0.67474	0.26152	0.34604		223	0.91405	0.84356	0.19083
174	0.67895	0.27213	0.34895		224	0.91915	0.85387	0.18238
175	0.68319	0.28288	0.35162		225	0.92425	0.86406	0.17378
176	0.68746	0.29375	0.35404		226	0.92935	0.87411	0.16504
177	0.69175	0.30475	0.35621		227	0.93445	0.88404	0.15616
178	0.69606	0.31586	0.35812		228	0.93954	0.89382	0.14715
179	0.70041	0.32708	0.35978		229	0.94463	0.90347	0.13800
180	0.70477	0.33841	0.36119		230	0.94971	0.91297	0.12874
181	0.70916	0.34982	0.36237		231	0.95479	0.92233	0.11935
182	0.71358	0.36132	0.36327		232	0.95985	0.93154	0.10984
183	0.71802	0.37291	0.36392		233	0.96491	0.94061	0.10023
184	0.72249	0.38456	0.36429		234	0.96996	0.94954	0.09052
185	0.72697	0.39629	0.36440		235	0.97500	0.95831	0.08071
186	0.73149	0.40808	0.36424		236	0.98002	0.96691	0.07079
187	0.73602	0.41992	0.36380		237	0.98504	0.97534	0.06076
188	0.74060	0.43186	0.36311		238	0.99004	0.98356	0.05061
189	0.74532	0.44415	0.36214		239	0.99503	0.99158	0.04033
190	0.75005	0.45644	0.36092		240	1.00000	0.99937	0.02993
191	0.75479	0.46874	0.35943					
192	0.75954	0.48104	0.35769					
193	0.76432	0.49334	0.35569					
194	0.76910	0.50565	0.35344					
195	0.77391	0.51795	0.35093					
196	0.77874	0.53025	0.34818					
197	0.78358	0.54254	0.34517					
198	0.78844	0.55482	0.34193					
199	0.79331	0.56707	0.33845					
200	0.79821	0.57931	0.33472					

Table 15: Cylinder Shape

ID	s	x	y		ID	s	x	y
1	0.00000	1.00000	0.00000		52	0.51000	0.00099	-0.03140
2	0.01000	0.99901	0.03140		53	0.52000	0.00394	-0.06267
3	0.02000	0.99606	0.06267		54	0.53000	0.00886	-0.09369
4	0.03000	0.99114	0.09369		55	0.54000	0.01571	-0.12434
5	0.04000	0.98429	0.12434		56	0.55000	0.02447	-0.15451
6	0.05000	0.97553	0.15451		57	0.56000	0.03511	-0.18406
7	0.06000	0.96489	0.18406		58	0.57000	0.04759	-0.21289
8	0.07000	0.95241	0.21289		59	0.58000	0.06185	-0.24088
9	0.08000	0.93815	0.24088		60	0.59000	0.07784	-0.26791
10	0.09000	0.92216	0.26791		61	0.60000	0.09549	-0.29389
11	0.10000	0.90451	0.29389		62	0.61000	0.11474	-0.31871
12	0.11000	0.88526	0.31871		63	0.62000	0.13552	-0.34227
13	0.12000	0.86448	0.34227		64	0.63000	0.15773	-0.36448
14	0.13000	0.84227	0.36448		65	0.64000	0.18129	-0.38526
15	0.14000	0.81871	0.38526		66	0.65000	0.20611	-0.40451
16	0.15000	0.79389	0.40451		67	0.66000	0.23209	-0.42216
17	0.16000	0.76791	0.42216		68	0.67000	0.25912	-0.43815
18	0.17000	0.74088	0.43815		69	0.68000	0.28711	-0.45241
19	0.18000	0.71289	0.45241		70	0.69000	0.31594	-0.46489
20	0.19000	0.68406	0.46489		71	0.70000	0.34549	-0.47553
21	0.20000	0.65451	0.47553		72	0.71000	0.37566	-0.48429
22	0.21000	0.62434	0.48429		73	0.72000	0.40631	-0.49114
23	0.22000	0.59369	0.49114		74	0.73000	0.43733	-0.49606
24	0.23000	0.56267	0.49606		75	0.74000	0.46860	-0.49901
25	0.24000	0.53140	0.49901		76	0.75000	0.50000	-0.50000
26	0.25000	0.50000	0.50000		77	0.76000	0.53140	-0.49901
27	0.26000	0.46860	0.49901		78	0.77000	0.56267	-0.49606
28	0.27000	0.43733	0.49606		79	0.78000	0.59369	-0.49114
29	0.28000	0.40631	0.49114		80	0.79000	0.62434	-0.48429
30	0.29000	0.37566	0.48429		81	0.80000	0.65451	-0.47553
31	0.30000	0.34549	0.47553		82	0.81000	0.68406	-0.46489
32	0.31000	0.31594	0.46489		83	0.82000	0.71289	-0.45241
33	0.32000	0.28711	0.45241		84	0.83000	0.74088	-0.43815
34	0.33000	0.25912	0.43815		85	0.84000	0.76791	-0.42216
35	0.34000	0.23209	0.42216		86	0.85000	0.79389	-0.40451
36	0.35000	0.20611	0.40451		87	0.86000	0.81871	-0.38526
37	0.36000	0.18129	0.38526		88	0.87000	0.84227	-0.36448
38	0.37000	0.15773	0.36448		89	0.88000	0.86448	-0.34227
39	0.38000	0.13552	0.34227		90	0.89000	0.88526	-0.31871
40	0.39000	0.11474	0.31871		91	0.90000	0.90451	-0.29389
41	0.40000	0.09549	0.29389		92	0.91000	0.92216	-0.26791
42	0.41000	0.07784	0.26791		93	0.92000	0.93815	-0.24088
43	0.42000	0.06185	0.24088		94	0.93000	0.95241	-0.21289
44	0.43000	0.04759	0.21289		95	0.94000	0.96489	-0.18406
45	0.44000	0.03511	0.18406		96	0.95000	0.97553	-0.15451
46	0.45000	0.02447	0.15451		97	0.96000	0.98429	-0.12434
47	0.46000	0.01571	0.12434		98	0.97000	0.99114	-0.09369
48	0.47000	0.00886	0.09369		99	0.98000	0.99606	-0.06267
49	0.48000	0.00394	0.06267		100	0.99000	0.99901	-0.03140
50	0.49000	0.00099	0.03140		101	1.00000	1.00000	0.00000
51	0.50000	0.00000	0.00000					

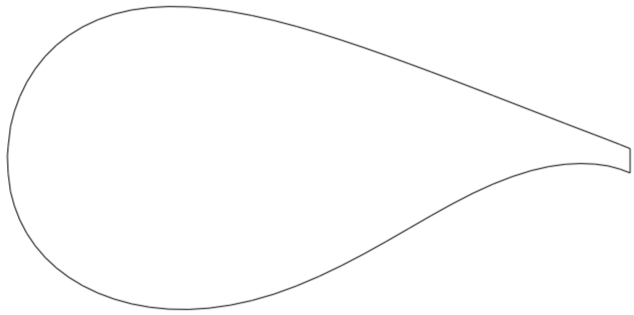

Figure 12: FFA-W3-480 Shape

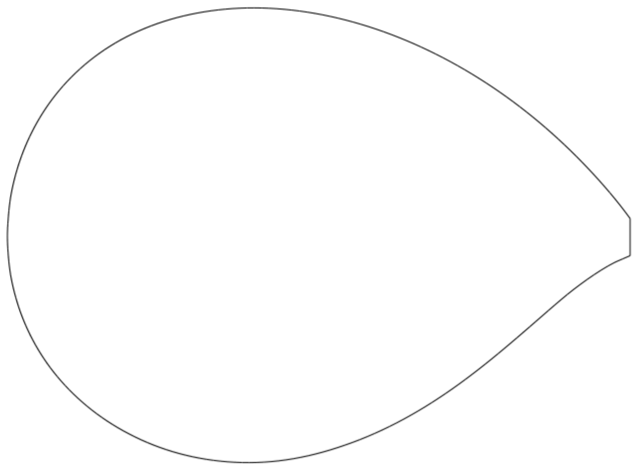

Figure 13: TC-72 Shape

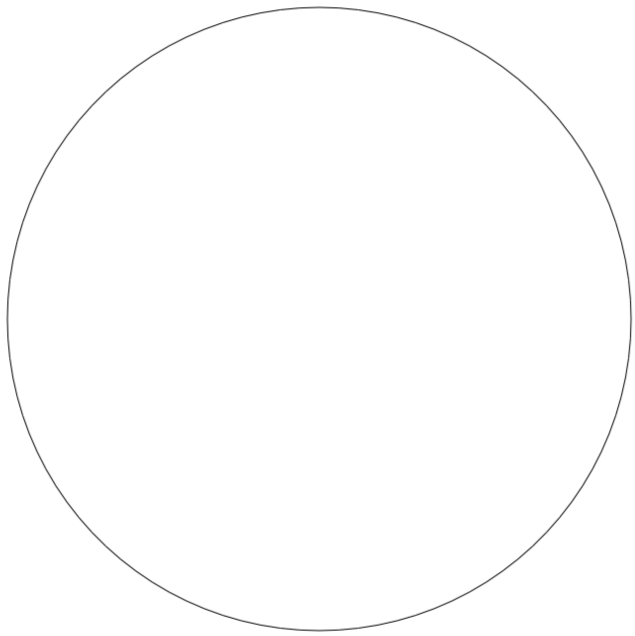

Figure 14: Cylinder Shape

Table 16: Wind Speed and Frequency

Wind Speed m/s	Cumulative Distribution
4.0	0.09703
5.0	0.10542
6.0	0.10657
7.0	0.10153
8.0	0.09185
9.0	0.06073
9.5	0.03623
10.0	0.03275
10.5	0.02930
11.0	0.02596
11.5	0.02276
12.0	0.02861
13.0	0.02906
14.0	0.02055
15.0	0.01401
16.0	0.01249
18.0	0.00739
20.0	0.00318
25.0	0.00033

3 The Optimization Problem

3.1 The objective

The optimization goal is to maximize the Annual Energy Production (AEP) for the rotor. The annual energy production is calculated based on the Weibull distribution for the wind speed. This distribution is given by a scale and shape parameter of $8m/s$ and 2 respectively. The density of air is $1.225kg/m^3$. For convenience, the cumulative distribution for the wind speed bins is given in Figure 16.

3.2 Wind turbine regulation

The rotor is expected to operate on a pitch regulated wind turbine platform. The optimization is suppose to find the pitch settings that produce the maximum power for the rotor when the target tip speed ratio cannot be achieved due to the RPM constraints without exceeded the maximum mechanical power. The upper limit of mechanical power is $10.6383MW$. It is expected that the blades are pitched to feather to shed power so the minimum pitch is given as $0.0°$. The rotational speed is controlled by targeting a tip speed ratio of 7.8 that is constant. There is a a minimum and maximum rotor speed of 6 and 9.6 RPM respectively for the platform. The target tip speed ratio is meant to be constant within this range.

3.3 Design variables

The design variables for this problem are the chord, twist and airfoil distribution along the whole blade except at the root. There are no constraints on the chord and twist distribution. However, there are constraints on how the airfoils should be blended. The airfoil distribution is altered by varying the weights in Table 3 or Table 2. The user should only use the airfoils given here in this report. The weights should sum to 1 for each station along the blade. Only interpolate between 2 airfoils in adjacent columns in Tables 3 or 2. This in effect reduces the airfoil distribution to a one dimensional design parameter.

3.4 Geometric constraints

The inner 80% of the blade has a minimum absolute thickness constraint to ensure the blade is thick enough for structural reinforcement. This constraint is given in Table 17. This is shown graphically in Figure 15. The absolute thickness is the product of the blended relative thickness and chord. The blended relative thickness is based on the airfoil blending weights.

Table 17: Absolute Thickness Constraint

S	Absolute Thickness $[m]$
0.00000	5.38000
0.03057	5.36300
0.06222	5.12887
0.09487	4.69181
0.12841	4.16284
0.16274	3.58395
0.19772	3.05710
0.23321	2.66154
0.26907	2.37277
0.30515	2.14436
0.34128	1.95137
0.37731	1.78626
0.41309	1.64011
0.44846	1.49563
0.48328	1.35340
0.51741	1.21399
0.55073	1.07789
0.58312	0.94557
0.61449	0.81742
0.64476	0.69379
0.67385	0.57494
0.70172	0.46110
0.72833	0.35242
0.75364	0.24901
0.77766	0.15091
0.80038	0.05811

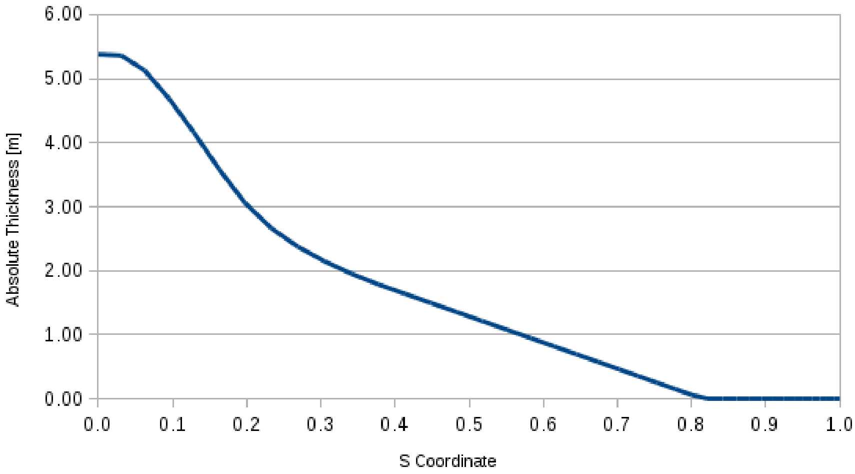

Figure 15: Absolute Thickness Constraint

3.5 Rotor load constraints

There is a thrust and root flap-wise bending moment constraint on the rotor. To limit the effect of code differences, the user is expected to simulate the initial design with their tools. The user can increase the thrust by 14%. Similarly the user can increase the root flap-wise bending moment by 11%.

3.6 Operating points

Researchers should evaluate the constraints and the AEP with the wind speeds given in Table 16. It is assumed that the wind is uniform, with no shear, turbulence or yaw error.

4 Reporting Results

The IEA Task 37 ask the participants to apply their optimization tools to this problem and prove their results to Michael McWilliam at **mimc@dtu.dk**. This section describes what information and how the researchers should specify. The requested information can be grouped into 4 parts. The first part asks researcher to describe their tools and optimization algorithms in section 4.1. The third part asks researchers to report their initial performance, optimal design and optimal performance in section 4.2. This study aims to quantify the effort required to apply numerical design optimization, researchers are asked to provide this information in section 4.3.

4.1 Reporting Tools

Researchers are asked to report the details on the their simulation tools. This comparison is open to all researchers with any aerodynamic model, however it is expected that most researchers will use Blade Element Momentum (BEM) type models. If this is the case, researchers are asked to provide the following details. Otherwise, if researchers used different models they are asked to give a similarly detailed description of their models.

- Does the BEM model solve the angular momentum equations?
- Does the model use a tip-loss model, and what model do you use?
- Does the model use a turbulent wake model and what model do you use?
- Does your model include any other correction models?
- Although this should not affect results, we are also interested in the following:
  - Do you use a dynamic wake model?
  - Do you use a dynamic stall model?
- Please describe anything unique about your set of tools?

Researchers are asked to report the following about the tools they use for the optimization:

- How did you parameterize the design (*e.g.* directly, splines or other)?
- How many design variables did you use?
- What optimization algorithm did you use?
- Please give the following details on your optimization algorithm:
  - Please give any relevant tuning parameters (*e.g.* for genetic algorithms what was the mutation rate)?
  - What was the stopping criteria you used for your optimization?
- Describe the tools you used to couple your models to the optimization algorithm (*i.e.* the "glue" code)?
- If you used gradient based algorithms please answer the following:
  - What algorithm did you use to calculate your gradients?
  - What step-size did you use for relevant algorithms?
  - Did you have to tune your gradient algorithms?
  - If you used analytic gradients, did you use forward propagating algorithms (*i.e.* primal) or backwards propagating (*i.e.* adjoint equations)?
  - Describe any automatic differentiation packages you may have used?
- Please describe any alterations to the optimization problem that were required for you to apply your tools?
- Please describe anything that is unique or special about your optimization tools?

4.2 Reporting Optimization Results

To help facilitate the use of automated tools in compiling the overall results, we ask you to use either the excel template or the YAML file to report all your results. If you use the excel template please place your results in the indicated cells, any changes in the formatting may cause errors.

We ask researchers to report 4 sets of results described below. We ask researchers to simulate the initial design with their tools and provide those results so we can evaluate differences in the tools. The remaining results pertain to the optimization solution. We want you to report the actual results so we can assess differences in the code. However, your results will be normalized according to your initial results and reported as relative changes in our publications.

- Researchers are asked to report the following for the wind speeds given in table 16 for the initial design:
  - Initial AEP
  - Initial mechanical power
  - Initial rotor thrust
  - Initial root flap-wise bending moment
  - Please provide the following detailed results for the initial design:
    * The distributed thrust and torsion forces along the blade
    * The normal and tangential flow velocity along the blade
    * If relevant the circulation distribution
- For your optimal solution please provide the following design variables:
  - Chord distribution vs. S
  - Twist distribution vs. S
  - Airfoil blending weights vs. S
  - Rotor rotational rate vs. wind speed
  - Blade pitch vs. wind speed
- Researchers are asked to report the following for the wind speeds given in table 16 for the optimal design:
  - Optimal AEP
  - Optimal mechanical power
  - Optimal rotor thrust
  - Optimal root flap-wise bending moment
  - Please provide the following detailed results for the optimal design:
    * The distributed thrust and torsion forces along the blade
    * The normal and tangential flow velocity along the blade
    * If relevant the circulation distribution
- Please describe the following results from the final optimization calculation:
  - Number of iterations
  - Number of function evaluations
  - The level of convergence
  - If relevant, the number of gradient evaluations
  - If relevant, the Lagrange multipliers for the constraints
  - If known, the norm of the Kuhn-Karush-Tucker condition achieved in the final solution

4.3 Reporting Effort

We would like to know the effort that was required to obtain your results. We want this information to understand the resources that are required to incorporate numerical design optimization in industrial design processes. We would like researchers to quantify the effort for tool development, conducting the optimization study and the computational resources. We expect that many researchers have already developed their many of the tools so please include this past effort in your estimates. The following list outlines the questions:

- What additional effort was required in the development of the simulation tools to make them appropriate for optimization. These are the following situations we are interested in, please estimated the man-hours and any challenges you faced in any of these tasks.
  - We are not interested in quantifying the effort that was required to develop the tool for pure simulation purposes, only the effort for adapting an existing tool for optimization.
  - Improving the robustness of the numerical algorithms in the simulation code
  - Eliminating any troublesome discontinuities or nonlinearities in the simulation code
  - Implementing analytic gradient algorithms in the simulation code
  - Making the simulation code faster specifically for the purpose of optimization
  - Any other challenges in adapting the simulation tools for optimization
- Please describe the effort required to couple the simulation code to the optimization algorithm. This includes the following tasks:
  - Wrapping the simulation code in the glue code
  - Wrapping the optimization algorithms in the glue code
  - Developing the code to automatically generate input files for the simulation tools
  - Developing the code to automatically extract the results from the simulation tools
  - Writing the code to perform any post-processing functions
  - Validating the optimization set-up
- Please describe the effort required to set-up and execute this optimization problem. It is expected that you may perform multiple studies to obtain the final results. We would like to know the total effort to obtain the final results and the effort required to set-up a single optimization calculation. We would like to know both the man-hours and the number of days that was required to obtain the final results. This includes the following tasks:
  - Performing any initial parameter studies or sensitivity studies
  - Preparing the input files for the optimization problem
  - Man-hours fixing any problems in the set-up for this specific problem
  - Tuning numerical gradients or optimization algorithms for this specific problem
  - Setting-up and executing any exploratory optimization studies
  - The effort to set-up a single optimization calculation
- Please report the computational effort that was required to obtain the final results. The previous questions were focused on the engineering effort, this part is more focused on the computational effort. Again we want to know the total effort for all the studies and the effort for a single optimization calculation.
  - Describe the computational resources you typically use for your optimization calculations
  - Describe any parallel execution techniques that you use in your optimization calculations
  - Describe the computational set-up you use for your optimization calculations, number of nodes, number of CPUs, the maximum wall time, *etc.*
  - Estimate the total CPU time[2] and Wall time[3] for all your studies combined
  - Estimate the CPU time and Wall time required for a single optimization calculation

[2] CPU time is the total amount of time used by all CPU's to perform the calculation. When executed in serial, this time is typically the same as the Wall time. When the calculation is executed in parallel, this time is typically the wall time multiplied by the number of processors. The CPU time does not always correspond to the Wall time for less efficient implementations where CPU's can be stalled waiting for various resources.

[3] The Wall time is the time elapsed from when the calculation starts to when it completed (*i.e.* how long you have to wait to get your results)

A Meeting Minutes with Clarifications

A.1 Meeting April 21st 2017

Attendance:

- Ervin
- Michael McWilliam
- Pietro
- Birger
- Ozlem

Agenda:

To briefly present the aerodynamic optimization test case and answer any questions.

Clarification Notes:

The main points of discussion in the meeting were on clarifying the aerodynamic optimization test case. The test case was communicated in a general format that we are hoping to standardize for future test cases where more complicated descriptions are required. However, this was a source of confusion. Despite the complexity of the test case description, the test case is actually quite simple. Here are the following points of clarification that were required:

Why 2 sets of airfoils? We wanted to make the test case applicable to both BEM/Lifting line and fully resolved panel/CFD codes. The former are based on airfoil coefficients where as the latter are based on the actual 3D geometry. The initial design is based on a modified DTU 10MW wind turbine. For that turbine there is not an established airfoil between 48% thickness and the cylinder. To help in the interpolation an intermediate airfoil was created. For the coefficients, a 60% thick airfoil was chosen. While for the geometry a 72% thick airfoil was chosen.

Do people need to calculate their own polars? The focus of the test case is on the planform design variables. A 2D Airfoil shape optimization case study may be created if people are interested. Accordingly any optimization based on models that rely primarily on coefficients should only use the coefficients. You should not calculate your own coefficients based on the profile shapes. The profile shapes are available for the special case where an analysis code must have the 3D geometry.

Why are the airfoils specified with blending weights? The idea behind the weights is to create a more general description for the airfoil selection. In future test cases one may have multiple airfoil families to choose from, where one family could be optimized more for aerodynamics another family for structural reasons and others for other reasons. In this scenario one cannot select airfoils based on relative thickness alone, so that is why this approach has been adopted.

Are we expected to optimize on the weights directly? No, part of the case study is to explore different parameterization schemes, so we encourage users to use the scheme they think is most appropriate. The weights are merely a scheme to describe the airfoil selection. Optimization based on blending weights directly introduce some challenges that need to be address: 1) The weights at any station should always sum to 1; 2) All individual weights should be between 0 and 1, negative and greater than unity weights do not make sense 3) One should only blend between airfoils of similar thickness. Any other blending does not make sense.

Can we optimize based on relative thickness? In this test case, there is only 1 family, so a relative thickness parameter could be used to select the families. The relative thickness is given in the spreadsheet.

Why is the blade shape description so complicated? Again, this case study is trying to establish a general way of specifying blade geometry for future case studies. In future case studies, the blades may have pre-bend or sweep. For these curved blades it is not completely clear how the profile shape and twist is applied to the blade. Thus, the scheme defines a local Frenet frame to describe cross section geometry. This description will be more relevant for more complicated geometry.

Do we need to consider this complicated geometric description? No, the blades in this test case are straight

Other points of clarification:

- Root cylinder size must be heald constant to attach to the hub
- No tilt, no coning
- The entire wind turbine structure should be treated as rigid, thus no elasticity is required and no mass parameters are required. For codes that assume flexible bodies, increase stiffness and decrease accordingly to achieve near rigid results.
- The thrust and bending moment constraints are evaluated by integrating the aerodynamic forces. Inertial, gravitational and other forces are ignored.

Something that is unique in this test case, is that we ask practitioners to report their effort. MDAO is an alternative engineering approach. For industry to adopt these practices they need to know the man-hours and the computational resources that are required. Thus, we would like to report this information as well.

Work-Plan:

The following participants have agreed they would provide results:

- Pietro
- Ozlem
- Birger
- DTU

I suspect these additional participants may also provide results:

- Curran Crawford from UVic
- Andrew Ning from BYU
- NREL
- CENER

The time line for providing results is as follows:

1. We ask that as many participants as possible try to give preliminary results in the first week of June 2017. The purpose here is to have multiple people apply the test case to confirm that everything is sufficiently specified and that there are no serious problems. Furthermore these preliminary results will be presented at the WESC 2017 in Lyngby Denmark.
2. The final deadline for optimization results will be August 15 2017. This should give most people enough time to provide results. The final comparison will be presented at the 4th biannual System Engineering Workshop 2017.