Best Practices for Wake Model and Optimization Algorithm Selection in Wind Farm Layout Optimization.

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This paper presents the results of two discrete case studies regarding the Wind Farm Layout Optimization (WFLO) problem. Case study (1) considers variations in optimization strategies for a simplified Bastankhah Gaussian wake model, while case study (2) studies trade offs in performance with variation in both physics model and optimization strategy selection. For (1), a supplied wake model outputs Annual Energy Production (AEP) given participant input turbine locations. For (2), participants calculate AEP using a wake model of their choice, while also implementing their preferred optimization method. Participant submissions for optimized turbine locations were then cross-compared through on another's wake models. Results for (1) show that gradient-based methods, on average, are superior to gradient-free methods in terms of discovering an optimal resultant AEP. A clear front-runner emerged with the SNOPT + WEC optimization method, which consistently discovered a comparative optimal AEP for each scenario. Results for (2) show that for small windfarms with few turbines, turbine placement on the wind farm boundary is superior, despite the majority of participant optimization methods failing to discover this. Conclusions for (2) were drawn from participant cross-comparison of results, but further study must be done using an LES analysis of participant turbine layouts.

I. Introduction

Optimizing turbine placement within a wind farm is a complex problem characterized by many local minima. The large number of inter-dependent variables involved in Wind Farm Layout Optimization (WFLO) create a design space that can quickly become intractable. We designed and conducted a set of case studies to discover the best practices in solving the WFLO problem.

Two approaches have been taken to simplify the WFLO problem, as described by Padron et.al.[1] The first approach aims at improving the quality of individual models for wind farm attributes, i.e., aerodynamics, atmospheric physics, turbine structures, etc. As they go on to state, "The second approach is to improve the optimization problem formulation, and the algorithms [used] to solve the optimization."[1]

To better model the aerodynamics of the waked airflow region of a wind turbine, complex computational methods such as such as Direct Numerical Simulations (DNS) or Large Eddy Simulations (LES) have been developed. But the computational time these require for full simulation can be prohibitive in multi-iterative optimizations. Simplified Engineering Wake Models (EWMs) make certain limiting physics assumptions, resulting in greatly reduced computational costs.[2] Yet these simpler, less accurate approximations can sometimes lead to inefficient recommendations for turbine placement, due to what can be inaccurate assumptions in specific wind farm scenarios.

Given a single EWM, optimization methods to select ideal turbine locations are limited by characteristics of the functions governing the model. For example, EWMs that define a discontinuous wind speed behind wind turbines cannot be effectively used with gradient-based optimization methods, and models for which gradients have not been calculated are limited to gradient-free algorithms, or gradient-based with finite difference derivatives. Additionally, within these limitations different optimization strategies have varying capacity to escape local optima in the pursuit of a global optimum.

To better understand the differences in EMW selection and optimization algorithm application, we have created two discrete case studies. These studies are designed to involve participants from many different research labs working on the WFLO problem. The first isolates optimization techniques for a single simplified EWM, the second observes the differences when combining variations in EWM selection and optimization method.

In our research, we have not found a study similar to this ever done before. Though papers have been published which survey the state of the Wind Farm Optimization (perhaps most notibly the one by Herbert and Acero[2]), our Case Studies are the first time international collaboration has been conducted to isolate optimization method and EWM selection to determine "best practices" when addressing the WFLO problem.

This work is done in support of the the International Energy Agency (IEA)'s Task 37. The IEA was created in 1974, and currently has 30 member countries. Its mission is "to ensure reliable, affordable and clean energy"[?] for those countries, and does so through four areas of focus:

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- Energy security
- Economic development
- Environmental awareness
- Engagement worldwide.

The IEA's Technology Collaboration Program (TCP) has a Working Party on Renewable Energy Technologies (REWP). REWP, itself, has a Wind Energy TCP, which is further subdivided into numbered tasks. These tasks cover individual concepts relative to wind energy.[7] For example Task 19 deals with Wind Energy in Cold Climates, Task 26 deals with the Cost of Wind Technology.

Our Case Studies are created in support of IEA's Wind Task 37 (Task37). Task37 coordinates international research activities centered around the analysis of wind power plants as holistic systems.[3] Though our case studies concentrate mainly on wake modelling optimization at the farm-level scale, our results still contribute to the "hollistic" knowledge of wind energy.

II. Methodology

Many factors affect reccomendations for superior turbine placement of a proposed wind farm. Two of the major factors[2] which we chose to study are 1) EWM characteristics and 2) optimization algorithm. We therefore designed two distinct case studies in an attempt to quantify the effects of alterations in each of these variables.

To isolate variability in the optimization method, we pre-coded a representative wake model as a control variable, and permit participants to use any optimization strategy they think will discover turbine locations which deliver the best Annual Energy Production (AEP) for the farm. This first Case Study is called the Optimization Only Case Study, and is described below in Section B.

Isolating EWM variability proves more complicated. An EWM's compatibility with gradient-based or gradient-free optimization methods dictate which algorithms can be applied. As such, designing a case study that restricts participants to a single optimization algorithm would unnecessarily limit the scope of EWMs studied. For this reason, our second case study permits not only participant selection of EWM, but also implemented optimization algorithm. It is called the Combined Physics Model/Optimization Algorithm Case Study, and is described below in Section C.

To enable production of useful data, both Case Studies require a model wind farm with characteristics that are simultaneously restrictive enough to maintain simplicity, yet general enough to maintain relevance to more complex and realistic problems. The wind farm scenarios we selected to meet this criteria, and other details relevant to both case studies are described below in Section A.

A. Common to Both Case Studies

1. Turbine

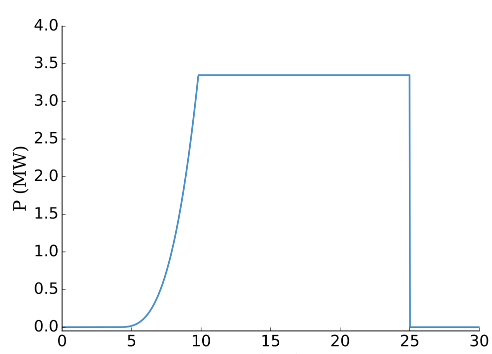
We used NREL's 3.35MW reference turbine in all wind farms. It's attributes are open source, and it is designed as a baseline for onshore wind turbine specifications.[4] The specifics of the turbine necessary for our simplified Gaussian wake model (used in Case Study 1) are located in Table 1:

Table 1: Attributes for NREL's 3.35MW onshore reference turbine

Rotor Diameter	130	m
Turbine Rating	3.35	MW
Cut-In Wind Speed	4	m/s
Rated Wind Speed	9.8	m/s
Cut-Out Wind Speed	25	m/s

Its power curve is defined as:

$$P(V) = \begin{cases} 0 & V < V_{cut\text{-}in} \\ P_{rated} \cdot \left(\frac{V - V_{cut\text{-}in}}{V_{rated} - V_{cut\text{-}in}} \right)^3 & V_{cut\text{-}in} \leq V < V_{rated} \\ P_{rated} & V_{rated} \leq V < V_{cut\text{-}out} \\ 0 & V \geq V_{cut\text{-}out} \end{cases}$$



2. Farm Geography

To focus on optimization method and EWM variability, as well as to avoid introducing too many unecessary variables, the wind farms for all scnearios are on flat and level terrain.

Boundary Shape

To reduce boundary impacts on farm design, we chose a radially-symmetric farm boundary. Doing otherwise may have tended to give optimal turbine locations where turbines cluster at the farm corners or protrusions.

Turbine Placement
Though a turbine's blade radius is permitted to extend beyond the farm boundary, turbine (x, y) hub locations are restricted to be on or within the boundary radius. Turbines are further constrained to be no less than two rotor diameters apart from any other turbine.

Farm Diameter
Farm diameter sizing for each scenario needs to be restrictive enough to avoid simply placing all turbines on the boundary, yet also permit meaningful turbine movement by the optimizers. The boundary radii were selected to get an average turbine spacing of 5 rotor diameters, with turbines placed in concentric rings. Past experience and intution tell us that this pattern of concentric ring placement tends to result in turbine layouts resulting in optimal AEP.

Example Layouts
These layouts are displayed below graphically in Fig. 1. Though primarily for Case Study 1, they are provided to participants of both studies in `.yaml` format for 4 reasons:

1. To provide an example of the `.yaml` schema formatting being developed by IEA Task 37, used to enable collection of results
2. To help participants visualize the different farm sizes and geography
3. To verify AEP calculation if participants implement the Case Study 1 wake model in a different language
4. To provide participants with an optimization starting point if desired

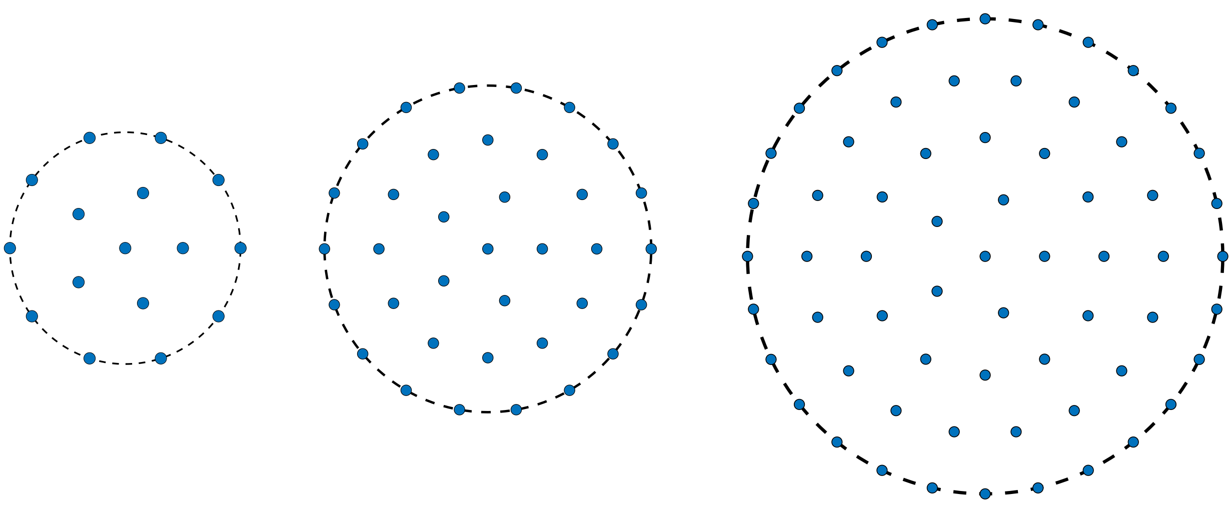


Figure 1: Example layouts for 16, 36, and 64 turbine farms

It is explicitly stated in the announcement document that these layouts are only examples. Participants are not required to use these as starting points for their optimizations, though they have the option to do so.

3. Wind Attributes

Again, to avoid introducing unecessary variables, the wind distribution frequency and wind speed are the same for all wind farm scenarios in both Case Studies.

Wind Speed
Freestream wind velocity is constant in all wind directions, at 9.8 m/s, regardless of turbine location or time of day. 9.8 m/s is used because it is the minimum wind speed for the NREL 3.35MW reference turbine to produce its rated power. Using this incoming wind velocity, turbine wakes will push air speeds down the turbine's power curve, and more local optima will be experienced by participant optimizers. If the incoming wind speed were too high (i.e. 13 m/s), some turbine layouts, though different in location, will produce the same power output.

Wind Direction Frequency
In creating these Case Studies, we desired a wind direction frequency distribution (displayed graphically in a wind rose) that meets two criteria:

1. Creates many local optima
2. Prevents dissimilar turbine layouts from reaching the same AEP values

A lack of local optima in a design space permits even inefficient optimizers to find a "best" result. In a design space where local optima are present, inferior designs are very likely. Such design spaces test the robustness of optimization methods. In our tests, uni-directional windroses often push turbines into a single row perpendicular to the major wind direction, hinitng at a relative lack of local optima. Omnidirectional roses that we tested tended towards a pattern of simply spreading the turbines out uniformly, also hinting at a lack of local optima. We therefore intentionally designed a windrose with relatively many local optima, in order to demonstrate the difference in capabilities of the various optimization algorithms used by participants.

If, on the other hand, many unique layouts produce high and similar AEP values, it would likewise not tell us much regarding the abilities of participant optimization methods. When many answers give superior results, algorithms are not incentivized to find one "best" result.

Therefore to acheive both of these criteria, we experimented with 4 different patterns of wind direction frequency:

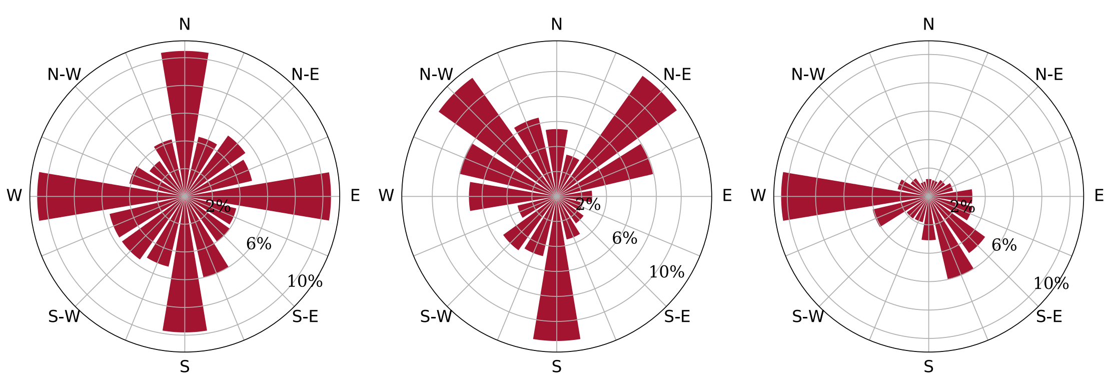
1. Bi-modal uniaxis (to simulate a canyon geography)
2. Quad-directional (experienced at some onshore airports)
3. Tri-direcitonal (experienced by some coastline geographies)
4. Bi-modal off-axis (experienced in both on- and off-shore locations)

Simulations were run using the simplified Gaussian wake model and the optimization package SNOPT. We ran each wind rose through 1000 random-start optimizations. All wind roses gave generally similar optimization results with AEP distributions following a bell-curve pattern. There were slight differences, however, in that:

1. Bi-modal uniaxis gave turbine arrangements that tended towards a straight line, perpendicular to the uniaxis of the wind. Since it met our first disqualifying criteria of a lack of local minima, it was not further analyzed.
2. Quad-directional gave turbine arrangements that tended towards a grid pattern. Since this too didn't have many local optima to trap optimizers, it was disqualified.
3. Tri-directional gave many gird-like arrangements that all had similar AEP values. Since it met our second disqualifying criteria of non-similar layouts producing similar AEP, it was not selected.
4. Bi-modal off-axis gave few results with high AEP values. We interpreted this to be indicitive of the presence of many local optima. Since this best met our selection criteria, it is the wind rose utilized for our Case Studies.

The wind roses are binned for 16 directions. The quad-directional, tri-directional, and bi-modal off-axis roses are dipectied in Fig. 2, and histograms of the 1000 random start optimizations are in Fig. 3.

Figure 2: Quad-directional, Tri-directional, and Bi-modal off-axis wind roses



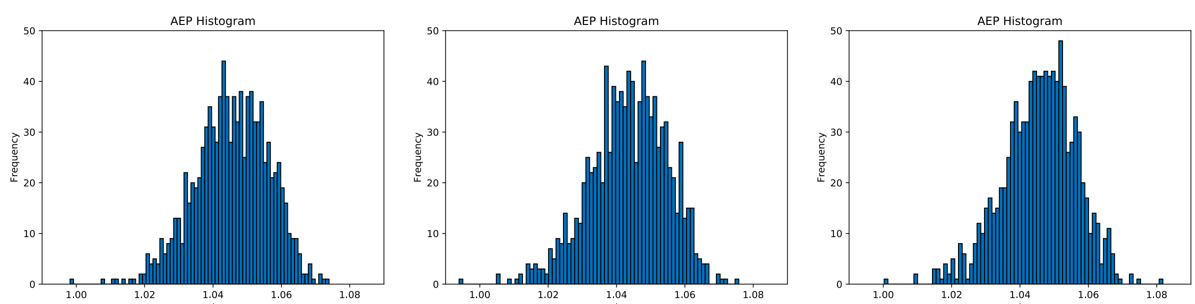


Figure 3: 1000 random start optimization runs for the Quad-, Tri-, and Bi-modal off-axis wind roses

After the above described study, we selected the bi-modal off axis wind frequency distribution for our Case Studies, the rightmost windrose in Fig. 2. A greater magnitude in the radial direction from the origin indicates a higher frequency from that cardinal direction.

4. *Data File Type*

One request made by members of IEA Task 37 work package (WP) ten (X) was to implement the IEA 37 WP X's ontology open source `.yaml` schema for all necessary data. This included:

- Farm turbine attributes
- Farm turbine locations
- Farm wind frequency and wind speeds

There exists a separate IEA37 working group tasked with refining the precise schema of these items, whose work is outside the scope of our specific Case Studies. Though a current work in progress, we implemented the most recent iteration and adapted them to our scenarios as necessary.

The above mentioned data schema format is supplied to participants of both cases. They are:

1. `iea37-windrose.yaml` - binned wind frequency for both Case Studies, in `.yaml` format
2. `iea37-335mw.yaml` - data for reference turbine used in both Case Studies, in `.yaml` format

Further examples for reporting turbine locations in `.yaml` schema, as well as a Python parser for all these schema was made available to Case Study participants. Since the use of the example layouts and Python `.yaml` parser is more applicable to the Optimization Only Case Study, they are described in Section B.

B. Optimization Only Case Study

The purpose of this Case Study is to determine the best optimization practices for WFLO, using a single representative EWM.

1. *Wake Model*

A simplified version of Bastankhah's Gaussian wake model[5,6] is used, since it is compatible with both gradient-based and gradient-free methods, and is computationally inexpensive in comparison to LES and DNS methods. This wake model is described by the following equations:

$$\frac{\Delta U}{U_\infty} = \left(1 - \sqrt{1 - \frac{C_T}{8\sigma_y^2/D^2}}\right) \exp\left(-0.5\left(\frac{y-\delta}{\sigma_y}\right)^2\right) \tag{1}$$

Where $\frac{\Delta U}{U_\infty}$ is the wake velocity deficit, $C_T = \frac{8}{9}$ and is the thrust coefficient, $y - \delta$ is the distance of the point of interest from the wake center in the cross-stream horizontal direction, D is the turbine diameter, and σ_y is the standard deviation of the wake deficit in the cross-stream horizontal direction as defined in Eq. (2):

$$\sigma_y = (k_y \cdot x) + \frac{D}{\sqrt{8}} \tag{2}$$

In Eq. (2), x is the downstream distance from the turbine generating the wake to the turbine of interest, and D is the turbine diameter. k_y is determined as a function of turbulence intensity (I). In this case study turbulence intensity is treated as a very small constant of 0.075, and we therefore used a coresponding k_y of 0.0324555.[6,7]

Increasing turbulence intensity has numerous effects and draws attention away from the main purpose of this Case Study, which is to observe the differences of optimization strategies. For the wake model we use (given in Eq. (1)), increasing the turbulence intensity widened the wake cone, but second and third order effects are unknown. As such, this first IEA Task 37 set of Case Studies uses a very low intensity in an attempt to minimize the considered variables.

2. *Farm Sizes*

Variability in wind farm size (and thus number of design variables) affect optimization algorithm performance. To account for this, 3 wind farm sizes are specified in Case Study 1: 16, 36, and 64 turbines. Inclusion of 3 farm sizes is to observe how increased complexity correlates to convergence time and algorithm performance, to demonstrate trends of scalability in optimization algorithms.

The turbine numbers are selected as perfect squares which roughly double in size. Perfect squares are used to permit even grid turbine arrangements, if desired.

Example `.yaml` schema is provided to all participants, to help understand the format with which they will need to report their optimal turbine locations. These files are:

- `iea37-ex16.yaml` - 16 turbine scenario example layout
- `iea37-ex36.yaml` - 36 turbine scenario example layout
- `iea37-ex64.yaml` - 64 turbine scenario example layout

These example layouts are depicted graphically, previously in Fig. 1.

3. *Supplied Code*

To enable participation in this Case Study, we created and supplied a pre-coded Python module. This module includes:

- Turbine charactersitics, wind frequency, and wind speed in IEA 37 WP X's `.yaml` schema
- Example turbine layouts for each farm size (in `.yaml` format), displayed graphically in Fig. 1
- Python parsers of the `.yaml` schema, included in the Appendix
- Python target function to calculate AEP (given `.yaml` turbine locations and farm attributes)

We selected the programming language Python since it is open source and widely used by researchers in the industry.

Participant alteration to our specific code implementation, or replication of our model in another language, was permitted if needed for compatibility with participant optimization methods. This is with the understanding, however, that final wind farm layouts would be evaluated with the original Python code that we provided.

C. Combined Physics Model/Optimization Algorithm Case Study

The intent of this Case Study is to assess not only the optimization methods measured by Case Study 1, but also the effects that different physics model approximations have on turbine location recommendations.

Case Study 2 differs from the previous one in that 1) no wake model is provided, and 2) only a single wind farm size is to be optimized. Participants are free to chose their preferred EWM and optimization method combination.

Unlike Case Study 1, participant reported AEP is not comparable, since different EWMs (which account for different physics phenomena) are used to calculate them. To help with this, we conducted a cross-comparison of results between participants. For the cross-comparison, each participant's proposed optimal turbine locations in the standardized `.yaml` format was published to the other Combined Case Study participants. Each participant would then use their own wake model to calculate the AEP of the other partici[ant's proposed farm layouts. From this portion of the Case Study, we hope to learn if any participant's results are seen as superior by other EWMs.

1. *LES Comparison*

Due to the difficulty in comparability of results between EWMs, we will also run all participant-reported optimized turbine locations through an LES for comparison. With the inherent bias each EWM has for its own optimized locations removed, reported turbine locations will be measured using the same simulation tool for a comparative AEP. The LES we will use is produced by NREL and called the Simulator fOr Wind Farm Applications (SOWFA). Unfortunately, due to the computional time requirements, all participant submissions were not able to be run through SOWFA before the writing of this document.

2. *Farm Attributes*

The wind farm size for the Combined Case Study is limited to 9 turbines, in order to limit the LES computation time requirements when assessing results. The previously described method under **Farm Diameter** was used to determine the boundary distance, and the wind rose and wind speed are the same as Case Study 1.

III. Results

A. Case Study 1: Optimization Only

Each participant ran the optimization algorithm of their choosing using our suppied AEP target function, or a functional equivalent in another language. Since there exists a great deal of variability in hardware, participants also reported processor speed, function calls, number of cores utilized, and amount of RAM installed in their system when finding their optimized results. The AEP results and rankings are given below in Tables 2 to 4.

There were 10 submissions for the Optimization Only Case Study. One participant submitted twice, using a different optimization method for each submission. These two submissions are treated as if from different participants. For anonimity, each submission is assigned a number. We will refer to each submission below by this participant number (i.e. par1, ..., par10, etc.).

1. *Data*

Tables 2 to 4 display the final AEP data of all participant proposed optimal turbine layouts. The Python module we supplied which uses the simplified Gaussian wake model was used for all AEP calculations. Submissions are ranked from highest to lowest resultant AEP values, with participant number (par#) and whether a gradient-based (Yes) or gradient0free (No) optimization method was used.

Table 2: 16 turbine scenario participant results

Rank	AEP	par#	Grad.
1	418924.4064	4	Yes
2	414141.2938	5	Yes
3	412251.1945	8	Yes
4	411182.2200	1	Yes
5	409689.4417	2	No
6	408360.7813	10	Yes
7	402318.7567	3	No
8	392587.8580	7	No
9	388758.3573	6	No
10	388342.7004	9	No
ex.	366941.5712	11	N/A

Table 3: 36 turbine scenario participant results

Rank	AEP	par#	Grad.
1	863676.2993	4	Yes
2	851631.931	10	Yes
3	849369.7863	2	No
4	846357.8142	8	Yes
5	844281.1609	1	Yes
6	828745.5992	3	No
7	820394.2402	5	Yes
8	813544.2105	9	No
9	777475.7827	7	No
10	776000.1425	6	No
ex.	737883.0985	11	N/A

Table 4: 64 turbine scenario participant results

Rank	AEP	par#	Grad.
1	1513311.194	4	Yes
2	1506388.415	2	No
3	1480850.976	10	Yes
4	1476689.663	1	Yes
5	1455075.608	3	No
6	1425678.143	8	Yes
7	1422268.714	9	No
8	1364943.008	6	No
9	1336164.550	5	Yes
10	1332883.433	7	No
ex.	1294974.298	11	N/A

General Trends

Participants were given 4 calendar weeks to code, process, and finalize a turbine placement that delivered their proposed optimal AEP. All participants are volunteers, and their participation was conducted outside of normal work or educational responsibilities. This 4-week optimization window factors to be 0.2% of typical wind farm's 20-year lifespan,[2] and we submit that optimizations (even if needing the full extent of this timeframe) would be worth the time they require, if able to produce superior results. However, from the participant self-reported wall-times, none of the optimizations took nearly that long.

For this reason, we discount required wall time as a distiguishing factor, and use resultant AEP as our main metric for measuring algorithm effectiveness. Though factors like processing time and computing power do vary, we consider algorithms that produce results anywhere within the 4 week announcement-to-call window to be acceptable for applications in industry.

As a general trend, gradient-based methods performed better in discovering a relative optima, especially for smaller farm sizes. Some gradient-based algorithms improved in comparative performance as the number of design variables increase (par10, par3), while others degrade (par5, par8). Simultaneously, some gradient-free algorithms increase in effectiveness as design variables increase (par2, par3), while others compete for worst comparative performance, regardless of farm size (par6, par7, par9).

Despite these multi-variate results, one clear front-runner does emerge. par4's algorithm, regardless of wind farm size, consistently discovers turbine placement that delivers an AEP superior to all other participants. A summary of par4's method is included in a following section.

Also of note, as the number of design variables increases, the relative disparity between proposed optimal AEPs likewise diverge. For the 16 turbine case, the best result is 7.88% better than the worst. For the 36 and 64 cases, the best result is 11.45% and 13.54% better than the worst, respectively.

2. Analysis of Best Results

Gradient-based

For all 3 farm sizes, the superior method was implemented by par4, using a gradient-based method. Coded in Python and FORTRAN, it combined the Sparse Nonlinear OPTimizer (SNOPT)[8] with a sampling method called Wake Expansion Continuation (WEC).[6] Running 200 optimizations, par4 had one iteration start from the provided example layout, and the other 199 use randomized turbine starting locations within the farm boundary.

The WEC method is specifically designed to reduce the multi-modality found in wind farm layout optimization. In the cited paper,[6] it is also referred to as a Design Space Relaxation Optimization Process (DSROP). DSROP is a method of converting design spaces with many local minima into curves approaching convexity, allowing gradient-based optimizations to more easily find the global solution. An example of such "relaxation" to convexity is included in Figs. 4 and 5, reproduced for description.

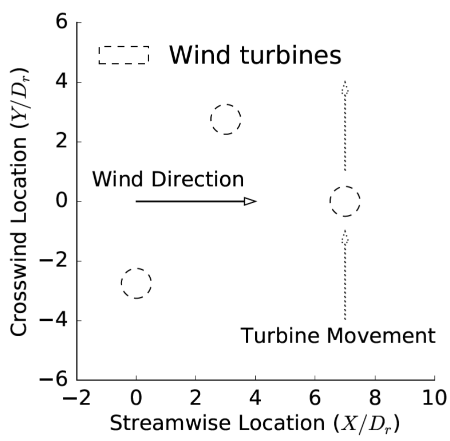


Figure 4: Simple design space used to demonstrate the effects of the relaxation factor, ξ, on the wind farm layout design space.[6]

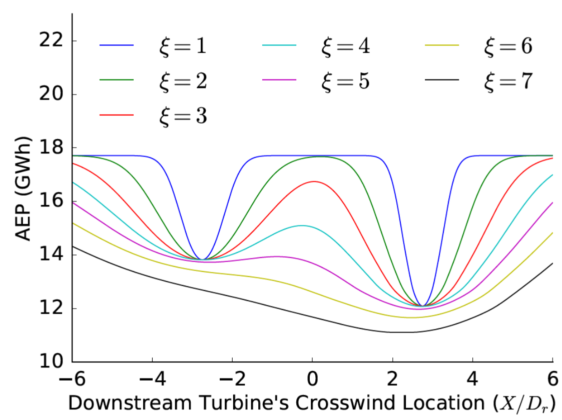


Figure 5: The impact of the wake relaxation factor, ξ. One turbine was moved across the wakes of two upstream turbines (see Fig. 4).[6]

Figs. 4 and 5 demonstrate the effects of the WEC method on a simple design space, relaxing the local optima into a more easily discovered global solution. As the author J.J. Thomas states, "Larger values of ξ allow the smaller local optima to disappear completely. Smaller values of ξ allow for more accurate wake widths but with an increase in the number and magnitude of local optima."[6] We suspect that this WEC sampling method for relaxing the multi-modality of the design space is the reason that par4's optimizatons found superior layouts to the other used methods.

Gradient-free

Of the gradient-free methods used, the superior particiant was par2. Futhermore, the relative performance increased as wind farm size increased. Out of the 10 participants, par2 ranked 5th, 3rd, then 2nd as the turbine sizes went from 16, to 36, to 64.

Programmed in Python, par2 used a "Preconditioned" Sequential Quadratic Programming (SQP) optimization method. SQP is an optimization method which breaks the problem into quadratic subproblems. To seed steps between generations, par2 took a starting layout and rotated it in $\pi/6$ steps. The best of the 12 resultant layouts was then taken as the warm-start for the next generation.

3. Analysis of Worst Results

Gradient-based

The worst performing gradient-based approach was different for each wind farm size. For the 16 turbine farm, par10's method was the poorest performing gradient-based method, but climed to the second best gradient method for the large sizes. For the 36 turbine farm, par1 was the poorest gradient-based method, but for the 64 turbine case, outperformed two other gradient-based methods. par5's attempt at the 64 sized farm was the worst gradient-based method performance, and almost was so even for all gradient-free methods.

par5 translated the target AEP function into MATLAB and used that language's built-in FMINCON() function to optimize turbine locations. For the 16 farm case, par5 did 1000 optimizations, each with randomized turbine starting locations. Due to computational time required, par5 did only 500 random starts for the 64 turbine case, and this poorer relative performance may be a result of the smaller sample size.

Gradient-free

Two participants using gradient-free methods (par6, par7) rotated positions for lowest relative AEP. They each used a different method, and will be described individually.

par6 used a multi-start partical swarm, with the interior point method, coded in MATLAB. par6 did 300 optimizations per farm size, with 20 swarm particles. If the minimum distance between turbines was violated at the end of an iteration, par6's algorithm randomised a new turbine location within the boundary and spacing constraints. The swarm algorith used an inertia weight 0.729, and social and cognitive weights of 1.49618. The initial turbine location population used only turbine coordinates satisfying the boundary and spacing constraints.

par7 used a genetic algorithm, coded in Python. The algorithm used a tournament selection method, with n=10 and elitism, where only the highest AEP layouts went on to seed the next generation. There were 25 generations each of size 500, using a mutation probability of 3%.

4. Conclusion

No worst-practices pattern can be concluded from the 3 cases, as the poorest performers all had vastly different methods for reaching their results.

In terms of best-practices, for the 3 farm sizes we tested, par4's strategy of using the SNOPT optimizer combined with the WEC relaxation method consistently delivered superior layouts.

Though par4 consistently found the superior AEP relative to the other participants, par2's results demonstrate a trend closing the gap as the number of design variables increased. For the 16 turbine case, par4 was 2.5% better than par2's results. For the 36 and 64 cases, par4 was 1.68% and 0.46% better, respectively. It should be noted, however, that at the current average U.S. rate[9] of roughly ¢13.3 for a kWh (or $133 per MWh), the income difference between the AEPs of par4 and par2 in the 64 turbine case, though only 0.46%, equates to a difference of a little under 1 million U.S. dollars.

Since par2's SQP method steadily closed the gap, a future study should test even larger wind farm sizes. This could determine if the SQP algorithm will eventually outperform the SNOPT + WEC method when a certain number of design variables are reached, or if there is an upper limit or convergence to this trend.

B. Case Study 2: Combined

For the Combined Wake Model and Optimization Method Seletion Case Study, each participant ran both the optimization algorithm of their choosing as well as their choce of wake model target function. There were no restrictions on programming language for either the wake model or optimization algorithm, but results of optimal turbine layouts were to be submitted in the `.yaml` format supplied in the Case Study 1 examples.

Results were to be adjudicated on two criteria:

1. An LES analysis of each layout
2. A cross-comparison of layouts between participants.

There were 5 participant submissions for the Combined Case Study. All 5 also participated in Case Study 1 (though were not required to do so) so we assigned them the same participant numbers from that Case Study. i.e., par1 - par5 are the same for both Case Study 1 and Case Study 2.

1. Data

Due to time and computing resource constraints, the authors were unable to run all submitted participant layouts through an LES. With each participant using a different wake model, calculated AEP values cannot be fairly compared between participants. Case Study 2 was constructed mainly for this LES wake model evaluation, in order to gauge which simplified model is most accurate when compared to a higher-cost computational model. Without this LES analysis, a key piece of adjudication is lacking. The LES analysis will be conducted in the near future, and results will be published.

Despite not having LES data for each participant layout, the cross-comparison does displays some interesting trends. The following tables show how each participant's wake models ranked the proposed optimal turbine layouts for the other 4 participants. Each participant's ranking of their own layout is highlighted in **bold**. The last column in the table is the participant number of the layout being cross-compared (cc-par#). So participant 4's analysis of participant 2's layout would be found in the par4 table, with 2 in the cc-par# column.

par1	Rank	AEP	cc-par#
	1	262350.319	4
	2	262282.416	5
	3	**260722.295**	**1**
	4	260640.906	3
	5	248215.024	2

par2	Rank	AEP	cc-par#
	1	250464.9732	4
	2	250249.0259	5
	3	247812.0522	3
	4	240309.5850	1
	5	**236342.799**	**2**

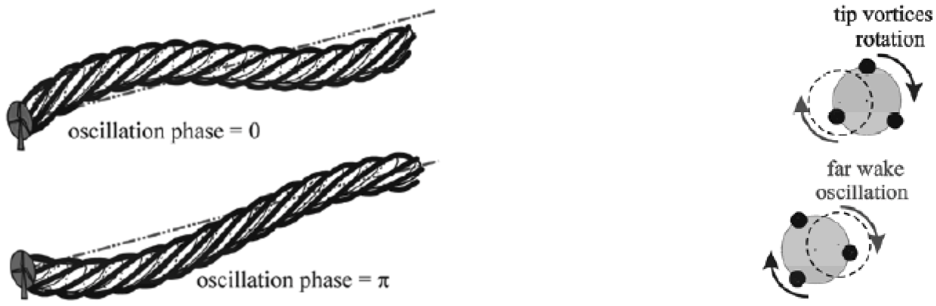
par3	Rank	AEP	cc-par#
	1	247109.5234	5
	2	246942.3767	4
	3	**245659.4124**	**3**
	4	242431.5431	2
	5	237548.6622	1

par4	Rank	AEP	cc-par#
	1	**257790.1924**	**4**
	2	257663.4068	5
	3	255063.8201	3
	4	251776.7157	1
	5	239612.8223	2

par5	Rank	AEP	cc-par#
	1	**251771.9067**	**5**
	2	251697.7126	4
	3	249829.2199	3
	4	246503.8323	1
	5	239482.6767	2

General Trends

We expected each participant to rank their own layout as superior to the others. Each wake model accounts for different fluids phenomena, and what one wake model considers an optimal layout, another may not. An example of this would be if one participant used Larsen's Dynamic Meandering Wake (DMW),[10] depicted in Fig. 6. A turbine placed downstream, slightly offset to miss the wake oscilations accounted for in DMW would, under another wake model not accounting for this "meander" (such the Jensen's model[11]), feel the full brunt of the wake, and deliver a sub-optimal AEP.



(a) DMW depicted 3-dimensionally

(b) DMW in the Trefftz plane

Figure 6: Sketch of Larsen's Dynamic Meandering Wake (DMW) behind a wind turbine.[10]

Unexpectedly, only par4 and par5 found their own layouts to be superior to the other participants. Furthermore, all other participants also found par4 and par5's layouts superior to their own, though to varying degrees. 3 particiants (including self-reporting) found par4 to have the highest AEP producing layout. The other two participants found par5 to have the highest AEP producing layout.

2. *Analysis of Best Results*

Within expectations, par4 and par5 ranked their own layouts superior to all other participant results. Two correlations are important to note regarding par4 and par5. Firstly, both used variations of the same wake model. par5 used the simplified Gaussian wake model supplied for Case Study 1. par4 used a more intricate version of the Gaussian wake model, combined with the wake model created by Niayifar and Porté-Agel,[7] supplemented by the WEC method described earlier. par4 also accounted for partial wake, shear, ambient turbulence intensity, and local turbulence intensity. None of these factors were accounted for by par5. The second factor to note is that despite using very similar wake models, par4 and par5 used different gradient-based optimization algorithms that nonetheless reached very similar conclusions.

As can be seen in the visual depictions included later in Section IV, par4 and par5 found nearly identical optimal turbine placements. Though pictorally appearing identical, the actual coordinates do indeed differ, enough so to result in different AEP calculations shown in the tables above.

Without LES data, the conclusions able to be drawn from the cross-comparison analysis are limited. That both par4 and par5 were found by the other participant wake models to have superior placement could be a result of either a more efficient optimization method, or a better coupling between optimization method and wake model. That these minima existed within the other wake models (resulting in a higher computed AEP by those models) yet were nevertheless undiscovered in their optimizations, is inconclusive in telling us which it is.

par4 and par5 used similar wake models, but very different optimization methods. par5 did all coding in MATLAB, did 1000 random starts, and used MATLAB's built in FMINCON() (which uses a finite difference method to find gradients) to optimize for a minimum. par4 used a combination of Python, FORTRAN, and Bash, and ran 1 ordered with 199 random starts for 200 optimizations altogether. par4 used SNOPT's SQP algorithm as chosen optimizer.

Of note, from trends seen above in Case Study 1, par5's optimization methods demostrate superior performance for small design variable sizes, but comparitively degrades as the windfarm size increases. The superior performance of this wake model and optimization method combination for this small farm may not be reproducable for farms of even larger sizes than those tested.

3. *Analysis of Worst Results*

Again, conlusions able to be drawn from the cross-comparison analysis are limited due to the lack of LES data. Suprisingly, however, par2's results were found by every other participant to be inferior, even by par2. As noted earlier, this could be a result of either poor wake model pairing with optimization method, or simply an optimization method shortcoming. Using data from Case Study 1, par2's optimization method shows inferior results for small sample sizes, but increases in comparative performance as wind farm size and number of design variables increase. That data leads us to believe the inferior performance here is a product of the wind farm size, and not a poor pairing of wake model with optimization method. However, both the LES analysis and attempts at farms of larger sizes would need to be analyzed to find a definitive conclusion.

Discounting the two universal top performers (par4 and par5), par1's results were more in line with what we expected from the cross-comparison. Namely, that par1 found its layout superior to the others, but that the others did not find it so.

4. *Conclusion*

A weakness voiced by participants of earlier case studies are scenarios where non-novel, and simplistic layouts (such as all turbines on the boundary border) are optimal. The small farm radius with few turbines given for this case study seems to have unknowingly fallen into this category. What is interesting, however, is that only two of the five participants found optimal layouts on the boundary border. Though the other participant wake models judged this border placement as superior when provided this answer, it is interesting that their optimization algorithm and wake model pairing did not find this result on their own. This could be a result of inferior optimization methods, lack of sufficient iterations, lack of sufficient wall time, or a combination of factors.

Border placement of turbines in concentric rings was supplied for the three wind farm sizes in Case Study 1, created from intuition. As an unintended validation, the optimal layouts for this smaller farm size followed the same pattern, despite being results of random starting locations. This confirms the hypothesis that, for small farm sizes, border placement and concentric rings will tend to deliver optimal farm AEP.

IV. Conclusion

Results from Case Study 1 show that par4's optimizaton of SNOPT + WEC delivers superior results for the tested wind farms with 16, 36, and 64 turbines. While information on this method is continuing to be produced, the initial paper written by J.J. Thomas[6] describes this method. par2's gradient-free method shows a trend of increased performance that may surpass SNOPT + WEC for wind farms of sizes larger than 64, but this must be tested to be proven.

Case Study 2 demonstrates that, for wind farms of small area with few turbines, placement on the wind farm boundary delivers superior AEP. Shortcomings in participant pairings of optimization methods and wake models generally failed to discover this, however. The lesson learned here is to either train researcher intuition to use such layouts as warm starts, or improve optimization methods so that automated optimizers can discover this themselves. LES analysis needs to be conducted of participant submissions from Case Study 2, for further conclusions to be drawn.

Though we are happy with the level of participation in the Case Studies, a larger participant sample size with different methods may provide more informative data, or display other novel and superior methods. To refine our data collection process, we plan on running another round of results for these Case Studies in the near future.

For the life of me, I can't think of any other points to hit in the conclusion that aren't already covered above. Any suggestions on things to say in this section would be appreciated.

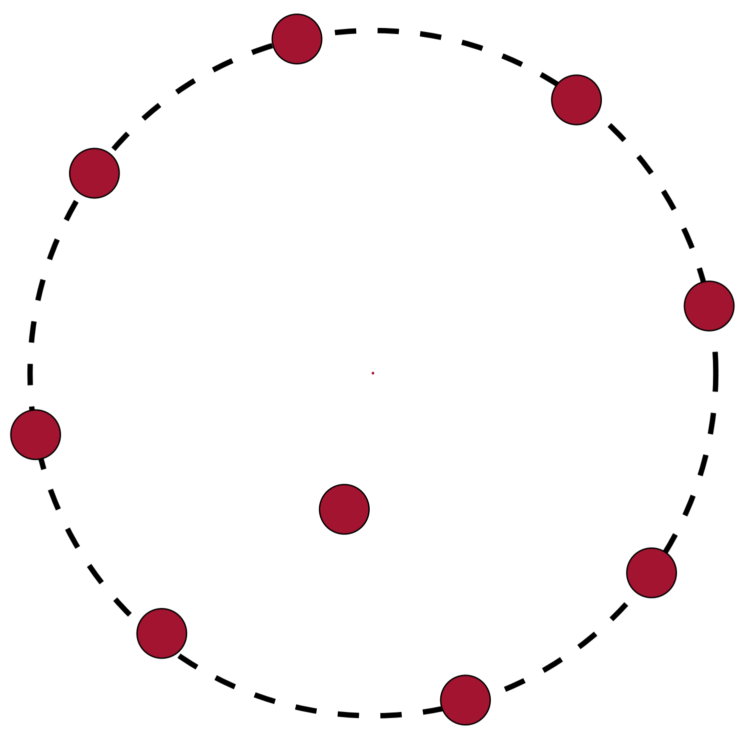
Appendix

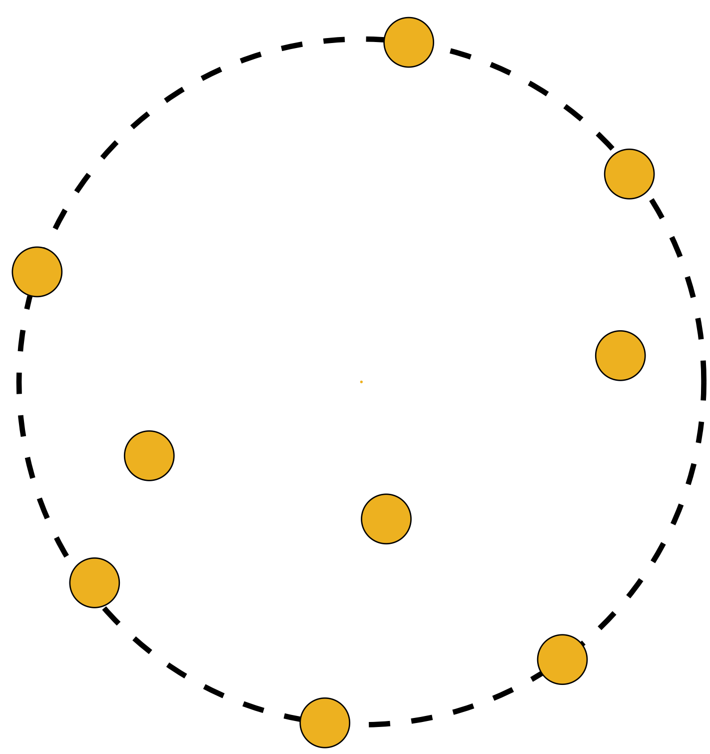
Acknowledgments

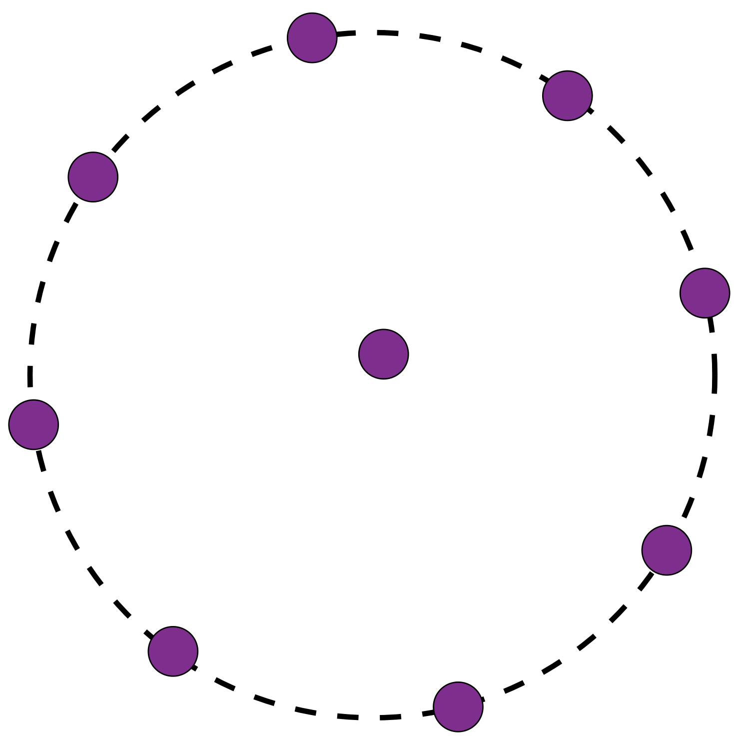
This work is funded by Brigham Young University.

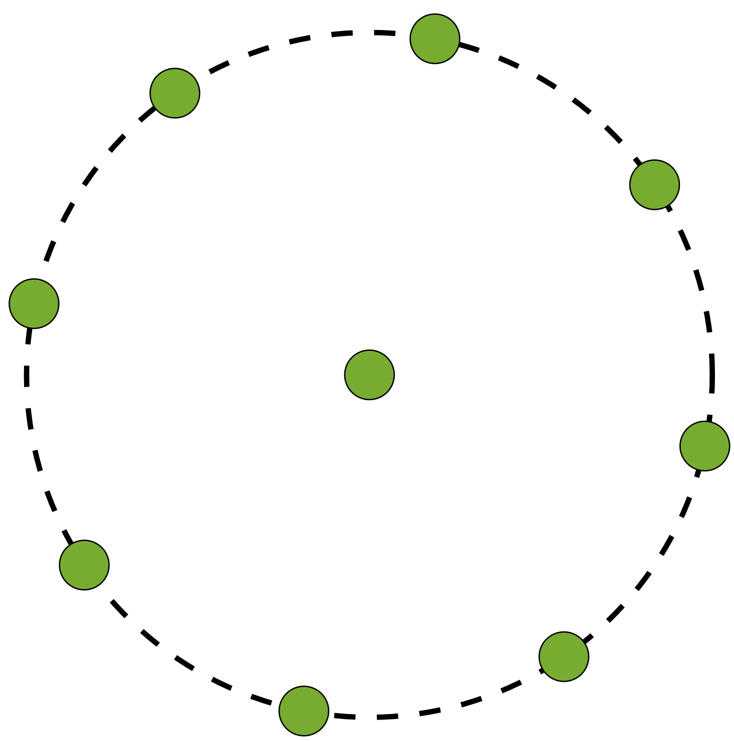
Incude a participant information list? (not assigned numbers, just individuals and sponsoring institutions)

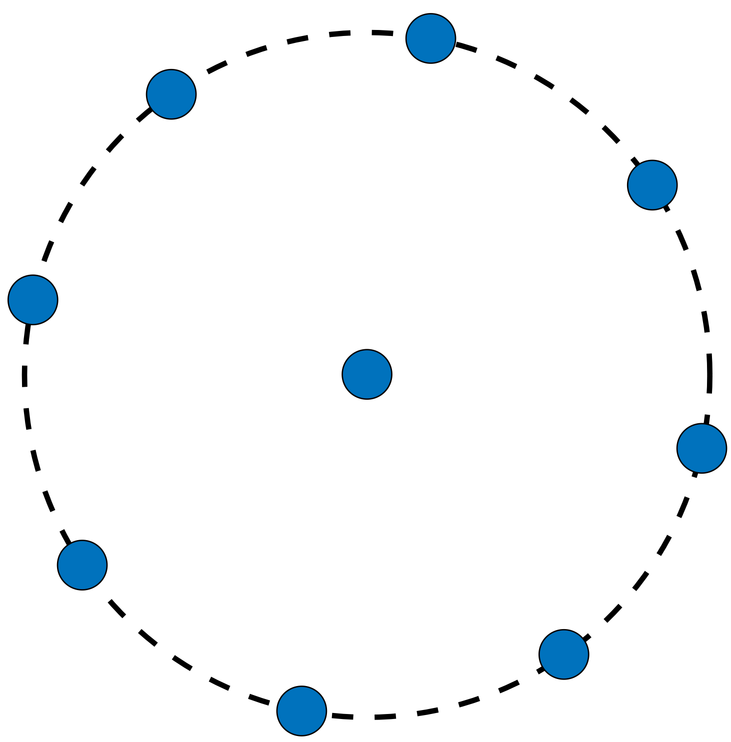
Pictoral Representations of Case Study 2's Participant Submissions











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