Best Practices for Wake Model and Optimization Algorithm Selection in Wind Farm Layout Optimization

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This paper presents the results of two case studies regarding the wind farm layout optimization problem. We asked a general audience to take part in the studies that we designed, and 9 individuals participated. Case study 1 considers variations in optimization strategies for a given simple Gaussian wake model. Participants were given a wake model which outputs annual energy production (AEP) with given wind turbine locations. Each participant used an optimization method of their choosing to find an optimal wind farm layout. Case study 2 looks at trade-offs in performance with variation in both physics model and optimization strategy. For case study 2, participants calculate AEP using a wake model of their choice, while also implementing their chosen optimization method. Participant submissions for optimized turbine locations were then cross-compared, by recalculating the AEP using every other participants' wake model. Results for case study 1 show that the best optimal wind farm layouts in this study were achieved by participants that used gradient-based optimization methods. A clear front-runner emerged with the SNOPT+WE timization method, which consistently discovered a higher AEP for each scenario. Results for case study 2 show that for small wind farms with few turbines, turbine placement on the wind farm boundary is superior. Conclusions for case study ${\bf 2}$ were drawn from participant cross-comparison of results.

I. Introduction

OPTIMIZING turbine placement within a wind farm is a complex problem characterized by many local minima. The large number of inter-dependent variables involved in wind farm layout optimization (WFLO) create a design space that is difficult to solve reliably. In this study, we designed and conducted a set of case studies to discover superior practices in solving the WFLO problem.

Two approaches have been taken to simplify the WFLO problem, as described by Iron 1. The first approach aims at improving the quality of individual models for wind farm attributes, i.e., aerodynamics, atmospheric physics, turbine structures, etc. The second approach is to improve formulating the optimization problem, as well as the algorithms used to solve the optimization.

To better model the aerodynamics of the waked airflow region of a wind turbine, complex computational methods such as direct numerical simulations (DNS) or large eddy simulations (LES) have been developed. But since these methods use the full Navier-Stokes equations, the computational time these require for full simulation can be prohibitive. Simplified engineering wake models (EWMs) respond to this weakness by making certain limiting physics assumptions, which result in greatly reduced computational costs.² Yet these simpler, less accurate, approximations may lead to inefficient recommendations for turbine placement, due to what can be inaccurate assumptions in specific wind farm scenarios.

Given a single EWM, optimization methods to select ideal turbine locations may be limited by characteristics of the functions governing the model. For example, EWMs that define a discontinuous wind speed behind wind turbines cannot be effectively used with gradient-based optimization methods. Conversely, EWMs for which gradients have not been calculated are limited to gradient-free algorithms, or gradient-based algorithms using finite difference derivatives. Furthermore, within these limitations, different optimization strategies have varying capacity to avoid local optima in the pursuit of a global optimum. Factors such as step size or the "temperature" allowances in simulated annealing optimization methods can permit algorithms to escape local concavities in a design space in order to pursue a superior optimum.

To better understand the effects of EWMs and optimization algorithms, we have created two case studies. We solicited participant involvement from many different research labs and private companies in industry currently working on both general optimization methods, as well as methods specific to solving the WFLO problem. The first case study isolates optimization techniques for a single simplified $EWM_{\overline{\lambda}}$ the second case study observes the differences when combining variations in the EWM and optimization method.

Though papers have been published which surest the state of the wind farm optimization (perhaps one of the most notable written by Herbert and Acetor eviewing the current methodologies in the field²), our

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case studies are the first time an international collaboration has been conducted to comparatively analyze optimization methods and EWM selection on a representative WFLO problem.

Our case studies are created in support of the International Energy Agency's (IEA)'s Wind Task 37 (IEA37). IEA37 coordinates international research activities centered around the analysis of wind power plants as holistic systems.³ Though our case studies concentrate mainly on wake modelling optimization at the farm-level scale, our results still contribute to what IEA37 terms a 'hollistic' proach³ to wind energy.

II. Methodology

To enable production of useful data, our case studies require a model wind farm with characteristics that are simultaneously restrictive enough to maintain simplicity, yet general enough to maintain relevance to more complex and realistic problems. The wind farm scenarios we selected to meet this criteria, and other details relevant to this project as a whole, are described below in Section A.

Many factors affect recommendations for superior turbine placement of a proposed wind farm. The two major factors we chose to study are 1) EWM characteristics and 2) optimization algorithm.² We designed two case studies in an attempt to quantify the effects of each of these choices.

For the first case study, in which the goal was to isolate variability in the optimization method, we precoded a representative wake model as a control variable, and permitted participants to use any optimization strategy to alter turbine locations which would deliver the best annual energy production (AEP) for the farm. This is called case study 1, and is described below in Section B.

Isolating EWM variability proves more complicated. An EWM's compatibility with gradient-based or gradient-free optimization methods dietate which algorithms can be applied. As such, designing a case study that restricts participants to a single optimization algorithm would unnecessarily limit the scope of EWMs studied. For this reason, our second case study permits not only participant selection of EWM $_{\overline{\imath}}$ but also implemented optimization algorithm. It is called case study $2_{\overline{\imath}}$ and is described below in Section C.

A. Common to Both Case Studies

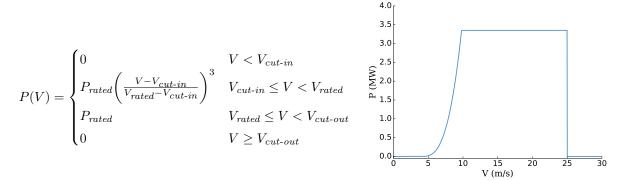
1. Wind Turbine Specifications

We used IEA's 3.35MW reference turbine in all wind farms. It's attributes are open source, and it is designed as a baseline for onshore wind turbine specifications. The specifics of the turbine necessary for our simplified Gaussian wake model (used in case study 1) are located in Table 1:

Table 1: Attributes for NREL's 3.35MW on shore reference turbine

Rotor Diameter	130	\mathbf{m}
Turbine Rating	3.35	MW
Cut-In Wind Speed	4	m/s
Rated Wind Speed	9.8	m/s
Cut-Out Wind Speed	25	m/s

Its power curve is defined as:



2. Farm Geography

To focus on optimization method and EWM variability, as well as to avoid introducing too many unecessary variables, the wind farms for all scnearios are on flat and level terrain.

2.a Boundary Shape

To reduce boundary impacts on farm design, we chose a radially symmetric farm boundary.

2.b Turbine Placement

Turbine (x, y) hub locations are restricted to be on or within the boundary radius. Turbines are further constrained to be no less than two rotor diameters apart from any other turbine.

2.c Farm Diameter

Farm diameter sizing for each scenario needed to be restrictive enough to avoid simply placing all turbines on the boundary, yet also permit meaningful turbine movement by the optimizers. Although the participants were not required to use the starting layouts that we provided, we tried to provide reasonable starting layouts by dispersing the turbines as much as possible in an orderly way. This was done by placing turbines in evenly spaced concentric rings. The boundary radii of the various wind farms we defined were selected to permit turbine placement in concentric rings with an average turbine spacing of five rotor diameters.

3. Wind Attributes

The wind distribution frequency and wind speed are the same for all wind farm scenarios in both case studies.

3.a Wind Speed

Freestream wind velocity is constant in all wind directions, at 9.8 m/s, regardless of turbine location or time of day. This wind speed is used because it is our utilized turbine's rated wind speed. Using this incoming wind velocity increases power production variability experienced by the farm, since any turbine wake will push air speeds down the turbine's power curve. With greater variability in the resultant produced power, more local optima will be experienced by participant optimizers.

3.b Wind Direction Frequency

A lack of local optima in a design space permits even ineffective optimizers to find a "best" result. In a design space where local optima are present, inferior designs are very likely. Such design spaces test the robustness of optimization methods. The selection of wind rose is a major factor in the frequency and magnitude of local optima resulting from turbine placement.

We selected a wind rose with a frequency distribution, binned for 16 directions. When we

We selected a wind rose with noff-axis wind frequency distribution, binned for 16 directions. When we tested this wind rose against 1000 randomized starting turbine locations, it gave few optimized results with relatively high AEP values. We interpreted this to be indicative of the presence of many local optima. The wind rose we used is depicted in Fig. 1, in polar coordinates. In this figure, a greater magnitude in the radial direction from the origin indicates a higher wind frequency from that cardinal direction.

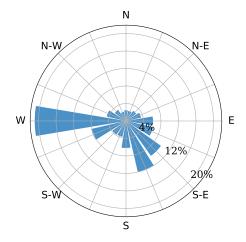


Figure 1: The wind frequency distribution for our case studies

B. Case Study 1: Optimization Only

The purpose of this case study is to determine the best optimization practices for WFLO, using a single representative EWM. We selected a generalized wake model able to be used by both gradient-based and gradient-free optimization algorithms, that was also computationally inexpensive in comparison to LES and DNS methods.

1. Wake Model

The wake model selected for case study 1 is a simplified version of Bastankhah's Gaussian wake model.^{5,6} This wake model is described by the following equations:

$$\frac{\Delta U}{U_{\infty}} = \left(1 - \sqrt{1 - \frac{C_T}{8\sigma_y^2/D^2}}\right) \exp\left(-0.5\left(\frac{y - \delta}{\sigma_y}\right)^2\right) \tag{1}$$

Where $\frac{\Delta U}{U_{\infty}}$ is the wake velocity deficit, $C_T = 8/9$ is the thrust coefficient, $y - \delta$ is the distance of the point of interest from the wake center in the cross-stream horizontal direction, D is the turbine diameter, and σ_y is the standard deviation of the wake deficit in the cross-stream horizontal direction as defined in Eq. (2):

$$\sigma_y = (k_y \cdot x) + \frac{D}{\sqrt{8}} \tag{2}$$

In Eq. (2), x is the downstream distance from the turbine generating the wake to the turbine of interest, and D is the turbine diameter. The variable k_y is determined as a function of turbulence intensity (I). In

this case study turbulence intensity $\frac{1}{1000}$ eated as a constant of 0.075 (reasonable for an off-shore scenario), and we therefore used a coresponding $\frac{1}{1000}$ of 0.0324555.6,7

Increasing turbulence intensity has numerous effects and draws attention away from the main purpose of this case study plich is to observe the differences of optimization strategies. For the wake model we use (given in Eq. (1) creasing the turbulence intensity plened the wake coput second and third order effects are unknown. As such, this first IEA Task 3 copies of Case Studies a very low intensity in an attempt to minimize the considered variables.

2. Farm Sizes

Variability in wind farm size (and thus number of design variables) affect optimization algorithm performance. To account for this, three wind farm sizes are specified in case study 1: 16, 36, and 64 turbines. These had farm boundary radii of 1300 m, 2000 m, and 3000 m respectively, determined in the manner described previously in Section 2. Justine numbers are selected as perfect squares which roughly double in size. Perfect squares are used to permit even grid turbine arrangements, if desired.

$\it 3. \quad Supplied \ Code$

To enable participation in this case study, we provided a link to a GitHub repository which included files that had:

- Turbine characterizes, wind frequency, and wind speed in IEA 37's .yaml schema
- Example turbine layouts for each farm size (in .yaml format)
- Python parsers of the .yaml schema
- Python target function to calculate AEP (given .yaml turbine locations and farm attributes)

We selected the programming language Python since it is widely used by researchers in the industry.

Participant alteration to our specific code implementation, or replication of our model in another language, was permitted if needed for compatibility with participant optimization methods. This is with the understanding, however, that final wind farm layouts would be evaluated with the original Python code that we provided.

C. Case Study 2: Combined Physics Model/Optimization Algorithm

The intent of this case study is to assess not only the optimization methods measured by case study 1, but also the effects that different physics model approximations have on turbine location recommendations.

case study 2 differs from the previous one in that 1) no wake model is provided, and 2) only a single wind farm size is to be optimized. Participants are free to choose their preferred EWM and optimization method combination.

Unlike case study 1, participant reported AEP is not comparable, since different EWMs are used to calculate them. To help with this, we conducted a cross-comparison of results between participants. For the cross-comparison, each participant's proposed optimal turbine locations in the standardized .yaml format was published to the other combined case study participants. Each participant then used their own wake model to calculate the AEP of the other participant's proposed farm layouts. From this portion of the case study, we hope to learn if any participant's results are seen as superior by other EWMs.

1. Farm Attributes

The wind farm size for the Combined case study is limited to nine turbines. We did this to limit the computation time requirements when assessing results in a standardized LES, discussed later in Section IV. We used the previously described method under **Farm Diameter** to determine the boundary radius, and the wind rose and wind speed are the same as case study 1.



III. Results

A. Case Study 1: Optimization Only

Each participant ran the optimization algorithm of their choosing using our supplied AEP function, or a functional equivalent in another language. Since there exists a great deal of variability in hardware, participants also reported processor speed, function calls, number of cores used, and total their system when finding their optimized results. The AEP results and rankings are given below in Tables 2 to 4.

There were 10 submissions for case study 1. One participant submitted twice, using a different optimization method for each submission. For anonimity, each submission is assigned a number. We will refer to each submission below by this submission number (i.e., sub1, ..., sub10, etc.).

^ahttps://github.com/byuflowlab/iea37-wflo-casestudies.git

1. Data

Tables 2 to 4 display the final AEP data of all participant proposed optimal turbine layouts. The Python module we supplied which uses the simplified Bastankhah wake model was used for all AEP calculations. Submissions are ranked from highest to lowest return at AEP values, with submission number (sub#), whether using a gradient-based (G) or gradient-free (GF) optimization method, and the percentage increase (Increase) from the provided example layout's AEP.

Table 2: 16 turbine scenario participant results

Rank	Algorithm	$\mathrm{sub} \#$	Grad.	AEP	Norm.
1	SNOPT+WEC	4	G	418924.4064	100.00 %
2	fmincon	5	G	414141.2938	98.86~%
3	SNOPT	8	\mathbf{G}	412251.1945	98.41~%
4	Sparse Nonlinear Optimizer (SNOPT)	1	G	411182.2200	98.15~%
5	Preconditioned Sequential Quadratic Programming	2	\mathbf{G}	409689.4417	97.80~%
6	Multistart Interior-Point	10	G	408360.7813	97.48~%
7	Full Pseudo-Gradient Approach	3	GF	402318.7567	96.04~%
8	Basic Genetic Algorithm	7	GF	392587.8580	93.71~%
9	Simple Particle Swarm Optimization	6	GF	388758.3573	92.80~%
10	Simple Pseudo-Gradient Approach	9	GF	388342.7004	92.70~%
11	(Example Layout)	-	-	366941.5712	87.59~%

Table 3: 36 turbine scenario participant results

Rank	Algorithm	$\mathrm{sub}\#$	Grad.	AEP	Norm.
1	SNOPT+WEC	4	G	863676.2993	100.00 %
2	Multistart Interior-Point	10	G	851631.9310	98.61~%
3	Preconditioned Sequential Quadratic Programming	2	G	849369.7863	98.34~%
4	SNOPT	8	G	846357.8142	97.99~%
5	Sparse Nonlinear Optimizer (SNOPT)	1	G	844281.1609	97.75~%
6	Full Pseudo-Gradient Approach	3	GF	828745.5992	95.96~%
7	fmincon	5	G	820394.2402	94.99~%
8	Simple Pseudo-Gradient Approach	9	GF	813544.2105	94.20~%
9	Basic Genetic Algorithm	7	GF	777475.7827	90.02~%
10	Simple Particle Swarm Optimization	6	GF	776000.1425	89.85~%
11	(Example Layout)	-	-	737883.0985	85.44~%

Table 4: 64 turbine scenario participant results

Rank	Algorithm	$\mathrm{sub}\#$	Grad.	AEP	Norm.
1	SNOPT+WEC	4	G	1513311.1936	100.00 %
2	Preconditioned Sequential Quadratic Programming	2	G	1506388.4151	99.54~%
3	Multistart Interior-Point	10	G	1480850.9759	97.86~%
4	Sparse Nonlinear Optimizer (SNOPT)	1	G	1476689.6627	97.58~%
5	Full Pseudo-Gradient Approach	3	GF	1455075.6084	96.15~%
6	SNOPT	8	G	1445967.3772	95.55~%
7	Simple Pseudo-Gradient Approach	9	GF	1422268.7144	93.98~%
8	Simple Particle Swarm Optimization	6	GF	1364943.0077	90.20~%
9	fmincon	5	G	1336164.5498	88.29~%
10	Basic Genetic Algorithm	7	GF	1332883.4328	88.08~%
11	(Example Layout)	-	-	1294974.2977	85.57~%

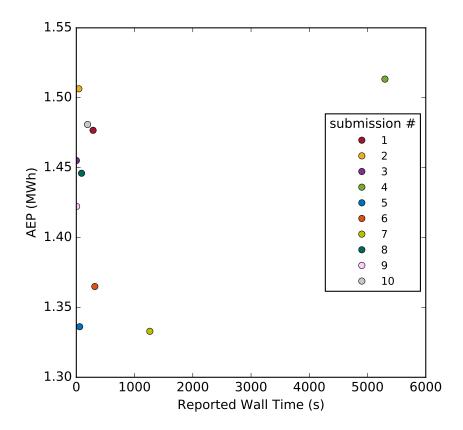


Figure 2: AEP vs reported wall time for each submitted optimization method of the 64 turbine scenario

1.a General Trends

As a general trend, gradient-based methods performed better in discovering a relative optima, especially for smaller farm sizes. Some gradient-based algorithms improved in comparative AEP ranking as the number of design variables increase (sub10, sub3), while others degrade (sub5, sub8). Simultaneously, one gradient-free algorithm increases in effectiveness as design variables increase (sub3), while others compete for lowest comparative performance, regardless of farm size $(sub6, \underline{sub7}, sub9)$.

Despite these multi-variate results, one clear front-uper does emerge. Regardless of wind farm size, sub4's algorithm, consistently discovered turbine placement that delivers an AEP superior to all other participants. A summary of sub4's method is included in a following section.

Also of note, as number of design variables increases, the relative disparity between proposed optimal AEPs likewise diverge. For the 16 turbine case. highest result is 7.88% better than the lowest. For the 36 and 64 cases, the highest result is 11.45% and 13.54% better than the lowest, respectively.

1.b AEP vs Wall Time

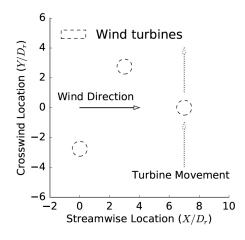
Fig. 2 is a plot of AEP from each submission's turbine layout, verses reported wall time for the 64 turbine case, per optimization. Note that in the call for participation, we may not have stressed enough that time and computational expense were important metrics in this study. Therefore, it is possible that many of these reported times were not measured during optimization runs and may have been estimates. Under the premise that these reported times are accurate, Fig. 2 shows the SNOPT+WEC method, though reporting the higest AEP turbine arrangement, also may have taken much more time than some other approaches.

More testing could be done to determine if the other methods, if permitted to run longer, would discover an optimum better than that discovered by par5, or if by nature of the algorithms, no matter the time limit permitted to run, there are some optima unattainable by their formulations.

2. Analysis of Best Results

For all 3 farm sizes, the superior method was implemented by subusing a gradient-based method. Coded in Python and FORTRAN, it combined the Sparse Nonlinear OPTimizer (SNOPT)⁸ with a method called Wake Expansion Continuation (WEC).⁶ Running 200 optimizations, sub4 had one optimization run start from the provided example layout, and the other 199 use randomized turbine starting locations within the farm boundary.

The WEC method is specifically designed to reduce the multi-modality found in wind farm layout optimization. In the cited paper, it is a method of converting design spaces with many local minima into curves approaching convexity, allowing gradient-based optimizations to more easily find the better solutions. An example of such "relaxation" to convexity is included in Figs. 3 and 4, reproduced with permission.



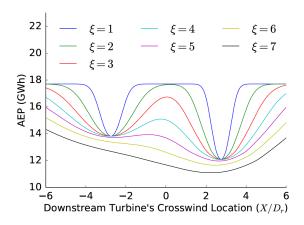


Figure 3: Simple design space used to demonstrate the effects of the relaxation factor, ξ , on the wind farm layout design space.⁶

Figure 4: The impact of the wake relaxation factor, ξ . One turbine was moved across the wakes of two upstream turbines (see Fig. 3).⁶

Figs. 3 and 4 demonstrate the effects of the WEC method on a simple design space, relaxing the local optima into a more easily discovered global solution. As the authors Thomas and Ning state, "Larger values of ξ allow the smaller local optima to disappear completely. Smaller values of ξ allow for more accurate wake widths but with an increase in the number and magnitude of local optima." We suspect the WEC method for reducing the multi-modality of the design space is the reason that sub4's optimized found superior layouts to the other used methods.

3. Discussion

For the 3 farm sizes we tested, sub4's strategy of using the SNOPT optimizer combined with the WEC relaxation method consistently delivered superior layouts.

Though sub4 consistently found the superior AEP relative to the other participants, sub2's results demonstrate a trend closing the gap as the number of design variables increased. For the 16 turbine case, sub4 was 2.5% better than sub2's results. For the 36 and 64 cases, sub4 was 1.68% and 0 better, respectively. It should be noted, however, that at the current average U.S. rate⁹ of roughly closed for a kWh (or \$133 per MWh), the income difference between the AEPs of sub4 and sub2 in the 64 turbine case, though only 0.46%, equates to a difference of a little under 1 million U.S. dollars per year. Since sub2's Preconditioned Sequential Programming (PSQP) method steadily closed the gap, a future

Since sub2's Preconditioned Sequential Programming (PSQP) method steadily closed the gap, a future study should test even larger wind farm sizes. This could determine if the PSQP algorithm will eventually outperform the SNOPT+WEC method when a certain number of design variables are reached, or if there is an upper limit or convergence to this trend.

B. Case Study 2: Combined Physics Model/Optimization Algorithm

For case study 2, each participant ran both the optimization algorithm and wake model of their choosing. There were no restrictions on programming language for either the wake model or optimization algorithm, but results of optimal turbine layouts were to be submitted in the .yaml format supplied in the case study 1 examples.

There were five participant submissions for case study 2. All five participants also submitted for case study 1 (though were not required to do so) so we assigned their submissions the same numbers from that case study. i.e., sub1 - sub5 are from the same individual participants for both case study 1 and case study 2. With each participant using a different wake model, AEP values reported by participants cannot be fairly compared. Results were judged on a cross-comparison of layouts between participants.

1. Data

The cross-comparison displays some interesting trends. The following tables show how each submission's wake models ranked the proposed optimal turbine layouts for the other 4 submission. Each submission's ranking of their own layout is in **bold**. The last column in the table is the submission number of the layout being cross-compared (cc-sub#). So submission 4's analysis of submission 2's layout would be found in the *sub4* table, with 2 in the cc-sub# column. The last column is the percentage increase (Inc.) In the reporting submission's submitted layout. A negative value here indicates a worse AEP.



Rank	Wake Model	Algorithm	AEP	cc-sub#	Increase
1	Simplified Bastankhah	fmincon	262350.319	4	0.624 %
2	Bastankhah	SNOPT+WEC	262282.416	5	0.598~%
3	FLORISSE3D	SNOPT	260722.295	1	0.000~%
4	Bastankhah	Full Pseudo-Gradient Approach	260640.906	3	-0.031 $\%$
5	Park2	PSQP	248215.024	2	-4.797~%

sub2:

Rank	Wake Model	Algorithm	AEP	$\operatorname{cc-sub}\#$	Increase
1	Simplified Bastankhah	fmincon	250464.9732	4	5.975 %
2	Bastankhah	SNOPT+WEC	250249.0259	5	5.884~%
3	Bastankhah	Full Pseudo-Gradient Approach	247812.0522	3	4.853~%
4	FLORISSE3D	SNOPT	240309.5850	1	1.678~%
5	Park2	PSQP	236342.799	2	0.000~%

sub3:

Rank	Wake Model	Algorithm	AEP	$\operatorname{cc-sub}\#$	Increase
1	Bastankhah	SNOPT+WEC	247109.5234	5	0.590 %
2	Simplified Bastankhah	fmincon	246942.3767	4	0.522~%
3	Bastankhah	Full Pseudo-Gradient Approach	245659.4124	3	0.000~%
4	Park2	PSQP	242431.5431	2	-1.314 $\%$
5	FLORISSE3D	SNOPT	237548.6622	1	-3.302~%

sub4:

Rank	Wake Model	Algorithm	AEP	$\operatorname{cc-sub}\#$	Increase
1	Simplified Bastankhah	fmincon	257790.1924	4	0.000 %
2	Bastankhah	SNOPT+WEC	257663.4068	5	-0.049 $\%$
3	Bastankhah	Full Pseudo-Gradient Approach	255063.8201	3	-1.058 $\%$
4	FLORISSE3D	SNOPT	251776.7157	1	-2.333~%
5	Park2	PSQP	239612.8223	2	-7.051 %

$\mathrm{sub}5$:

Rank	Wake Model	Algorithm	AEP	$\operatorname{cc-sub} \#$	Increase
1	Bastankhah	SNOPT+WEC	251771.9067	5	0.000 %
2	Simplified Bastankhah	fmincon	251697.7126	4	-0.029 $\%$
3	Bastankhah	Full Pseudo-Gradient Approach	249829.2199	3	-0.772 $\%$
4	FLORISSE3D	SNOPT	246503.8323	1	-2.092 $\%$
5	Park2	PSQP	239482.6767	2	-4.881 $\%$

1.a General Trends

We expected each participant to rank their own layout as superior to the others. Each wake model accounts for different fluids phenomena, and what one wake model considers an optimal layout, another may not. An example of this is if one EWM predicts a wake deficit due to some factor such as vorticity or turbulence. A turbine placed downstream under the such a model would, under a more simplistic wake model not accounting for this phenomena (such the Jensen's model¹⁰), feel the full brunt of the wake, and deliver a sub-optimal AEP.

Unexpectedly, only sub4 and sub5 found their own layouts to be superior to the other participants. Furthermore, all other participants also found sub4 and sub5's layouts superior to their own, though to varying degrees. Three participants (including sub4) found sub4 to have the highest AEP producing layout. The other two participants found sub5 to have the highest AEP producing layout.

2. Analysis of Best Results

Within expectations, sub4 and sub5 ranked their own layouts superior to all other participant results. Two correlations are important to note regarding sub4 and sub5. Firstly, both used variations of the same wake model. From case study 1, sub5 used the simplified Gaussian wake model previously described. Though sub4 also used the Gaussian wake model, sub4 combined it with the model created by Niayifar and Porté-Agel, supplemented by the WEC method described earlier. Furthermore, sub4 also accounted for partial wake, shear, ambient turbulence intensity, and local turbulence intensity. None of these factors were accounted for by sub5. The second factor to note is that despite using very similar wake models, sub4 and sub5 used different gradient-based optimization algorithms that nonetheless reached very similar conclusions.

As can be seen in the visual depictions included later in Section IV, sub4 and sub5 found nearly identical optimal turbine placements. Though pictorally appearing identical, the actual coordinates do indeed differ, enough so to result in different AEP calculations shown in the tables above.

Without LES data, the conclusions able to be drawn from the cross-comparison analysis are limited. That both sub4 and sub5 were found by the other participant wake models to have superior placement could be a result of either a more efficient optimization method, or a better coupling between optimization method and wake model. That these minima existed within the other wake models (resulting in a higher computed AEP by those models) yet were nevertheless undiscovered in their optimizations, is inconclusive in telling us which it is.

Both sub4 and sub5 used similar wake models, but very different optimes in hods. Coding in MATLAB, sub5 did 1000 random starts, and used MATLAB's built in FMINCON() (which uses a finite difference method to find gradients) to optimize for a minimum. Using a combination of Python and FORTRAN, sub4 ran 1 ordered with 199 random starts for 200 optimizations altogether. SNOPT's SQP algorithm was sub4's implemented optimizer.

Of note, from trends seen above in case study 1, sub5's optimization methods demostrate superior performance for small design variable sizes, but comparitively degrades as the windfarm size increases. The superior performance of this wake model and optimization method combination for this small farm may not be representative of performance on larger wind farms.



A weakness voiced by participants of earlier case studies are scenarios where non-novel, and simplistic layouts (such as all turbines on the boundary border) are optimal. The small farm radius with few turbines given for this case study seems to have fallen into this category. What is interesting, however, is that three of the five participants were trapped in local optima, and proved blind to optima others found, using different physics approximations and optimization methods. Many factors could have led to these shortfalls (i.e., inferior optimization methods, lack of sufficient iterations, lack of sufficient wall time, etc...) and further testing would need to be done to discover which factors majorly contributed to the outcome.

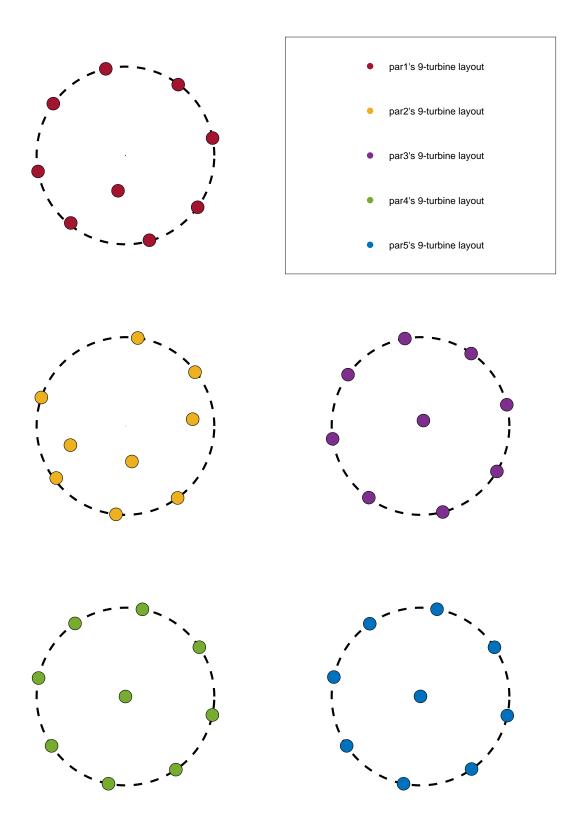
IV. Conclusion

Results from case study 1 show that sub4's use of SNOPT+WEC delivers superior results for the tested wind farms with 16, 36, and 64 turbines. While information on this method is continuing to be produced, the initial proper written by Thomas and Ning⁶ describes this method. Time differences displayed in Fig. 2 show that sub4-s took one of the comparatively longer amounts of time. However this required computational time difference must be counterbalanced with resultant AEP improvement. Profitability of the wind farm scenario this method is applied to must be considered to determine if the computational trade-off is worth the extra time it will require. Regarding sub2's PSQP method, though it shows a trend of increased performance that may surpass SNOPT+WEC for wind farms of sizes larger than 64, further testing is required to validate this pattern.

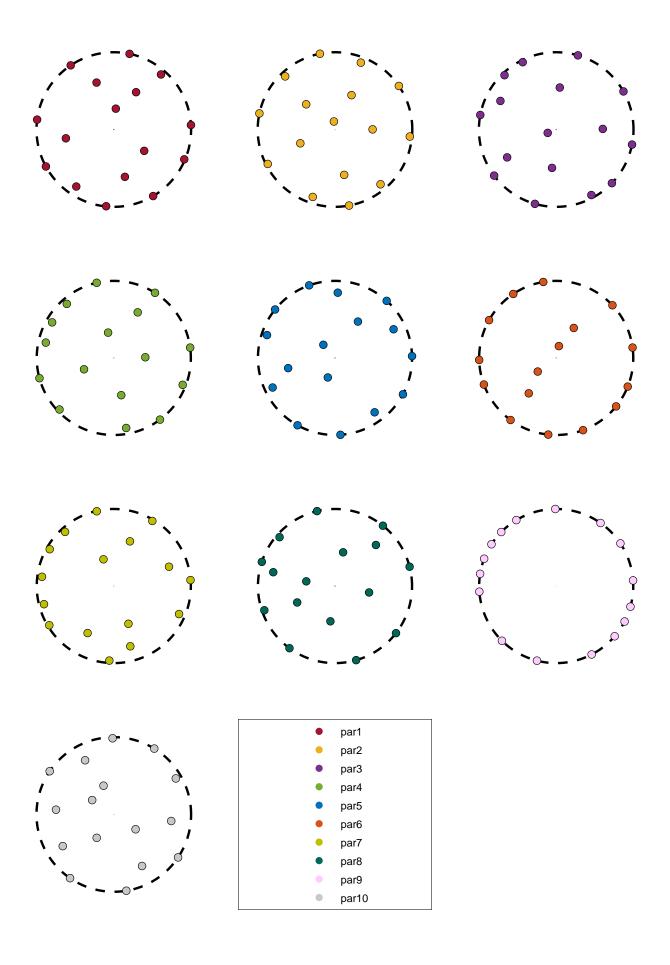
Case study 2 demonstrates that, for wind farms of small area with few turbines, placement on the wind farm boundary delivers superior AEP. Shortcomings in participant pairings of optimization methods were trapped in local optima, however. The lesson learned here is to either train researcher intuition to use such layouts as warm starts, or improve optimization methods so that automated optimizers can discover this themselves

Though we are happy with the level of participation in the Case Studies, a larger participant sample size with different methods may provide more informative data, or display other novel and superior methods. To refine our data collection process, we plan on running another round of results for these Case Studies in the near future.

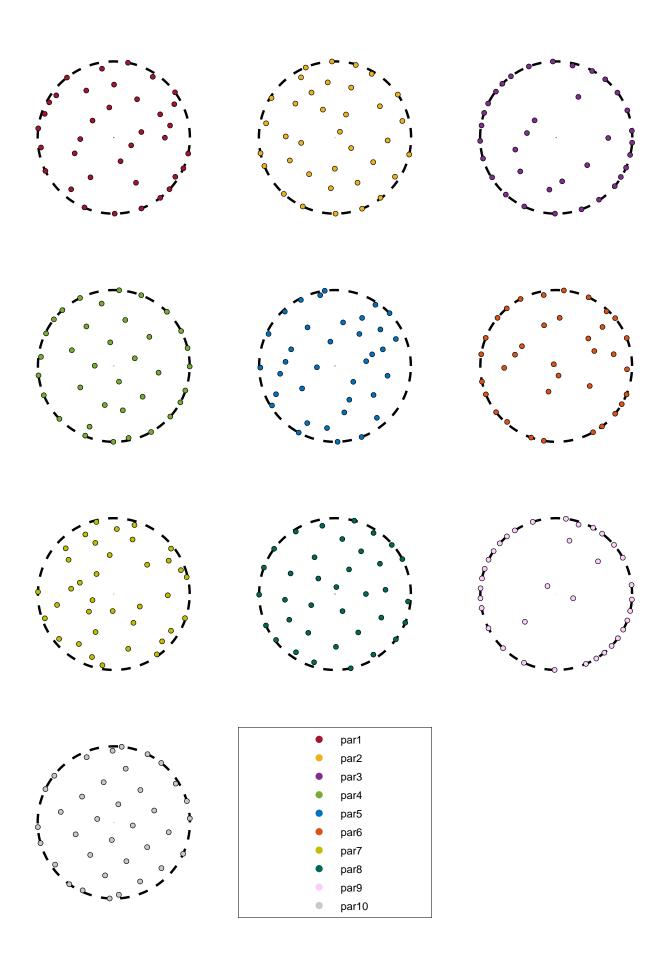
Due to the difficulty in comparing the results of different EWMs, we will run all participant-reported optimized turbine locations through an LES. With the inherent bias each EWM has for its own optimized locations removed, reported turbine locations will be measured using the same high fidelity simulation tool for a comparative AEP. Case study 2 was constructed mainly for this LES wake model evaluation, in order to gauge which simplified model is most accurate when compared to a higher-cost computational model. Due to time and computing resource constraints, the authors were unable to run all submitted participant layouts through an LES.

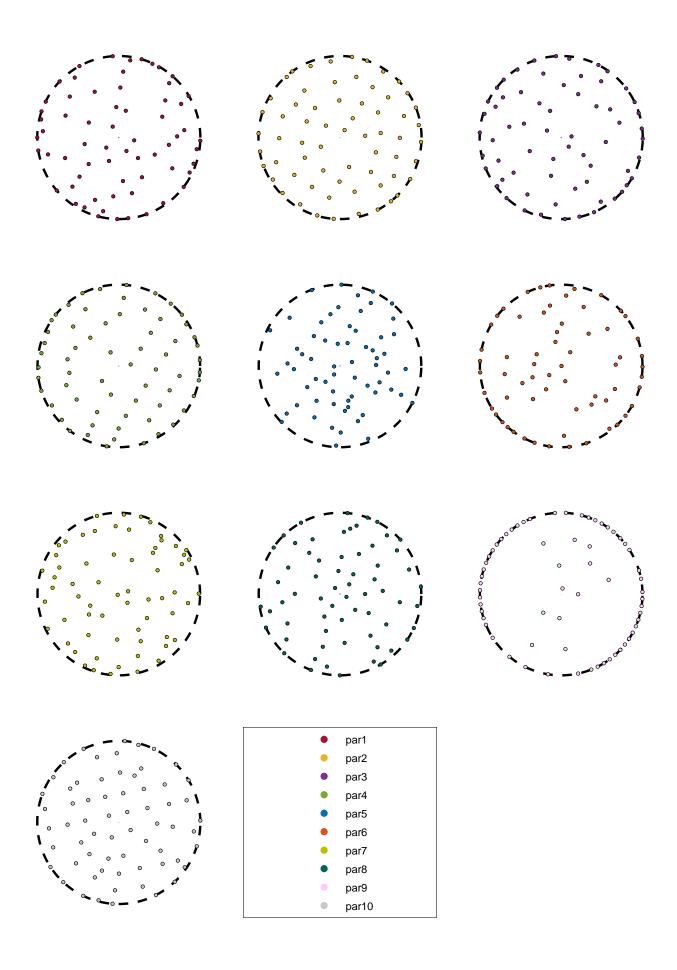


B. Pictoral Representations of Case Study 1's 16-Turbine Submissions



C. Pictoral Representations of Case Study 1's 36-Turbine Submissions





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