Best Practices for Wake Model and Optimization Algorithm Selection in Wind Farm Layout Optimization.

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This paper presents the results of two case studies regarding the Wind Farm Layout Optimization (WFLO) problem. Case study (1) considers variations in optimization strategies for a given simplified Bastankhah gaussian wake model, while case study (2) looks at trade offs in performance with variation in both physics model and optimization strategy selection. For case study (1), a supplied wake model outputs annual energy production (AEP) given participant supplied turbine locations. For case study (2), participants calculate AEP using a wake model of their choice, while also implementing their chosen optimization method. Participant submissions for optimized turbine locations were then cross-compared, by recalculating the AEP using every other participants' wake models. Results for case study (1) show that the best optimal wind farm layouts in this study were achieved by participants that used gradient-based optimization methods. A clear front-runner emerged with the SNOPT+WEC optimization method, which consistently discovered a comparatively optimal AEP for each scenario. Results for case study (2) show that for small windfarms with few turbines, turbine placement on the wind farm boundary is superior. Conclusions for case study (2) were drawn from participant cross-comparison of results, but further study will be done using an LES analysis of participant turbine layouts.

I. Introduction

Optimizing turbine placement within a wind farm is a complex problem characterized by many local minima. The large number of inter-dependent variables involved in Wind Farm Layout Optimization (WFLO) create a design space that is difficult to solve reliably In this study, we designed and conducted a set of case studies to discover the best practices in solving the WFLO problem.

Two approaches have been taken to simplify the WFLO problem, as described by Padron et.al.[1] The first approach aims at improving the quality of individual models for wind farm attributes, i.e., aerodynamics, atmospheric physics, turbine structures, etc. The second approach is to improve formulating the optimization problem, as well as the algorithms used to solve the optimization.[1]

To better model the aerodynamics of the waked airflow region of a wind turbine, complex computational methods such as such as direct numerical simulations (DNS) or large eddy simulations (LES) have been developed. But since these methods use the full Navier-Stokes formulations, the computational time these require for full simulation can be prohibitive.

Simplified engineering wake models (EWMs) respond to this weakness by mak certain limiting physics assumptions, resulting in greatly reduced computational costs.[2] Yet these simpler, less accurate, approximations may lead to inefficient reccomendations for turbine placement, due to what can be inaccurate assumptions in specific wind farm scenarios.

Given a single EWM, optimization methods to select ideal turbine locations may be limited by characteristics of the functions governing the model. For example, EWMs that define a discontinuous wind speed behind wind turbines cannot be effectively used with gradient-based optimization methods, and models for which gradients have not been calculated are limited to gradient-free algorithms, or gradient-based with finite difference derivatives. Additionally, within these limitations different optimization strategies have varying capacity to avoid local optima in the pursuit of a global optimum.

To better understand the affects of EWM and optimization algorithm selection, we have created two case studies. We solicited involvement from participants from many different research labs, working on both general optimization methods, as well as methods specific to solving the WFLO problem. The first case study isolates optimization techniques for a single simplified EWM, the second case study observes the differences when combining variations in EWM and optimization method.

Though papers have been published which survey the state of the Wind Farm Optimization (perhaps most notibly a paper by Herbert and Acero[2]), our case studies are the first time international collaboration has been conducted to isolate optimization method and EWM selection.

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Our case studies are created in support of the International Energy Agency's (IEA)'s Wind Task 37 (IEA37). IEA37 coordinates international research activities centered around the analysis of wind power plants as holistic systems.[3] Though our case studies concentrate mainly on wake modelling optimization at the farm-level scale, our results still contribute to what IEA37 terms a "hollistic" approach[3] to wind energy.

II. Methodology

To enable production of useful data, our case studies would require a model wind farm with characteristics that are simultaneously restrictive enough to maintain simplicity, yet general enough to maintain relevance to more complex and realistic problems. The wind farm scenarios we selected to meet this criteria, and other details relevant to this project as a whole, are described below in Section A.

Many factors affect reccomendations for superior turbine placement of a proposed wind farm. The two major factors we chose to study are 1) EWM characteristics and 2) optimization algorithm.[2] We designed two case studies in an attempt to quantify the effects of each of these choices.

For the first case study, in which the goal was to isolate variability in the optimization method, we pre-coded a representative wake model as a control variable, and permit participants to use any optimization strategy they think will discover turbine locations which deliver the best Annual Energy Production (AEP) for the farm. This is called the Optimization Only case study (or simply case study 1), and is described below in Section B.

Isolating EWM variability proves more complicated. An EWM's compatibility with gradient-based or gradient-free optimization methods dictate which algorithms can be applied. As such, designing a case study that restricts participants to a single optimization algorithm would unnecessarily limit the scope of EWMs studied. For this reason, our second case study permits not only participant selection of EWM, but also implemented optimization algorithm. It is called the Combined Physics Model/Optimization Algorithm case study, or case study 2, and is described below in Section C.

A. Common to Both Case Studies

1. *Wind Turbine Specifications*

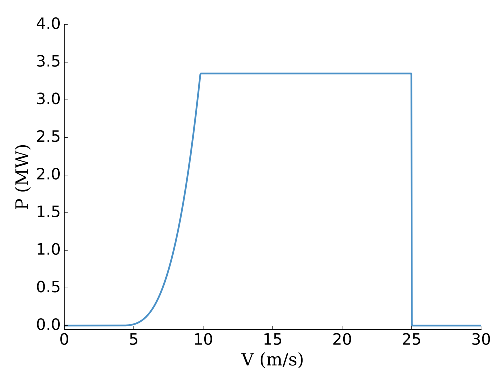
We used IEA's 3.35MW reference turbine in all wind farms. It's attributes are open source, and it is designed as a baseline for onshore wind turbine specifications.[4] The specifics of the turbine necessary for our simplified Gaussian wake model (used in Case Study 1) are located in Table 1:

Table 1: Attributes for NREL's 3.35MW onshore reference turbine

Rotor Diameter	130	m
Turbine Rating	3.35	MW
Cut-In Wind Speed	4	m/s
Rated Wind Speed	9.8	m/s
Cut-Out Wind Speed	25	m/s

Its power curve is defined as:

$$P(V) = \begin{cases} 0 & V < V_{\textit{cut-in}} \\ P_{\textit{rated}} \cdot \left(\frac{V - V_{\textit{cut-in}}}{V_{\textit{rated}} - V_{\textit{cut-in}}}\right)^3 & V_{\textit{cut-in}} \le V < V_{\textit{rated}} \\ P_{\textit{rated}} & V_{\textit{rated}} \le V < V_{\textit{cut-out}} \\ 0 & V \ge V_{\textit{cut-out}} \end{cases}$$



2. *Farm Geography*

To focus on optimization method and EWM variability, as well as to avoid introducing too many unecessary variables, the wind farms for all scnearios are on flat and level terrain.

2.a Boundary Shape

To reduce boundary impacts on farm design, we chose a radially-symmetric farm boundary.

2.b Turbine Placement

Turbine (x, y) hub locations are restricted to be on or within the boundary radius. Turbines are further constrained to be no less than two rotor diameters apart from any other turbine.

2.c Farm Diameter

Farm diameter sizing for each scenario needed to be restrictive enough to avoid simply placing all turbines on the boundary, yet also permit meaningful turbine movement by the optimizers. We tried to provide reasonable starting layouts by dispersing the turbines as much as possible in an orderly way, and did so by laying out turbines placed in concentric rings. evenly spaced. Using this as a starting point from which participants could further optimize, the boundary radii were selected to permit turbine placement in concentric rings with an average turbine spacing of 5 rotor diameters apart.

3. Wind Attributes

The wind distribution frequency and wind speed are the same for all wind farm scenarios in both Case Studies.

3.a Wind Speed

Freestream wind velocity is constant in all wind directions, at 9.8 m/s, regardless of turbine location or time of day. 9.8 m/s is used because it is the rated wind speed. Using this incoming wind velocity, turbine wakes will push air speeds down the turbine's power curve, and more local optima will be experienced by participant optimizers. If the incoming wind speed were too high (i.e. 13 m/s), more turbine layouts, though different in location, will produce the same power output.

3.b Wind Direction Frequency

A lack of local optima in a design space permits even ineffective optimizers to find a "best" result. In a design space where local optima are present, inferior designs are very likely. Such design spaces test the robustness of optimization methods. The selection of wind rose is a major factor in the frequency and magnitude of local optima resulting from turbine placement.

We selected a wind rose with an off-axis wind frequency distribution, binned for 16 directions. When we tested this wind rose against 1000 randomized starting turbine locations, it gave few optimized results with relatively high AEP values. We interpreted this to be indicitive of the presence of many local optima.

Depicted in Fig. 1 in polar coordinates, a greater magnitude in the radial direction from the origin indicates a higher wind frequency from that cardinal direction.

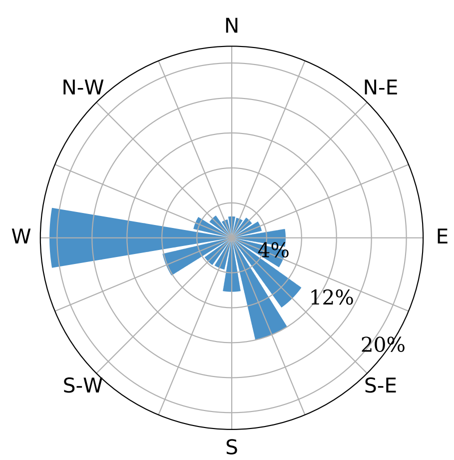


Figure 1: The wind frequency distribution for our case studies

B. Optimization Only Case Study

The purpose of this Case Study is to determine the best optimization practices for WFLO, using a single representative EWM.

1. Wake Model

A simplified version of Bastankhah's Gaussian wake model[5,6] is used, since it is compatible with both gradient-based and gradient-free methods, and is computationally inexpensive in comparison to LES and DNS methods. This wake model is described by the following equations:

$$\frac{\Delta U}{U_\infty} = \left(1 - \sqrt{1 - \frac{C_T}{8\sigma_y^2/D^2}}\right) \exp\left(-0.5\left(\frac{y-\delta}{\sigma_y}\right)^2\right) \tag{1}$$

Where $\frac{\Delta U}{U_\infty}$ is the wake velocity deficit, $C_T = \frac{8}{9}$ and is the thrust coefficient, $y - \delta$ is the distance of the point of interest from the wake center in the cross-stream horizontal direction, D is the turbine diameter, and σ_y is the standard deviation of the wake deficit in the cross-stream horizontal direction as defined in Eq. (2):

$$\sigma_y = (k_y \cdot x) + \frac{D}{\sqrt{8}} \tag{2}$$

In Eq. (2), x is the downstream distance from the turbine generating the wake to the turbine of interest, and D is the turbine diameter. k_y is determined as a function of turbulence intensity (I). In this case study turbulence intensity is treated as a constant of 0.075 (reasonable for an off-shore scenario), and we therefore used a coresponding k_y of 0.0324555.[6,7]

Increasing turbulence intensity has numerous effects and draws attention away from the main purpose of this Case Study, which is to observe the differences of optimization strategies. For the wake model we use (given in Eq. (1)), increasing the turbulence intensity widened the wake cone, but second and third order effects are unknown. As such, this first IEA Task 37 set of Case Studies uses a very low intensity in an attempt to minimize the considered variables.

2. Farm Sizes

Variability in wind farm size (and thus number of design variables) affect optimization algorithm performance. To account for this, 3 wind farm sizes are specified in Case Study 1: 16, 36, and 64 turbines. These had farm boundary radii of 1300 m, 2000 m, and 3000 m respectively, determined in the manner described previously in Section 2. Inclusion of 3 farm sizes is to observe how increased complexity correlates to convergence time and algorithm performance.

The turbine numbers are selected as perfect squares which roughly double in size. Perfect squares are used to permit even grid turbine arrangements, if desired.

3. Supplied Code

To enable participation in this Case Study, we provided a link to a GitHub repository which included files that had:

- Turbine charactersitics, wind frequency, and wind speed in IEA 37's `.yaml` schema
- Example turbine layouts for each farm size (in `.yaml` format)
- Python parsers of the `.yaml` schema
- Python target function to calculate AEP (given `.yaml` turbine locations and farm attributes)

We selected the programming language Python since it is widely used by researchers in the industry.

Participant alteration to our specific code implementation, or replication of our model in another language, was permitted if needed for compatibility with participant optimization methods. This is with the understanding, however, that final wind farm layouts would be evaluated with the original Python code that we provided.

C. Combined Physics Model/Optimization Algorithm Case Study

The intent of this case study is to assess not only the optimization methods measured by case study 1, but also the effects that different physics model approximations have on turbine location recommendations.

Case study 2 differs from the previous one in that 1) no wake model is provided, and 2) only a single wind farm size is to be optimized. Participants are free to choose their preferred EWM and optimization method combination.

Unlike case study 1, participant reported AEP is not comparable, since different EWMs are used to calculate them. To help with this, we conducted a cross-comparison of results between participants. For the cross-comparison, each participant's proposed optimal turbine locations in the standardized `.yaml` format was published to the other combined case study participants. Each participant then used their own wake model to calculate the AEP of the other participant's proposed farm layouts. From this portion of the Case Study, we hope to learn if any participant's results are seen as superior by other EWMs.

1. Farm Attributes

The wind farm size for the Combined Case Study is limited to 9 turbines. We did this to limit the computation time requirements when assessing results in a standardized LES, discussed later in Section C. We used the previously described method under **Farm Diameter** to determine the boundary radius, and the wind rose and wind speed are the same as case study 1.

III. Results

A. Case Study 1: Optimization Only

Each participant ran the optimization algorithm of their choosing using our supplied AEP function, or a functional equivalent in another language. Since there exists a great deal of variability in hardware, participants also reported processor speed, function calls, number of cores used, and total RAM installed in their system when finding their optimized results. The AEP results and rankings are given below in Tables 2 to 4.

There were 10 submissions for the optimization only case study. One participant submitted twice, using a different optimization method for each submission. For anonimity, each submission is assigned a number. We will refer to each submission below by this submission number (i.e. sub1, ..., sub10, etc.).

1. *Data*

Tables 2 to 4 display the final AEP data of all participant proposed optimal turbine layouts. The Python module we supplied which uses the simplified Bastankhah wake model was used for all AEP calculations. Submissions are ranked from highest to lowest resultant AEP values, with submission number (sub#), whether using a gradient-based (Yes) or gradient-free (No) optimization method, and the percentage increase (Inc.) from the provided example layout's AEP.

Table 2: 16 turbine scenario participant results

Rank	Algorithm	Grad.	sub#	AEP	Norm.
1	SNOPT+WEC	Yes	4	418924.4064	100.00 %
2	FMinCon()	Yes	5	414141.2938	98.86 %
3	SNOPT	Yes	8	412251.1945	98.41 %
4	Sparse Nonlinear Optimizer (SNOPT)	Yes	1	411182.2200	98.15 %
5	Preconditioned Sequential Quadratic Programming	No	2	409689.4417	97.80 %
6	Multistart Interior-Point	Yes	10	408360.7813	97.48 %
7	Full Pseudo-Gradient Approach	No	3	402318.7567	96.04 %
8	Basic Genetic Algorithm	No	7	392587.8580	93.71 %
9	Simple Particle Swarm Optimization	No	6	388758.3573	92.80 %
10	Simple Pseudo-Gradient Approach	No	9	388342.7004	92.70 %
11	(Example Layout)	-	-	366941.5712	87.59 %

Table 3: 36 turbine scenario participant results

Rank	Algorithm	sub#	Grad.	AEP	Norm.
1	SNOPT+WEC	4	Yes	863676.2993	100.00 %
2	Multistart Interior-Point	10	Yes	851631.9310	98.61 %
3	Preconditioned Sequential Quadratic Programming	2	No	849369.7863	98.34 %
4	SNOPT	8	Yes	846357.8142	97.99 %
5	Sparse Nonlinear Optimizer (SNOPT)	1	Yes	844281.1609	97.75 %
6	Full Pseudo-Gradient Approach	3	No	828745.5992	95.96 %
7	FMinCon()	5	Yes	820394.2402	94.99 %
8	Simple Pseudo-Gradient Approach	9	No	813544.2105	94.20 %
9	Basic Genetic Algorithm	7	No	777475.7827	90.02 %
10	Simple Particle Swarm Optimization	6	No	776000.1425	89.85 %
11	(Example Layout)	-	-	737883.0985	85.44 %

Table 4: 64 turbine scenario participant results

Rank	Algorithm	sub#	Grad.	AEP	Norm.
1	SNOPT+WEC	4	Yes	1513311.1936	100.00 %
2	Preconditioned Sequential Quadratic Programming	2	No	1506388.4151	99.54 %
3	Multistart Interior-Point	10	Yes	1480850.9759	97.86 %
4	Sparse Nonlinear Optimizer (SNOPT)	1	Yes	1476689.6627	97.58 %
5	Full Pseudo-Gradient Approach	3	No	1455075.6084	96.15 %
6	SNOPT	8	Yes	1445967.3772	95.55 %
7	Simple Pseudo-Gradient Approach	9	No	1422268.7144	93.98 %
8	Simple Particle Swarm Optimization	6	No	1364943.0077	90.20 %
9	FMinCon()	5	Yes	1336164.5498	88.29 %
10	Basic Genetic Algorithm	7	No	1332883.4328	88.08 %
11	(Example Layout)	-	-	1294974.2977	85.57 %

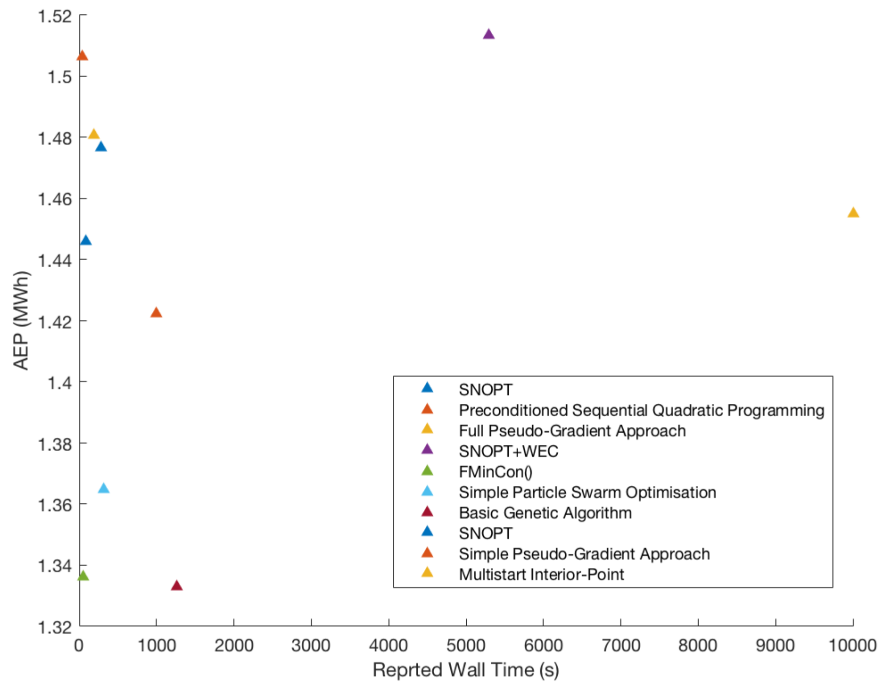


Figure 2: AEP vs reported wall time for each submitted optimization method of the 64 turbine scenario

1.a General Trends

Participants were given 4 calendar weeks to complete the case studies. All participants are volunteers, and their participation was conducted outside of normal work or educational responsibilities. This 4-week optimization window factors to be 0.2% of typical wind farm's 20-year lifespan,[2] and we submit that optimizations (even if needing the full extent of this timeframe) would be worth the time they require, if able to produce superior results. However, from the participant self-reported wall-times, none of the optimizations took nearly that long.

For this reason, we discounted required wall time as a distiguishing factor, and use resultant AEP as our only metric for measuring algorithm effectiveness. Though factors like processing time and computing power do vary, we consider algorithms that produce results anywhere within the 4 week announcement-to-call window to be acceptable for applications in industry.

As a general trend, gradient-based methods performed better in discovering a relative optima, especially for smaller farm sizes. Some gradient-based algorithms improved in comparative AEP ranking as the number of design variables increase (*sub10*, *sub3*), while others degrade (*sub5*, *sub8*). Simultaneously, some gradient-free algorithms increase in effectiveness as design variables increase (*sub2*, *sub3*), while others compete for lowest comparative performance, regardless of farm size (*sub6*, *sub7*, *sub9*).

Despite these multi-variate results, one clear front-runner does emerge. Regardless of wind farm size, *sub4*'s algorithm, consistently discovered turbine placement that delivers an AEP superior to all other participants. A summary of *sub4*'s method is included in a following section.

Also of note, as the number of design variables increases, the relative disparity between proposed optimal AEPs likewise diverge. For the 16 turbine case, the highest result is 7.88% better than the lowest. For the 36 and 64 cases, the highest result is 11.45% and 13.54% better than the lowest, respectively.

1.b AEP vs Wall Time

Fig. 2 is a plot of AEP from each submission's turbine layout, vs participant reported wall time for the 64 turbine case, per optimization. It shows the SNOPT+WEC method, though reporting the higest AEP turbine arrangment, also took orders of magnitude longer than some other approaches.

More testing must be done to determine if the other methods, if permitted to run longer, would discover an optimum better than that discovered by *par5*, or if by nature of the algorithms, no matter the time limit permitted to run, there are some optima unattainable by their formulations.

2. Analysis of Best Results

2.a Gradient-based

For all 3 farm sizes, the superior method was implemented by *sub4*, using a gradient-based method. Coded in Python and FORTRAN, it combined the Sparse Nonlinear OPTimizer (SNOPT)[8] with a method called Wake Expansion Continuation (WEC).[6] Running 200 optimizations, *sub4* had one optimization run start from the provided example layout, and the other 199 use randomized turbine starting locations within the farm boundary.

The WEC method is specifically designed to reduce the multi-modality found in wind farm layout optimization. In the cited paper,[6] it is a method of converting design spaces with many local minima into curves approaching convexity, allowing gradient-based optimizations to more easily find the better solutions. An example of such "relaxation" to convexity is included in Figs. 3 and 4, reproduced with permission.

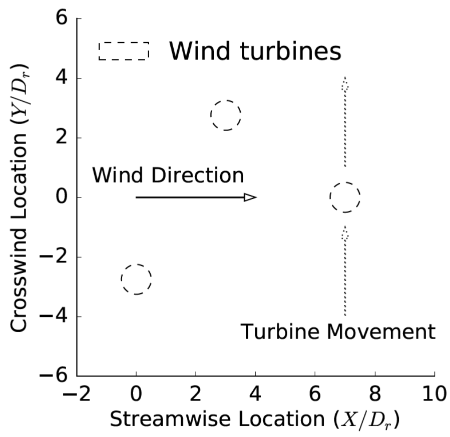


Figure 3: Simple design space used to demonstrate the effects of the relaxation factor, ξ, on the wind farm layout design space.[6]

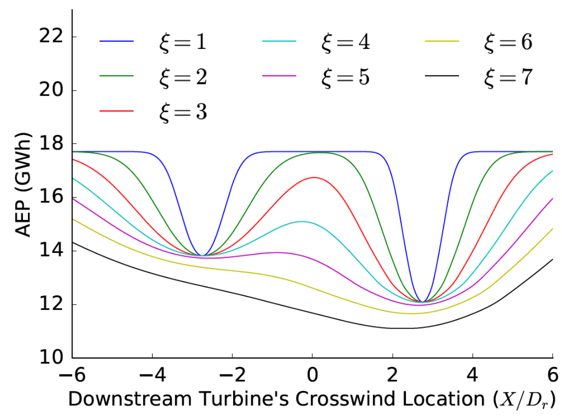


Figure 4: The impact of the wake relaxation factor, ξ. One turbine was moved across the wakes of two upstream turbines (see Fig. 3).[6]

Figs. 3 and 4 demonstrate the effects of the WEC method on a simple design space, relaxing the local optima into a more easily discovered global solution. As the authors Thomas and Ning state, "Larger values of ξ allow the smaller local optima to disappear completely. Smaller values of ξ allow for more accurate wake widths but with an increase in the number and magnitude of local optima."[6] We suspect that the WEC method for reducing the multi-modality of the design space is the reason that *sub4*'s optimizatons found superior layouts to the other used methods.

2.b Gradient-free

Of the gradient-free methods used, the superior particiant was *sub2*. Futhermore, the relative performance increased as wind farm size increased. Out of the 10 participants, *sub2* ranked 5th, 3rd, then 2nd as the turbine sizes went from 16, to 36, to 64.

Programmed in Python, *sub2* used a "Preconditioned" Sequential Quadratic Programming (SQP) optimization method. SQP is an optimization method which breaks the problem into quadratic subproblems. To seed steps between generations, *sub2* took a starting layout and rotated it in $\pi/6$ steps. The best of the 12 resultant layouts was then taken as the warm-start for the next generation.

3. Analysis of Worst Results

3.a Gradient-based

The worst performing gradient-based approach was different for each wind farm size. For the 16 turbine farm, *sub10*'s method was the poorest performing gradient-based method, but climed to the second best gradient method for the large sizes. For the 36 turbine farm, *sub1* was the poorest gradient-based method, but for the 64 turbine case, outperformed two other gradient-based methods. For the 36 turbine farm, *sub5*'s attempt was the worst gradient-based method performance, and poorer than almost all of the gradient-free methods as well.

Not using the Python implementation we supplied, *sub5* translated the target AEP function into MATLAB and used that language's built-in FMINCON() function to optimize turbine locations. For the 16 farm case, *sub5* did 1000 optimizations, each with randomized turbine starting locations. Due to computational time required, *sub5* did only 500 random starts for the 64 turbine case, and this poorer relative performance may be a result of the smaller sample size.

3.b Gradient-free

Two participants using gradient-free methods (*sub6*, *sub7*) rotated positions for lowest relative AEP. They each used a different method, and will be described individually.

Coding in MATLAB, *sub6* used a multi-start partical swarm, with the interior point method. For each farm size *sub6* did 300 optimizations, with 20 swarm particles. If the minimum distance between turbines was violated at the end of an iteration, *sub6*'s algorithm randomized a new turbine location within the boundary and spacing constraints. The swarm algorithm used an inertia weight 0.729, and social and cognitive weights of 1.49618. The initial turbine location population used only turbine coordinates satisfying the boundary and spacing constraints.

Coding in Python, *sub7* used a genetic algorithm. The algorithm used a tournament selection method, with n=10 and elitism, where only the highest AEP layouts went on to seed the next generation. There were 25 generations each of size 500, using a mutation probability of 3%.

4. Discussion

No worst-practices pattern can be concluded from the 3 cases, as the poorest performers all had vastly different methods for reaching their results.

In terms of best-practices, for the 3 farm sizes we tested, *sub4*'s strategy of using the SNOPT optimizer combined with the WEC relaxation method consistently delivered superior layouts.

Though *sub4* consistently found the superior AEP relative to the other participants, *sub2*'s results demonstrate a trend closing the gap as the number of design variables increased. For the 16 turbine case, *sub4* was 2.5% better than *sub2*'s results. For the 36 and 64 cases, *sub4* was 1.68% and 0.46% better, respectively. It should be noted, however, that at the current average U.S. rate[9] of roughly ¢13.3 for a kWh (or \$133 per MWh), the income difference between the AEPs of *sub4* and *sub2* in the 64 turbine case, though only 0.46%, equates to a difference of a little under 1 million U.S. dollars per year.

Since *sub2*'s SQP method steadily closed the gap, a future study should test even larger wind farm sizes. This could determine if the SQP algorithm will eventually outperform the SNOPT + WEC method when a certain number of design variables are reached, or if there is an upper limit or convergence to this trend.

B. Case Study 2: Combined

For the combined wake model and optimization method seletion case study, each participant ran both the optimization algorithm of their choosing as well as their choce of wake model target function. There were no restrictions on programming language for either the wake model or optimization algorithm, but results of optimal turbine layouts were to be submitted in the `.yaml` format supplied in the case study 1 examples.

There were 5 participant submissions for the combined case study. All 5 participants also submitted for case study 1 (though were not required to do so) so we assigned their submissions the same numbers from that case study. i.e., *sub1 - sub5* are from the same individual participants for both case study 1 and case study 2.

With each participant using a different wake model, AEP values reported by participants cannot be fairly compared. Results were judged on a cross-comparison of layouts between participants.

1. Data

The cross-comparison does displays some interesting trends. The following tables show how each submission's wake models ranked the proposed optimal turbine layouts for the other 4 submission. Each submission's ranking of their own layout is in **bold**. The last column in the table is the submission number of the layout being cross-compared (cc-sub#). So submission 4's analysis of submission 2's layout would be found in the *sub4* table, with 2 in the cc-sub# column. The last column is the percentage increase (Inc.) from the reporting submission's submitted layout. A negative value here indicates a worse AEP.

sub1:

Rank	Wake Model	Algorithm	AEP	cc-sub#	Inc.
1	Simplified Bastankhah	FMinCon()	262350.319	4	0.624 %
2	Bastankhah	SNOPT+WEC	262282.416	5	0.598 %
3	FLORISSE3D	SNOPT	**260722.295**	**1**	0.000 %
4	Bastankhah	Full Pseudo-Gradient Approach	260640.906	3	-0.031 %
5	Park2	PSQP	248215.024	2	-4.797 %

sub2:

Rank	Wake Model	Algorithm	AEP	cc-sub#	Inc.
1	Simplified Bastankhah	FMinCon()	250464.9732	4	5.975 %
2	Bastankhah	SNOPT+WEC	250249.0259	5	5.884 %
3	Bastankhah	Full Pseudo-Gradient Approach	247812.0522	3	4.853 %
4	FLORISSE3D	SNOPT	240309.5850	1	1.678 %
5	Park2	PSQP	**236342.799**	**2**	0.000 %

sub3:

Rank	Wake Model	Algorithm	AEP	cc-sub#	Inc.
1	Bastankhah	SNOPT+WEC	247109.5234	5	0.590 %
2	Simplified Bastankhah	FMinCon()	246942.3767	4	0.522 %
3	Bastankhah	Full Pseudo-Gradient Approach	**245659.4124**	**3**	0.000 %
4	Park2	PSQP	242431.5431	2	-1.314 %
5	FLORISSE3D	SNOPT	237548.6622	1	-3.302 %

sub4:

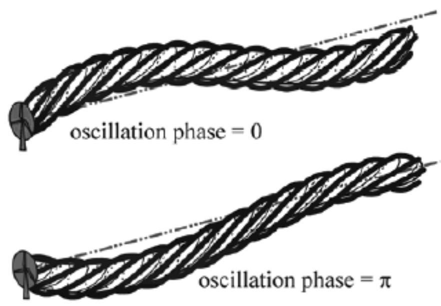
Rank	Wake Model	Algorithm	AEP	cc-sub#	Inc.
1	Simplified Bastankhah	FMinCon()	**257790.1924**	**4**	0.000 %
2	Bastankhah	SNOPT+WEC	257663.4068	5	-0.049 %
3	Bastankhah	Full Pseudo-Gradient Approach	255063.8201	3	-1.058 %
4	FLORISSE3D	SNOPT	251776.7157	1	-2.333 %
5	Park2	PSQP	239612.8223	2	-7.051 %

sub5:

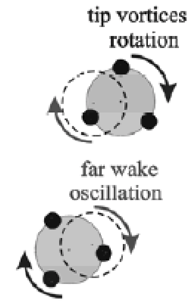
Rank	Wake Model	Algorithm	AEP	cc-sub#	Inc.
1	Bastankhah	SNOPT+WEC	**251771.9067**	**5**	0.000 %
2	Simplified Bastankhah	FMinCon()	251697.7126	4	-0.029 %
3	Bastankhah	Full Pseudo-Gradient Approach	249829.2199	3	-0.772 %
4	FLORISSE3D	SNOPT	246503.8323	1	-2.092 %
5	Park2	PSQP	239482.6767	2	-4.881 %

1.a General Trends

We expected each participant to rank their own layout as superior to the others. Each wake model accounts for different fluids phenomena, and what one wake model considers an optimal layout, another may not. An example of this would be if one participant used Larsen's Dynamic Meandering Wake (DMW),[10] depicted in Fig. 5. A turbine placed downstream, slightly offset to miss the wake oscillations accounted for in DMW would, under another wake model not accounting for this "meander" (such the Jensen's model[11]), feel the full brunt of the wake, and deliver a sub-optimal AEP.



(a) DMW depicted 3-dimensionally



(b) DMW in the Trefftz plane

Figure 5: Sketch of Larsen's Dynamic Meandering Wake (DMW) behind a wind turbine.[10]

Unexpectedly, only *sub4* and *sub5* found their own layouts to be superior to the other participants. Furthermore, all other participants also found *sub4* and *sub5*'s layouts superior to their own, though to varying degrees. Three particiants (including self-reporting) found *sub4* to have the highest AEP producing layout. The other two participants found *sub5* to have the highest AEP producing layout.

2. Analysis of Best Results

Within expectations, *sub4* and *sub5* ranked their own layouts superior to all other participant results. Two correlations are important to note regarding *sub4* and *sub5*. Firstly, both used variations of the same wake model. From case study 1, *sub5* used the simplified gaussian wake model previously described.[5,6] Though *sub4* also used the gaussian wake model,[5] *sub4* combined it with the model created by Niayifar and Porté-Agel,[7] supplemented by the WEC method described earlier. Furthermore *sub4* also accounted for partial wake, shear, ambient turbulence intensity, and local turbulence intensity. None of these factors were accounted for by *sub5*. The second factor to note is that despite using very similar wake models, *sub4* and *sub5* used different gradient-based optimization algorithms that nonetheless reached very similar conclusions.

As can be seen in the visual depictions included later in Section 2, *sub4* and *sub5* found nearly identical optimal turbine placements. Though pictorally appearing identical, the actual coordinates do indeed differ, enough so to result in different AEP calculations shown in the tables above.

Without LES data, the conclusions able to be drawn from the cross-comparison analysis are limited. That both *sub4* and *sub5* were found by the other participant wake models to have superior placement could be a result of either a more efficient optimization method, or a better coupling between optimization method and wake model. That these minima existed within the other wake models (resulting in a higher computed AEP by those models) yet were nevertheless undiscovered in their optimizations, is inconclusive in telling us which it is.

Both *sub4* and *sub5* used similar wake models, but very different optimization methods. Coding in MATLAB, *sub5* did 1000 random starts, and used MATLAB's built in FMINCON() (which uses a finite difference method to find gradients) to optimize for a minimum. Using a combination of Python and FORTRAN, *sub4* ran 1 ordered with 199 random starts for 200 optimizations altogether. SNOPT's SQP algorithm was *sub4*'s implemented optimizer.

Of note, from trends seen above in case study 1, *sub5*'s optimization methods demostrate superior performance for small design variable sizes, but comparitively degrades as the windfarm size increases. The superior performance of this wake model and optimization method combination for this small farm may not be representative of performance on larger wind farms.

3. Analysis of Worst Results

Again, conlusions drawn from the cross-comparison analysis are limited due to the lack of LES data. Suprisingly, however, *sub2*'s results were found by every other participant to be inferior, even by *sub2*. As noted earlier, this could be a result of either poor wake model pairing with optimization method, or simply an optimization method shortcoming. Using data from case study 1, *sub2*'s optimization method shows inferior results for small sample sizes, but increases in comparative performance as wind farm size and number of design variables increase. That data leads us to believe the inferior performance here is a product of the wind farm size, and not a poor pairing of wake model with optimization method. However, both the LES analysis and attempts at farms of larger sizes would need to be done to find a definitive conclusion.

Discounting the two universal top performers (*sub4* and *sub5*), *sub1*'s results were more in line with what we expected from the cross-comparison. Namely, that *sub1* found its layout superior to the others, but that the others did not find it so.

4. *Discussion*

A weakness voiced by participants of earlier case studies are scenarios where non-novel, and simplistic layouts (such as all turbines on the boundary border) are optimal. The small farm radius with few turbines given for this case study seems to have fallen into this category. What is interesting, however, is that three of the five participants were trapped in local optima, and proved blind to optima others found, using different physics approximations and optimization methods. Many factors could have led to these shortfalls (i.e. inferior optimization methods, lack of sufficient iterations, lack of sufficient wall time, etc...) and further testing would need to be done to discover which factors majorly contributed to the outcome.

IV. Conclusion

A. Case Study 1

Results from Case Study 1 show that sub4's optimizaton of SNOPT+WEC delivers superior results for the tested wind farms with 16, 36, and 64 turbines. While information on this method is continuing to be produced, the initial paper written by Thomas and Ning[6] describes this method. Though further testing is required for validation, *sub2*'s gradient-free method shows a trend of increased performance that may surpass SNOPT+WEC for wind farms of sizes larger than 64.

B. Case Study 2

Case study 2 demonstrates that, for wind farms of small area with few turbines, placement on the wind farm boundary delivers superior AEP. Shortcomings in participant pairings of optimization methods were trapped in local optima, however. The lesson learned here is to either train researcher intuition to use such layouts as warm starts, or improve optimization methods so that automated optimizers can discover this themselves.

C. Future Work

1. *Sample Size*

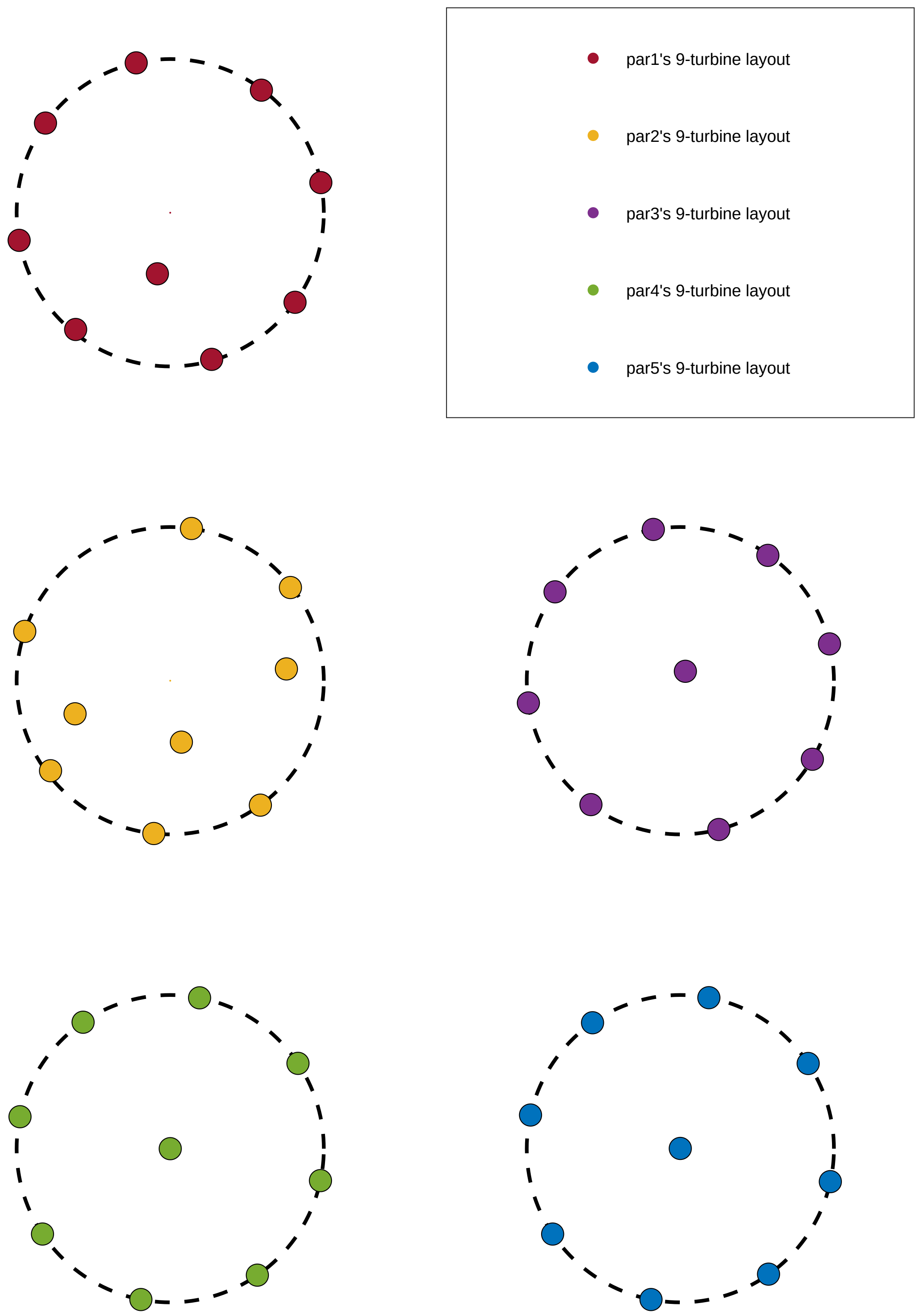
Though we are happy with the level of participation in the Case Studies, a larger participant sample size with different methods may provide more informative data, or display other novel and superior methods. To refine our data collection process, we plan on running another round of results for these Case Studies in the near future.

2. *LES Comparison*

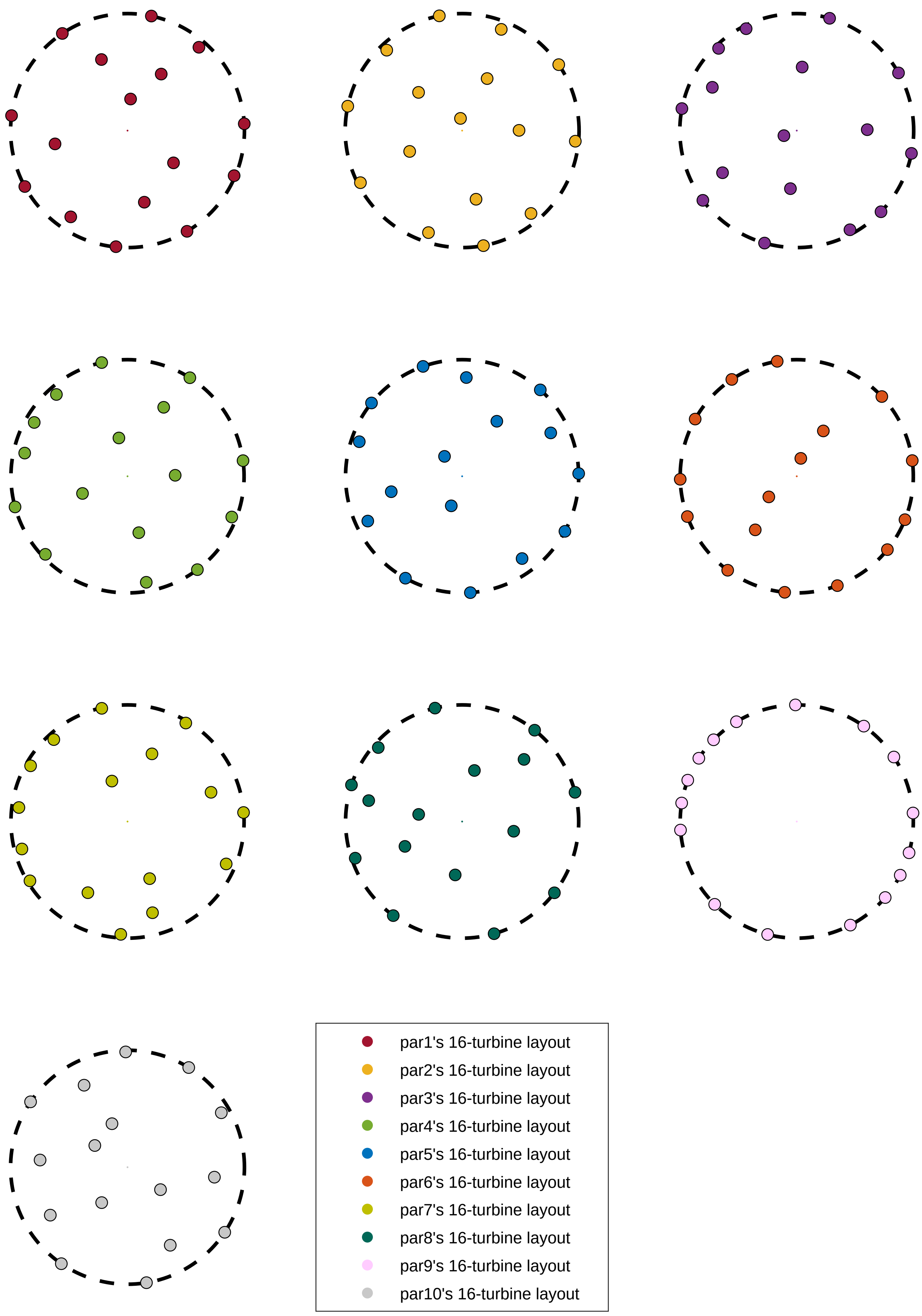
Due to the difficulty in comparability of results between EWMs, we will also run all participant-reported optimized turbine locations through an LES. With the inherent bias each EWM has for its own optimized locations removed, reported turbine locations will be measured using the same simulation tool for a comparative AEP. Case Study 2 was constructed mainly for this LES wake model evaluation, in order to gauge which simplified model is most accurate when compared to a higher-cost computational model. Due to time and computing resource constraints, the authors were unable to run all submitted participant layouts through an LES.

Without this LES analysis, a key piece of adjudication is lacking. The LES we will use is produced by NREL and called the Simulator fOr Wind Farm Applications (SOWFA). The LES analysis will be conducted in the near future, and results will be published.

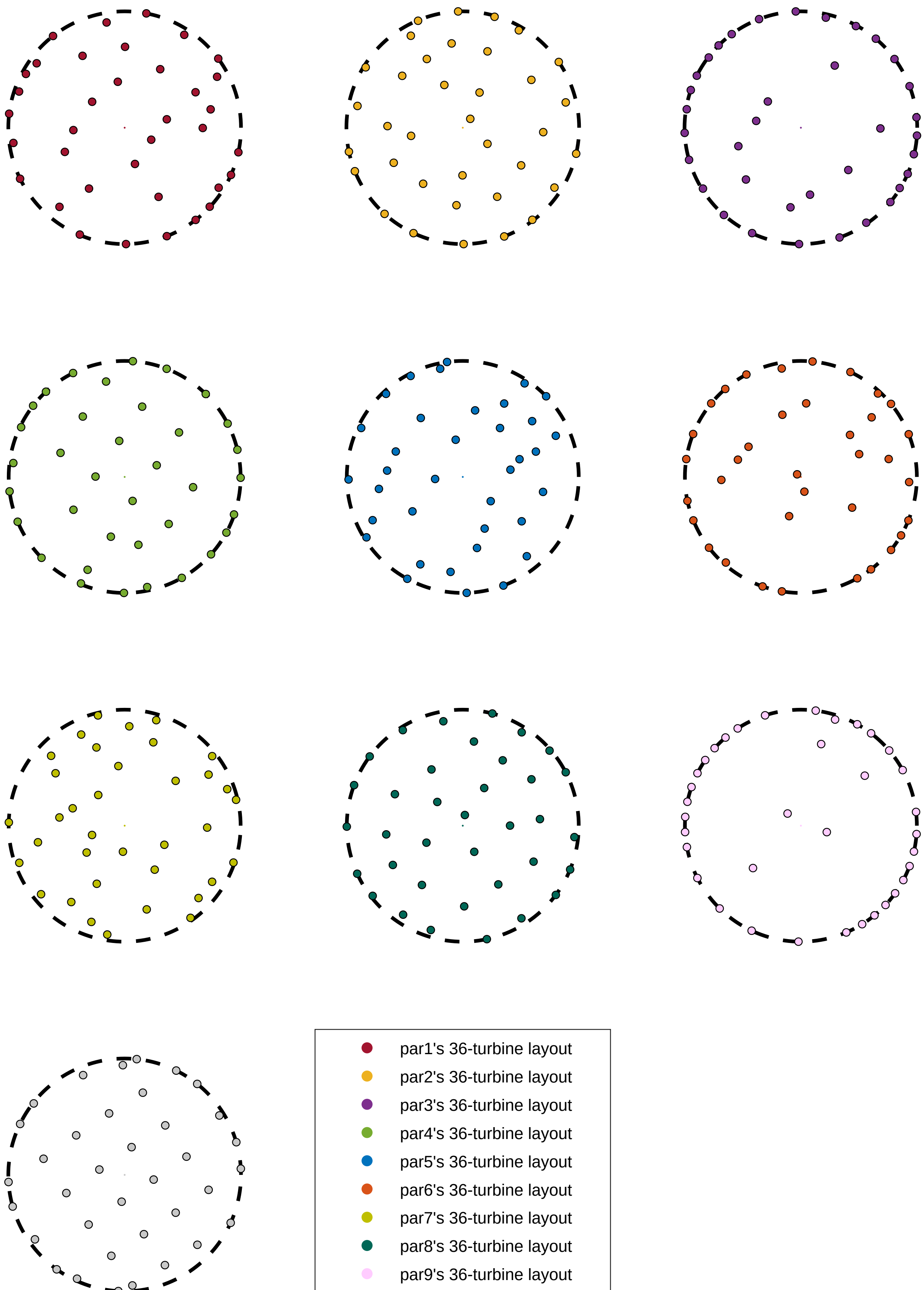
A. Pictoral Representations of Case Study 2's Participant Submissions



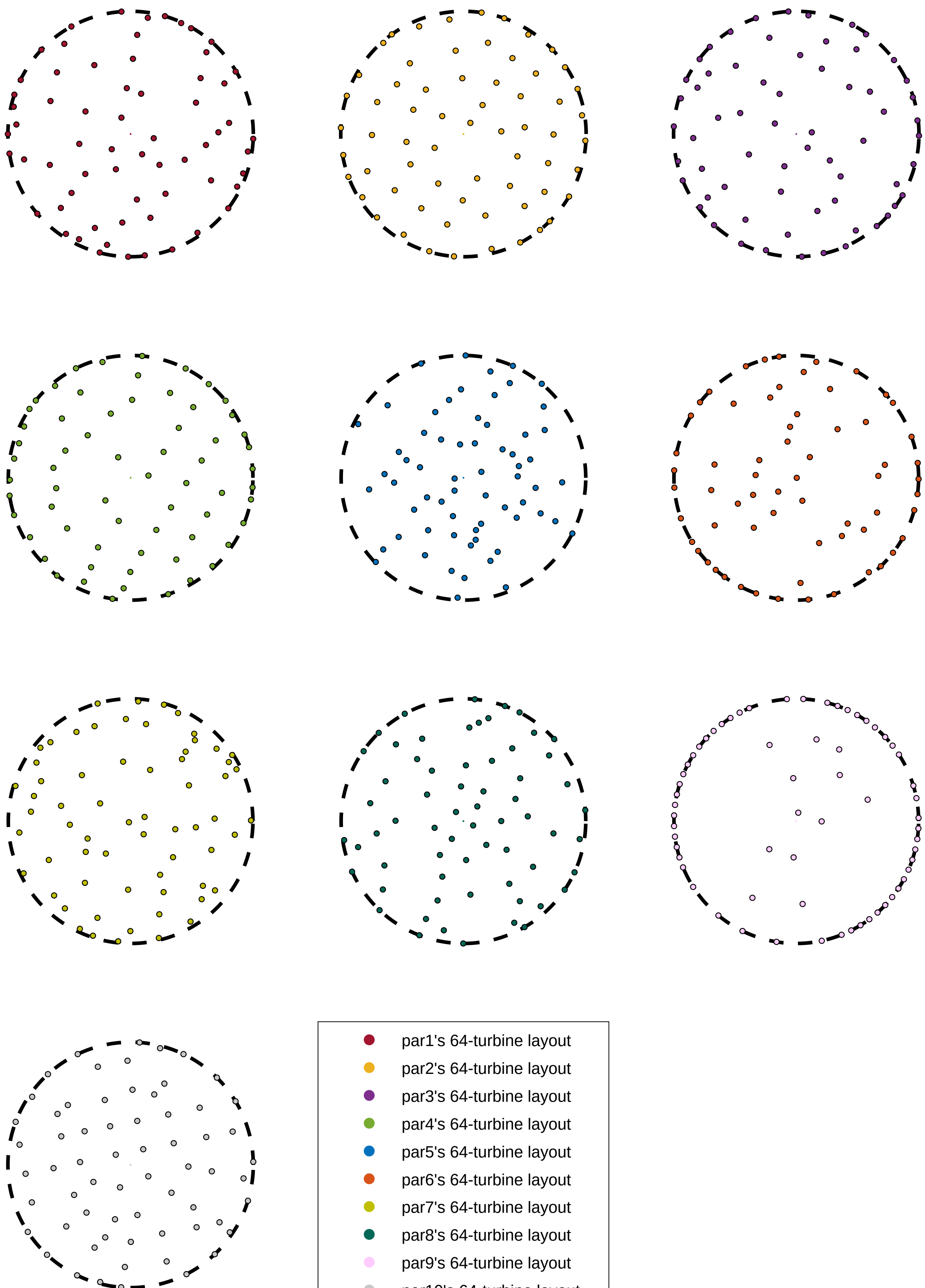
B. Pictoral Representations of Case Study 1's 16-Turbine Submissions



C. Pictoral Representations of Case Study 1's 36-Turbine Submissions



D. Pictoral Representations of Case Study 1's 64-Turbine Submissions



References

[1]Padrón, A. S., Thomas, J., Stanley, A. P. J., Alonso, J. J., and Ning, A., "Polynomial Chaos to Efficiently Compute the Annual Energy Production in Wind Farm Layout Optimization," *Wind Energy Science*, Jan. 2018, (in review).

[2]Herbert-Acero, J. F., Probst, O., Réthoré, P.-E., Larsen, G. C., and Castillo-Villar, K. K., "A Review of Methodological Approaches for the Design and Optimization of Wind Farms," *Energies*, 2014, pp. 23.

[3]McWilliam, M. K., Zahle, F., and Dykes, K., "IEA Task 37 on System Engineering in Wind Energy The Aerodynamic Only Optimization Case Study," May 2017.

[4]Bortolotti, P., Dykes, K., Merz, K., Sethuraman, L., and Zahle, F., "IEA Wind Task 37 on System Engineering in Wind Energy, WP2 - Reference Wind Turbines," Tech. rep., National Renewable Energy Laboratory (NREL), Golden, CO., May 2018.

[5]Bastankhah, M. and Porté-Agel, F., "Experimental and theoretical study of wind turbine wakes in yawed conditions," *J. Fluid Mech.*, Vol. 806, 2016, pp. 506–541.

[6]Thomas, J. J. and Ning, A., "A Method for Reducing Multi-Modality in the Wind Farm Layout Optimization Problem," *Journal of Physics: Conference Series*, Vol. 1037, The Science of Making Torque from Wind, Milano, Italy, jun 2018, p. 10.

[7]Niayifar, A. and Porté-Agel, F., "Analytical Modeling of Wind Farms: A New Approach for Power Prediction," *Energies*, September 2016.

[8]Gill, P., Murray, W., and Saudners, M., "SNOPT: an SQP algorithm for large-scale constrained optimization," *SIAM Review*, Vol. 47, 2005, pp. 99–131.

[9]"Worth of kWh per state," November 2018.

[10]Larsen, G. C., Madsen, A., Helge; Bingöl, F., Mann, J., Ott, S., Sørensen, J., Okulov, V., Troldborg, N., Nielsen, M., Thomsen, K., Larsen, T. J., and Mikkelsen, R., "Dynamic wake meandering modeling," Tech. rep., Technical University of Denmark, 2007.

[11]Jensen, N., "A Note on Wind Generator Interactions," Tech. rep., RISO National Laboratory, 1983.