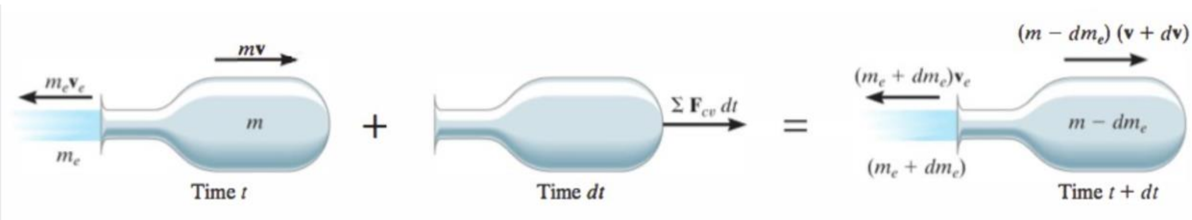
Rocket Launch into Low Earth Orbit

Introduction:

The system under consideration is a variable mass system. The system models the dynamic behaviours of a rocket being launched into a low earth orbit (approx. 400km). Rockets are propulsive systems which burn fuel which is ejected at high velocities from the back of the rocket to achieve thrust. This loss in mass by the ejection of the fuel can be considered by its effective change in momentum within the system of the rocket and the ejected mass figure (1). The rocket of mass (m) and velocity (v) ejects mass (m_e) with an exhaust velocity (v_e) at an instant of time. Hence by applying the conservation of momentum to the system, the sum of the forces in the direction of v over time plus the momentum of the rocket minus the momentum of the exhaust is equal to the change in the rocket mass multiplied by the change in the rocket velocity, minus the change in the ejected mass multiplied by the exhaust velocity. By then dividing this by the change in time dt and ignoring the higher-order terms the end result for the variable mass system is:

$$\sum F = m\frac{dv}{dt} - (v + v_e)\frac{dm_e}{dt} \qquad [1]$$

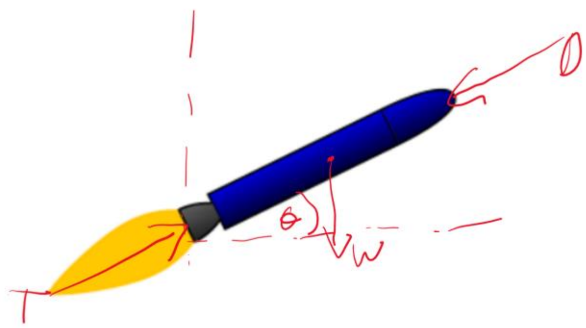
This is the sum of the forces is equal to the mass multiplied by the acceleration minus the relative exhaust velocity multiplied by the rate of fuel consumption. Where the relative exhaust velocity multiplied by the rate of fuel consumption can be considered as the thrust of the ricket. In order to apply this variable mass system to a real world application other forces need to be accounted for. The other forces acting on a rocket during its launch include the weight force and the air resistance or drag. A rockets trajectory to low earth orbit is not a vertical path. For the rocket to achieve an orbit it must be travelling parallel to the surface of earth. Hence the rocket must travel at an angle theta to reach a horizontal trajectory. By breaking the above equation down into horizontal and vertical components reduced by cos(theta) and sin(theta) respectively and calculated over the time of flight, the instantaneous masses, forces, accelerations, velocities, positions, distances and angles can be obtained and plotted to illustrate the dynamic kinetic and kinematic behaviour of the rocket launch as a variable mass system.



(Figure 1: Illustration of conservation of momentum of variable mass system [1])

System:

An example of a variable mass system is a rocket launching into low earth orbit. The following equations of motion are developed and used to model this variable mass system.



(Figure 2: Free Body Diagram of rocket launch)

$$\sum F = m\frac{dv}{dt} - (v + v_e)\frac{dm_e}{dt}$$

$$\sum F = -W - D$$

$$T - W - D = \sum F$$

Where :

$$(v + v_e)\frac{dm_e}{dt} = uc = T\ (Thrust\ (N))$$

W=weight force (N)= mg

$t_0 = 0$ s

$\Delta t = 0 \cdot 1$ s

$t_f = 510\ s$ [3]

Cd = 0.4 [4] Drag coefficient

Rho_0=1.225 [5] air density at sea level (kg/m^3)

A = pi*3.7^2 [2] cross sectional area of rocket (m^2)

G=6.67408*10^-11 [6] Gravitation Constant (m^3/kg.s^2)

M_earth=5.9722*10^24 [6] Mass of the Earth (kg)

R_earth=6.371*10^6 [6] Radius of Earth (m)

M_0=500000 [2] Initial mass (kg)

M_final=22800 [2] Payload to low earth orbit (kg)

M_fuel=M_0-M_final Mass used to reach orbit (kg)

u=5500 [5] [7] Relative exhaust velocity (m/s)

c= M_fuel/t_f Rate of change of mass (kg/s)

$m = m_0 - ct$ [1]

$g = \frac{Gm_{earth}}{(r_{earth}+y)^2}$ [6]

$\rho = \rho_0 e^{\left(\frac{-y}{8000}\right)}$ [5]

$D = 0.5Cd\rho Av^2$ =Drag force (N) [4]

$$\sum Fx = T\cos\theta - W\cos\theta - D\cos\theta$$

$$\sum Fy = T\sin\theta - W\sin\theta - D\sin\theta$$

$$Ax = \sum Fx/m$$

$$Ay = \sum Fy/m$$

$$vx_f = vx_i + Ax\Delta t$$

$$vy_f = vy_i + Ay\Delta t$$

$$x_f = x_i + vx\Delta t$$

$$y_f = y_i + vy\Delta t$$

$$Distance\_x_f = Distance\_x_i + abs(vx\Delta t)$$

$$Distance\_y_f = Distance\_y_i + abs(vy\Delta t)$$

$$Distance = \sqrt{Distance\_x^2 + Distance\_y^2}$$

If y<9000m

$\theta = 90$ (Initially launched vertically)

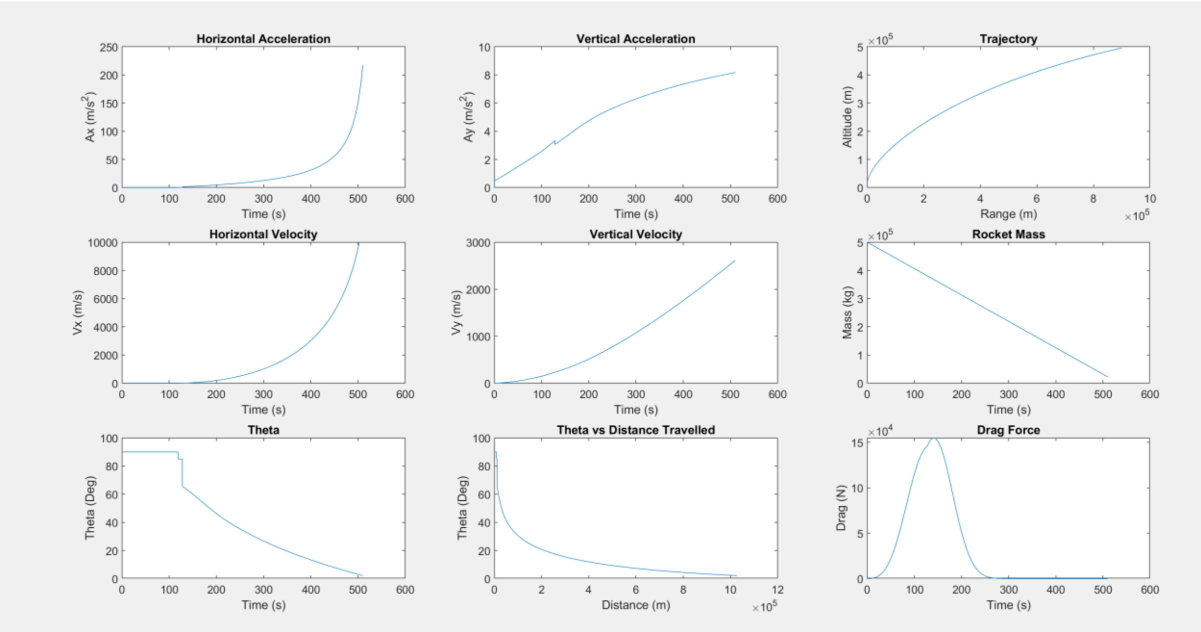
If 11000m >=y>=9000m

$\theta = 85$ (Gravity turn once the rocket has passed most of the thickness of the atmosphere [8])

If y>11000

$\theta = \arctan\left(\frac{g}{A_x + A_y}\right)$ (Change in theta after the gravity turn due to the weight force)

Results:



(Figure 3: A 3x3 subplot of various calculated variables during the rocket launch)

Discussion:

The results from the numerical modelling of the rocket launch in figure(3) show that the rocket reaches an altitude of approx. 500km and a horizontal velocity of approx. 10,000m/s after a burn time of 8mins 30sec [3]. The expected results from the research were that the rocket would reach an altitude of approx. 400km and a horizontal velocity of approx. 7700m/s[9] after an 8mins 30sec burn time. With 400km being the approximate distance of the intended low earth orbit and 7700m/s being the necessary velocity to maintain this orbit. The calculated results were close to these expected results which illustrates that the numerical model has a reasonable accuracy. Also, the angle theta reaches close to zero at the completion of the launch which is what is intended as the rocket must travel close to parallel to the ground in order to maintain a low earth orbit. The slope of the vertical acceleration begins to decrease whilst the horizontal acceleration increases which is what is expected as the rocket begins to travel parallel to the ground in orbit. The shape of the curve of the drag force is as expected, as the altitude increases the atmosphere thins and becomes less dense resulting in less drag on the rocket.

Whilst this model appears to provide a reasonable approximation of a rocket launch as a variable mass system there are some significant errors and oversights within the model. There are significant errors with the parameters used as they are mostly rough approximations (with the exception of some more exact values such as the air density at sea level and gravitation constant etc.) and come from various different sources which are not specific for a single rocket type. These variations can propagate significant errors through the calculations. Whilst care has been taken to attain accurate instantaneous results such as the change in the air density and gravity, there are inherent oversights in the model for variables such as the relative exhaust velocity and rate of change of mass and hence the thrust force as being constants. For actual systems, these variables would not be constant and would change to acquire optimal trajectory and velocities. Other oversights include the use of stages which is not included in this model. Staged rockets are used in order to minimize the mass of the rocket throughout the launch. As most of the mass of the rocket is the mass of the fuel and the structure holding the fuel and most of the structure is only there to carry this fuel it is obvious that getting rid of that structure as the fuel is used will reduce a lot of the weight of the rocket. This means that less fuel is needed to propel the final stage of the rocket into orbit and gain the necessary velocity, resulting in major cost savings. As the initial mass and the mass of the payload used in this model are taken from the data for the SpaceX falcon 9 rocket which is staged, there is a clear error in the thrust calculation as the mass of the structure dropped at each stage will not contribute much to the thrust. The falcon 9 rocket also carries enough fuel to bring the first stage safely back to the ground after stage separation, hence the mass of this additional fuel will not contribute to the thrust of the payload but has been calculated as thrust. To help slightly minimize these errors the initial mass provided by the SpaceX data was rounded down from 550,000kg to 500,000kg. The result of these errors will mostly likely be the cause of the higher altitude and horizontal velocity calculated in this model.

References

[1]"Week 11: Lectures and Tutorial: AMME2500 AMME9500 Engineering Dynamics", *Canvas.sydney.edu.au*, 2021. [Online]. Available: https://canvas.sydney.edu.au/courses/30664/pages/week-11-lectures-and-tutorial?module_item_id=1054109. [Accessed: 02- Jun- 2021].

[2]"SpaceX - Falcon 9", *SpaceX*, 2021. [Online]. Available: https://www.spacex.com/vehicles/falcon-9/. [Accessed: 03- Jun- 2021].

[3]"SpaceX- Launches", *SpaceX*, 2021. [Online]. Available: https://www.spacex.com/launches/. [Accessed: 03- Jun- 2021].

[4]"Basic of Space Flight: Aerodynamics", *Braeunig.us*, 2021. [Online]. Available: http://www.braeunig.us/space/aerodyn_wip.htm . [Accessed: 28- May- 2021].

[5]"Variable Mass Systems: The Rocket Equation", *Ocw.mit.edu*, 2021. [Online]. Available: https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-07-dynamics-fall-2009/lecture-notes/MIT16_07F09_Lec14.pdf. [Accessed: 28- May- 2021].

[6]"Gravity of Earth - Wikipedia", *En.wikipedia.org*, 2021. [Online]. Available: https://en.wikipedia.org/wiki/Gravity_of_Earth. [Accessed: 01- Jun- 2021].

[7]"Electric Propulsion", *Web.stanford.edu*, 2021. [Online]. Available: https://web.stanford.edu/~cantwell/AA284A_Course_Material/AA284A_Resources/Jahn%20and%20Choueiri,%20Electric%20Propulsion%20Encyclopedia%20of%20Physical%20Science%20and%20Technology%202002.pdf. [Accessed: 01- Jun- 2021].

[8]"Gravity turn - Wikipedia", *En.m.wikipedia.org*, 2021. [Online]. Available: https://en.m.wikipedia.org/wiki/Gravity_turn. [Accessed: 01- Jun- 2021].

[9]"Low Earth orbit - Wikipedia", *En.wikipedia.org*, 2021. [Online]. Available: https://en.wikipedia.org/wiki/Low_Earth_orbit. [Accessed: 01- Jun- 2021].