Hypothesis Testing pt. 2

More on Null Distributions and Test-statistics

Grinnell College

Review

Hypothesis Testing: formal technique for answering a question with two competing possibilities

Null Hypothesis: represents a skeptical view or a perspective of no difference

- H_0: 'parameter' $=$ (some value)

Alternate Hypothesis: what the researchers actually want to show with the study

- H_A: 'parameter' [$<$ / $>$ / $\neq$] (some value)
- choose the sign to match the research question

Review

This is the general outline we will follow for Hypothesis Testing

1. Define hypotheses
2. Simulate what the parameter looks like under H_A
3. See how our statistic compares to this
4. Compute a p-value
   - do the results look unlikely if H_A is true?
5. Interpretations / Conclusions

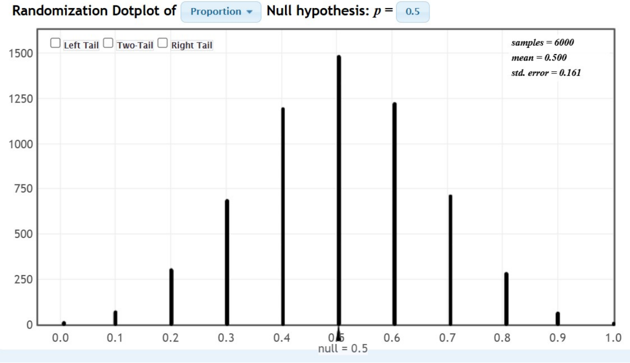
Note: Points 2 and 3 can be combined with the creation of a 'test-statistic'

Null Distribution

The distribution of the statistics if the null hypothesis is true

- simulates what the null hypothesis looks like
- use this to compute p-values

We looked at the coin-flip scenario.



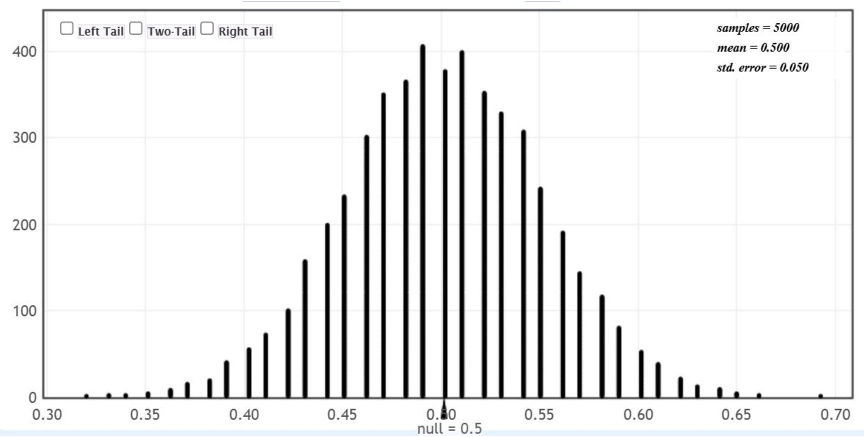
Null Distribution

What if we flipped 100 coins instead of 10

- Would getting $\hat{p} = .8$ be more common or less common?

Null Distribution

This represents 'H_0: p $=$ 0.5', when the sample size (# of flips) is 100



- What do we see?
- Are large or small values of $\hat{p}$ more/less common?

Null Distribution

The **null distribution** will look very similar to the sampling distribution stuff we saw before

- this time it is simulating the null hypothesis
- is **not** tell us what the population parameter looks like
- for means and proportions this looks like a Normal curve

These distributions looks Normal when certain conditions are met. Very similar to what we had when we were using confidence intervals.

Null Distribution – Proportion

Conditions for a proportion hypothesis test:

- Random Sample
  - this doesn't affect our null distr. but makes sure answers are accurate
- $np_0 > 10$
- $n(1 - p_0) > 10$

With the conditions met, the null distribution for $\widehat{p}$ looks like this:

$$\widehat{p} \sim N(p_0, \sqrt{\frac{p_0(1-p_0)}{n}})$$

Note: we have p_0's in the distribution because we are simulating what the null hypothesis looks like

Test Statistics

$$\widehat{p} \sim N(p_0, \sqrt{\frac{p_0(1-p_0)}{n}})$$

The test-statistic saves us from having to plot this Normal distribution every time. We will *standardize* $\hat{p}$ to make the distribution simpler.

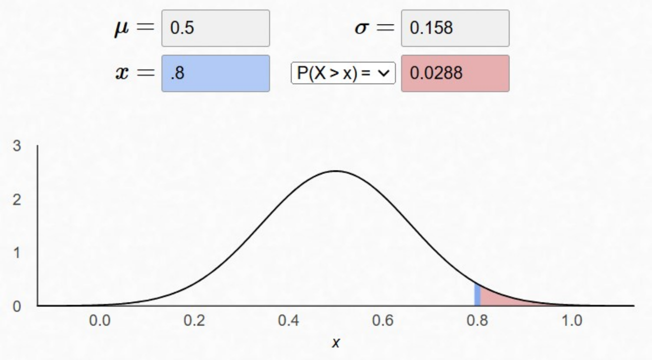
$$\underbrace{\frac{\widehat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}}_{\text{Test Statistic}} \sim N(0, 1)$$

- the whole term on the left is the Test-statistic
- we can compute this value and we know it follows N(0,1)

Coin Flip Example

For our scenario of 10 flips...

- $\hat{p} = 8/10 = 0.8$
- $p_0 = 0.5$
- $\sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(1-0.5)}{10}} = 0.158$

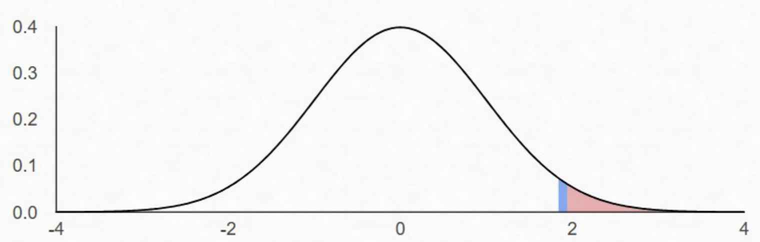


- can use Null distr. and $\hat{p}$ to get p-value

Coin Flip Example

For our scenario of 10 flips...

- $\hat{p} = 8/10 = 0.8, \; p_0 = 0.5$
- $Z = \frac{\widehat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.8-0.5}{\sqrt{\frac{0.5(1-0.5)}{10}}} = 1.897$



- can use N(0,1) and Test-statistic Z to get p-value

Test-statistics

From here on out, we will place almost all of our emphasis on Test-statistics instead of elaborately detailing what the null distribution should look like.