Odds and Risk

Grinnell College

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Review

Probabilities

- Unions (possibility of either event)
- Intersections (2 events at same time)
  - Disjoint (2 events *can't* happen at same time)
- Conditionals (one event has already happened)
  - Independence (do conditions add extra info?)

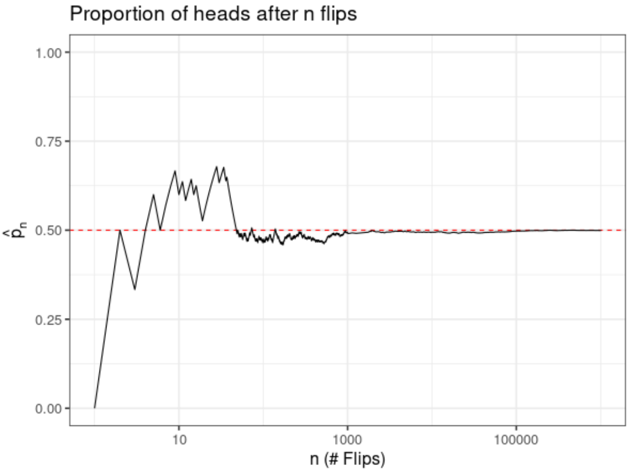
Lots of Probability math

Outline for Today

- Introduce odds (another likelihood comparison)
- Odds ratios
- Relative Risk

Law of Large Numbers

As our sample size increases, the empirical probability of something happening approaches the true probability (only holds when trials cannot influence each other)



Odds and Probability

When dealing with a *binary* event, we often speak in terms of **odds**, a *ratio* of "number of successes" to "number of failures"

$$\# \text{ success} : \# \text{ failure}$$

This is distinct from the idea of **probabilities**, which give a ratio of the "number of successes" to the number of possible outcomes

$$\begin{aligned} \# \text{ success} &: \# \text{ total outcomes} \\ &: \# \text{ success} + \# \text{ failure} \end{aligned}$$

Odds

Suppose we have a 6-sided die, and we are interested in rolls that land on either 1 or 2 (note how we have turned six distinct outcomes into two "events").

$$\text{Die} = \{1, 2, 3, 4, 5, 6\}$$

- The *probability* of rolling a 1 or 2 is $1/3$
  1. There are 6 possible outcomes
  2. There are 2 possible successes
  3. Probability is $2/6 = 1/3$
- The *odds* of rolling a 1 or 2 are 2:4 (or 1:2)
  1. There are 2 possible successes
  2. There are 4 possible failures
  3. The odds of success are 2:4 (or 1:2)

Odds Examples

The order of the event and non-event in this table matters for our calculations:

	Event	Non-Event
Exposure	A	B
No Exposure	C	D

- The odds of an event for the exposure group are A:B (or A/B)
- The odds of an event for the no exposure group are C:D (or C/D)

The **odds ratio** for these groups is then the ratio of their odds:

$$OR = \frac{A : B}{C : D} = \frac{A/B}{C/D} = \frac{A \times D}{B \times C}$$

Why Ratios?

Situation 1:

	Event	Non-Event
Exposure	6	2
No Exposure	3	2

Situation 2:

	Event	Non-Event
Exposure	103	2
No Exposure	100	2

1. Difference in odds for each situation?
2. Ratio of odds for each situation?

Event vs Non-Event

Which column is our "Event" changes how we report our results

Case 1:

	Survive	Death
Treatment	12	6
Placebo	5	10

Case 2:

	Death	Survive
Treatment	6	12
Placebo	10	5

Group Rows

The same is true for which group is in the first row

Case 1:

	Survive	Death
Treatment	12	6
Placebo	5	10

Case 2:

	Survive	Death
Placebo	5	10
Treatment	12	6

Odds Ratio Summary

- Odds and probabilities
- Column/row order matters
- Odds ratios
- $OR > 1$, $OR = 1$, $OR < 1$
  - $OR = 1$ implies no association. Why?

Example 1

A report published in 1988 summarizes results of a Harvard Medical School clinical trial determining effectiveness of asprin in preventing heart attacks in middle-aged male physicians

	Myocardial Infarction	
Treatment Status	Attack	No Attack
Placebo	189	10,845
Asprin	104	10,933

- Odds of having a heart attack for placebo:
- Odds ratio for treatment and infarction:
- Associated?

Example 2

The table below shows the results for drivers and passengers in auto accidents in Florida in 2008, according to whether or not the individual was wearing a seat belt.

Sealt-Belt Use	Injury	
	Fatal	Nonfatal
No	1085	55,623
Yes	703	441,239

- *Probability* of wearing seatbelt conditional on fatality status:
- *Odds* of fatality conditional on seat-belt use:
- Associated?

Relative Risk

Just like looking at odds ratios, we can look at probability ratios. These are often called **relative risk**.

- again, the order of events matters

Sealt-Belt Use	Injury	
	Fatal	Nonfatal
No	1085	55,623
Yes	703	441,239

relative risk of fatality for no-seat-belt use:

$$\frac{\textbf{P(Fatality if Seat-Belt Use = No)}}{\textbf{P(Fatality if Seat-Belt Use = Yes)}} = \frac{1085/(1085+55623)}{703/(703+441239)} = 12.02$$

- Prob. of Fatality is roughly 12 times *higher* for the no-seat-belt group

relative risk of fatality for seat-belt use:

$$\frac{\textbf{P(Fatality if Seat-Belt Use = Yes)}}{\textbf{P(Fatality if Seat-Belt Use = No)}} = \frac{703/(703+441239)}{1085/(1085+55623)} = .083$$

- Prob. of Fatality is .083 times *less* for the seat-belt group