Confidence Intervals

What can we do with Standard Error?

Grinnell College

Review

Normal distribution

- unimodal + symmetric bell-curve
- probabilities

Central Limit Theorem:

1. If variable X has mean μ and std.dev. σ, and
2. If the number of observations in the sample (n) is large
3. then the sampling distribution for $\overline{X}$ (sample mean) is Normal with mean μ and standard error $\sigma/\sqrt{n}$.

$$\overline{X} \sim \mathrm{N}(\mu, \sigma^2/n)$$

Review – Sources of Variation

Pop. Standard Deviation: Description of the variability in our *population*. It is often denoted σ

Sample Standard Deviation: Description of the variability in our *observations* (sample). It is often denoted **s**

Standard Error: Description of variability in our *estimates* of a parameter (such as the mean). We will denote standard error as SE, with $SE = \sigma/\sqrt{n}$, where n is the number of observations in our sample

Outline

We saw that the statistic is not going to be exactly equal to the parameter

- sampling bias
- sampling variability

So... we can't just provide a single value for our estimate of the parameter

- We also need to quantify how far away our guess is

This is why we came up with the *standard error (SE)*, now we need to figure out how to use it.

Goal: We are going to spend today learning how to estimate population means

Example – COVID Vaccines

According to the U.S. Census Bureau, as of October 11, 2021:

"83.3% (+/- 0.5%) of U.S. adults 18 years and older have received at least one dose of a COVID-19 vaccine." This is based on a representative sample of civilians aged 18 and over. "Margins of error shown at 90% confidence."

What does margin of error mean?

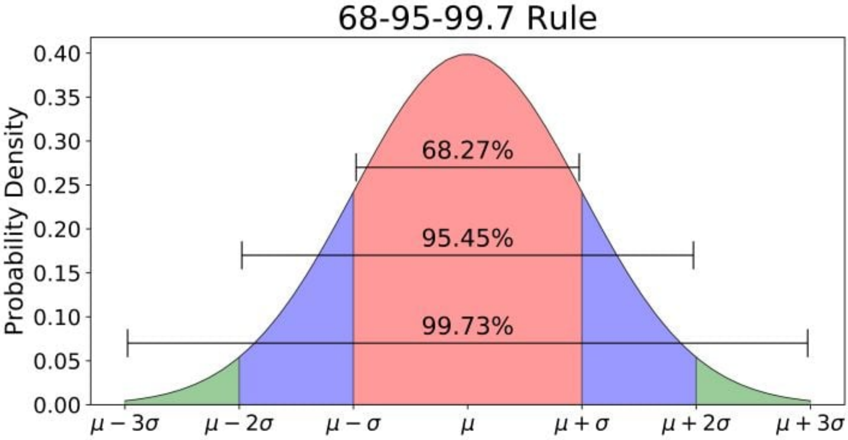
Intervals

For the rest of these slides, our goal is to determine the mean of a *population*.

We cannot rely on only our **point estimate** $\overline{X}$, but perhaps we can find a range of reasonable values that looks like:

$$\text{Point Estimate} \pm \text{Margin of Error}$$

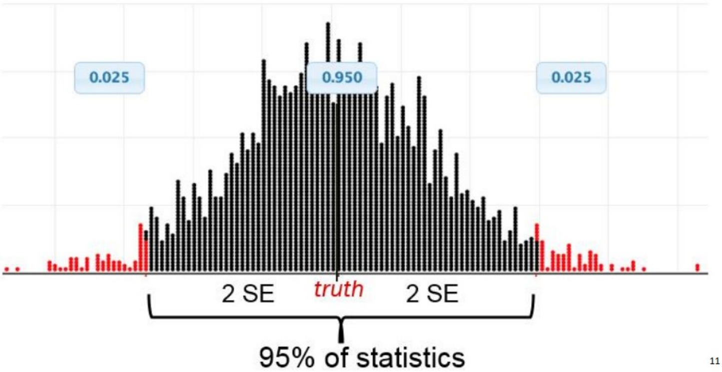
A good place to start:



Good place to start:

The sampling distribution for the sample mean looks N(μ, σ^2/n)

- Central Limit Theorem

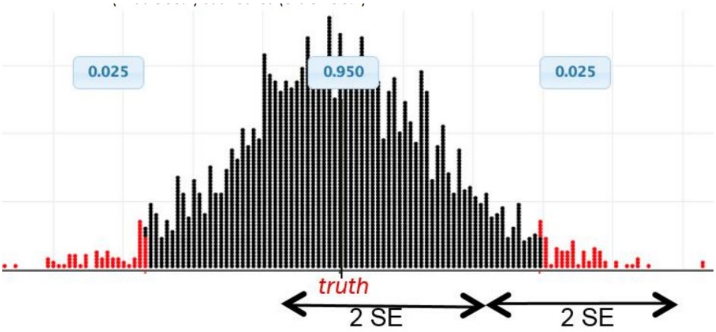


In the sampling distribution, 95% of statistics will be within 2$\times$SE of the pop. mean μ

Good place to start:

The sampling distribution for the sample mean looks N(μ, σ^2/n)

- Central Limit Theorem



Equivalently: The interval 'statistic $\pm$ 2$\times$SE' will contain μ for 95% of the statistics

Confidence Interval

We are going to call this interval a "95% Confidence Interval":

- 95% comes from the fact that 95% of statistics are within 2SE's of the mean
- Confidence refers to the fact that this is a range of plausible values for the parameter

Formula for a 95% confidence interval for estimating a pop. mean (μ) is:

$$\overline{x} \pm 2 \times \mathrm{SE}$$

Confidence Interval

Formula for a 95% confidence interval for estimating a pop. mean (μ) is:

$$\overline{x} \pm 2 \times \text{SE}$$

Margin of Error tells us how wide our interval is.

- ME = half the width (or length) of the interval
- for 95% CI $\rightarrow$ ME $= 2 \times$ SE $= 2\frac{\sigma}{\sqrt{n}}$

This makes our (final) formula for a 95% confidence interval for estimating a pop. mean (μ):

$$\overline{x} \pm 2 \times \frac{\sigma}{\sqrt{n}}$$

CI Interpretation

Remember our goal: we are trying to estimate the *population mean*

Confidence Interval Interpretation

- mention confidence level
- specify the values we got
- use context for the population mean when able

"We are 95% confident that (the population mean) is between (lower value) and (upper value)."

Example – Movie Budgets

Hollywood movie budget data: $\mu = 51.38$, $\sigma = 57.93$

From a sample of 50 movies we find $\overline{x} = 51.01$.

Construct a 95% Confidence Interval for the pop. mean movie budget.

Example – Movie Budgets

Hollywood movie budget data: $\mu = 51.38$, $\sigma = 57.93$

From a sample of 50 movies we find $\overline{x} = 45.65$.

Construct a 95% Confidence Interval for the pop. mean movie budget.

- When we know σ we can use the formula directly

$$\overline{x} \pm 2\frac{\sigma}{\sqrt{n}} \;\rightarrow\; 45.65 \pm 2 \times \frac{57.93}{\sqrt{50}} \;\rightarrow\; 45.65 \pm 16.39$$

$$(29.26,\ 62.04)$$

Example – Movie Budgets

The 95% CI for the population mean is (29.26, 62.04).

Interpretation:
We are 95% confident that (the population mean) is between (lower value) and (upper value).

Example – Movie Budgets

The 95% CI for the population mean is (29.26, 62.04).

Interpretation:
We are 95% confident that the population mean movie budget is between 29.26 and 62.04 million dollars.

Example – Fish Mercury Levels

Data was collected from 53 lakes in Florida. For each lake, the mercury level (parts per million) was computed for a large mouth bass.

Based on this sample of fish, the mean mercury level was 0.527 ppm with a standard deviation of .118

Construct a 95% CI for pop. mean:

- Issue: we don't know σ

Example – Fish Mercury Levels

Data was collected from 53 lakes in Florida. For each lake, the mercury level (parts per million) was computed for a large mouth bass.

Based on this sample of fish, the mean mercury level was 0.527 ppm with a standard deviation of 0.118

Construct a 95% CI for pop. mean:

$$\overline{x} \pm 2\frac{\sigma}{\sqrt{n}} \;\rightarrow\; \overline{x} \pm 2 \times \frac{s}{\sqrt{n}} \;\rightarrow\; 0.527 \pm 2 \times \frac{0.118}{\sqrt{53}}$$

$$(0.495, 0.560)$$

We are 95% confident that the true pop. mean mercury level of fish in Florida lakes is between 0.495ppm and 0.560ppm

More on "Confidence"

We are 95% confident that...

Let's really dig into what this 'confidence' part means

More on "Confidence"

A **confidence interval** is an interval that has the following properties:

- It is constructed according to a procedure or set of rules
- It is made with the intention of giving a plausible range of values for a *parameter* based on a *statistic*
- There is no probability associated with a confidence interval; *it is either correct or it is incorrect*

More on "Confidence"

Consider the confidence interval that we constructed in the movie budget example.

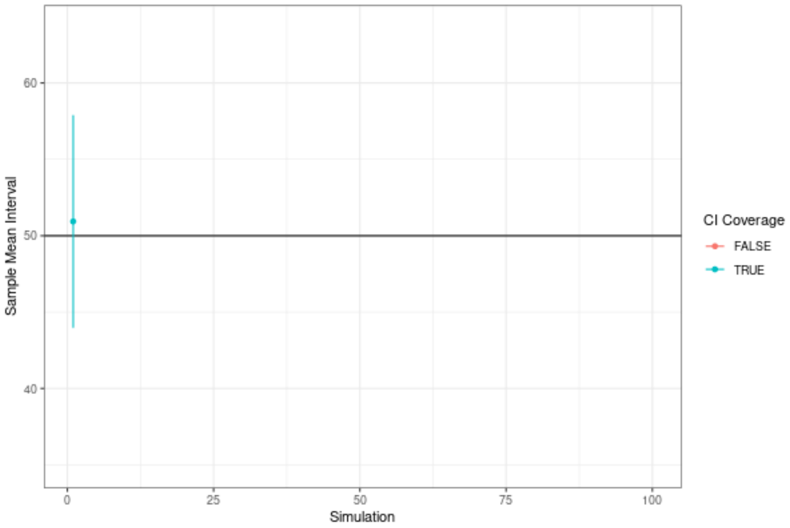
- It was constructed according to the procedure
  Point estimate $\pm$ Margin of Error
- It was made to present a reasonable range of values for the *parameter* μ as estimated by the *statistic* $\overline{X}$
- The interval was $(29.26, 62.04)$. As our true mean is $\mu = 51.38$, this interval *is* correct in the sense that it *contains* our true parameter

More on "Confidence"

When we say something has a 95% confidence interval, what we mean is:

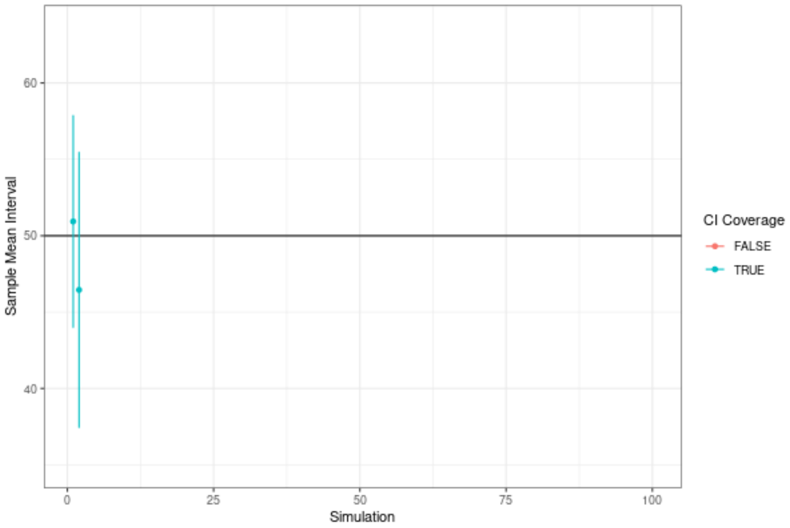
The process that constructed this interval has the property that, on average, it contains the true value of the parameter 95 times out of 100

Coverage



Sample Mean Interval
60
50
40
0
25
50
75
100
Simulation
CI Coverage
FALSE
TRUE

Coverage



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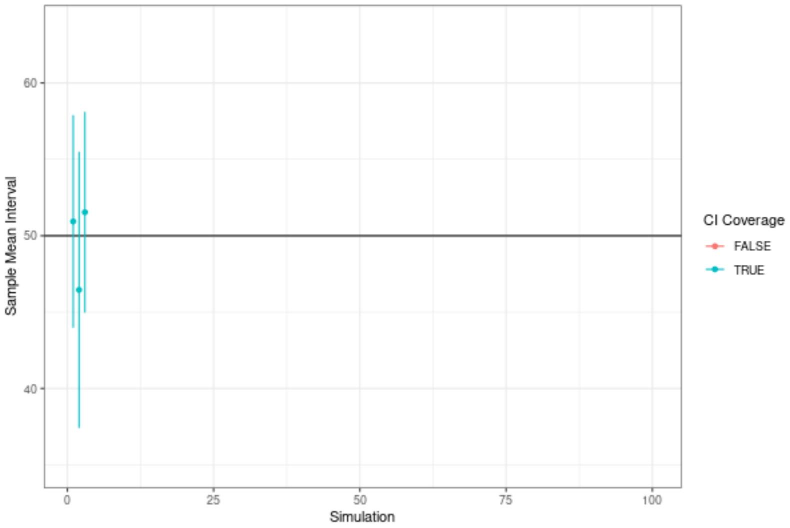
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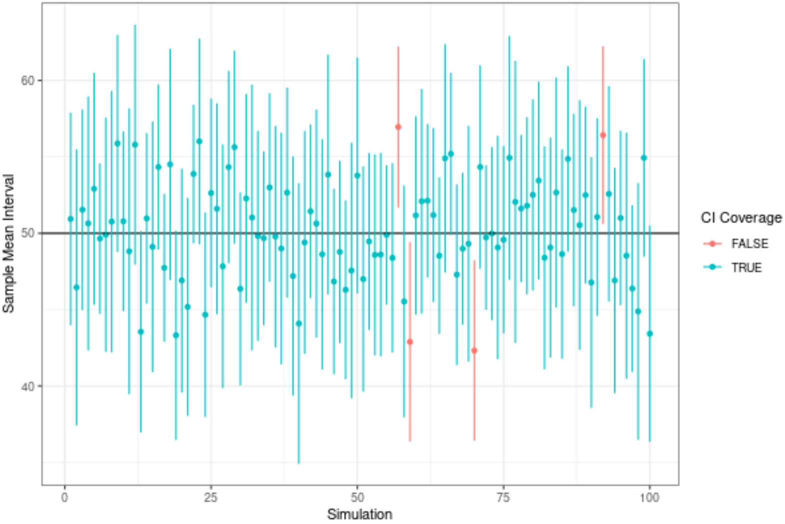

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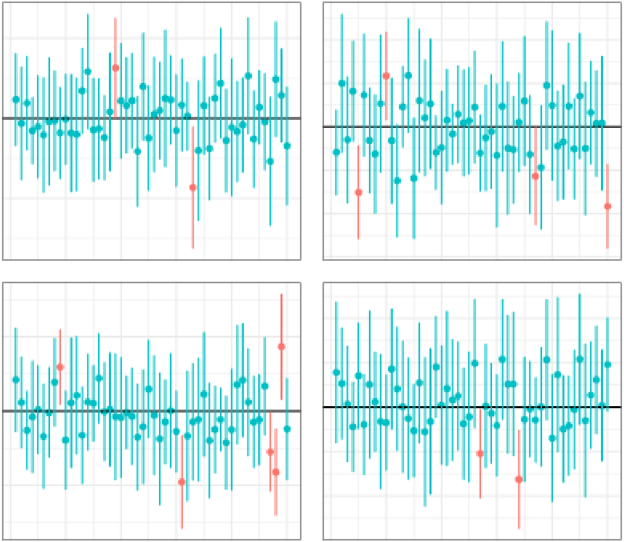


Confidence Intervals

To be absolutely clear: we will **never** know if the confidence interval we construct contains the true value of the parameter

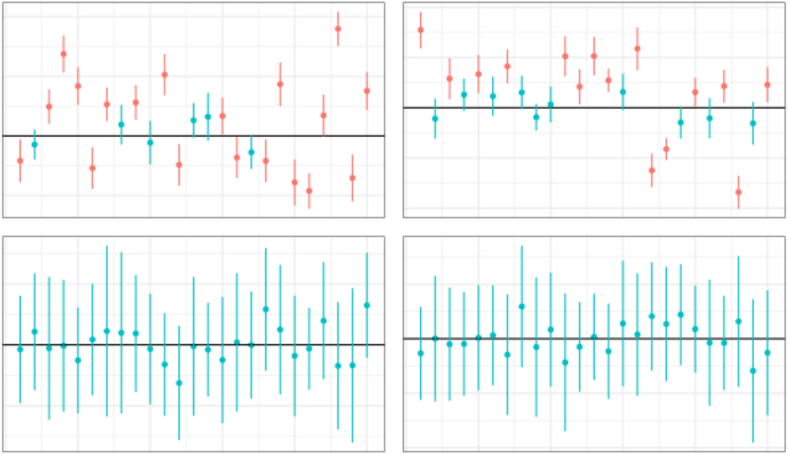
This is kind of like throwing a dart but never seeing the target

This is the nature of statistical inference

- we can describe properties of the *process* that created our intervals
- we can never conclusively speak about the interval itself

Confidence Intervals

It is also worth observing that we can *alter* our process to achieve different results. There is a tradeoff between how frequently we are correct and how much uncertainty we allow in our prediction



Common Misinterpretations

These are some common ways people interpret CI's that are absolutely **not** correct:

- A 95% confidence interval contains 95% of the data in the population.
- "I am 95% sure that the mean of the sample will fall within a 95% confidence interval for the mean."
- "95% of all sample means will fall within this 95% confidence interval."
- "The probability that the population parameter is in this particular 95% confidence interval is 0.95."

Generalizations

Confidence Intervals quantify sampling variability

- Range of plausible values for the statistic
- Range is determined by how much the statistics vary from sample to sample

Confidence Intervals **DO NOT** account for bias in the samples

- We will <u>never</u> be able to quantify the bias in our samples
- It is important to mention possible sources of bias in our final conclusions