Deriving Properties of Binary Black Hole from UV + Optical Spectrum with MCMC Simulations

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Finding BBH candidates : *Pros & Cons*

Variability *vs* Spectrum

Pros:
1. Simple and popular
2. Exist CRT data

Cons:
1. Time-consuming and painstaking
2. Fraudulent results from other drives (eg: RDW) with intended fitting
2. Some BBHs may not have variabilities

Pros:
1. Novel and intriguing
2. Less elusive and time-saving
3. Avoid complex discussion on the origin of variability

Cons:
1. Not ubiquitous features
2. Probably need further in-spection to break degene-racies

—— Spectrum is valuable to investigate in depth.

What I want to do…

➢ Conduct an MCMC Fitting like Yan (2015);

What I need…

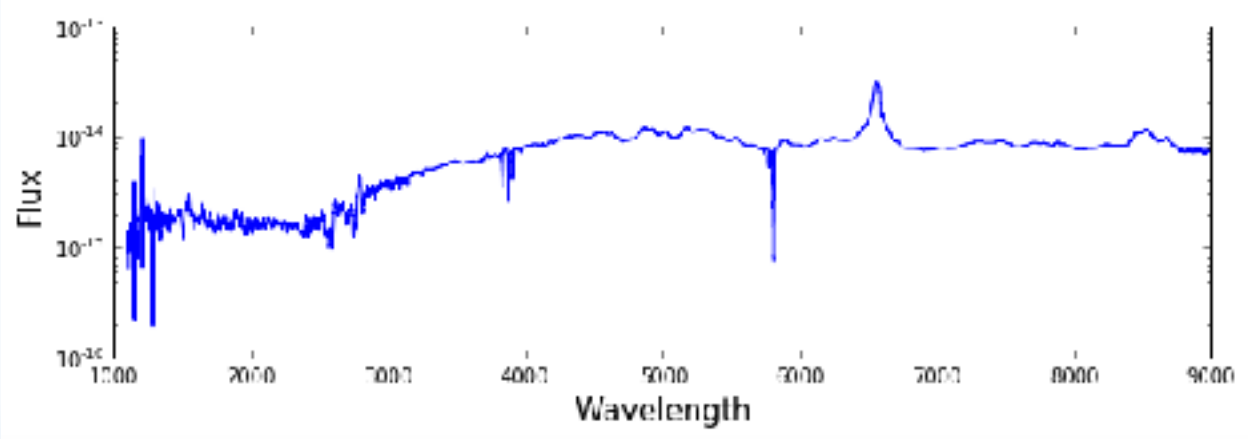
- Background on BBHs;
- Knowledge about Stochastic Process;
- Basic Bayesian Inference;
- Techniques about fitting process；

The first week

- Methods of model construction;
- Practical operative programming skills;

The second week

➢ Access to Mrk231 spectrum data (kindly provided by Mrs.Yan)



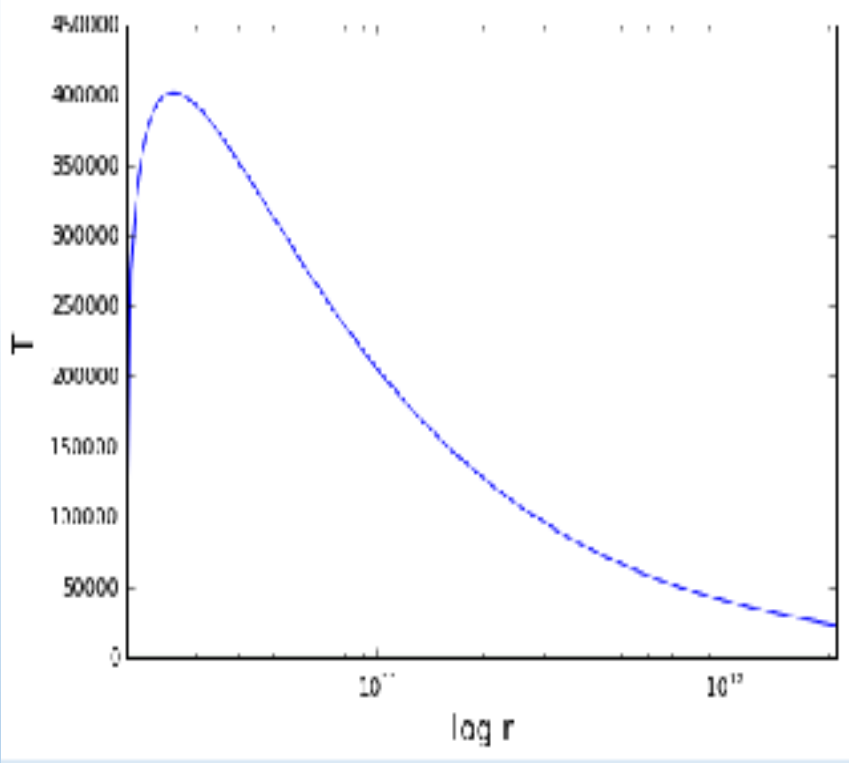
Flat in Optical and FUV with a clear drop off between 2500-4000 Å

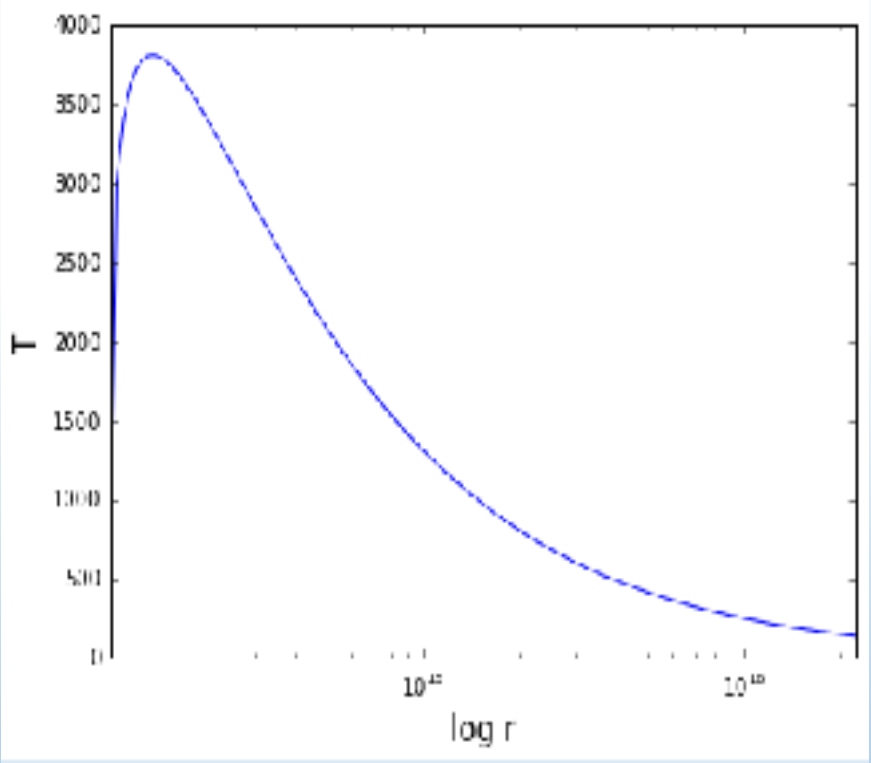
➢ Construct disk SED model

$$T_{\rm eff}(r) = \left[\frac{3GM_\bullet \dot{M}_{\rm acc}}{8\pi\sigma_{\rm B} r^3}\left(1-\sqrt{\frac{r_{\rm in}}{r}}\right)\right]^{1/4}, \quad F_\lambda = \int_{r_{\rm in}}^{r_{\rm out}} \frac{2hc^2\cos i/\lambda^5}{\exp\left[hc/\lambda k_{\rm B} T_{\rm eff}(r)\right]-1} dr,$$

As things does not change a lot in the case of circumbinary disk model, plus a rather complex issue about viscosity, I take Mrs. Yan's suggestion to use double standard thin disk model to construct the SED.

- Effective temperature distribution of disks

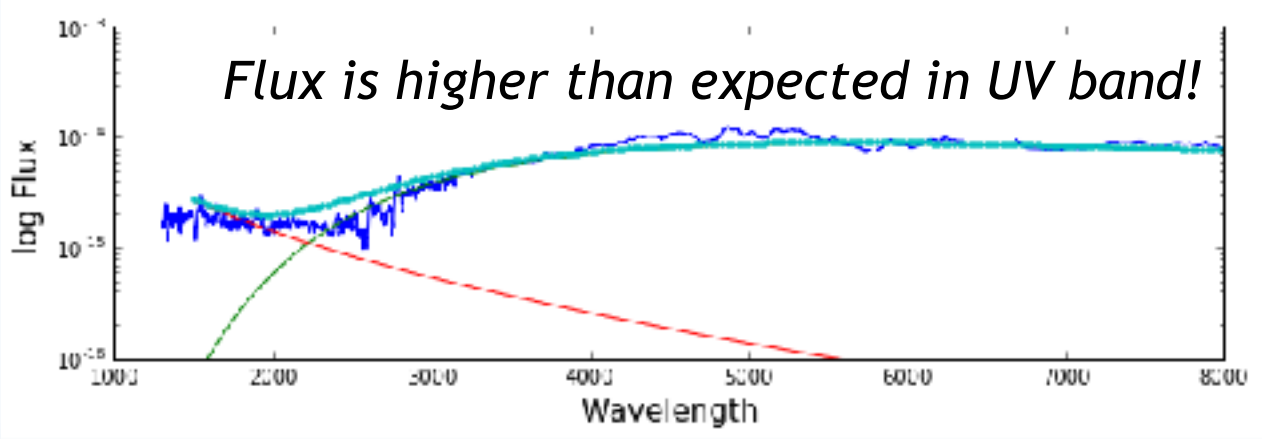




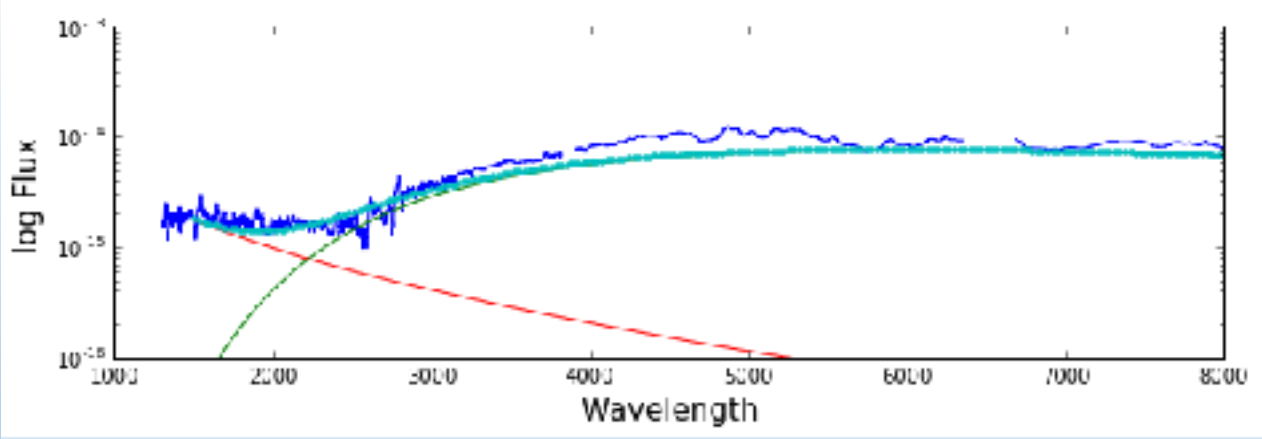
↑ Outer Circumbinary-disk

↑ Inner Mini-disk

- Problems during model construction



(Before extinction)



(After extinction)

Conclusion: Extinction matters a lot.

Here I use Calzetti(2000) and mask prominent absorption & emission lines.

➢ MCMC Fitting (with Python)

(† All Python codes are available on Github: github.com/NGC4676/LIU/tree/Python)

- **M-H algorithm**

Algorithm 6 Metropolis-Hastings 采样算法

1: 初始化马氏链初始状态 $X_0 = x_0$

2: 对 $t = 0, 1, 2, \cdots$, 循环以下过程进行采样

- 第 t 个时刻马氏链状态为 $X_t = x_t$, 采样 $y \sim q(x|x_t)$
- 从均匀分布采样 $u \sim Uniform[0, 1]$
- 如果 $u < \alpha(x_t, y) = \min\left\{\frac{p(y)q(x_t|y)}{p(x_t)p(y|x_t)}, 1\right\}$ 则接受转移 $x_t \to y$, 即 $X_{t+1} - y$
- 否则不接受转移，即 $X_{t+1} = x_t$

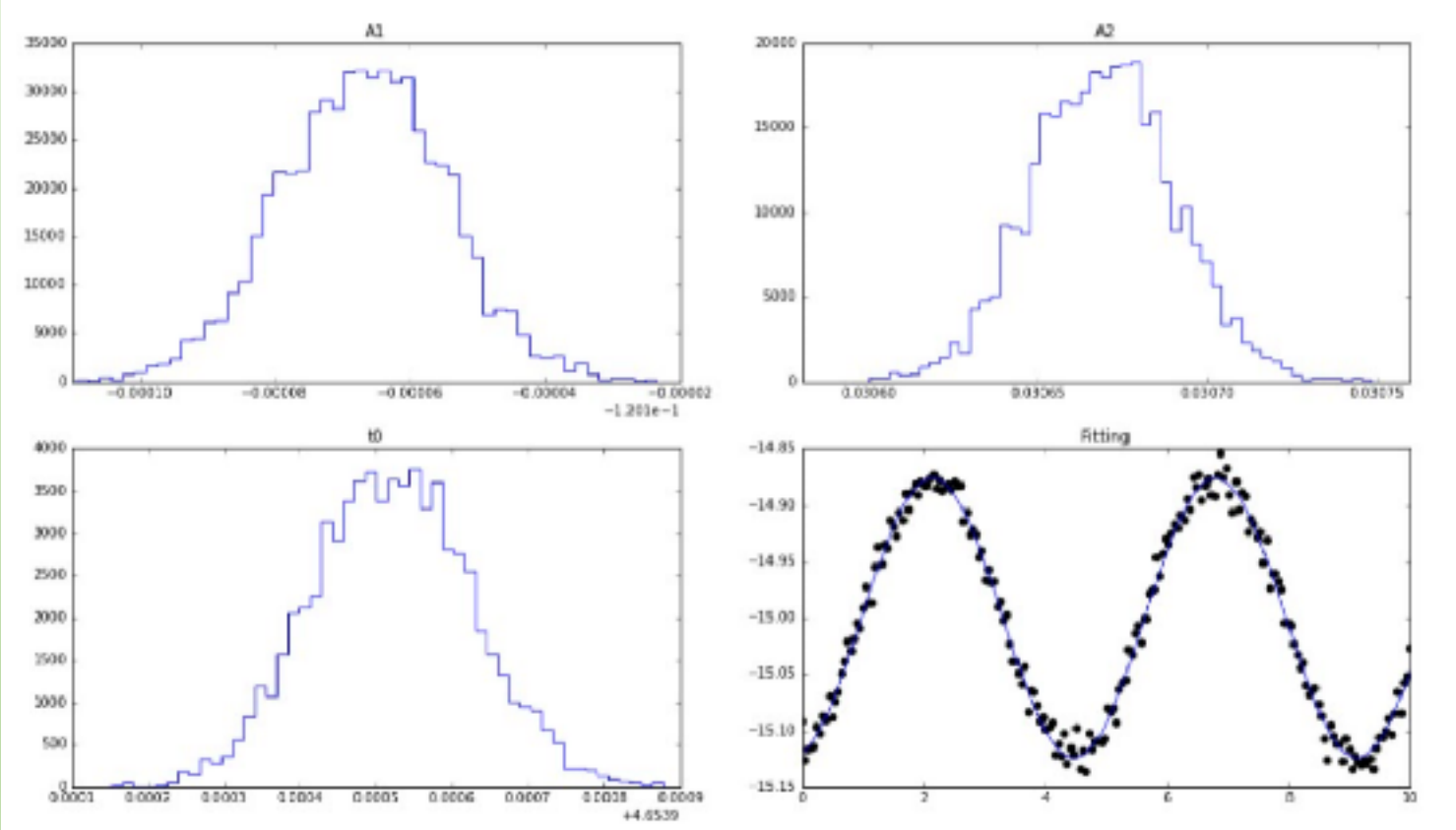
Avoid EXHAUSTIVE Grid method!

—— especially efficient with many parameters

- **A petit try on a simple case**

3 Parameters (A1, A2, t0)

50000 points； Burn-in = 10000； Thinned by 10



Succeed to fit the curve correctly!

- **Practical Fitting on Mrk-231 Spectrum**

8 Parameters with **uniform Priors**

† Total Mass ($M_{\cdot}$);

† Mass Ratio (q);

† Semi-major Axis (a BBH);

```
model   dict(a_UU1   a_UU1, q   q, M_t   M_t, M_s M_s,
             r_in = r_in, r_out = r_out,
             r_s_out = r_s_out, r_s_in = r_s_in,
             f_s_Edd = f_s_Edd, f_c_Edd = f_c_Edd,
             M_r_arr = M_r_arr, M_s_arr = M_s_arr,
             f_lamda = f_lamda, r_lamda = r_lamda,
             f_RS=f_RS, E_BV = E_BV, Extinction = Extinction,
             A = A, S = S, T = T)
```

† The Eddington Ratio of the inner mini-disk (f Edd,c) ;

† The Eddington Ratios of the outer circumbinary disk (f Edd,s) ;

† Ratio of the outer boundary of the inner disk to the mean Roche Radius (f r,s);

† Scale Factor (A);

† Extinction E(B–V);

Easy to Check

Easy to Track

Bottom-Top —— *Add parameters one by one*

- **Problems during MCMC Fitting**

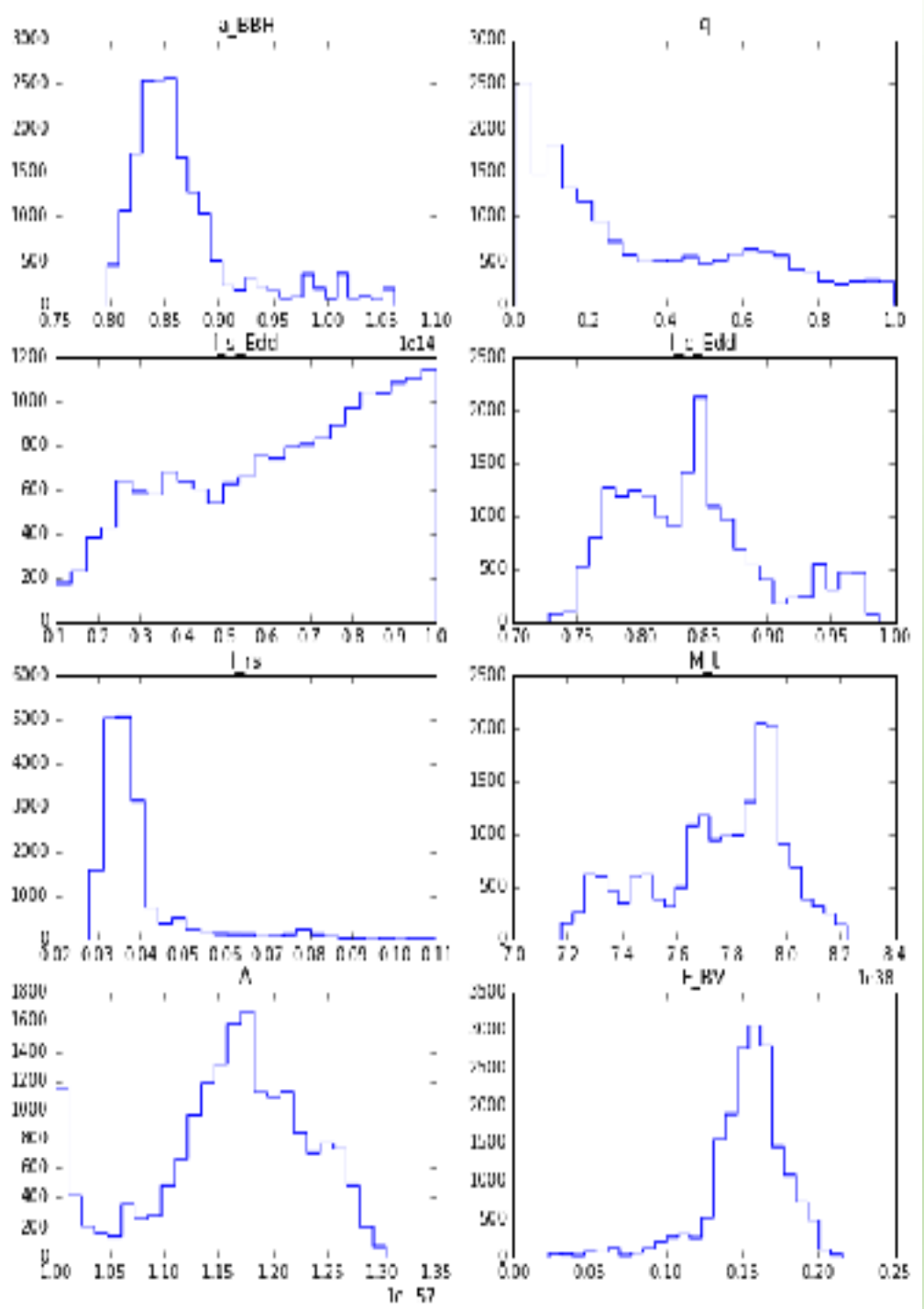
➔ Quad method cannot pass a whole list to the integration.

(temperature distribution along radius)

➔ Simpson method lacks accuracy and is sensitive to steps.

✳ **My Solution:**

Use Simpson method to fit the spectrum, and Quad method to plot the result, sacrificing a little accuracy to suffice the speed.



Run:

100000 points

Burn-in = 10000

Thinned by 10

5405.7 seconds

Output:

Marginalized probability distribution function (pdf , left panel) for 8 Params

➢ Limitations & Issues Deserve Attention

—— Do fitting parameters surely be true parame

1 Fitting results are sensitive to several non-physical factors:

- p Pick of Priors
- p Precision of fitting
- p Times of iteration
- p Resolution of the input

2 What if I found a multi-modal posterior?

3 How do I know that MCMC didn't get stuck in a local minimum?

4 How do I know that a chain has converged?

Always Be Careful When Using MCMC !

➢ Potential Improvements

- **A diagnosis for chain convergence**
  - ➔ Save running time
- **Criteria to tell whether one fitting is better than another**
  - ➔ Give more convincing result
- **Adapt the fitting for other models**
  - ➔ Find more details about the source
- **See if the method can be applied in other facets**
  - ➔ Look for a wider prospect

➢ 一些想法和感悟

Given an unfamiliar topic , how to establish the framework in a short time?

新接触一个领域、一项课题，怎么在最短的时间内迅速建立起一个框架，找到自己能做的事情，想做的事情，BrainStorm各种idea (这一过程是极为愉悦的)，构造自己的文献网络，寻找需要的数据，然后有导向地获取需要的知识（各种document）

—— 这是我此行最大的收获。

平时往往没有足够的时间花一整天功夫去想、去写、去改、去挖掘 —— 重要的是静下心。

遇到困难不要沮丧，科研问题总是举步维艰的 —— 坚持就有突破。

不要先想着依赖他人，你在一个问题上绊脚并爬起，将来你就是这个问题的专家。

At Last , Have Fun with Astronomy and Scientific Research !

Finally, appreciating for Prof. Lu ’s kindly guidance and Mrs. Yan ’s generous help !

(I enjoy those discussions with you very much !)