Note for Medlyn stomatal conductance model in FATES

1. Introduction

Previous versions of FATES calculated leaf stomatal resistance is using the Ball-Berry conductance model as described by (Collatz et al., 1991). We provide an alternative way to calculating stomatal conductance, Medlyn stomatal conductance model (Medlyn et al., 2011). The Medlyn model calculates stomatal conductance (i.e., the inverse of resistance) based on net leaf photosynthesis, the vapor pressure deficit, and the CO_2 concentration at the leaf surface. Leaf stomatal resistance is calculated as Eq. (1) with the information of symbols listed in Table 1.

$$\frac{1}{r_s} = g_s = g_0 \times \beta_{sw} + 1.6(1 + \frac{g_1}{\sqrt{D_s}})\frac{A_n}{C_s/P_{atm}} \quad (1)$$

Table 1. Information of symbols in Eq. (1)

Symbol	Standard name	Unit
r_s	Leaf stomatal resistance	s m^2 μmol^{-1}
g_s	Leaf stomatal conductance	μmol m^{-2} s^{-1}
g_0	Minimum stomatal conductance	μmol m^{-2} s^{-1}
β_{sw}	soil water stress	unitless
D_s	Vapor pressure deficit at the leaf surface	kPa
g_1	Slope for the relationship	$kPa^{0.5}$
A_n	Leaf net photosynthesis	μmol CO_2 m^{-2} s^{-1}
C_s	CO_2 partial pressure at the leaf surface	Pa
P_{atm}	Atmospheric pressure	Pa

The value for g_0 = 1000 μmol m^{-2} s^{-1} for all PFTs.

g_1 is a plant functional type (PFT) dependent parameter. According to the g_1 values and PFTs in CLM5, g_1 values for different PFTs for FATES model were listed in Table 2.

Table 2. Stomatal conductance slope parameters in Medlyn model

PFT	g_1 (kPa0.5)
Broadleaf evergreen tropical tree	4.1
Needleleaf evergreen extratrop tree	2.3
Needleleaf colddecid extratrop tree	2.3
Broadleaf evergreen extratrop tree	4.1
Broadleaf hydrodecid tropical tree	4.4
Broadleaf colddecid extratrop tree	4.4
Broadleaf evergreen extratrop shrub	4.7
Broadleaf hydrodecid extratrop shrub	4.7
Broadleaf colddecid extratrop shrub	4.7
Arctic C_3 grass	2.2
Cool C_3 grass	5.3
C_4 grass	1.6

2. Numerical implementation

Photosynthesis is calculated assuming there is negligible capacity to store CO_2 and water vapor at the leaf surface so that：

$$A_n = \frac{C_a - C_i}{(1.4r_b + 1.6r_s)P_{atm}} = \frac{C_a - C_s}{1.4r_b P_{atm}} = \frac{C_s - C_i}{1.6r_s P_{atm}} \quad (2)$$

The information of the symbols was listed in Table 3. The terms 1.4 and 1.6 are the ratios of diffusivity of CO_2 to H_2O for the leaf boundary layer resistance and stomatal resistance. The transpiration fluxes are related as Eq. (3) with the information of symbols listed in Table 3.

$$\frac{e_a - e_i}{r_b + r_s} = \frac{e_a - e_s}{r_b} = \frac{e_s - e_i}{r_s} \quad (3)$$

$$e_a = \frac{P_{atm} q_s}{0.622} \quad (4)$$

Table 3. Information of symbols in Eq. (2), (3), and (4)

Symbol	Standard name	Unit
C_a	Atmospheric CO_2 pressure	Pa
C_i	Internal leaf CO_2 partial pressure	Pa
r_b	Leaf boundary layer resistance	s m^2 μmol^{-1}
e_a	Vapor pressure of air	Pa
e_i	Saturation vapor pressure	Pa
e_s	Vapor pressure at the leaf surface	Pa
q_s	Specific humidity of canopy air	kg kg^{-1}

In the model, an initial guess of C_i is obtained assuming the ratio between C_i and C_a (0.7 for C_3 plants and 0.4 for C_4 plants) to calculate A_n based on the Farquhar photosynthesis model (Farquhar et al., 1980). Then Eq. (2) is solved for C_s:

$$C_s = C_a - 1.4 r_b P_{atm} A_n \quad (5)$$

e_s can be represented from Eq. (3) as:

$$e_s = \frac{e_a r_s + e_i r_b}{r_b + r_s} \quad (6)$$

Where e_i is a function of temperature.

Substitution of e_s into Eq. (1) (according to $D_s = e_i - e_s$) gives an expression for stomatal resistance (r_s) as a function of photosynthesis (A_n), given here in terms of conductance with $g_s = 1/r_s$ and $g_b = 1/r_b$

$$g_s^2 + b g_s + \mathrm{c} = 0 \quad (7)$$

Where

$$b = -[2(g_0 \times \beta_{sw} + \mathrm{d}) + \frac{(g_1 d)^2}{g_b D_a}] \quad (8)$$

$$c = (g_0 \times \beta_{sw})^2 + \left[2 g_0 \times \beta_{sw} + d\left(1 - \frac{{g_1}^2}{D_a}\right)\right] d \quad (9)$$

and

$$d = \frac{1.6 A_n}{C_s / P_{atm}} \quad (10)$$

$$D_a = \frac{e_i - e_a}{1000} \quad (11)$$

Stomatal conductance, as solved by Eq. (7), is the larger of the two roots that satisfies the quadratic equation. Values for C_i are given by

$$C_i = C_a - (1.4 r_b + 1.6 r_s) P_{atm} A_n \quad (12)$$

The equations for C_i , C_s , r_s , and A_n are solved iteratively until C_i converges. Iteration will be exited if convergence criteria is met or if at least five iterations are completed.

References

Collatz, G.J., Ball, J.T., Grivet, C., Berry, J.A., 1991. Physiological and environmental regulation of stomatal conductance, photosynthesis and transpiration: a model that includes a laminar boundary layer. Agricultural and Forest Meteorology 54: 107-136.

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