Neuroscience Graduate Program
2020 bootcamp

Sep. 21-22:
Modeling **neurons** and **networks**

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Outline: Exercises

Day 1 (Sep. 21st):

1a. The Hodgkin Huxley neuron

Voltage response to pulse input
Voltage response to DC input (f-I curve)

1b. From four to two dimensions

What are the three activation/inactivation HH variables (m, n, h) doing?
f-I curve upon replacement of m, n, h with one variable (FitzHugh-Nagumo model)

1c. From two to one dimension

f-I curve of a leaky integrate and fire neuron

Day 2 (Sep. 22nd):

2a. Response of a noise driven neuron

Inter-spike-interval (ISI) distribution of an integrate and fire neuron driven by noise

2b. A synapse is not a number

Effects of synapses with short-term depression/facilitation

Exercises

Exercise 1a. The Hodgkin Huxley neuron

- Simulate a HH neuron subject to a single pulse current injection.
  - Plot the voltage as a function of time for different current amplitudes.
  - What is the spike threshold?
  - Bonus: Does the spike threshold depend on the duration of the pulse?

- Simulate a HH neuron subject to a direct (constant) current injection.
  - Characterize the voltage response as function of the current amplitude.
  - Plot the f-I curve.

Useful resources:

Openworm HH tutorial

The openworm tutorial will guide you through writing code that implements the HH neuron.
You can follow that tutorial and take away the relevant parts, or write your own code.

Reading:

- Sec. 1 Chap. 4 of Neuronal Dynamics, Gerstner, et al.
- Brunel, N. (2010). Modeling point neurons: from Hodgkin-Huxley to integrate-and-fire. Computational Modeling Methods for Neuroscientists, 161-185.

Exercises

Exercise 1b. From four to two dimensions

- For the HH neuron in Exercise 1a,
  - Pick a current amplitude above the spiking threshold.
  - For each pair of variables V, m, n, h (6 pairs), plot one variable as a function of the other.
  - Identify pairs that are most strongly correlated.
    How can this identification help in reducing the neuron model's dimension?
  - Does the choice of current amplitude affect which pairs of variables are most correlated?

- Simulate a FitzHugh-Nagumo neuron model.
  - Plot the f-I curve.
  - Is it similar to what you got for the HH neuron?
  - Are there ways in which the two models are different?
    - Try to think of a scenario where it would be important to simulate the more complicated neuron model.

Useful resources:

Openworm HH tutorial. Note that the openwork HH tutorial already produces the four state variables. You can use the output of that code to produce the plots relevant to this exercise.
For the FitzHugh-Nagumo model, see hint in next page

Useful resources: Hint for simulating FitzHugh-Nagumo model
You have already simulated the Hodgkin-Huxley model: four coupled differential equations

$$C\frac{\mathrm{d}V}{\mathrm{d}t} = -\bar{g}_{\mathrm{K}} n^4 (V - V_{\mathrm{K}}) - \bar{g}_{\mathrm{Na}} m^3 h (V - V_{\mathrm{Na}}) - \bar{g}_{\mathrm{l}} (V - V_{\mathrm{l}}) + I$$
$$\frac{\mathrm{d}n}{\mathrm{d}t} = \alpha_n(V)(1-n) + \beta_n(V)n$$
$$\frac{\mathrm{d}m}{\mathrm{d}t} = \alpha_m(V)(1-m) + \beta_m(V)m$$
$$\frac{\mathrm{d}h}{\mathrm{d}t} = \alpha_h(V)(1-h) + \beta_h(V)h$$

The right hand side of each equation is the time derivative of the corresponding state variable. The python code computes this time derivative and puts it into a ODE solver.

To simulate the FitzHugh-Nagumo model, notice that the system of equations can be written in the same form. Here the system of equations is simpler: two ordinary differential equations.
You can adapt your existing code to simulate the FitzHugh-Nagumo neuron.

$$\frac{\mathrm{d}V}{\mathrm{d}t} = V - \frac{1}{3}V^3 - W + I$$
$$\frac{\mathrm{d}W}{\mathrm{d}t} = 0.08(V + 0.7 - 0.8W)$$

Exercises

Exercise 1c. From two to one dimension

- Simulate a leaky integrate and fire neuron, subject to direct current injection.
    - Plot the f-I curve.
        - What aspects of the f-I curves of Exercises 1a, 1b does it capture?
        - What aspects does it not capture?
        - Try to think of a scenario where it would be important to simulate the more complicated neuron model.

Useful resources:

[NMA, week 3, day 1, tutorial 1]
Section 1: implementation
Section 2: inputs
Section 3: f-I curve

Exercises

Exercise 2a. Response of a noise driven neuron

- Simulate a LIF neuron subject a noisy input current (using uncorrelated Gaussian noise).
  - Plot the f-I curve: *average*† spike frequency as a function of *average* input.
    Do this for a few different values of the standard deviation of the input noise.

  - Look for input mean + noise combinations that give the same frequency.

  - For these two pairs of mean and variance values, plot the inter-spike-interval distribution.
    Comment on how these distributions can be used to characterize the input statistics.

† Note that unlike yesterday, the spike count per unit time is variable. Averaging may require longer simulations.

Useful resources:
[NMA, week 3, day 1, tutorial 1]
Section 1: implementation
Section 2: inputs
Section 3: f-I curve

Exercises

Exercise 2b. A synapse is not a number (p. 1 of 2)

- Generate a random spike train with Poisson statistics and average frequency f = 100Hz:
  - Poisson statistics means that the ISI distribution is exponential.
  - The average ISI is the inverse of the frequency.

- Simulate a leaky integrate and fire (LIF) neuron driven by the spike train you generated.
  - Find a synaptic weight w such that the output firing rate is approximately 20Hz.
  - Bonus: explain how to use the results of Exercise 2a to make an educated guess for w.

Useful resources:
NMA, week 3, day 1, tutorial 1
Section 4: Poisson spike trains

Exercises

Exercise 2b. A synapse is not a number (p. 2 of 2)

- Following the NMA short term plasticity tutorial, compute the conductance of a synapse with short term depression. You can use the example spike train with regular intervals in the tutorial

Bonus: Exercise 2c.

- Now we will combine the results of Exercise 2a and 2b.
  If *w* is constant, it doesn't matter if there is one input neuron with *f* = 100Hz, or 10 input neurons each with *f* = 10Hz. Now we will study a scenario where *w* is not constant.
  - Generate 10 input spike trains with firing rate 10Hz.
  - For each spike train, compute the time dependence of the conductance as in Ex. 2b.
  - Compare the total conductance vs. time in the two scenarios (1×100Hz, 10×10Hz)
  - If you drive a LIF neuron with these inputs (as in Ex. 2a) does the short term depression make a difference?

Useful resources:
[NMA, week 3, day 1, tutorial 3]
Section 2: short term plasticity