Generalized linear models (GLM)

Tools for studying neural coding

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UCSD NGP bootcamp

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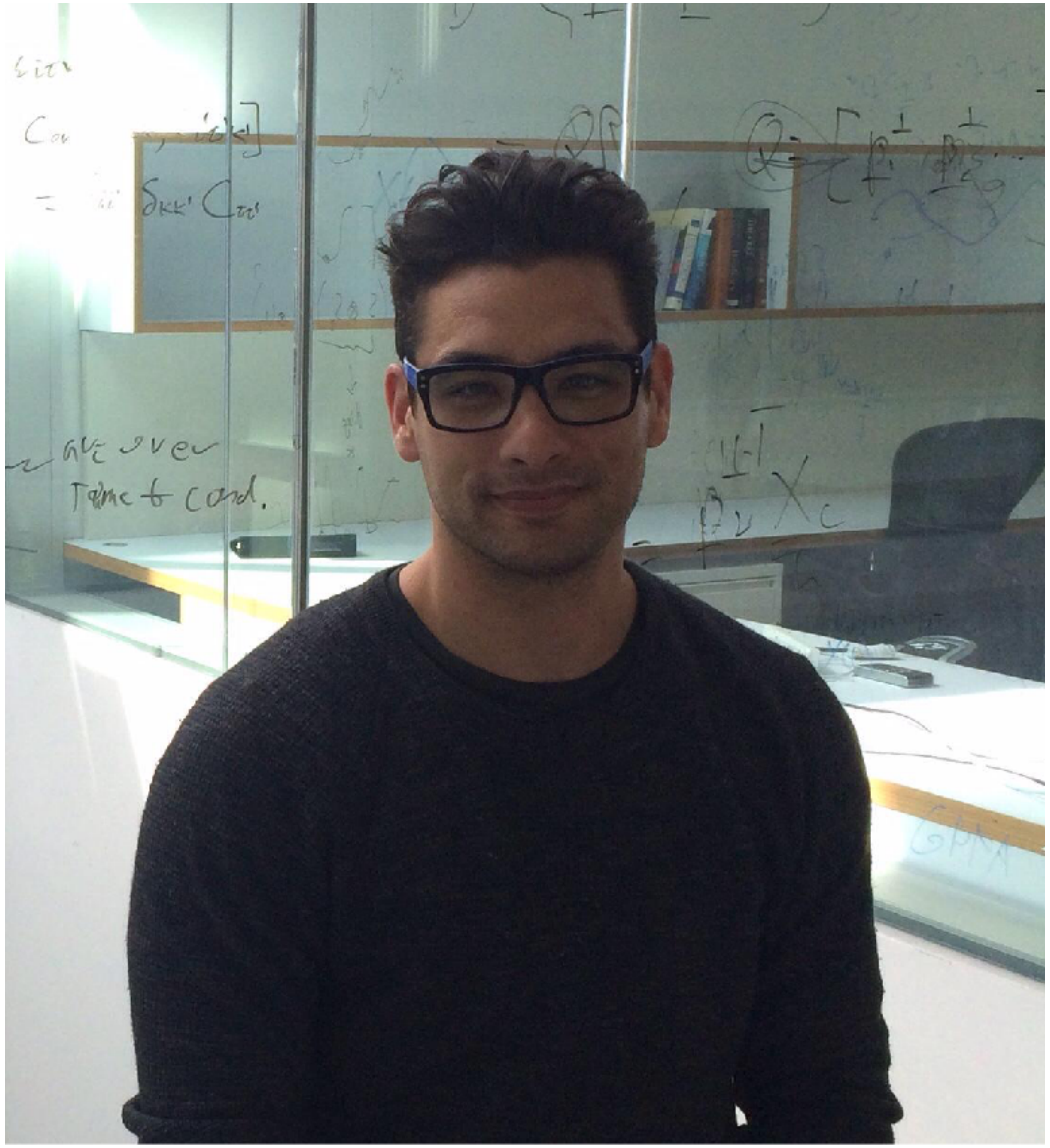
Who is Gal?

- Assistant Professor of Data Science
- Interested in how networks of neurons (real & artificial) learn
- Develops graph-based methods for processing and analysis of large-scale, high-dimensional data
- Will be discussing dimensionality reduction tomorrow

Who is Mikio?

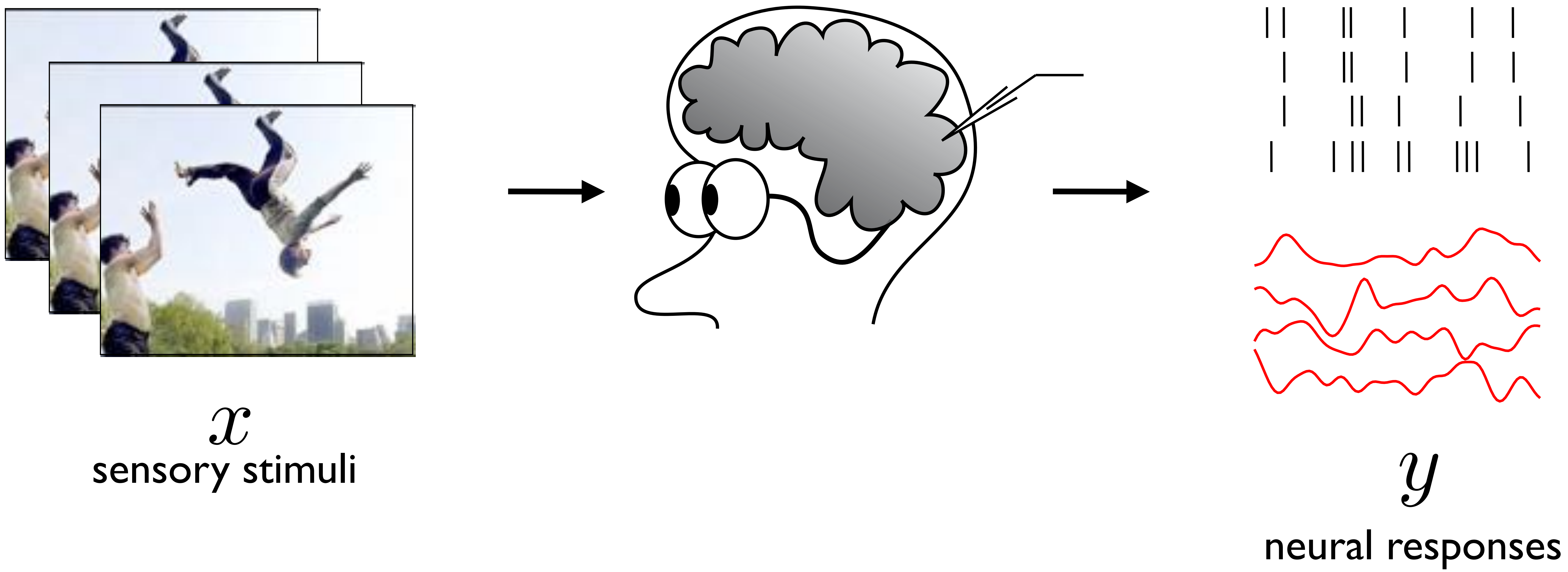
- Assistant Professor in Neurobiology & Data Science
- Interested in understanding how group of neurons coordinate their activity to implement computations
- Works on methods and theories for characterizing the ways that neurons coordinate.



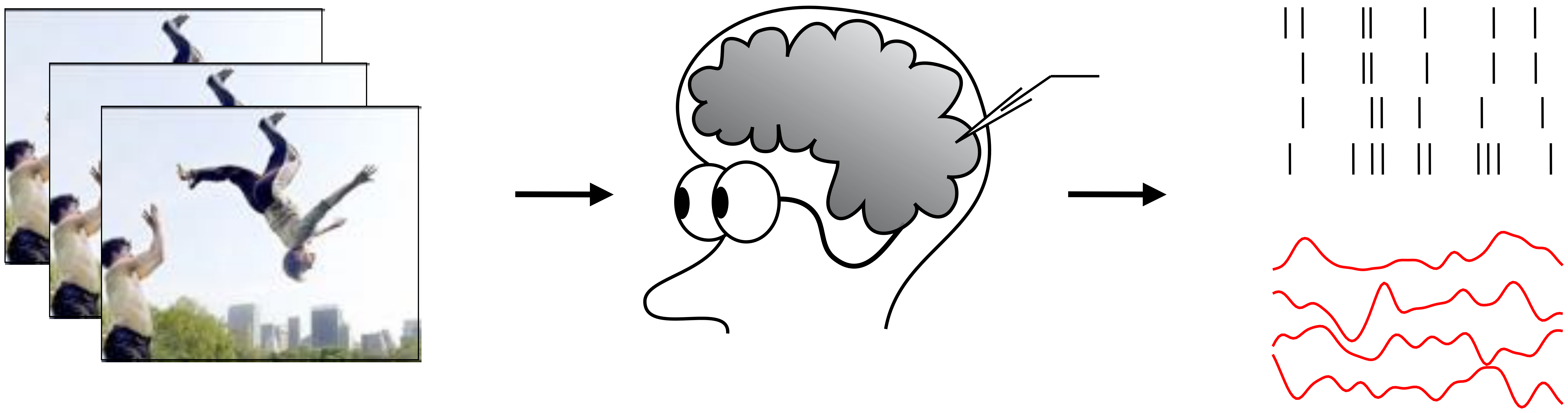
Agenda

- Neural coding
- What's a GLM?
- Common GLMs (Gaussian, Poisson, logistic)
- Tutorials

Neural coding



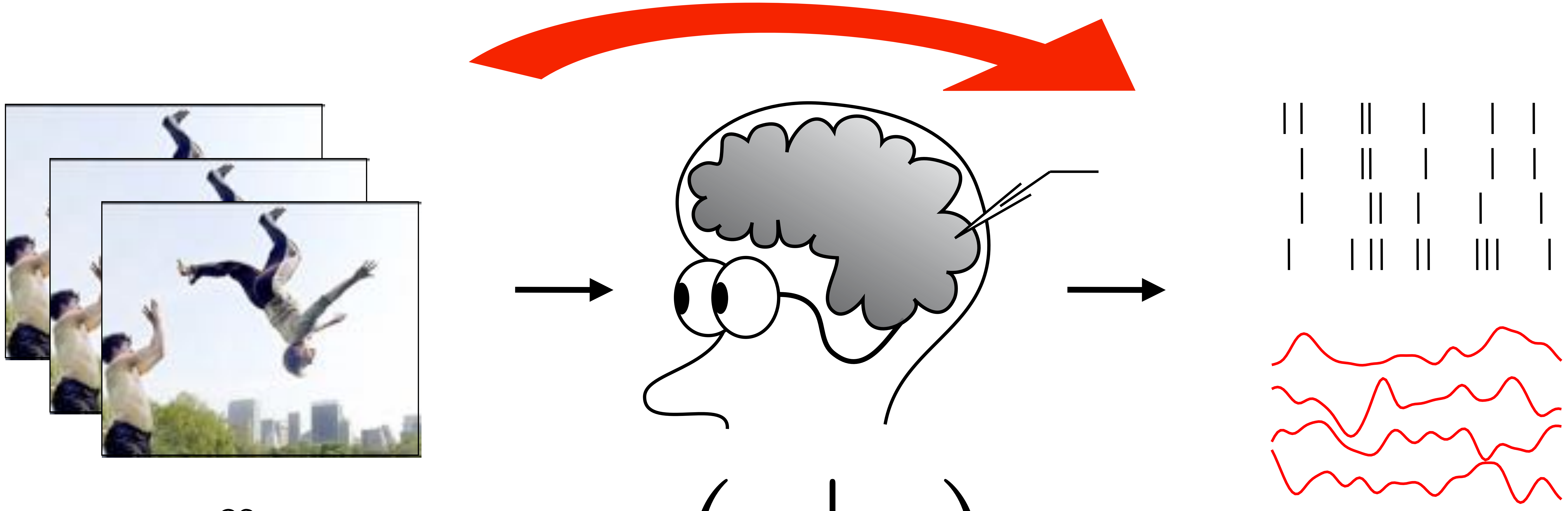
neural coding problem:
What is the relationship between external variables and neural activity?



x
sensory stimuli

y
neural responses

Encoding



x

external variables

$$p(y|x)$$

y

neural responses

encoding distribution

- probabilistic statement about relationship between *x* and *y*

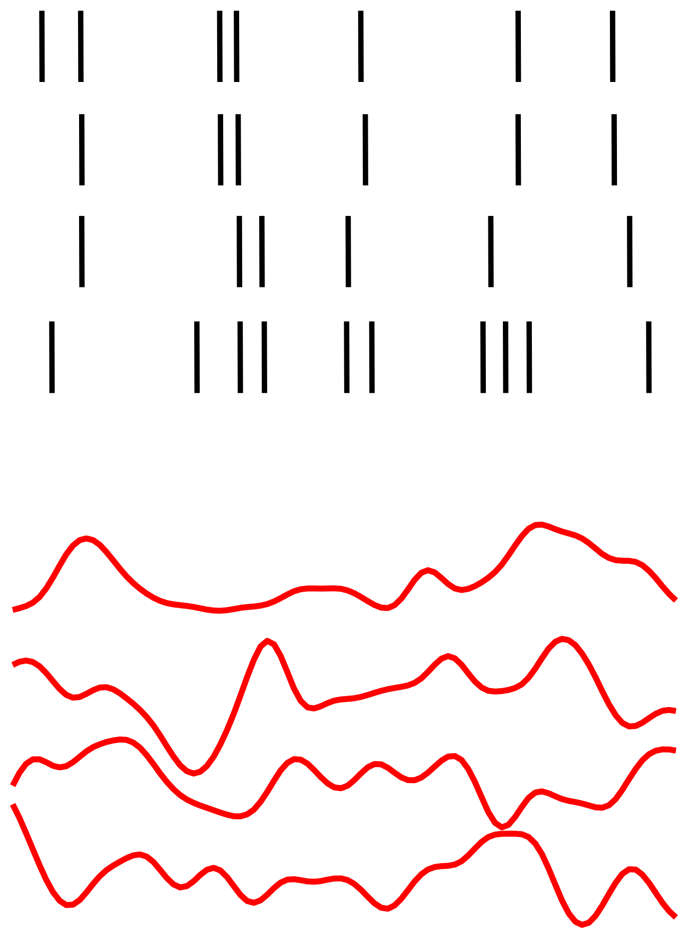
data analysis

x

external variables

encoding model

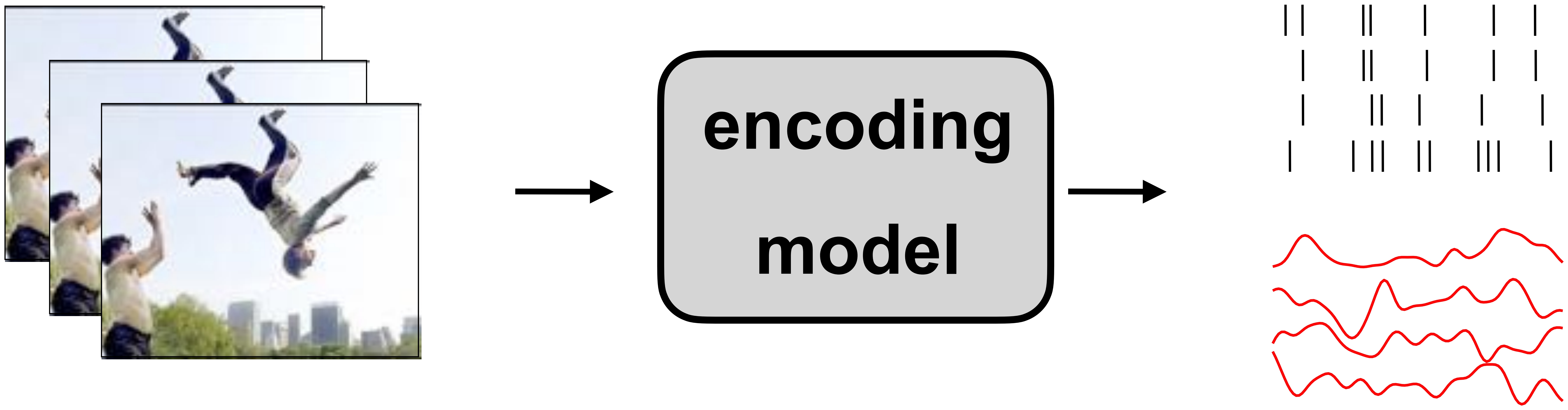
$$p_\theta(y|x) \approx p(y|x)$$



y

neural responses

Goal: find model that approximates the true encoding distribution

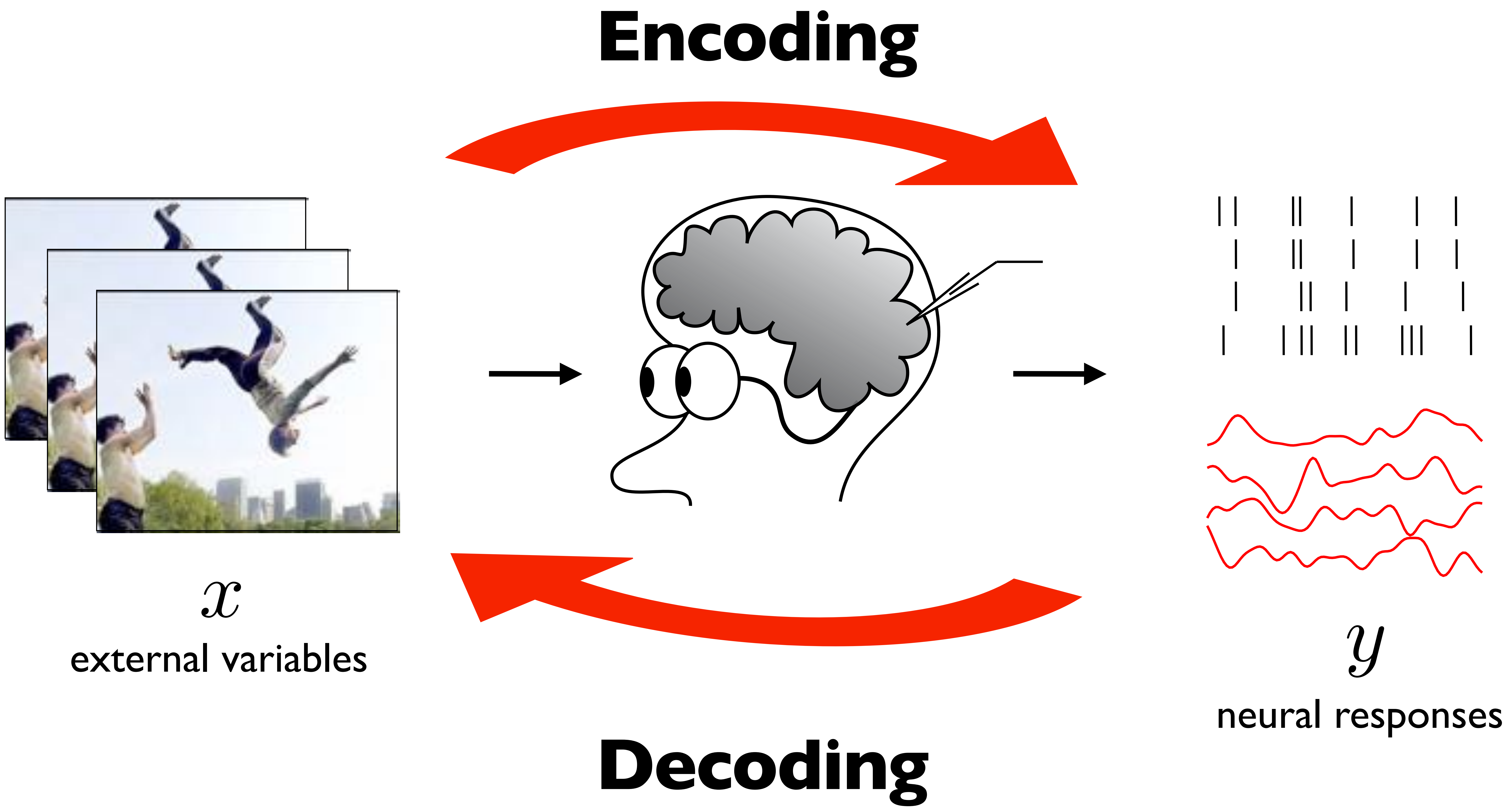


x
external variables

$$p_\theta(y|x) \approx p(y|x)$$

y
neural responses

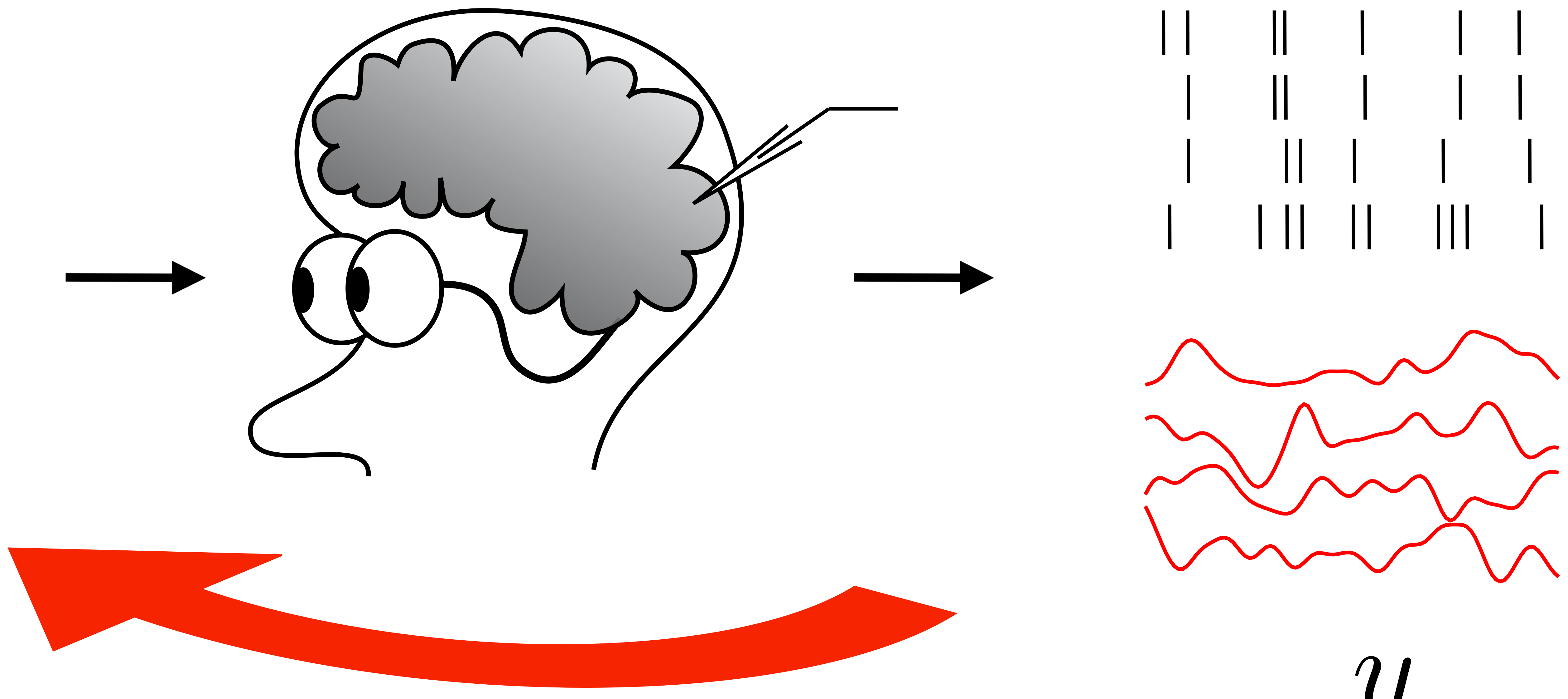
broadly: What is a good description of why a neuron(s) spike?


Encoding
x
external variables
y
neural responses
Decoding

read out neural activity to predict external variables “mind reading”

- “house” or “face”?
- what orientation?
- what action?

N neurons or voxels



x

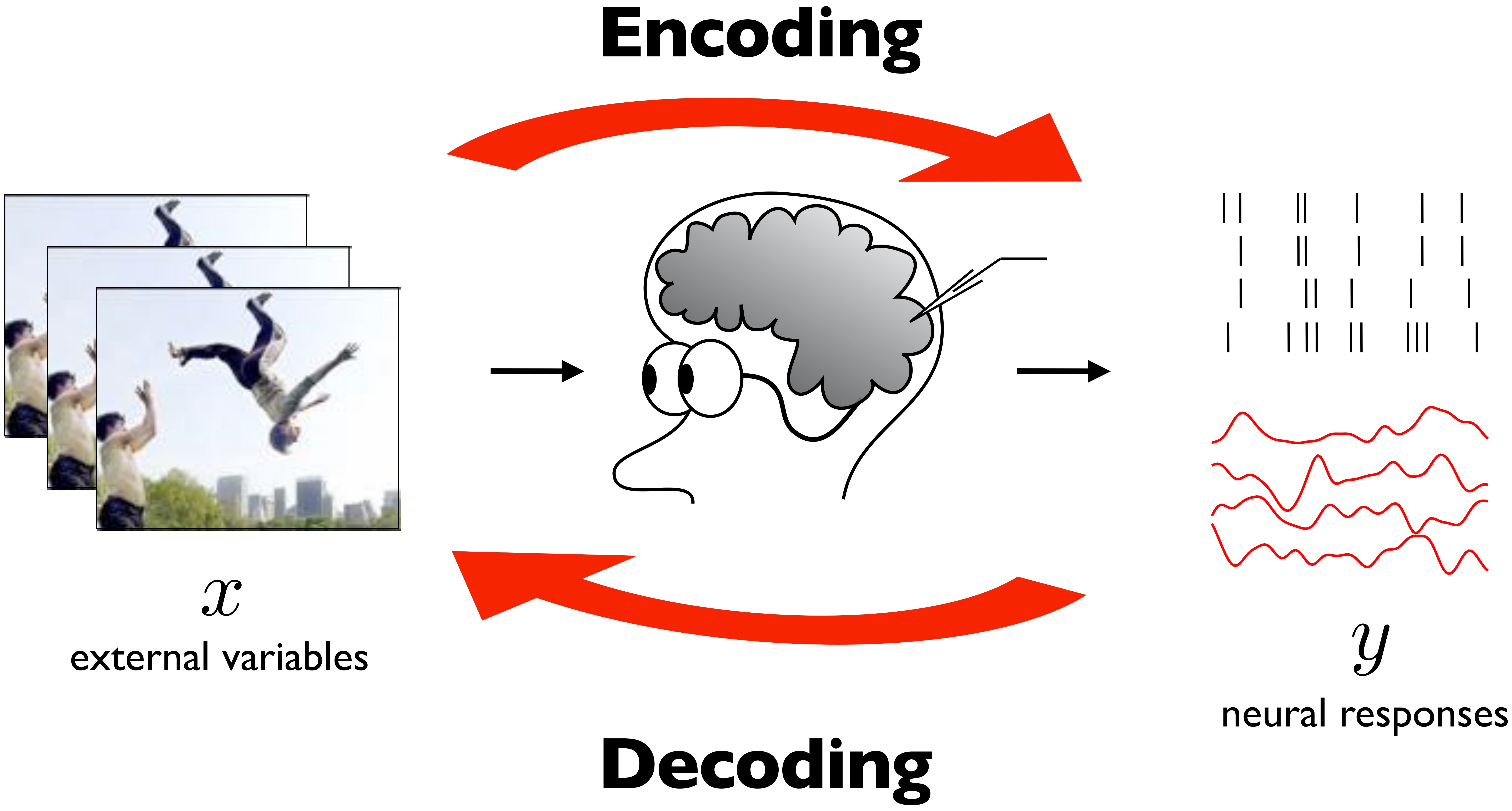
external variables

y

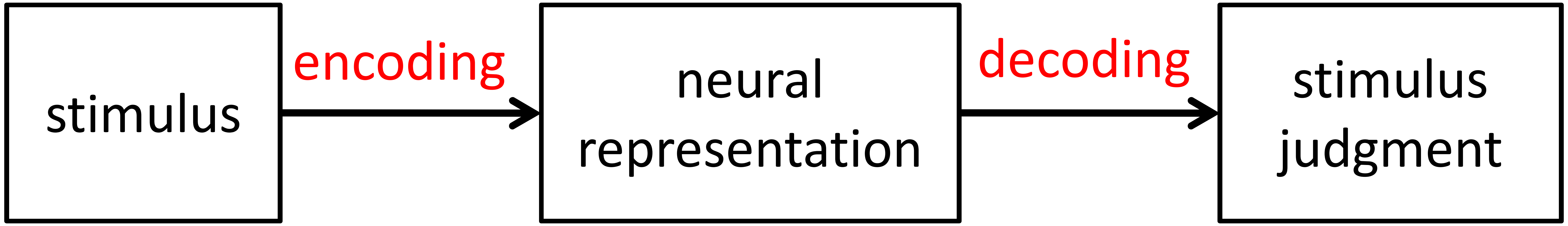
neural responses

Decoding

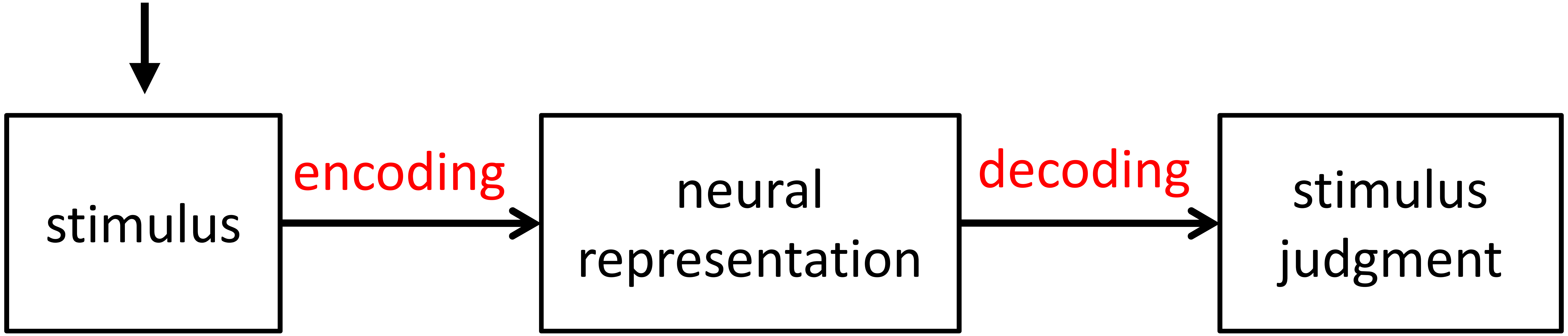
- constraints on readout by down-stream populations
- applications: eg. BCI motor prosthetics

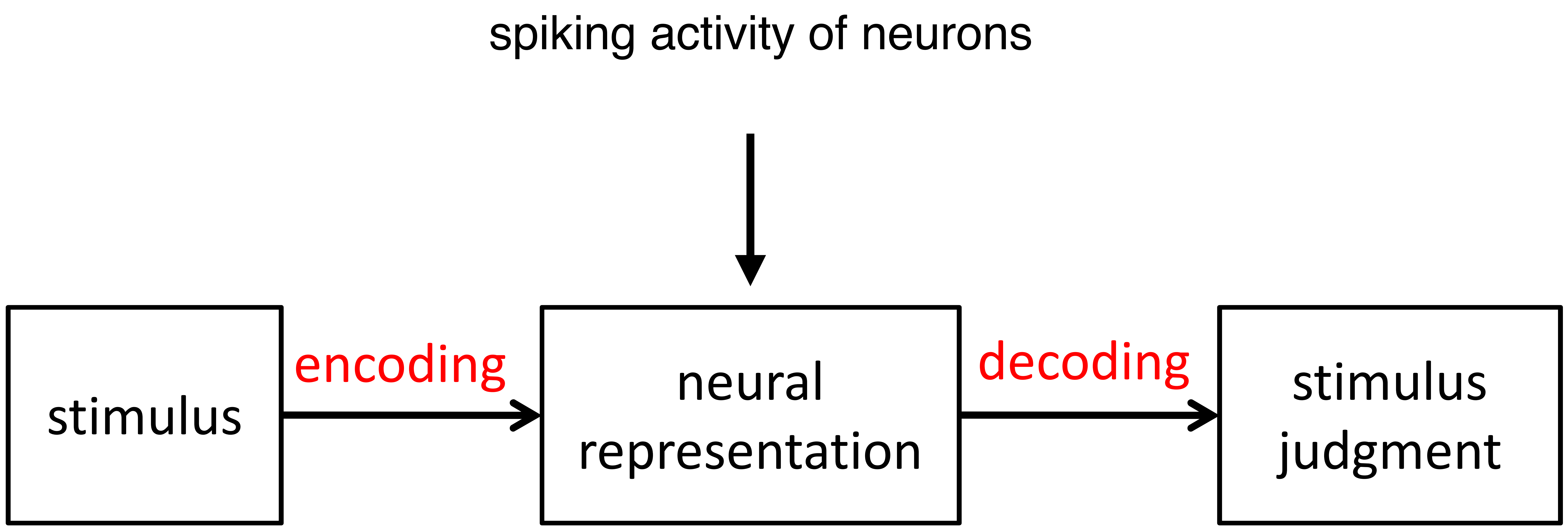

Encoding
x
external variables
y
neural responses
Decoding

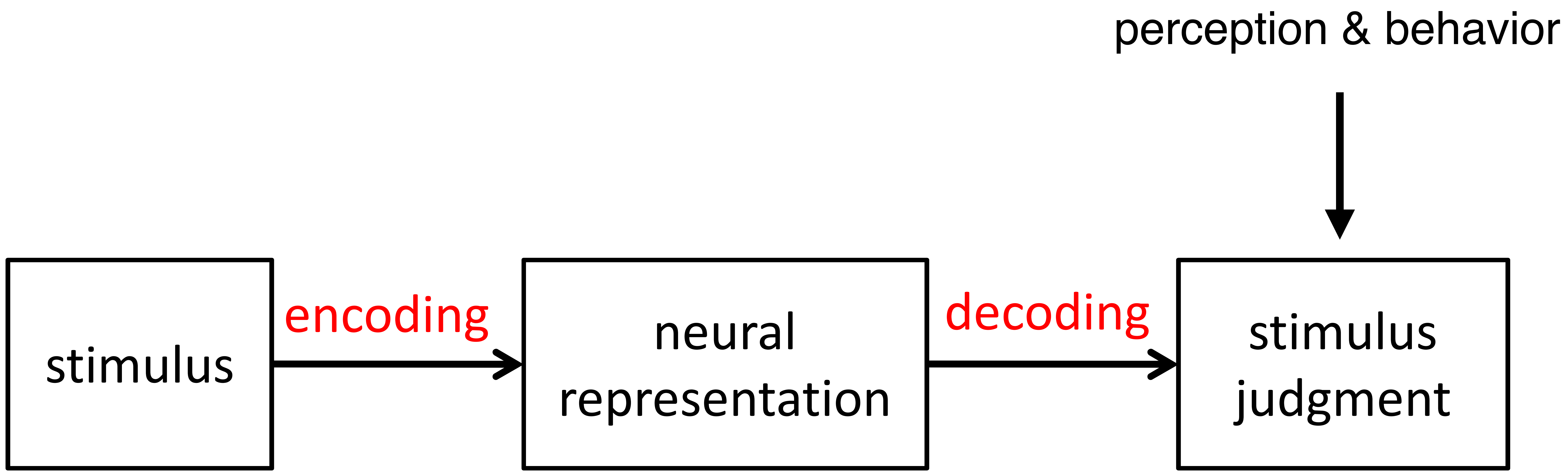
parallel between what the brain does and data analysis

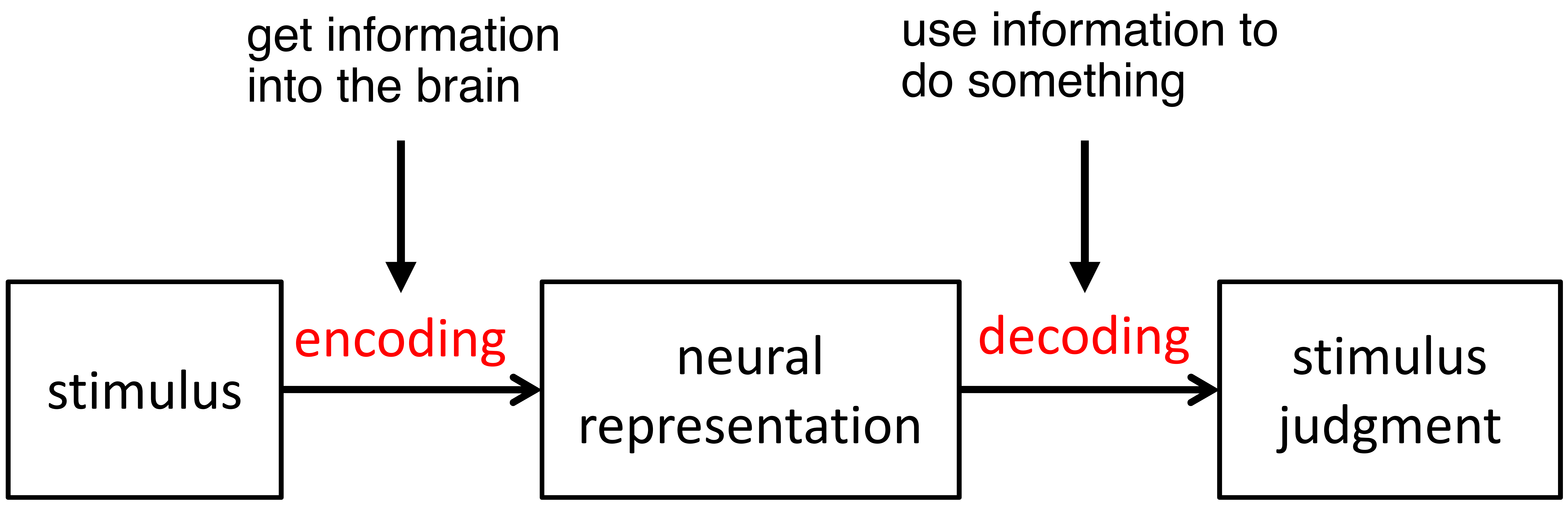


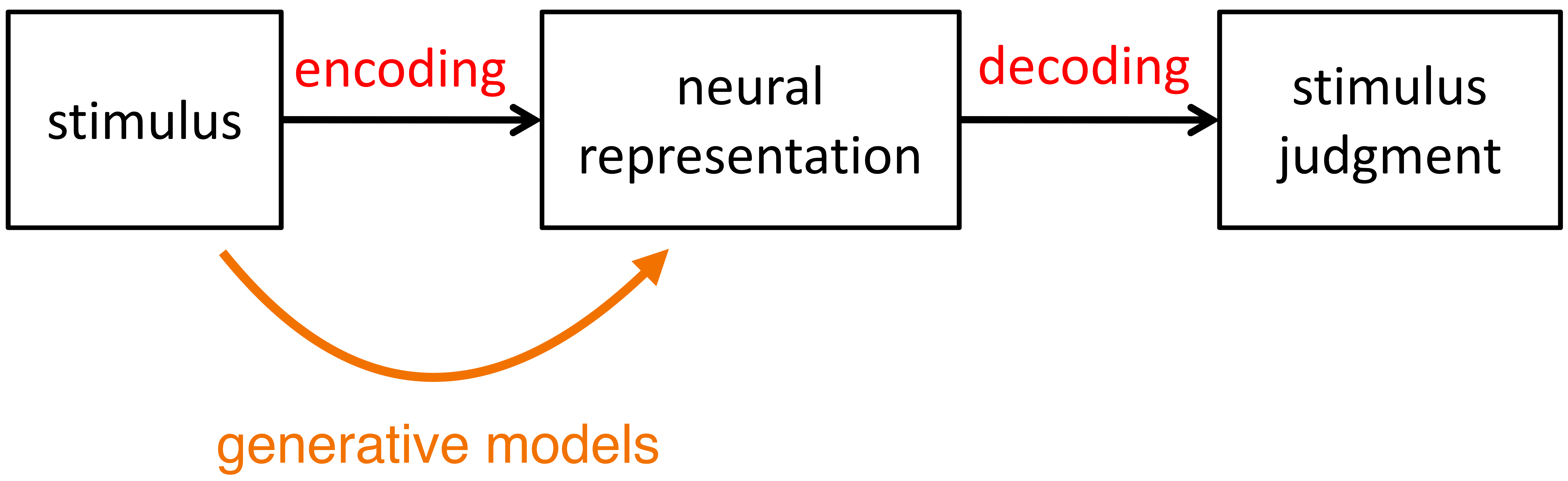
- orientation of contour
- direction of motion (self or object)
- number of students in class
- smell of coffee

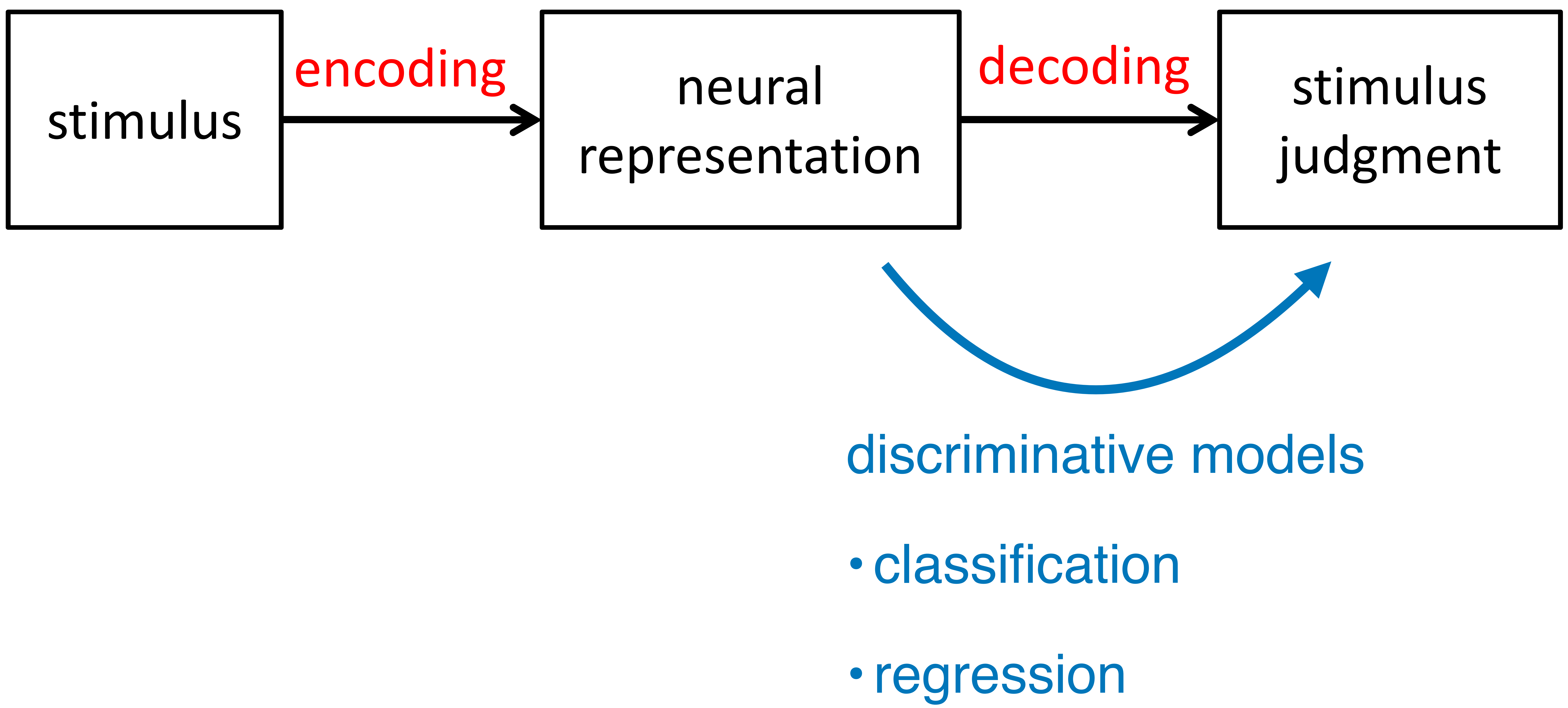


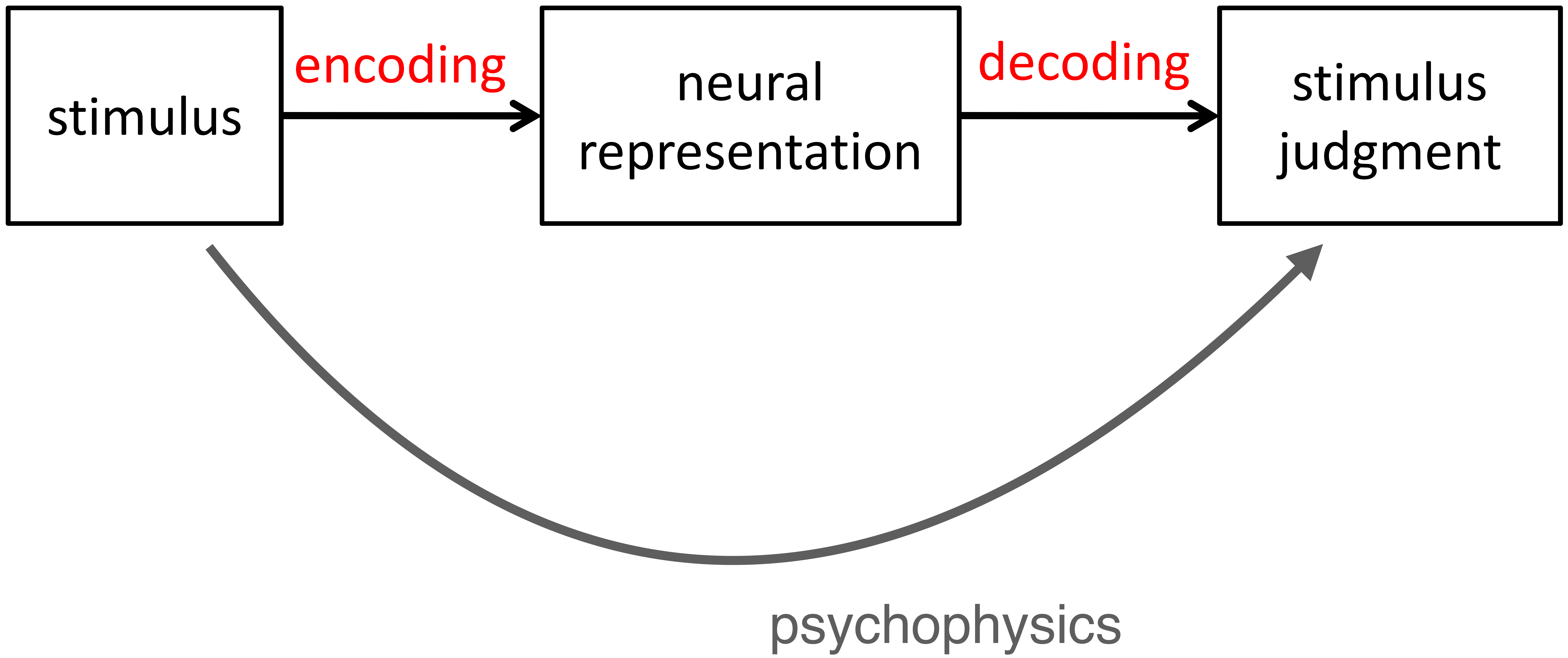

spiking activity of neurons
stimulus
encoding
neural representation
decoding
stimulus judgment

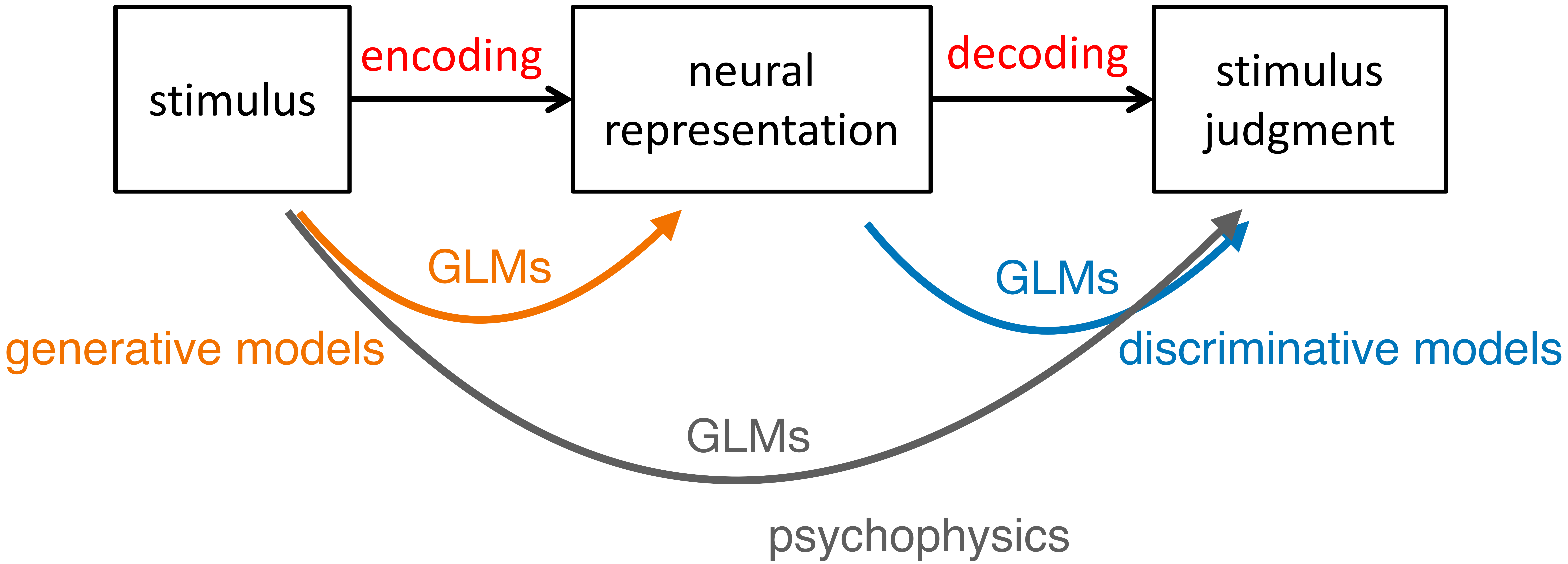

perception & behavior
stimulus
encoding
neural representation
decoding
stimulus judgment

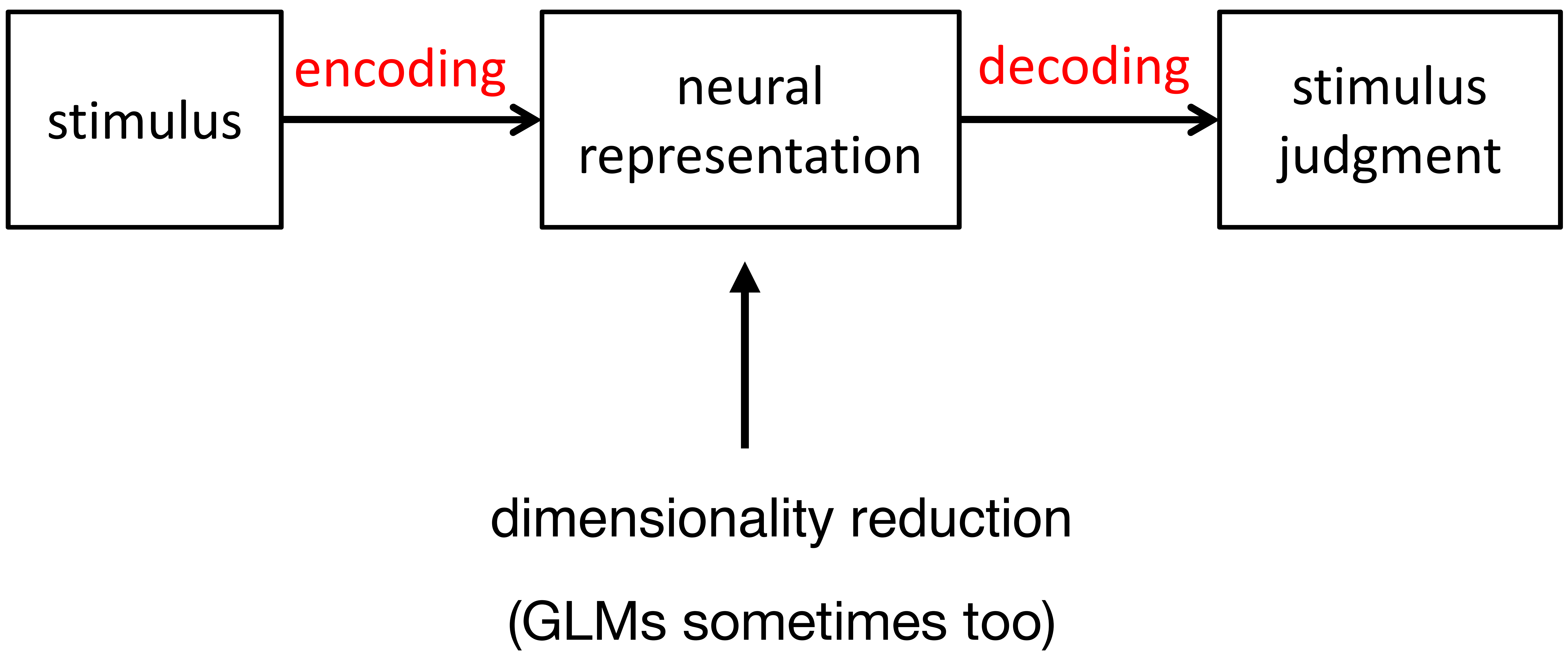

get information
into the brain
use information to
do something
stimulus
encoding
neural
representation
decoding
stimulus
judgment


stimulus
encoding
neural representation
decoding
stimulus judgment
generative models


stimulus
encoding
neural representation
decoding
stimulus judgment
discriminative models
• classification
• regression


stimulus
encoding
neural representation
decoding
stimulus judgment
psychophysics

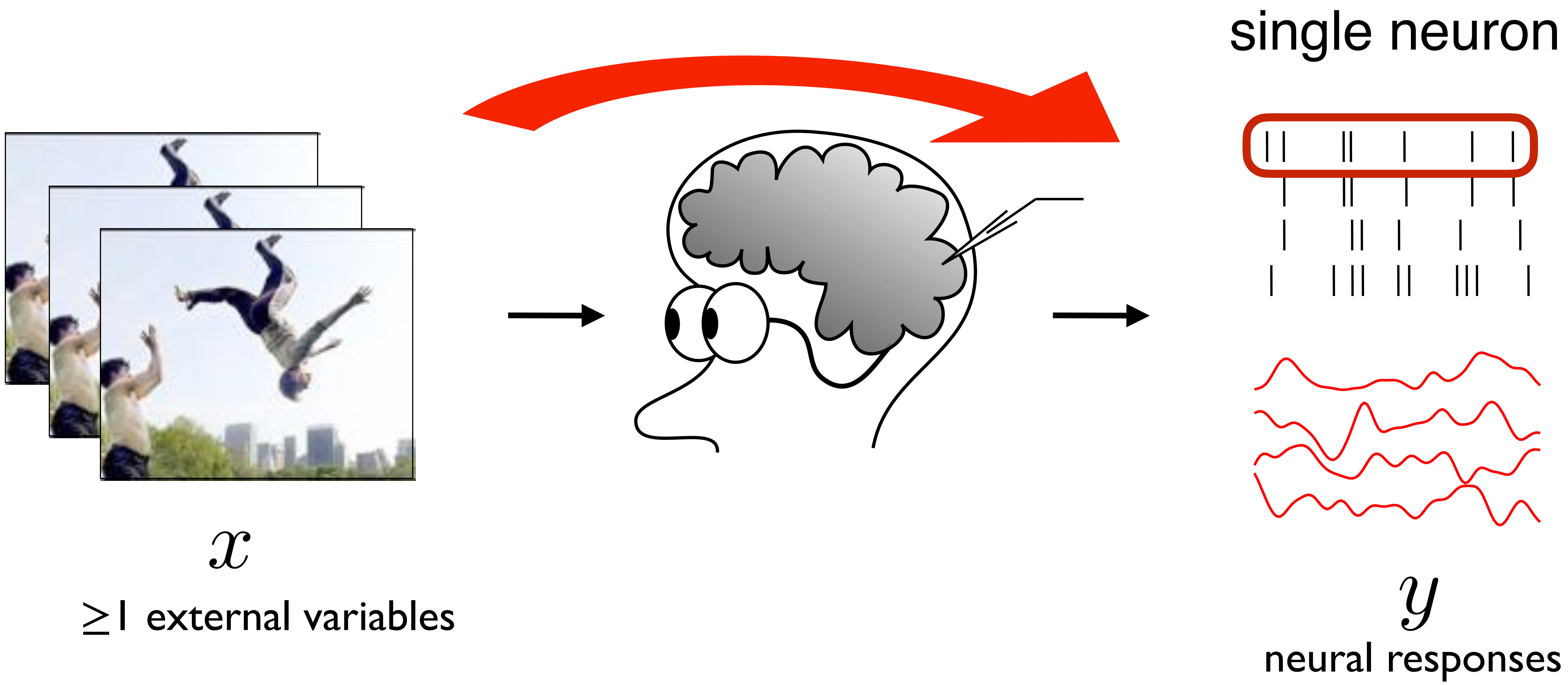

stimulus
encoding
neural representation
decoding
stimulus judgment
GLMs
GLMs
generative models
discriminative models
GLMs
psychophysics

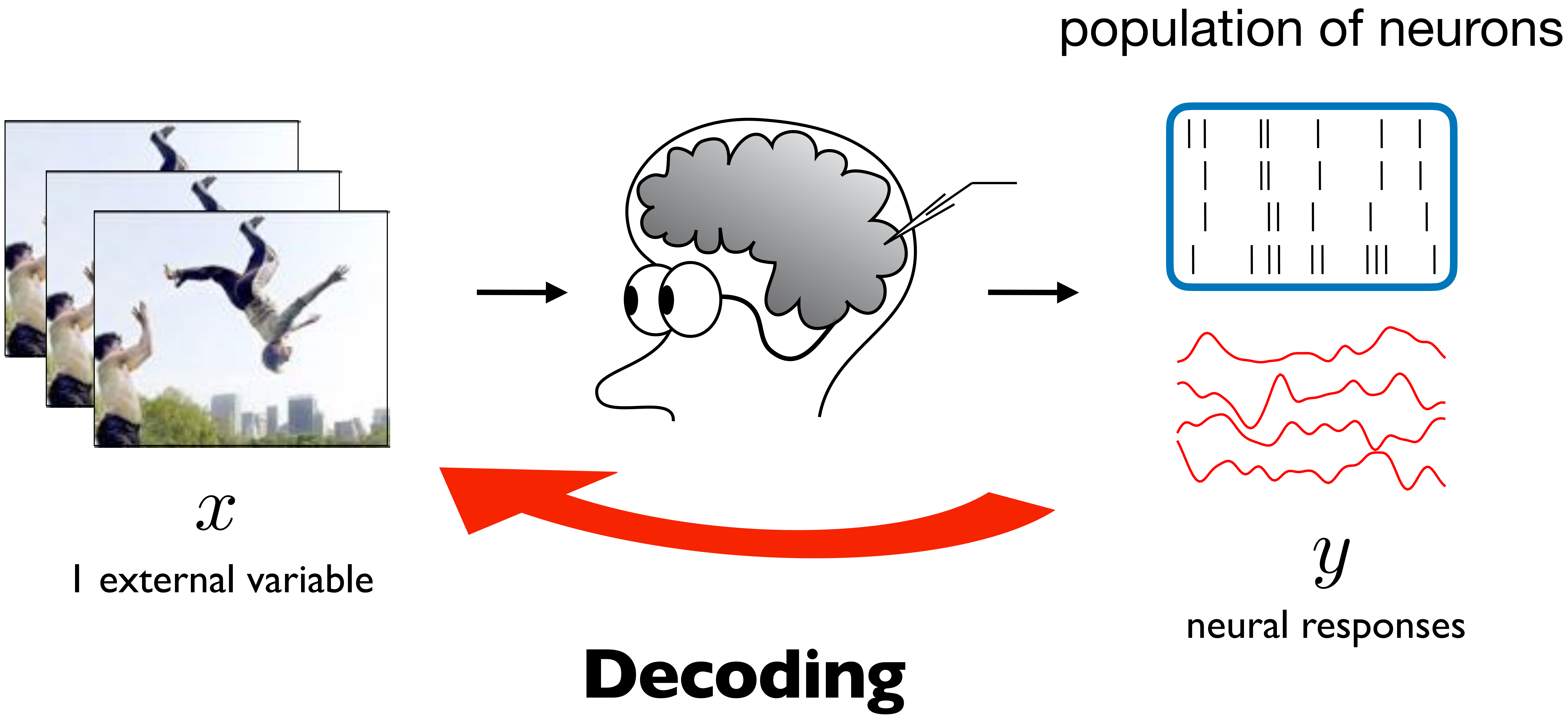

stimulus
encoding
neural representation
decoding
stimulus judgment
dimensionality reduction
(GLMs sometimes too)

GLM history

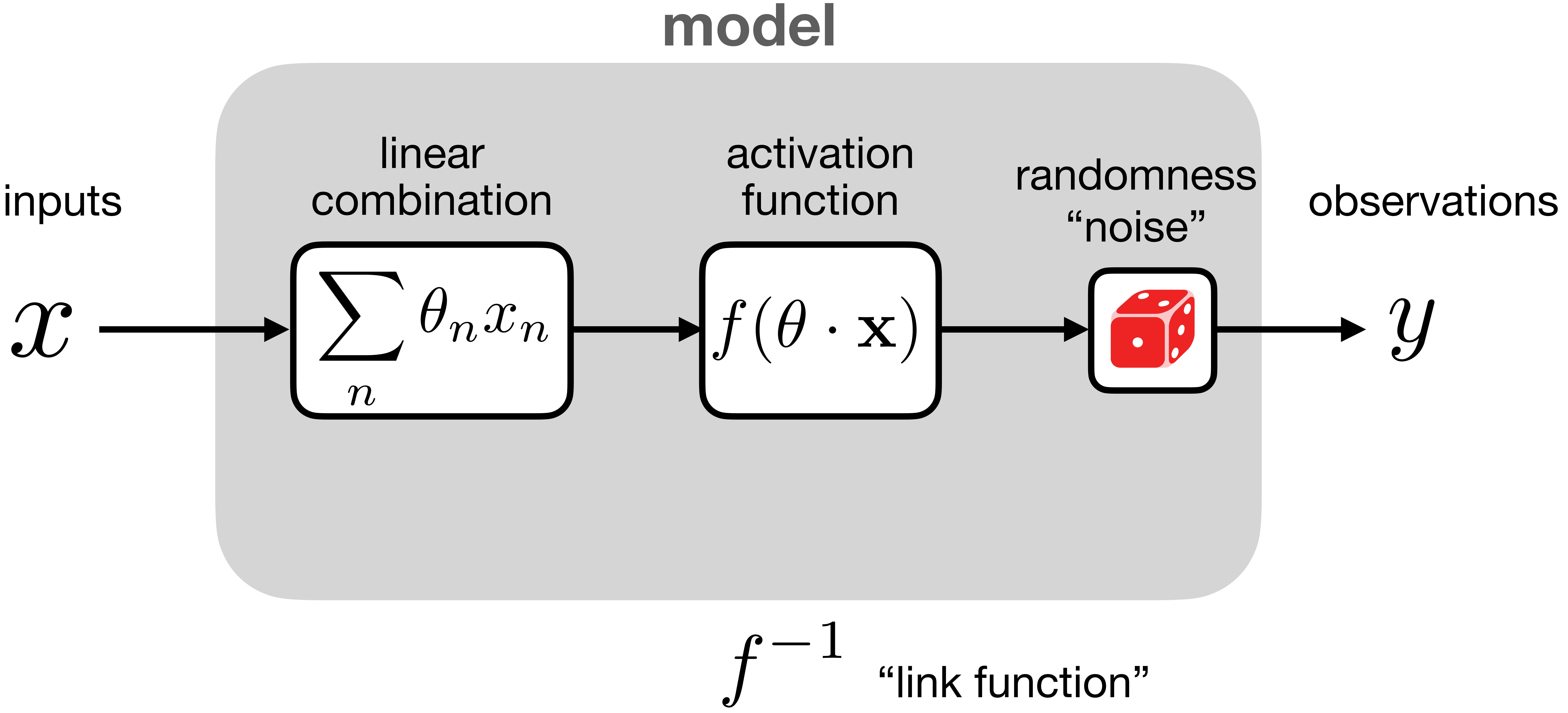
- Flexible & convenient way of testing and characterizing statistical relationships between “input” and “output” variables.
- Family of models developed in the early 70’s (Nelder & Wedderburn, 1972) to unify several types of regression
- Adoption in neuroscience - introduced independently by 2 groups, (nearly) simultaneously:
    - Simoncelli E, Paninski L, Pillow J and Schwartz O 2004 **Characterization of neural responses with stochastic stimuli** The Cognitive Neurosciences 3rd edn, ed M Gazzaniga (Cambridge, MA: MIT Press)
    - Truccolo W, Eden UT, Fellows MR, Donoghue JP, Brown EN.. **A point process framework for relating neural spiking activity to spiking history, neural ensemble, and extrinsic covariate effects**. *J Neurophysiol*. 2005;93(2):1074-1089. doi:10.1152/jn.00697.2004
- Related to feed-forward neural networks

Encoding



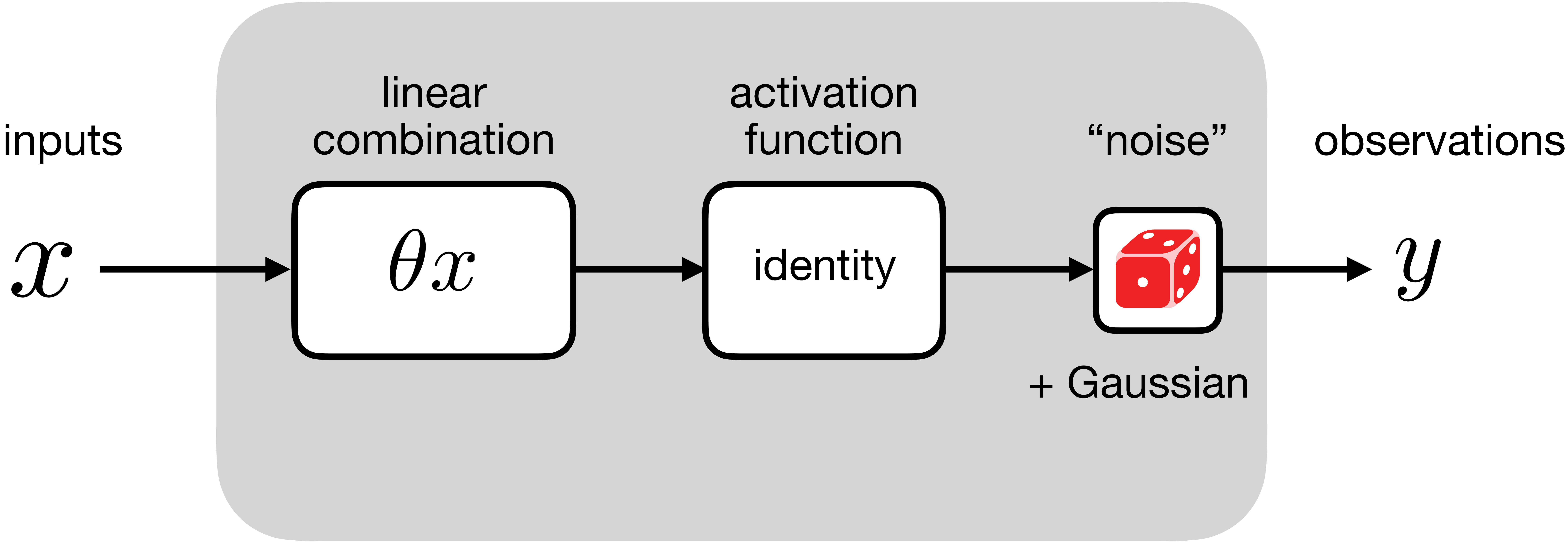

population of neurons
x
1 external variable
y
neural responses
Decoding

Structure of the GLM



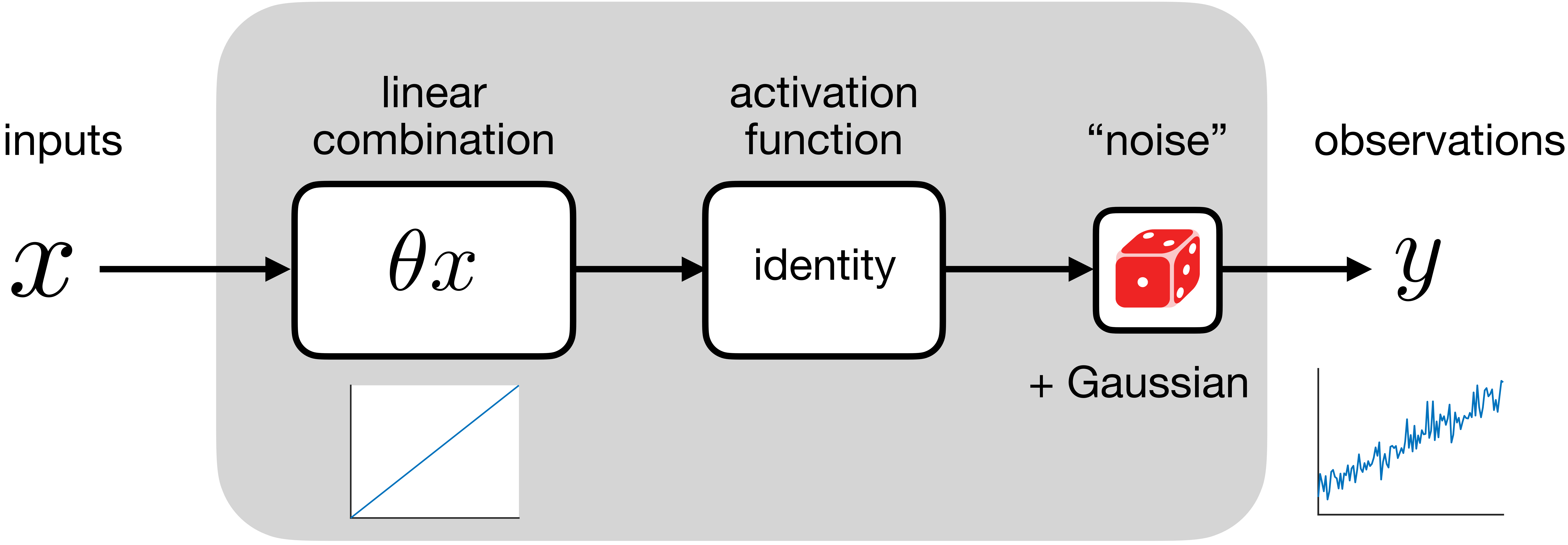
Simple linear regression

$$y = \theta x + \eta$$



Simple linear regression

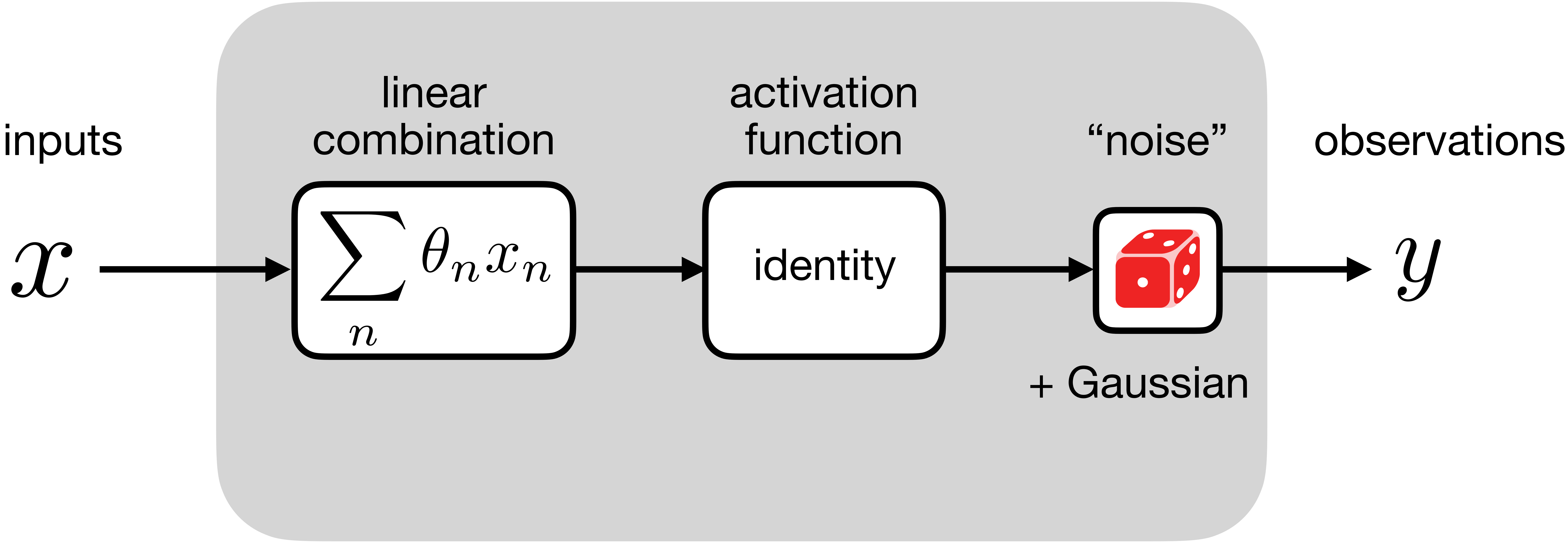
$$y = \theta x + \eta$$



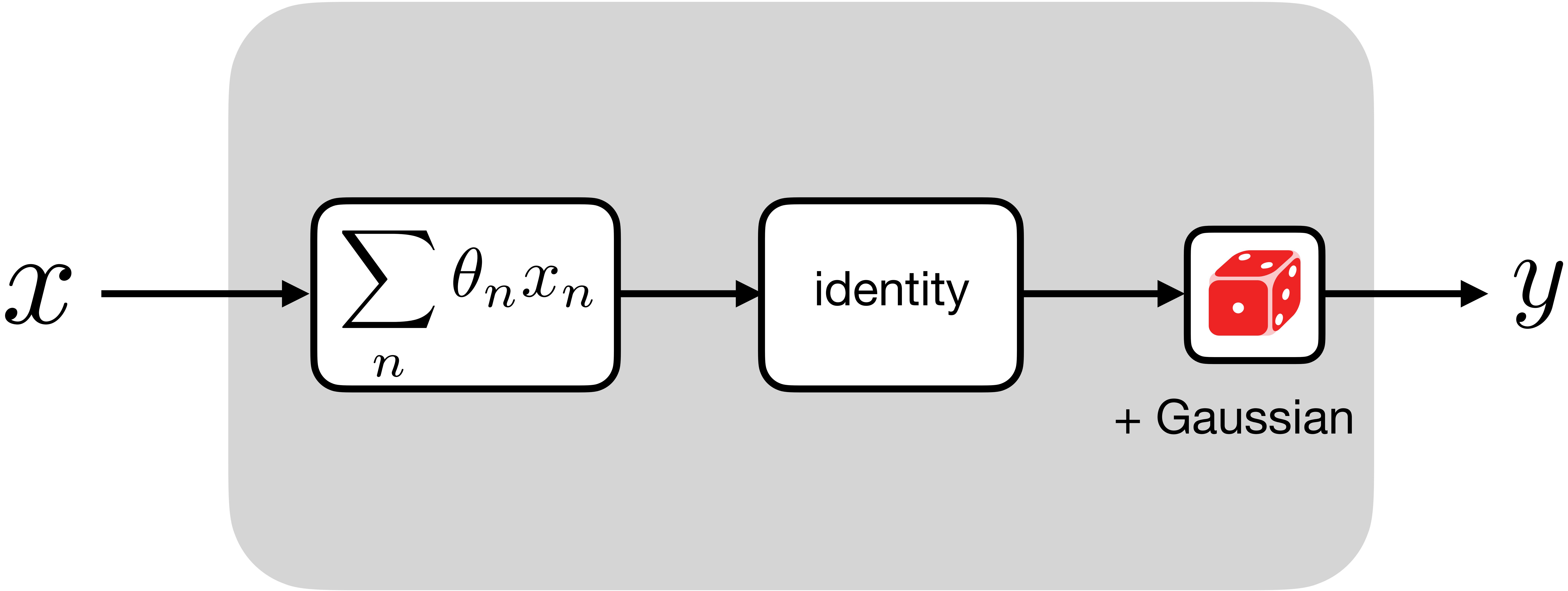
$$y \sim \mathcal{N}(\theta x, \sigma^2)$$

Multiple linear regression

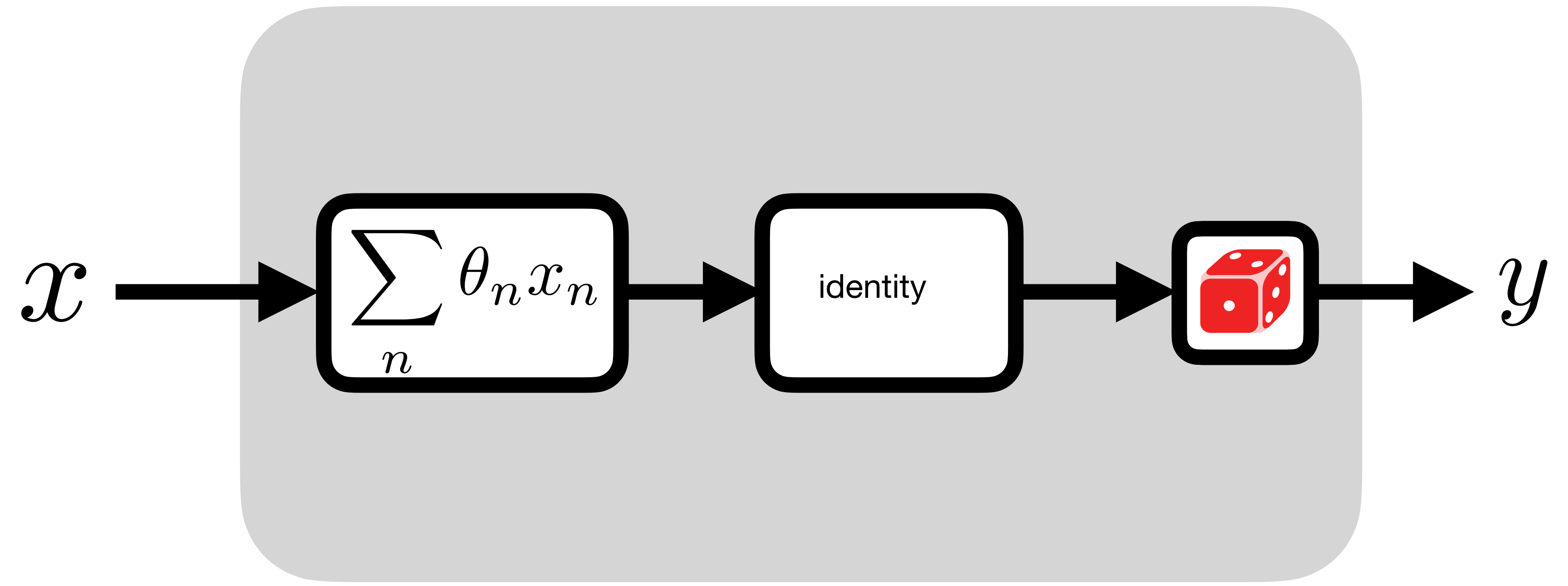
$$y = \sum_n \theta_n x_n + \eta$$



Multiple linear regression



Multiple linear regression



$$y \sim \mathcal{N}(\mu, \sigma^2)$$

$$p(y|\mu, \sigma^2) = \frac{1}{\sqrt{2\sigma^2}} \exp\left(-\frac{(\mu - y)^2}{\sigma^2}\right)$$

for Gaussian GLM

$$\boxed{\mu = \sum_n \theta_n x_n}$$

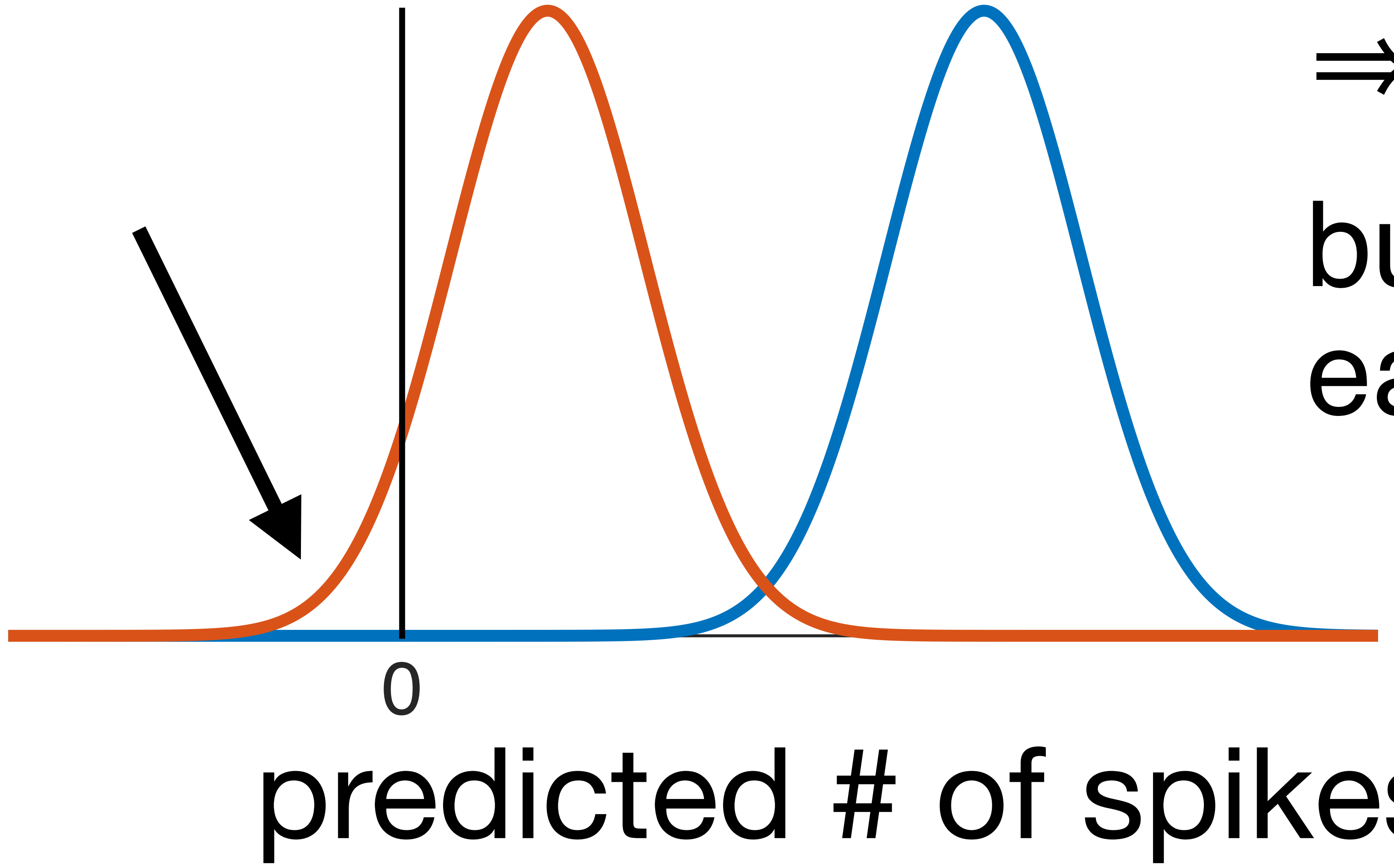
Gaussian variability

$$y|\mathbf{x} \sim \mathcal{N}(\mathbf{f}(\theta \cdot \mathbf{x}), \sigma^{\mathbf{2}})$$

$$y = f(\theta \cdot \mathbf{x}) + \eta$$

$$\eta \sim \mathcal{N}(0, \sigma^2)$$

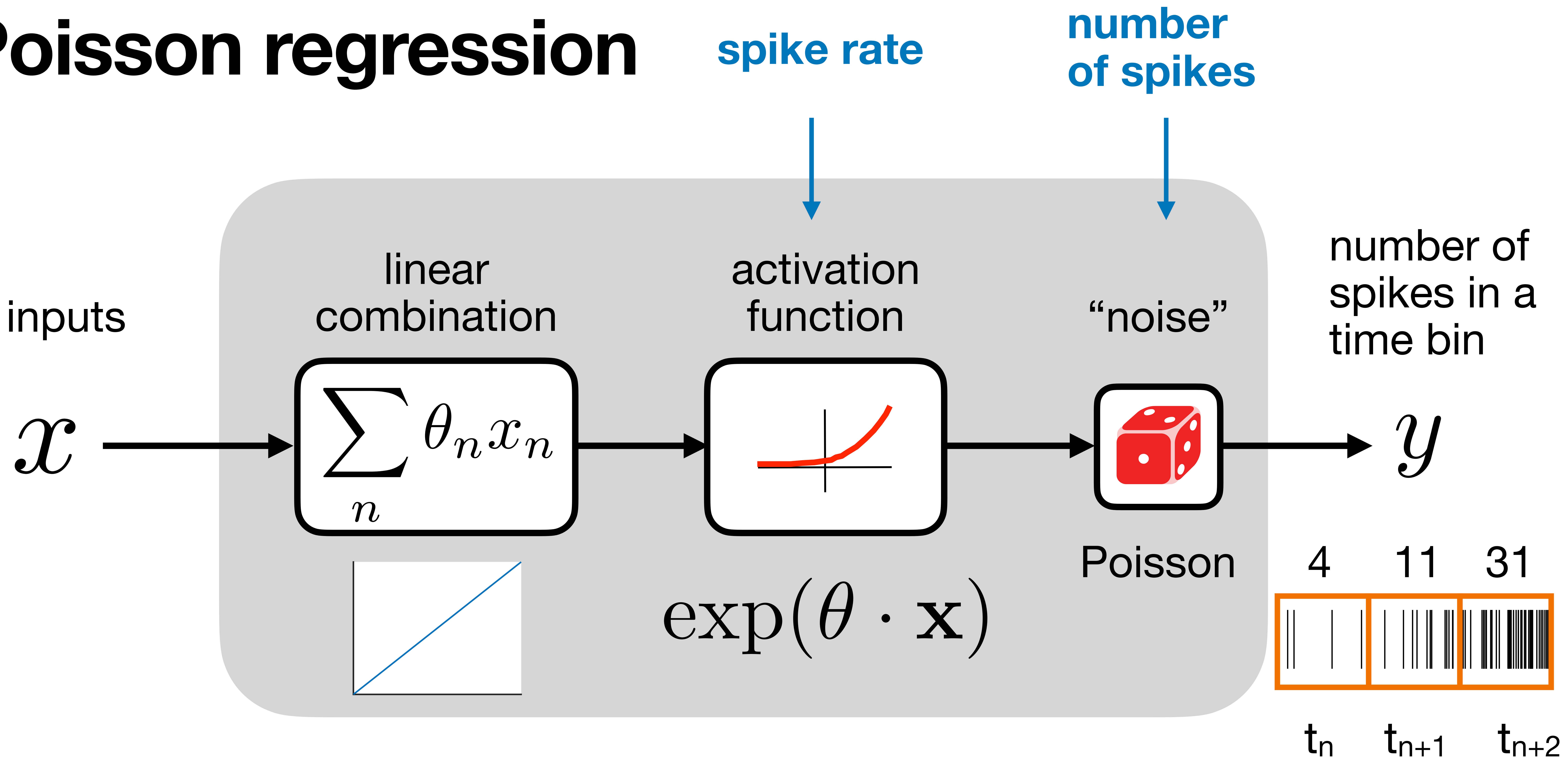
density at
negative spike
counts!



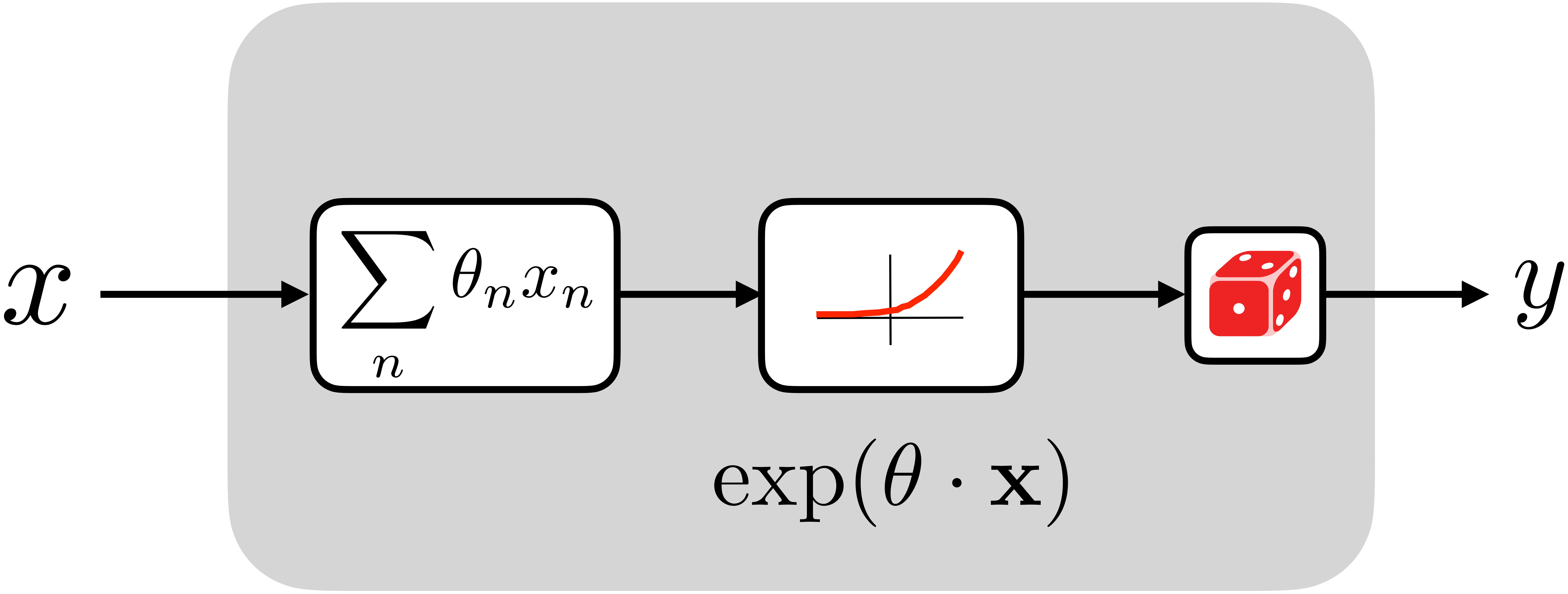
⇒model mismatch

but makes math
easy

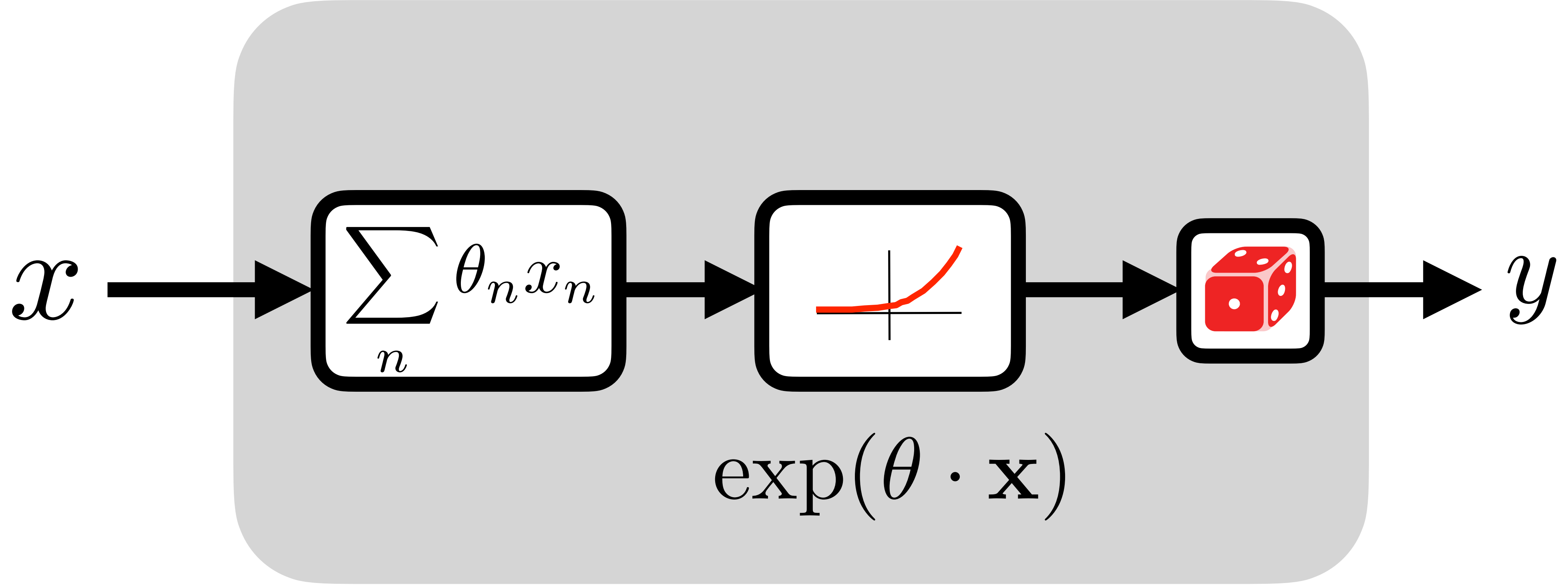
Poisson regression



Poisson regression

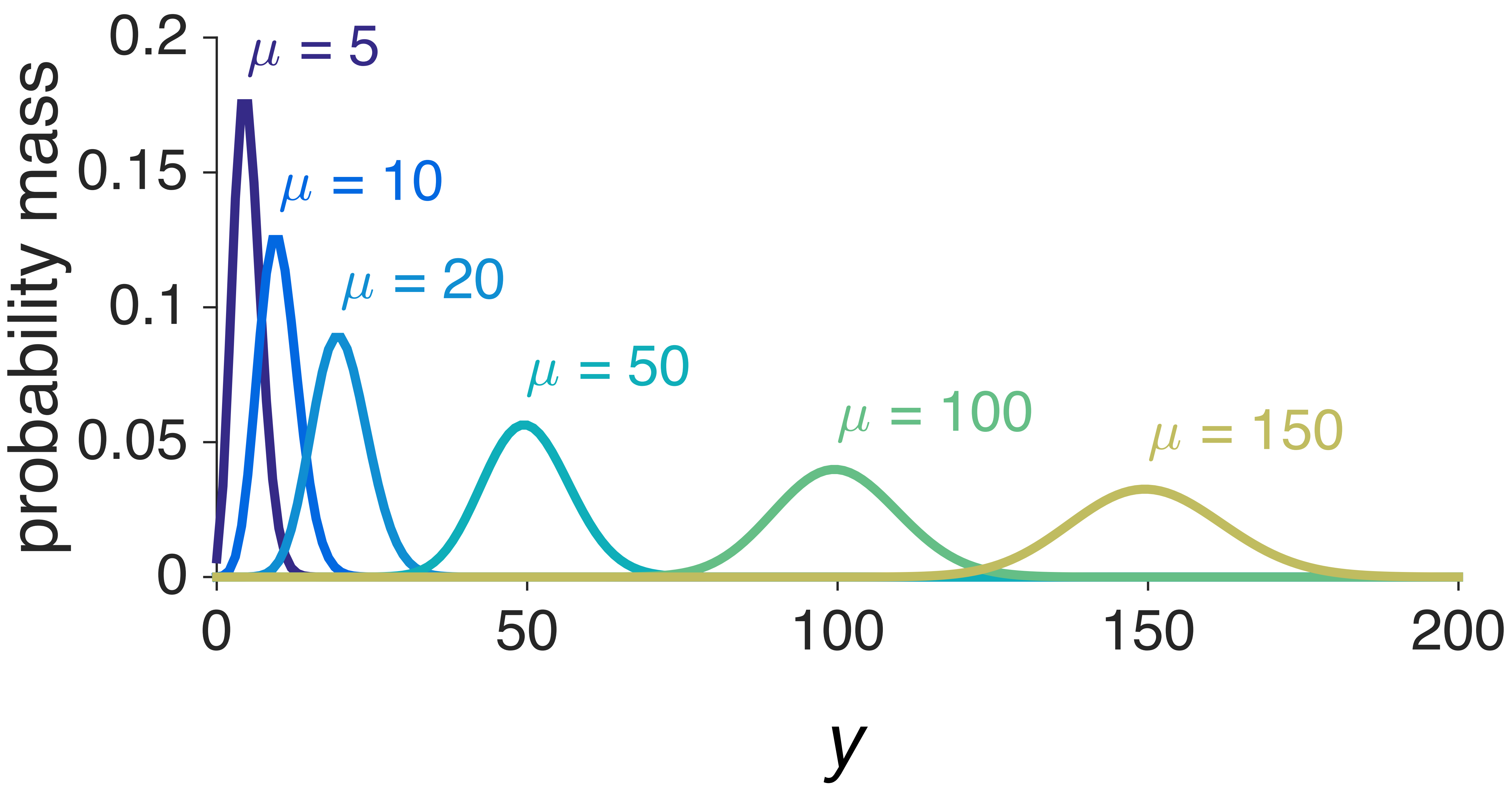


Poisson regression

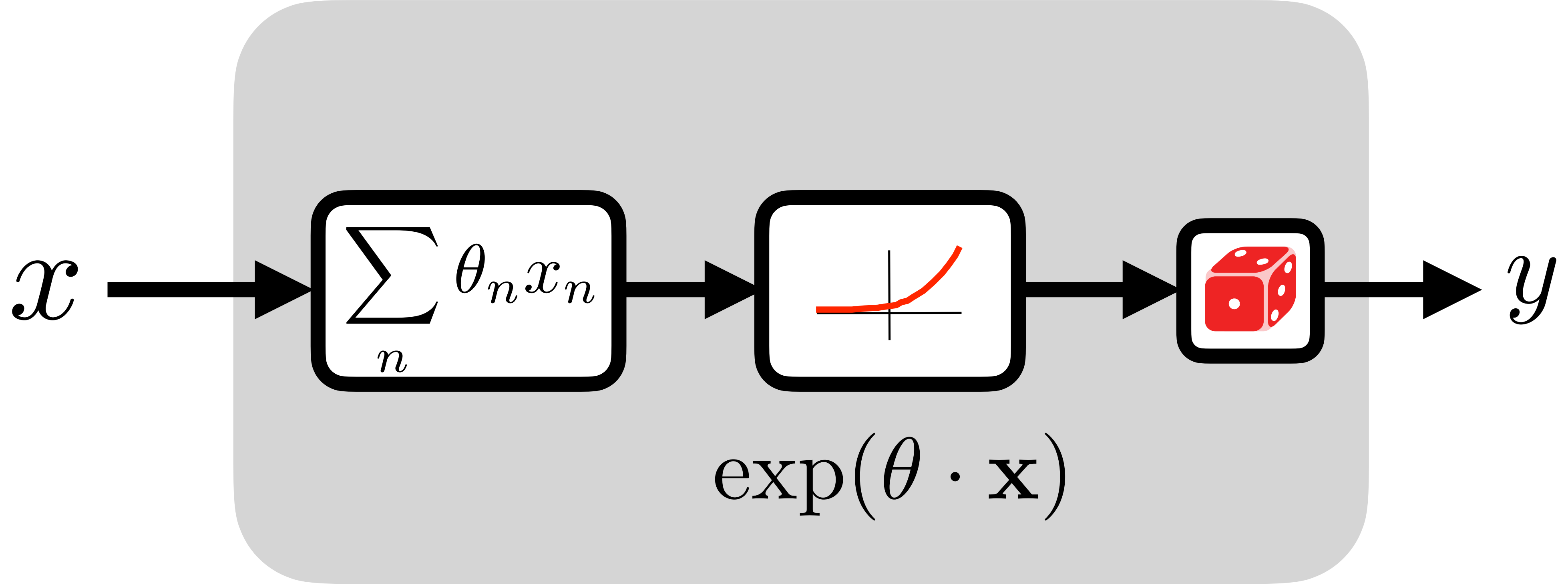


$$y \sim \mathrm{Poisson}(\lambda)$$

$$P(y|\lambda) = \frac{e^{-\lambda}\lambda^y}{y!}$$



Poisson regression



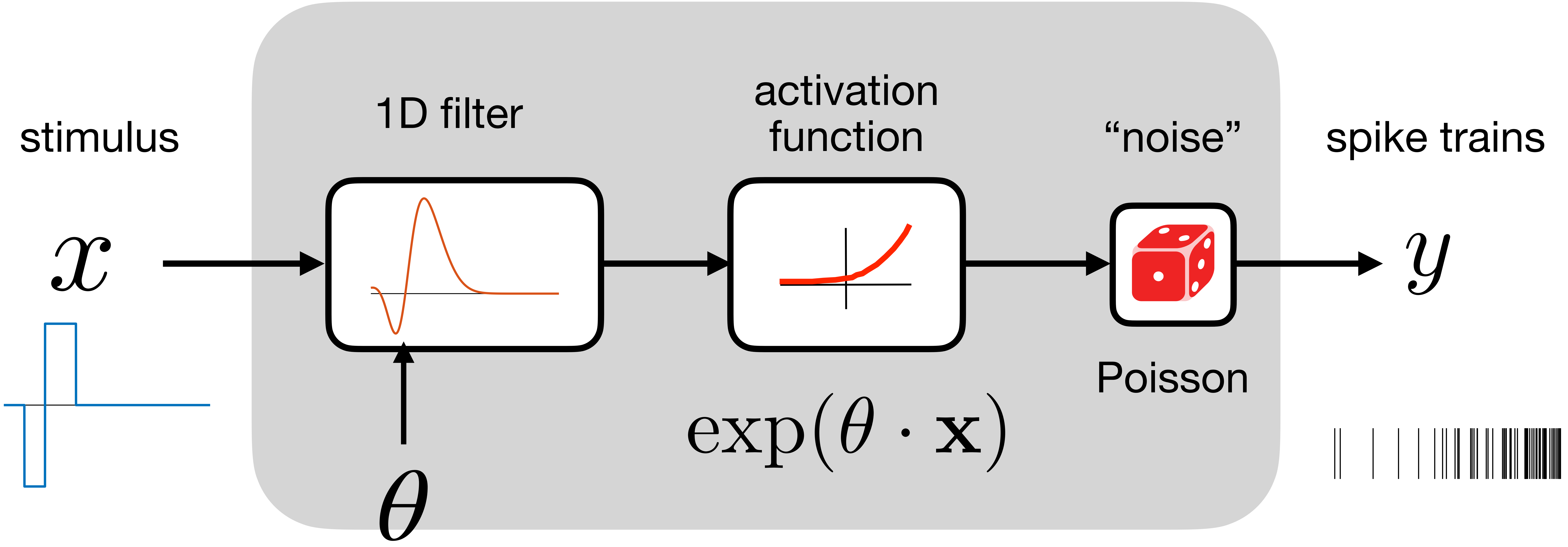
$$y \sim \mathrm{Poisson}(\lambda)$$

$$P(y|\lambda) = \frac{e^{-\lambda}\lambda^{y}}{y!}$$

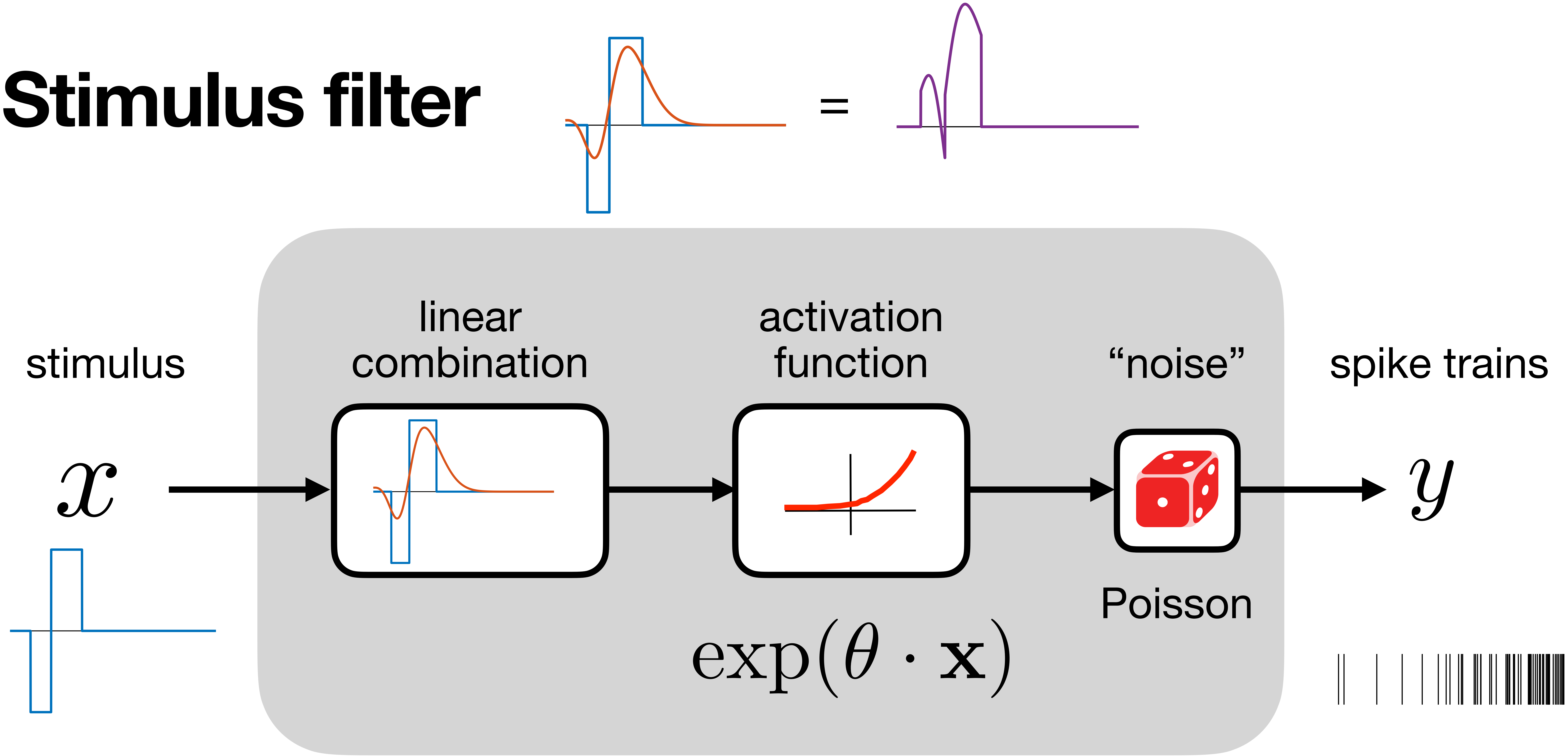
For Poisson GLM

$$\lambda = \exp\left(\sum_n \theta_n x_n\right)$$

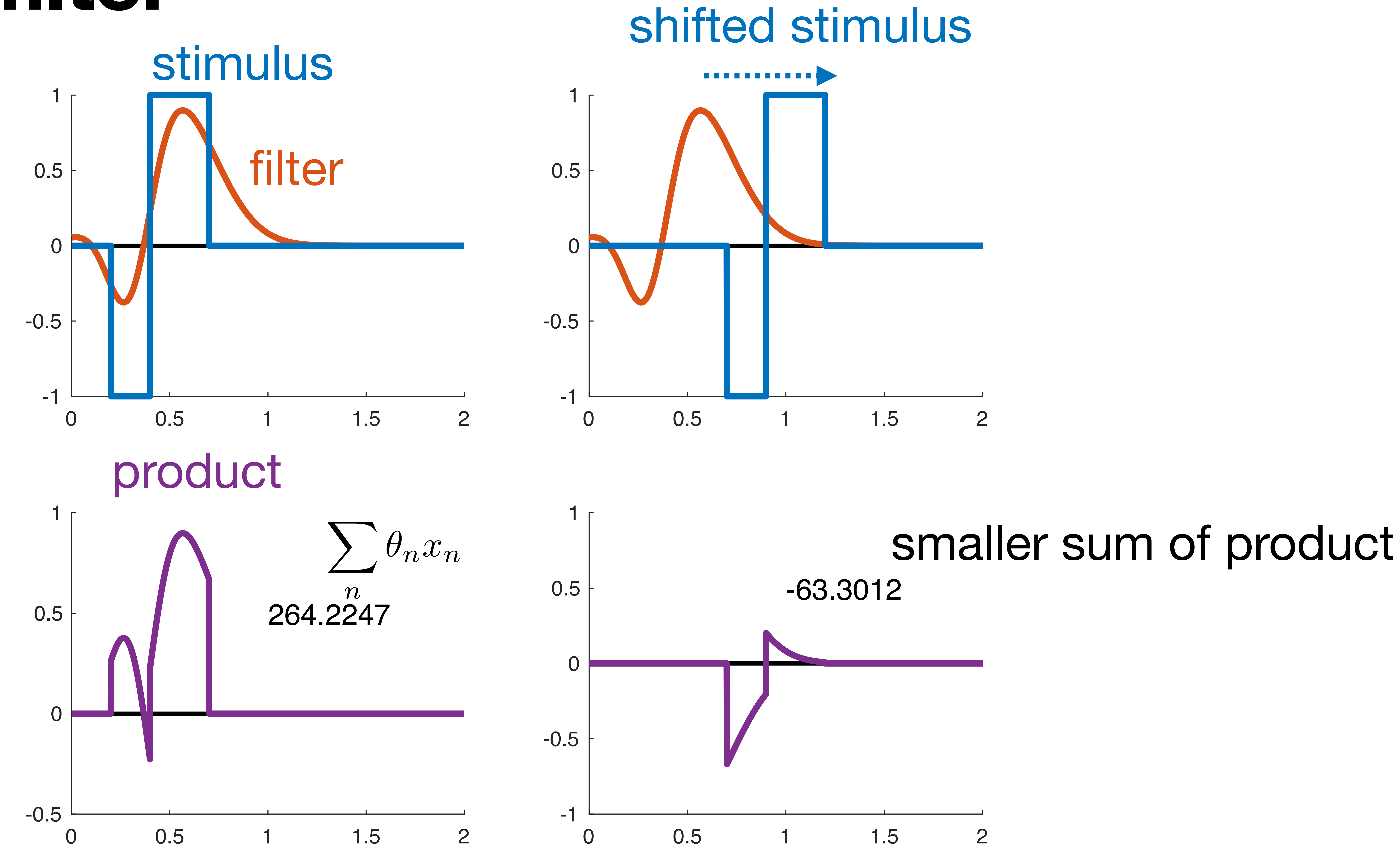
Stimulus filter



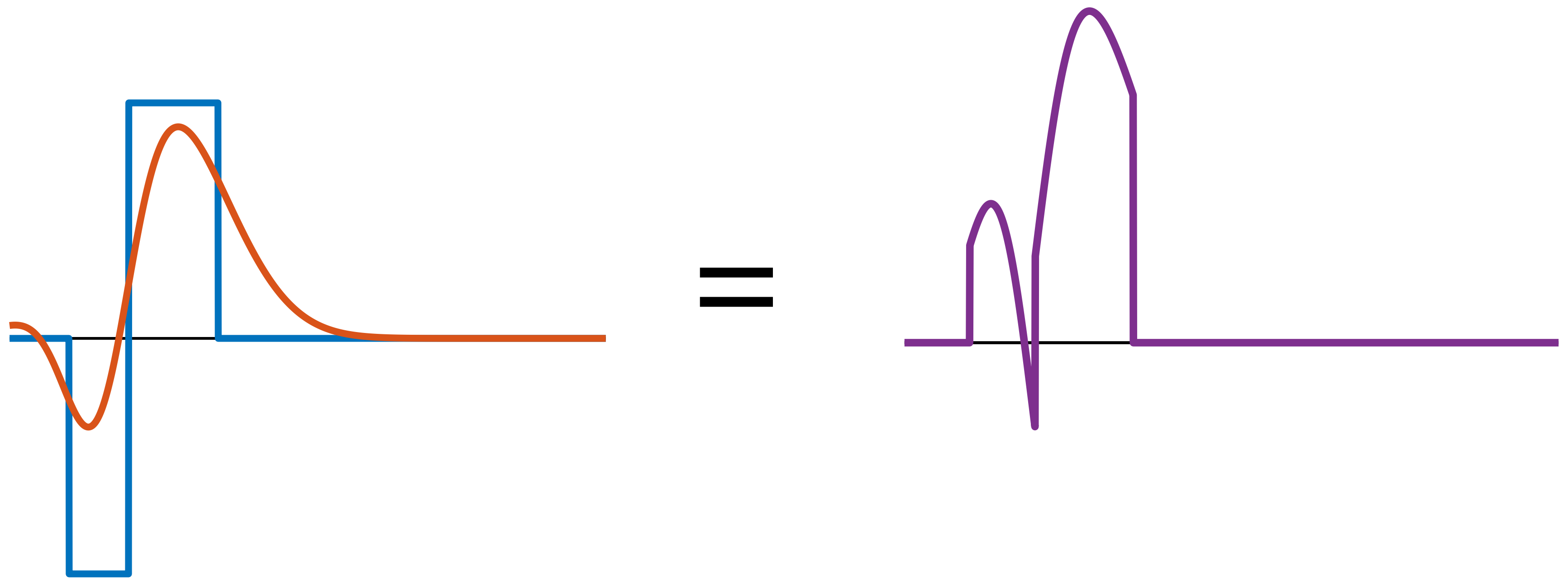
Stimulus filter

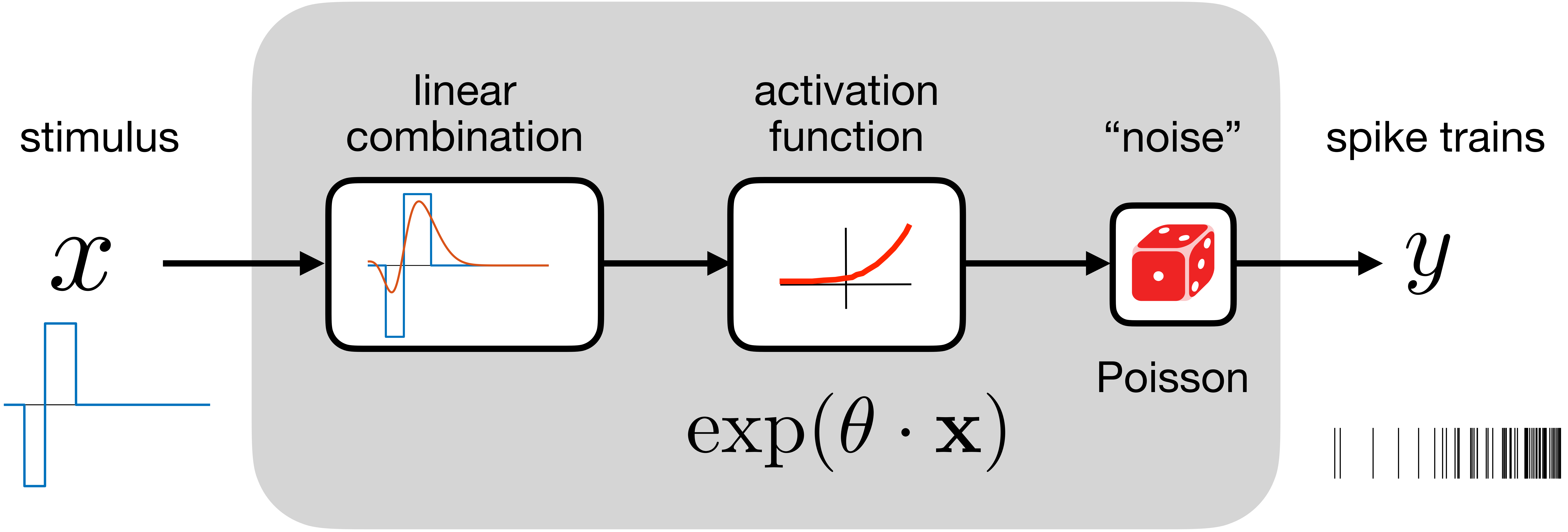


Stimulus filter

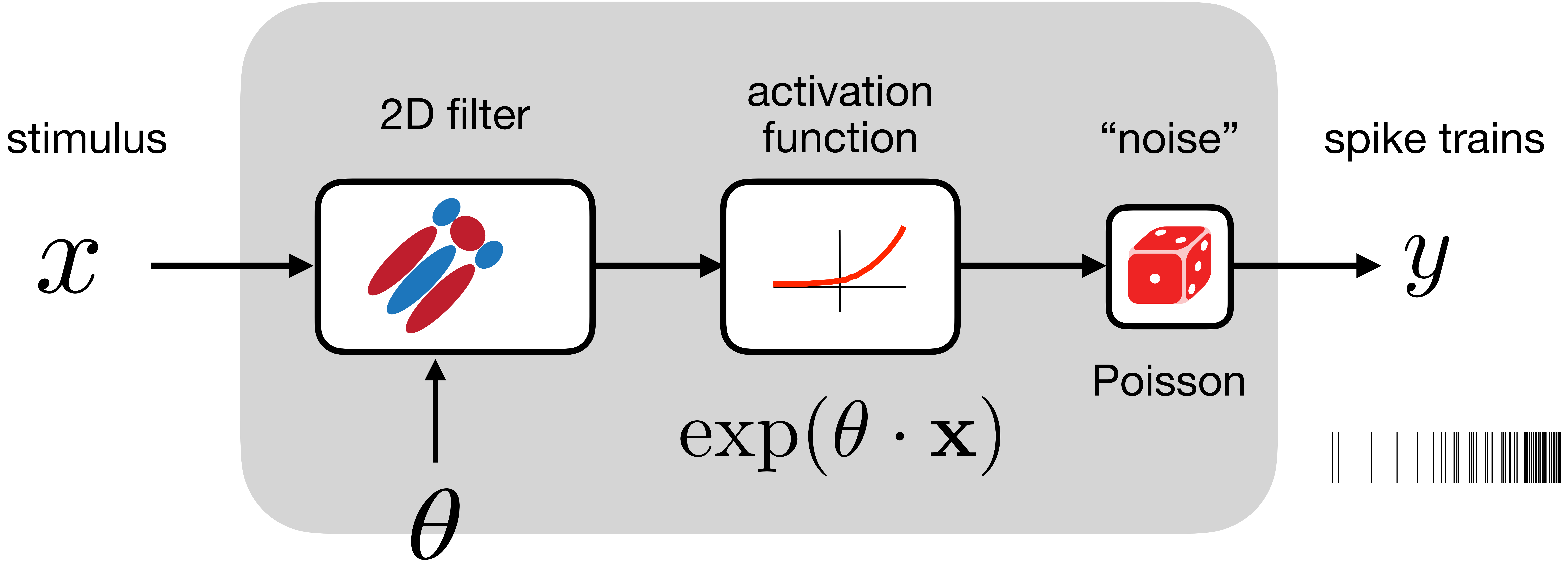


Stimulus filter

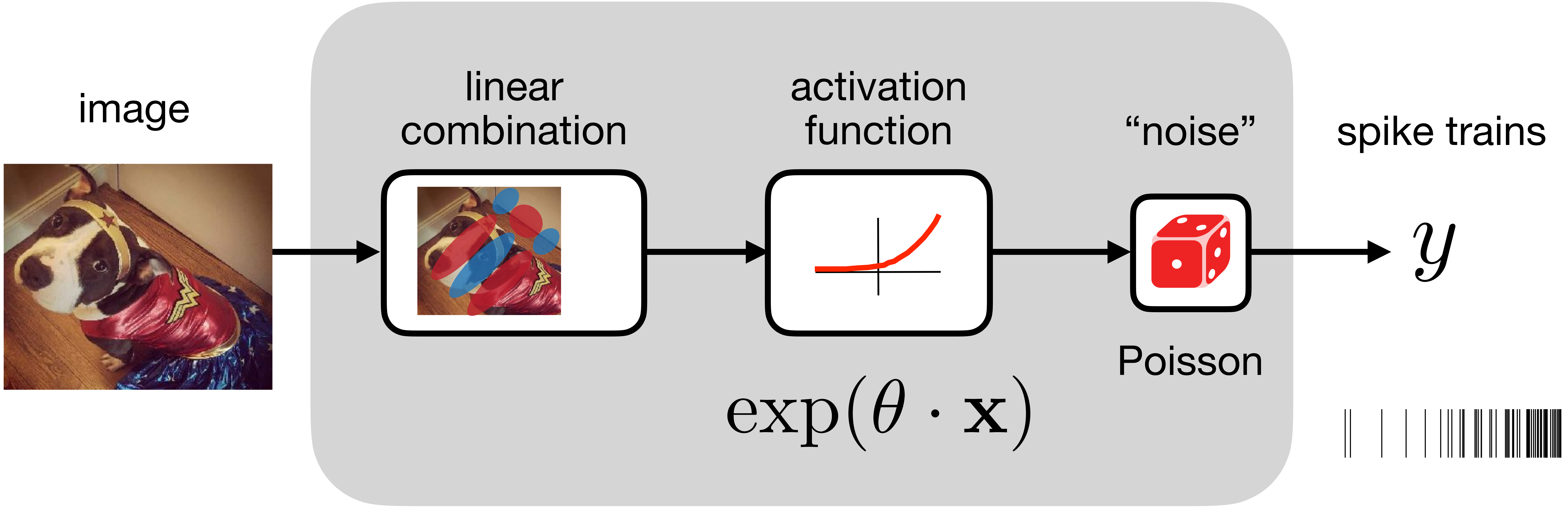




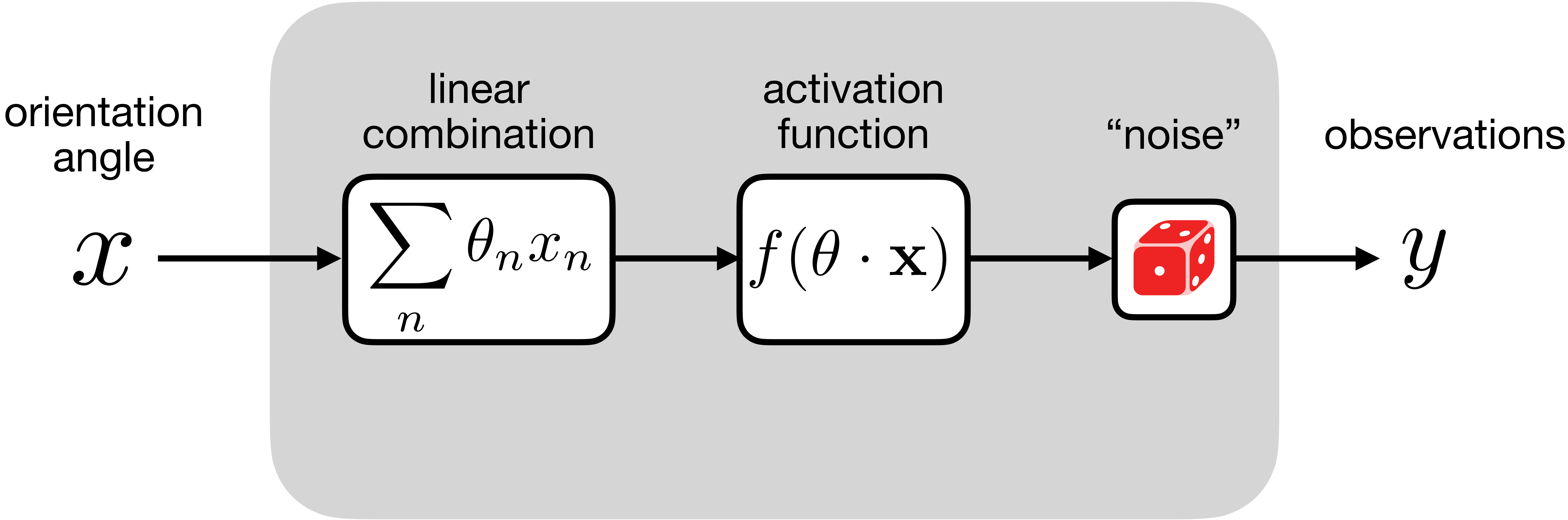
Stimulus filter



Stimulus filter

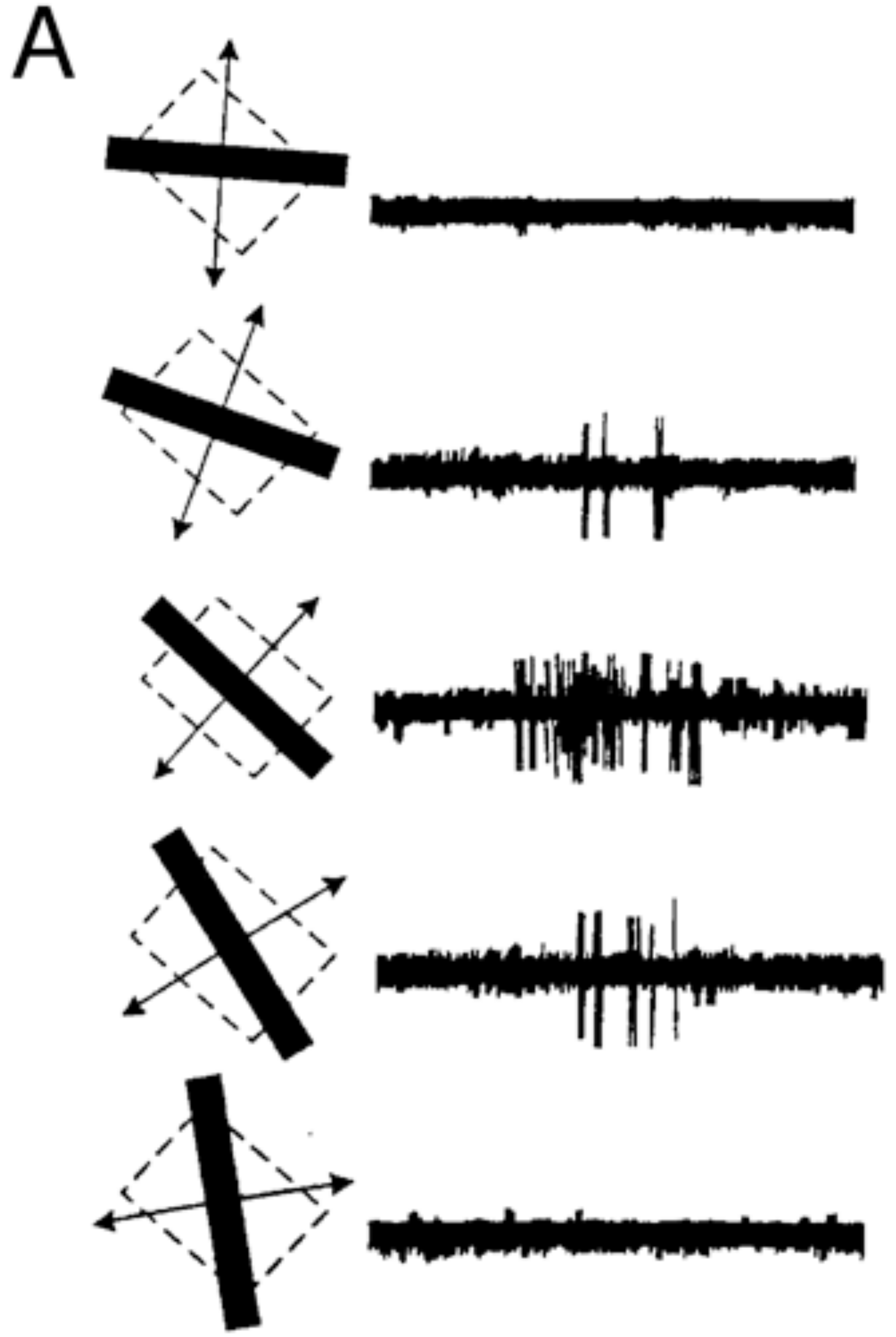


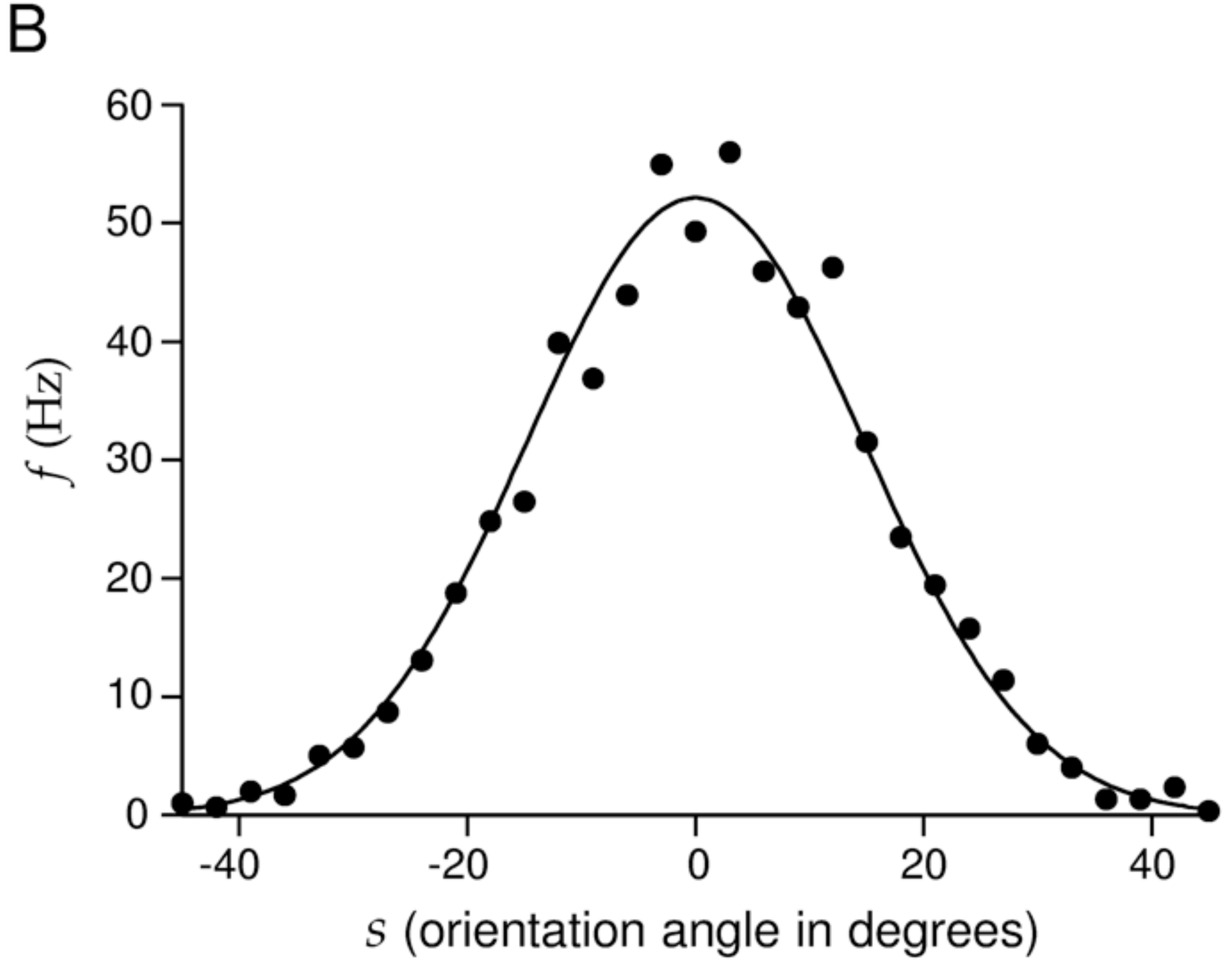
Nonlinear tuning (eg. cosine tuning)



Primary Visual Cortex
("V1")

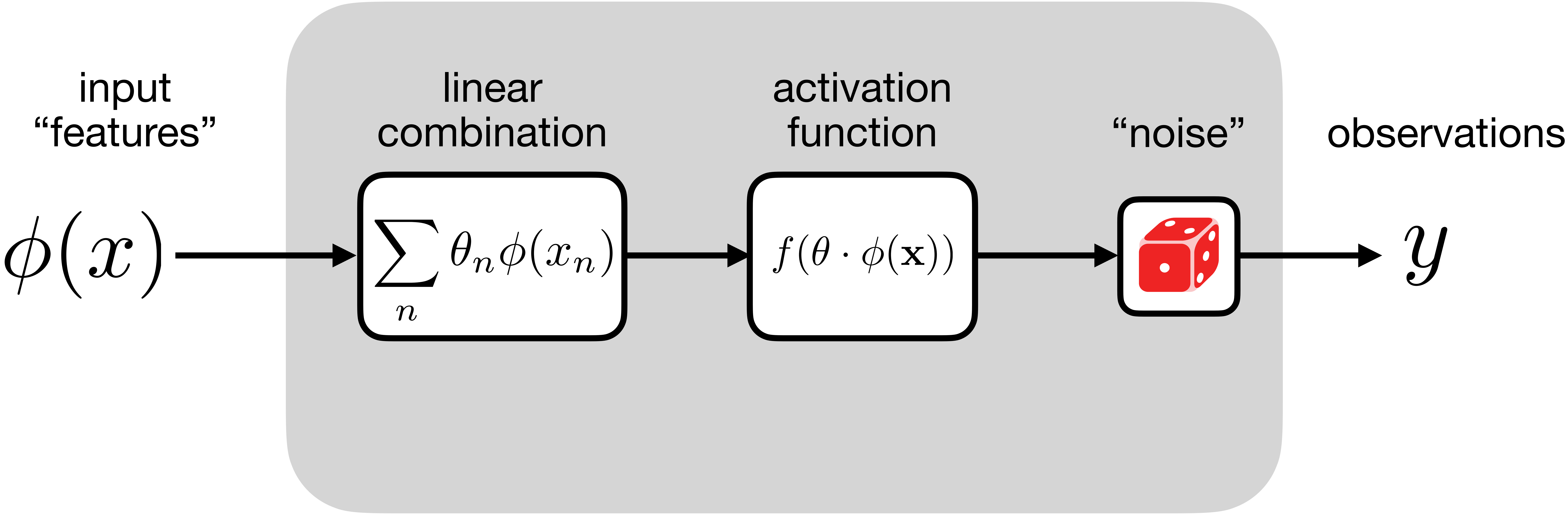
Tuning Curve
("Orientation tuned")



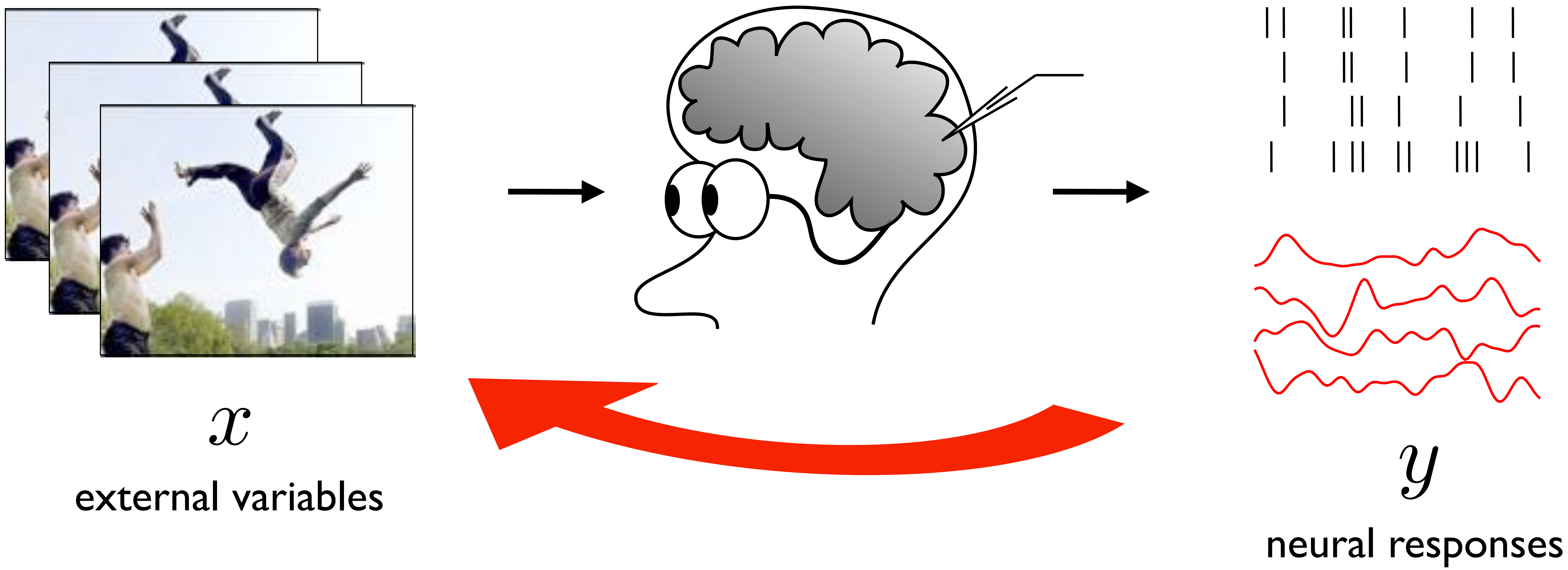


Hubel & Weisel, 1968

Nonlinear inputs



examples of $\phi(x)$: x^2 $\log(x)$ $\cos(x)$

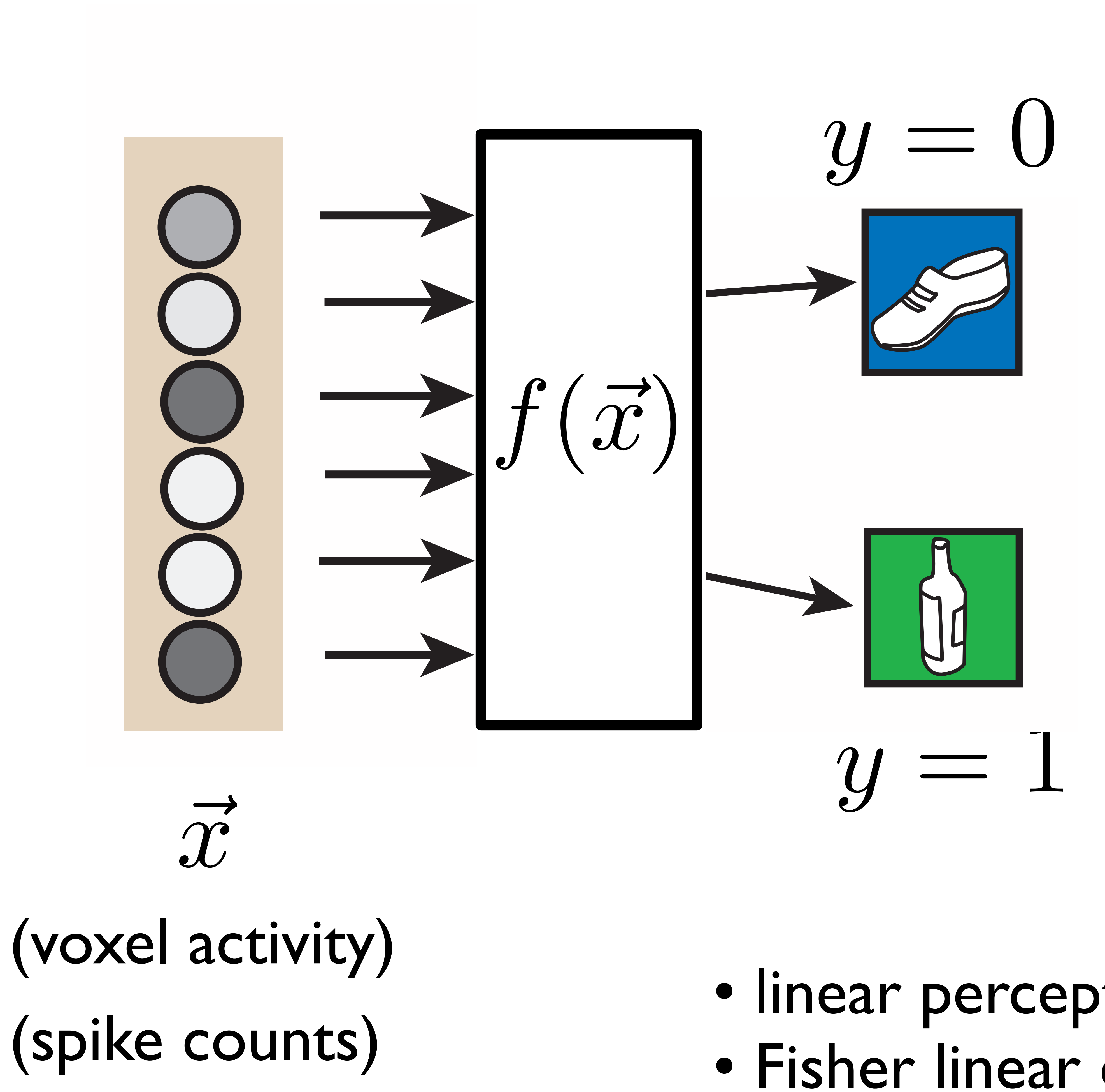


Decoding

- classification
- regression

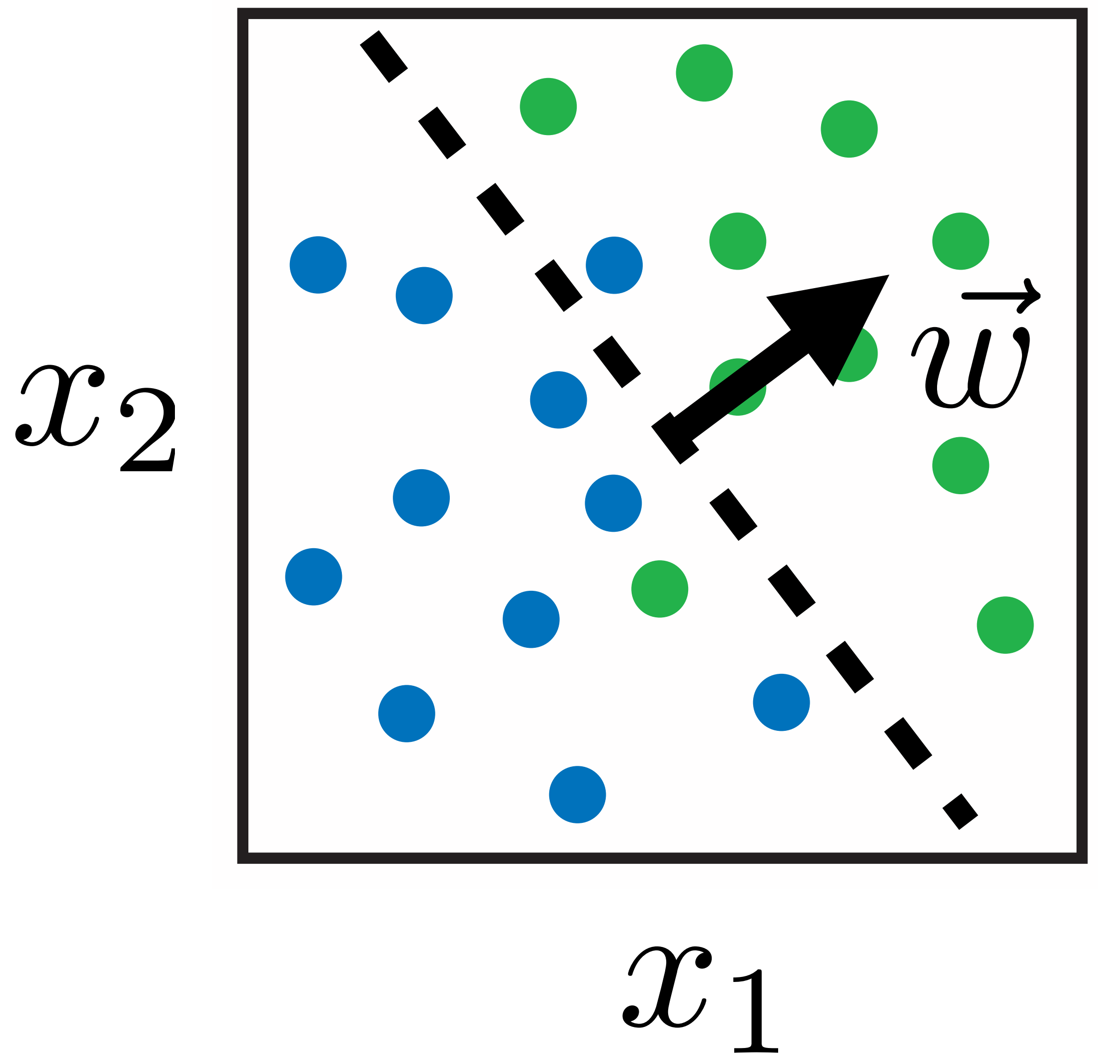
Classification

- mapping from vector input to discrete category



linear classifier

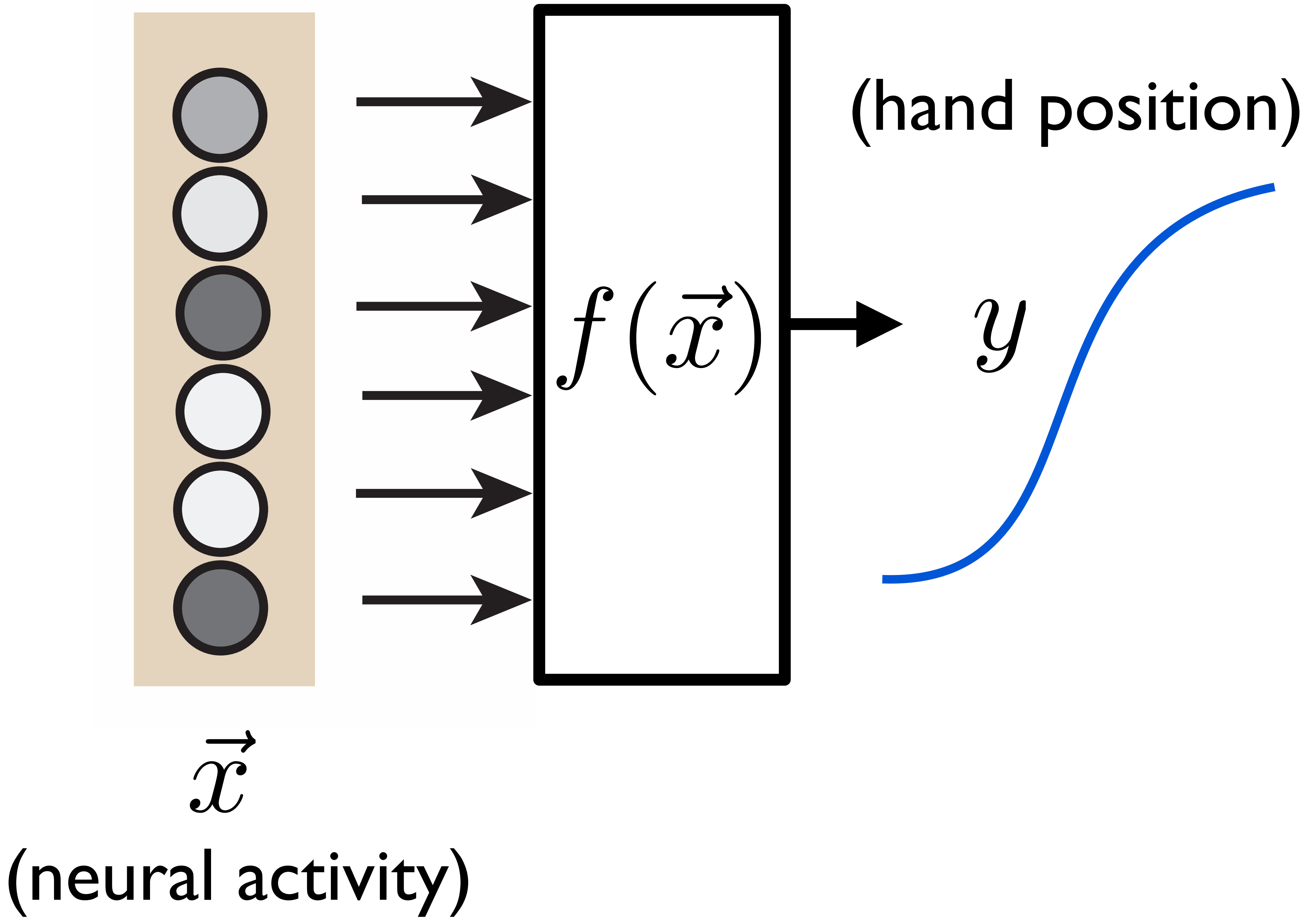
$$\vec{x} \cdot \vec{w} - b > 0$$



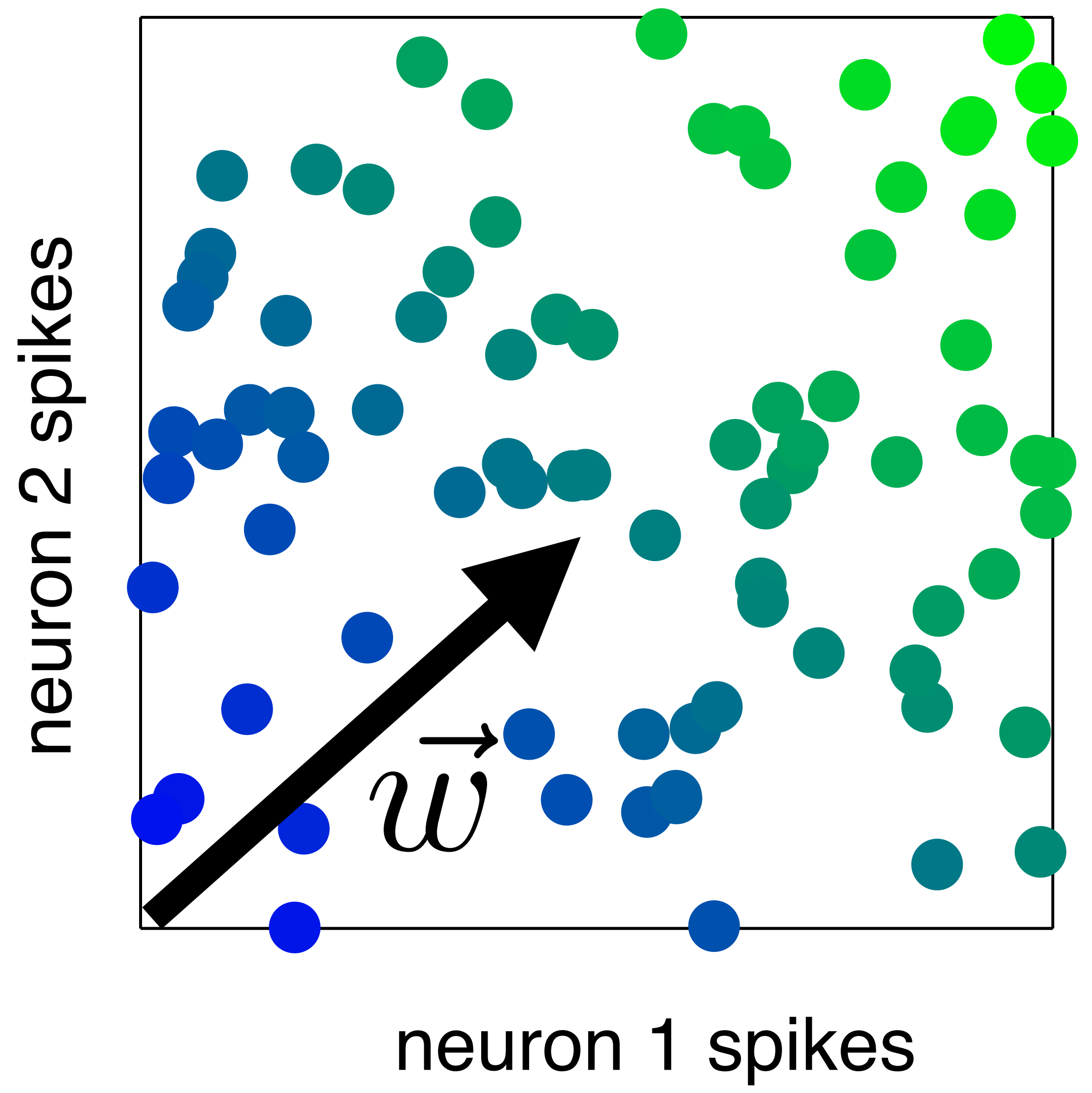
- linear perceptron
- Fisher linear discriminant
- support vector machine (SVM)

Regression

- output continuous instead of discrete



$$y = \vec{x} \cdot \vec{w}$$

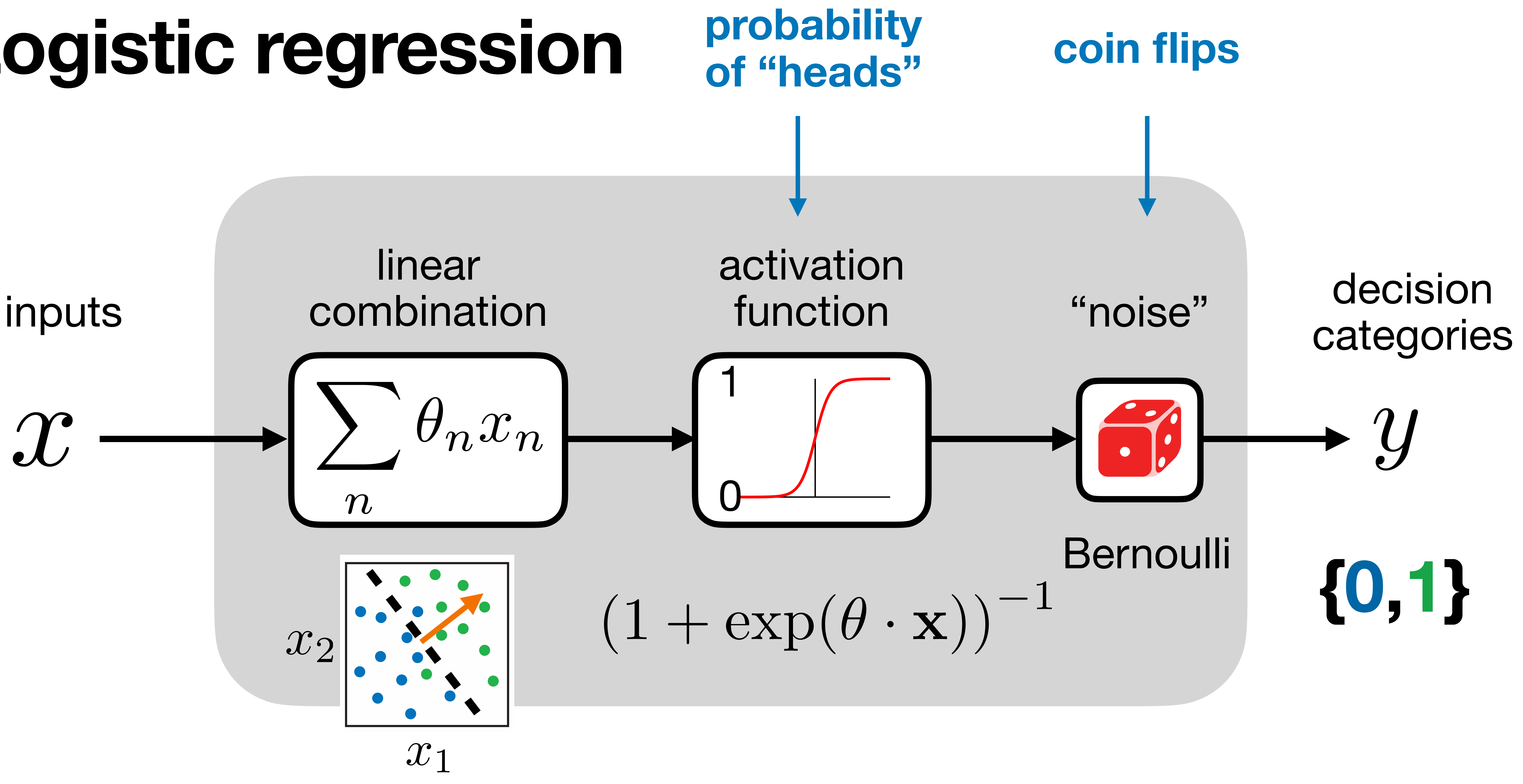


- can transform classification problems into regression problems ("logistic regression"):

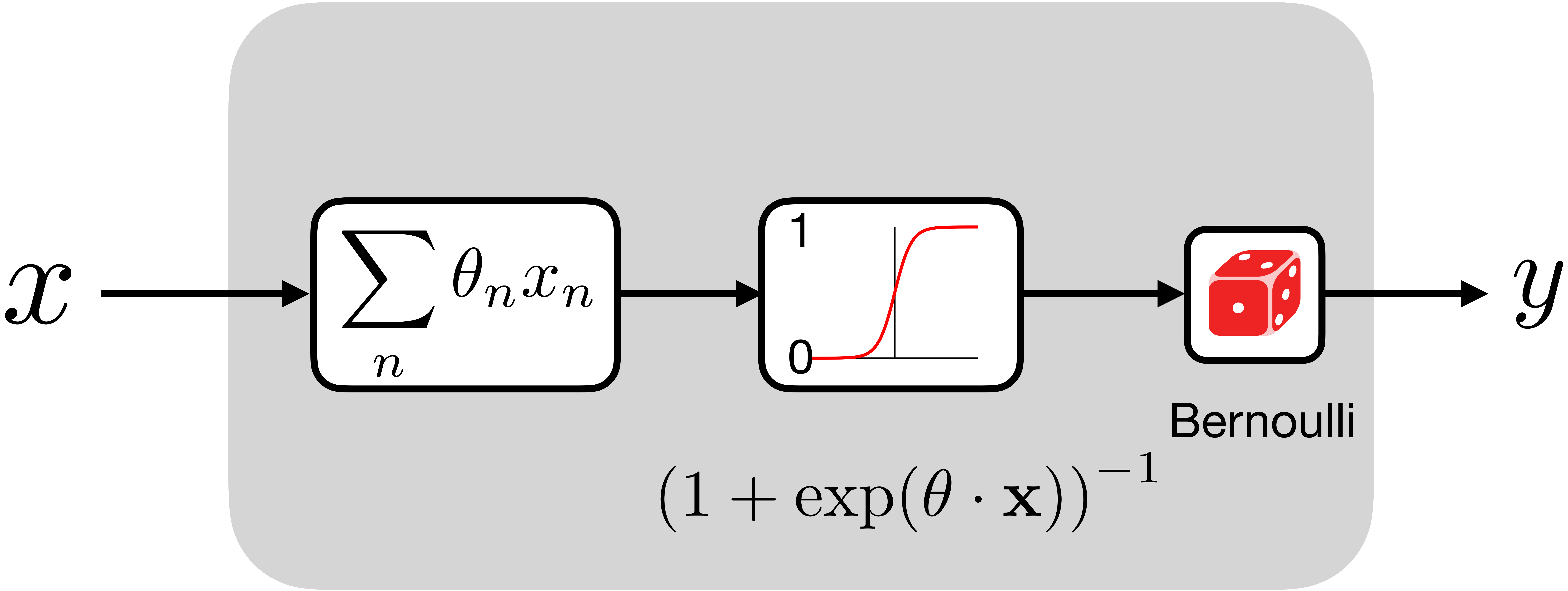
probability of being in category

$$p(y = 1) = f(\vec{x})$$

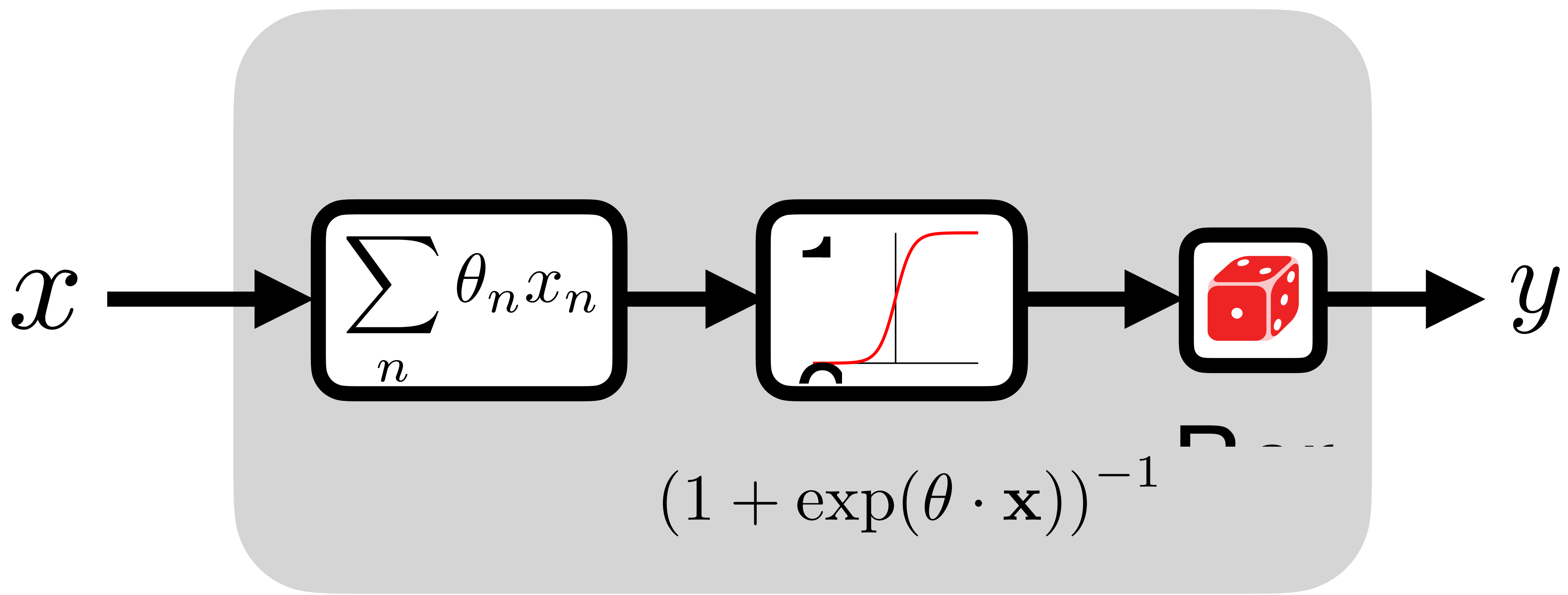
Logistic regression



Logistic regression



Logistic regression



$$y \sim \mathrm{Bernoulli}(q)$$

$$P(y|q) = q^y (1-q)^{1-y}$$

for logistic GLM

$$q = \frac{1}{1 + \exp\left(\sum_n \theta_n x_n\right)}$$

Today’s Tutorials

- Learning stimulus filters
  - Gaussian, nonlinear, and Poisson regression
  - Understand how to…
    - build a **design matrix**
    - use **optimization** software to learn parameters of a model
    - evaluate and communicate the consequences of different modeling choices
- Logistic regression
  - Build a classifier
  - Understand how to…
    - do logistic regression
    - evaluate classifier accuracy
    - do cross validation
  - We will skip r**egularization** for now and pick it up again on Friday

Thank you for listening!

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