Solution of the TEAM 13 problem

3d Non-Linear Magnetostatic Model

Valentin Hanser, TU Wien

January 9, 2019

1 Getting started

The script dependencies are:

- ngsolve
- numpy
- scipy
- matplotlib
- the .vol file of the geometry *./team13_mesh.vol*

If the geometry file is erroneous, execute the python script `./geometry.py`, which generates the necessary `.vol` file.

```
python3 geometry.py [-fullProblem True/False]
```

2 Problem Description

This script solves the T.E.A.M problem 13 (3-D Non-Linear Magnetostatic Model). The problem description can be found under the link.
The problem is based on the Maxwell equations with the corresponding boundary conditions

$$\nabla \times \mathbf{H} = \mathbf{J}, \qquad \mathbf{H} \times \mathbf{n} = \mathbf{0} \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0, \qquad \mathbf{B} \cdot \mathbf{n} = 0 \tag{2}$$

for magnetostatic phenomena. The second equation allows the introduction of a magnetic vector potential $\mathbf{A}$ as

$$\mathbf{B} = \nabla \times \mathbf{A}. \tag{3}$$

For the weak formulation

$$\int_\Omega \frac{1}{\mu(|\mathbf{B}|)} \nabla \times \mathbf{A} \cdot \nabla \times \mathbf{v} \, d\Omega = \int_{\Omega_c} \mathbf{J} \cdot \mathbf{v} \, d\Omega \tag{4}$$

is used, which considers the non-linear material relationship

$$\mathbf{H} = \frac{1}{\mu(|\mathbf{B}|)}\mathbf{B}. \tag{5}$$

Wherein $\mathbf{A}$ and $\mathbf{v}$ are the trial and the test function, Ω is the domain of interest and Ω_c is the domain of the coil.
For a unique solution, an additional regularisation term with a small $\varepsilon > 0$ has been added:

$$\int_\Omega \frac{1}{\mu(|\mathbf{B}|)} \nabla \times \mathbf{A} \cdot \nabla \times \mathbf{v} \, d\Omega + \int_\Omega \varepsilon \, \mathbf{A} \cdot \mathbf{v} \, d\Omega = \int_{\Omega_c} \mathbf{J} \cdot \mathbf{v} \, d\Omega \tag{6}$$

The magnetic flux density of the solution is presented in Fig. 1.

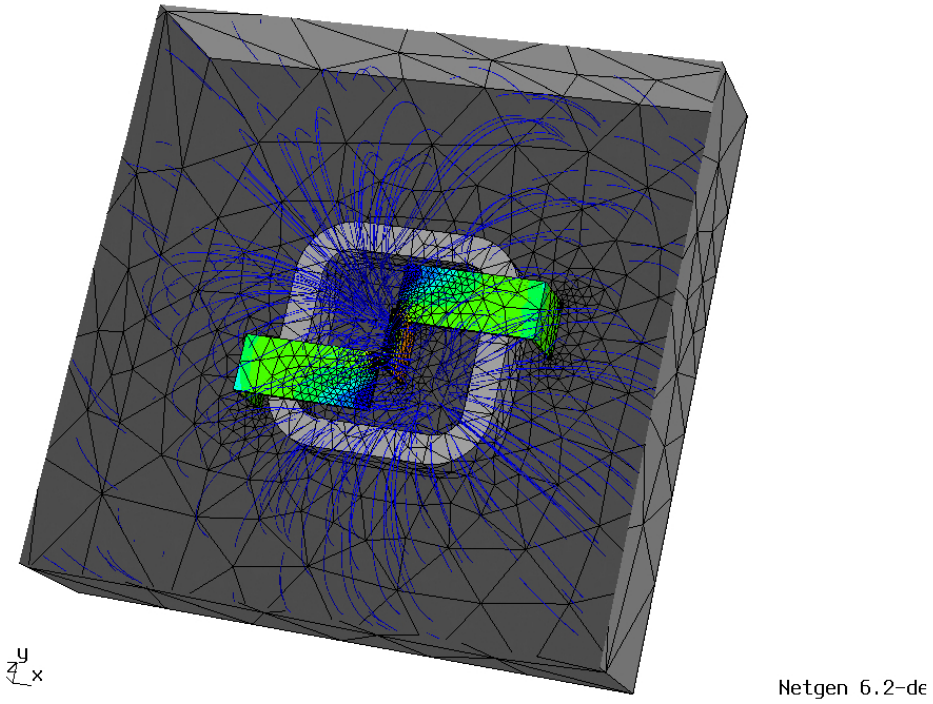


Figure 1: Illustration of the the magnetic flux density in the steel channels and magnetic field lines

3 Coding

3.1 Imports

Importing the packages *ngsolve, netgen, numpy, ...* enables all functionalities of this script. Additionally, the number of used threads is reduced and the permeability of vacuum μ_0 is set.

```
In [1]: from ngsolve import *
        from netgen.csg import *
        from ngsolve.internal import *
```

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib
import time

SetNumThreads(10)
import netgen.gui

mu0 = 4*np.pi*1e-7
```

3.2 Setting Arguments

First of all, the simulation parameters need to be set. The space order defines the used order of the trial function and the test function within the finite element space. The variable $\mathtt{I}_{ON}$ defines the value of Ampere-turns in the coil. Considering the Section 2, the value for the Ampere-turns is set to $1,000$ or $3,000$.

```
In [2]: space_order = 2
        ION = 1000                    # Ampere-turns 1000 or 3000
```

3.3 Meshing

Next, the available mesh is loaded. The command $Curve(5)$ determines the roundness of the mesh and is obligatory for the provided geometry with its curved corners.

```
In [3]: mesh = Mesh("./team13_mesh.vol")
        # mesh = Mesh("./team13_mesh_full.vol") # activate this to simulate the full problem
        mesh.Curve(5)

        # check domains
        val = {"corner_right_back":1 , "corner_left_back":1, "corner_left_front":1, "corner_righ
        domains = CoefficientFunction([val[mat] if mat in val.keys() else 0 for mat in mesh.GetM
        Draw(domains, mesh, "domains", draw_surf=False)

        # some viewing optioins
        ngsolve.internal.viewoptions.clipping.notdomain=1
        ngsolve.internal.visoptions.clipsolution="scal"
        ngsolve.internal.viewoptions.clipping.enable=1
        ngsolve.internal.viewoptions.clipping.ny=0
        ngsolve.internal.viewoptions.clipping.nz=-1
        ngsolve.internal.viewoptions.clipping.dist=-0.21
        ngsolve.internal.viewoptions.clipping.notdomain=1

        Rotate(0, 120)

        Redraw()
```

To check the correct assignment of the domains $\{air, iron, coil\}$, all corresponding subdomains are coloured accordingly. In Fig. 2 the coil is blue, the steel channels are red and the air is transperent. For a meaningful display a clipping plane was inserted.

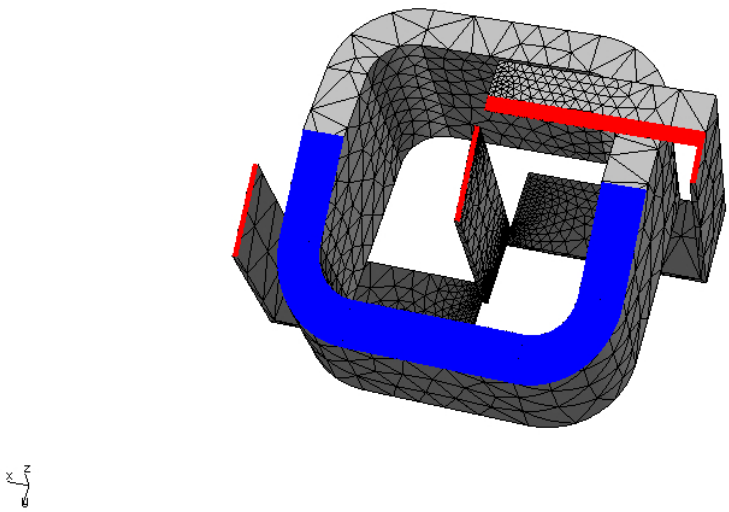


Figure 2: Illustration of the geometry

3.4 Finite Element Space

The finite element space $H(curl, \Omega)$ is selected and homogeneous Dirichlet boundary conditions are prescribed on the far boundary. Additionally * the solution, i.e. magnetic vector potential $\mathbf{A}$, is defined as `sol`, * trial and test function are introduced, * the *CoefficientFunction* `B` for the magnetic flux density is determined and * `B` is drawn.

```
In [4]: # create fe space
        fes = HCurl(mesh, order=space_order, dirichlet="outer", nograds=True)

        # magnetic vector potential as GridFunction
        sol = GridFunction(fes, "A")
        sol.vec[:] = 0

        # Test- and Trialfunction
        u = fes.TrialFunction()
        v = fes.TestFunction()

        # magnetic flux density
        B = curl(sol)
        Draw(B, mesh, "B", draw_surf=False)
```

3.5 Material Parameters

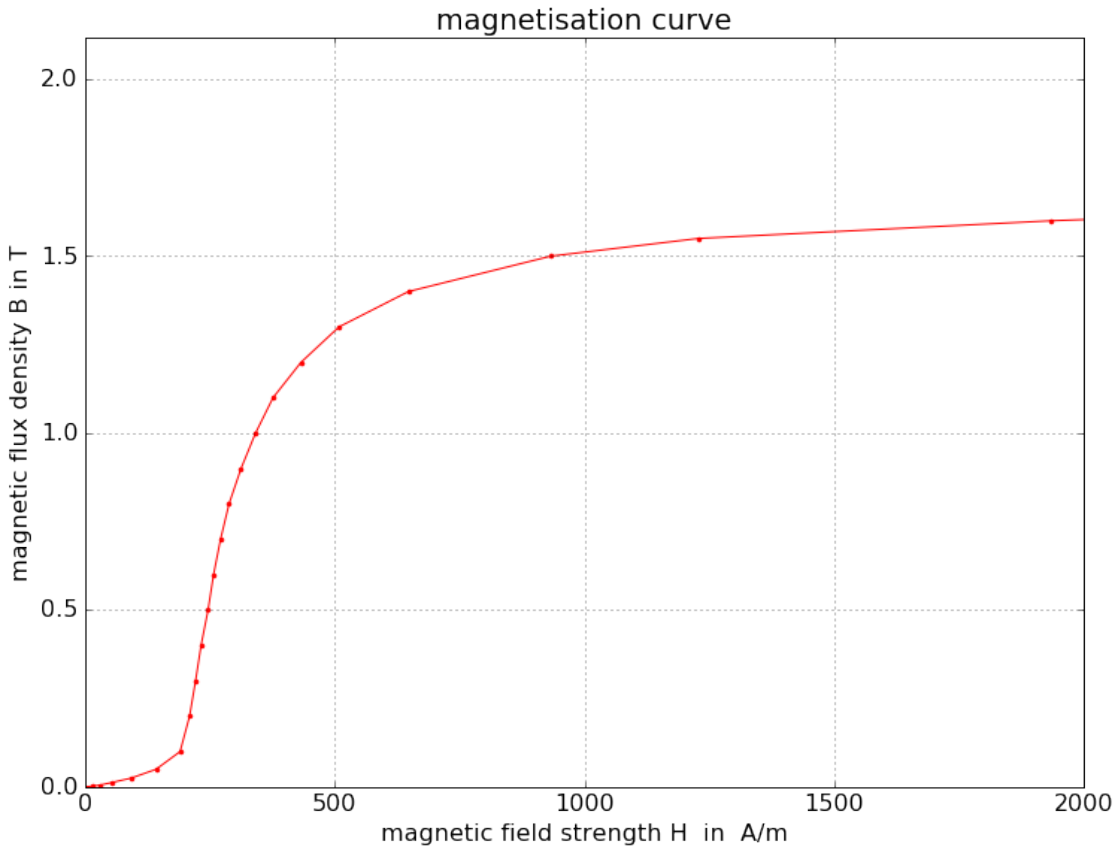
The non-linear magnetisation curve of the material is defined in the Section 2 .

```
In [5]: H_KL = [ -4.47197834e-13, 1.60000000e+01, 3.00000000e+01, 5.40000000e+01\
            , 9.30000000e+01, 1.43000000e+02, 1.91000000e+02, 2.10000000e+02\
```

```
, 2.22000000e+02, 2.33000000e+02, 2.47000000e+02, 2.58000000e+02\
, 2.72000000e+02, 2.89000000e+02, 3.13000000e+02, 3.42000000e+02\
, 3.77000000e+02, 4.33000000e+02, 5.09000000e+02, 6.48000000e+02\
, 9.33000000e+02, 1.22800000e+03, 1.93400000e+03, 2.91300000e+03\
, 4.99300000e+03, 7.18900000e+03, 9.42300000e+03, 9.42300000e+03\
, 1.28203768e+04, 1.65447489e+04, 2.07163957e+04, 2.55500961e+04\
, 3.15206135e+04, 4.03204637e+04, 7.73038295e+04, 1.29272791e+05\
, 1.81241752e+05, 2.33210713e+05, 2.85179674e+05, 3.37148635e+05\
, 3.89117596e+05, 4.41086557e+05, 4.93055518e+05, 5.45024479e+05\
, 5.96993440e+05, 6.48962401e+05, 7.00931362e+05, 7.52900323e+05\
, 8.04869284e+05, 8.56838245e+05, 9.08807206e+05, 9.60776167e+05\
, 1.01274513e+06, 1.06471409e+06, 1.11668305e+06, 1.16865201e+06\
, 1.22062097e+06, 1.27258993e+06, 1.32455889e+06, 1.37652785e+06\
, 1.42849682e+06, 1.48046578e+06, 1.53243474e+06, 1.58440370e+06\
, 1.63637266e+06, 1.68834162e+06, 1.74031058e+06, 1.79227954e+06\
, 1.84424850e+06, 1.89621746e+06, 1.94818643e+06, 2.00015539e+06\
, 2.05212435e+06, 2.10409331e+06, 2.15606227e+06, 2.20803123e+06\
, 2.26000019e+06]

B_KL = [  0.00000000e+00, 2.50000000e-03, 5.00000000e-03, 1.25000000e-02\
, 2.50000000e-02, 5.00000000e-02, 1.00000000e-01, 2.00000000e-01\
, 3.00000000e-01, 4.00000000e-01, 5.00000000e-01, 6.00000000e-01\
, 7.00000000e-01, 8.00000000e-01, 9.00000000e-01, 1.00000000e+00\
, 1.10000000e+00, 1.20000000e+00, 1.30000000e+00, 1.40000000e+00\
, 1.50000000e+00, 1.55000000e+00, 1.60000000e+00, 1.65000000e+00\
, 1.70000000e+00, 1.75000000e+00, 1.80000000e+00, 1.80000000e+00\
, 1.86530612e+00, 1.93061224e+00, 1.99591837e+00, 2.06122449e+00\
, 2.12653061e+00, 2.19183673e+00, 2.25714286e+00, 2.32244898e+00\
, 2.38775510e+00, 2.45306122e+00, 2.51836735e+00, 2.58367347e+00\
, 2.64897959e+00, 2.71428571e+00, 2.77959184e+00, 2.84489796e+00\
, 2.91020408e+00, 2.97551020e+00, 3.04081633e+00, 3.10612245e+00\
, 3.17142857e+00, 3.23673469e+00, 3.30204082e+00, 3.36734694e+00\
, 3.43265306e+00, 3.49795918e+00, 3.56326531e+00, 3.62857143e+00\
, 3.69387755e+00, 3.75918367e+00, 3.82448980e+00, 3.88979592e+00\
, 3.95510204e+00, 4.02040816e+00, 4.08571429e+00, 4.15102041e+00\
, 4.21632653e+00, 4.28163265e+00, 4.34693878e+00, 4.41224490e+00\
, 4.47755102e+00, 4.54285714e+00, 4.60816327e+00, 4.67346939e+00\
, 4.73877551e+00, 4.80408163e+00, 4.86938776e+00, 4.93469388e+00\
, 5.00000000e+00]
bh_curve = BSpline (2, [0]+list(B_KL), list(H_KL)) # [0] + is needed!
```

In [6]:
```
# create figure
plt.figure(1, figsize=[12, 9])
plt.clf()
plt.plot(H_KL, B_KL, '.-r')
plt.xlim(0, 2000)
plt.ylim(0, 2.12)
plt.grid()
```

```
plt.title("Magnetisation curve")
plt.xlabel("Magnetic Field Strength H  in  A/m")
plt.ylabel("Magnetic Flux Density B in T")
font = {'size'   : 16}
matplotlib.rc('font', **font)
plt.show(block=False)
```



The non-linear problem is solved by minimising the the functional

$$F(\mathbf{A}) = \int_{\Omega} w(|\nabla \times \mathbf{A}|)\, d\Omega - \int_{\Omega_c} \mathbf{J} \cdot \mathbf{A}\, d\Omega. \tag{7}$$

Therefore the magnetic energy density

$$w(B) = \int_0^B H(B')\, dB' \tag{8}$$

has to be set

```
In [7]: energy_dens = bh_curve.Integrate()     # to be minimised
```

3.6 Stiffness Matrix and Dirichlet Boundaries

The stiffness matrix is set on all regions and the regularisation term mentioned in the section Section 2 is added.

```
In [8]: a = BilinearForm(fes, symmetric=True)

        a += SymbolicBFI(1/mu0 * curl(u)*curl(v), definedon=~mesh.Materials("iron"))
        a += SymbolicEnergy(energy_dens(sqrt(1e-12+curl(u)*curl(u))), definedon=mesh.Materials("
        a += SymbolicBFI(1e-1*u*v)  # regularisation

        # preconditioner
        c = Preconditioner(a, type="direct", inverse="sparsecholesky")
```

The boundary condition

$$\mathbf{B} \cdot \mathbf{n} = 0 \tag{9}$$

is prescribed with the magnetic vector potential by

$$\nabla \times \mathbf{A} \cdot \mathbf{n} = 0 \tag{10}$$

$$\mathbf{A} = \mathbf{0} \qquad \text{on } \Gamma_{outer}. \tag{11}$$

3.7 Biot-Savart Field and Neumann Boundaries

In this section, the impressed current density in the coil is modelled. This has to be done carefully, since the orientation of the current has to be defined in the curved corners consistently.

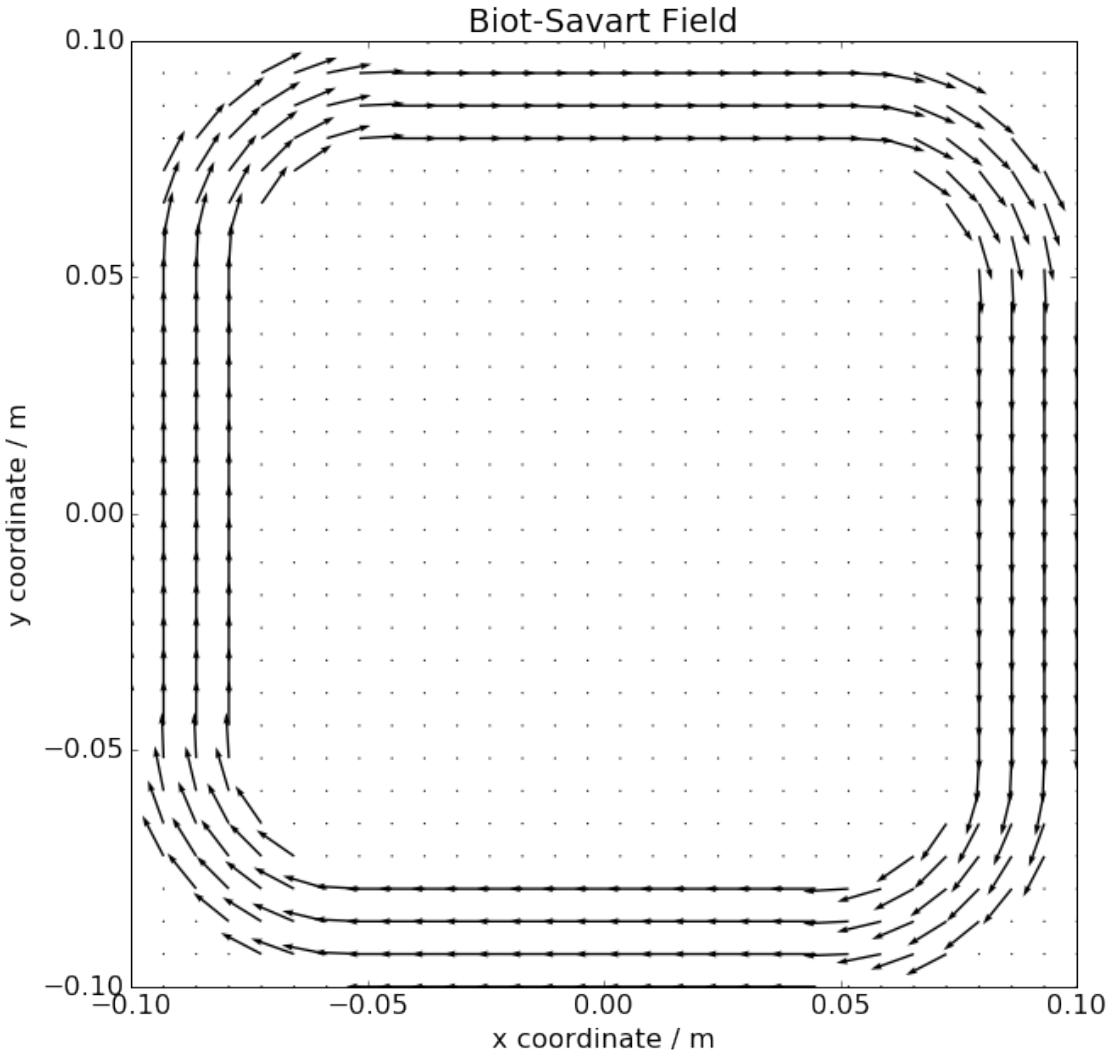
Without details:

```
In [9]: A = 2500*1e-6
        J0 = ION/A
        # +++ bricks +++
        J_brick_back = [1, 0]
        J_brick_front = [-1, 0]
        J_brick_left = [0, 1]
        J_brick_right = [0, -1]

        # +++ corners +++
        # right back
        x_right_back = x - 0.050
        y_right_back = y - 0.050
        r_right_back = (x_right_back**2 + y_right_back**2)**(1/2)
        J_corner_right_back = [1/r_right_back * y_right_back, -1/r_right_back * x_right_back]

        # left back
        x_left_back = x + 0.050
        y_left_back = y - 0.050
        r_left_back = (x_left_back**2 + y_left_back**2)**(1/2)
```

```
J_corner_left_back =  [1/r_left_back * y_left_back, -1/r_left_back * x_left_back]


# left front
x_left_front = x + 0.050
y_left_front = y + 0.050
r_left_front = (x_left_front**2 + y_left_front**2)**(1/2)
J_corner_left_front = [1/r_left_front * y_left_front, -1/r_left_front * x_left_front]

# right front
x_right_front = x - 0.050
y_right_front = y + 0.050
r_right_front = (x_right_front**2 + y_right_front**2)**(1/2)
J_corner_right_front = [1/r_right_front * y_right_front, -1/r_right_front * x_right_fron

# J
val={   "corner_right_back":J_corner_right_back, "corner_left_back":J_corner_left_back,
        "corner_left_front":J_corner_left_front, "corner_right_front":J_corner_right_fro
        "brick_back":J_brick_back, "brick_left":J_brick_left, \
        "brick_front":J_brick_front, "brick_right":J_brick_right}

J = J0 * CoefficientFunction([val[mat][0] if mat in val.keys() else 0 for mat in mesh.Ge
    J0 * CoefficientFunction([val[mat][1] if mat in val.keys() else 0 for mat in mesh.Ge

Draw(J, mesh, "J")
```

In [10]:
```
# figure
Nx = 30
Ny = 30
xi = np.linspace(-0.100, 0.100, Nx)
yi = np.linspace(-0.100, 0.100, Ny)

X, Y= np.meshgrid(xi, yi)
J_T = np.zeros([Nx, Ny, 2])

for i in range(Nx):
    for j in range(Ny):
        tmp = J(mesh(xi[i], yi[j], 0))
        J_T[i, j, 0] = tmp[0]
        J_T[i, j, 1] = tmp[1]
J_T[:, :, 0] = np.transpose(J_T[:, :, 0])
J_T[:, :, 1] = np.transpose(J_T[:, :, 1])
plt.figure(1, figsize=[10, 10])
plt.quiver(X, Y, J_T[:, :, 0], J_T[:, :, 1])
plt.xlabel("x coordinate / m")
plt.ylabel("y coordinate / m")
plt.title("Biot-Savart Field")
plt.xlim([-0.1, 0.1])
```

```
plt.ylim([-0.1, 0.1])
font = {'size'   : 16}
matplotlib.rc('font', **font)
plt.show(block=False)
```



The absolute value of the current density $|\mathbf{J}|$ is constant in the whole volume of the coil. The *CoefficientFunction* `J` is used for the right-hand side of the weak formulation.

```
In [11]: f = LinearForm(fes)
         f += SymbolicLFI(J*v)
```

3.8 Solving the Problem

The problem is iteratively solved by finding the minimum energy until an error falls bellow a specified limit.

```
In [12]: t_simulation = time.time()
         with TaskManager():
             f.Assemble()

             err = 1
             it = 1

             # create memories
             au = sol.vec.CreateVector()
             r = sol.vec.CreateVector()
             w = sol.vec.CreateVector()
             sol_new = sol.vec.CreateVector()

             while err > 1e-10:
                 print ("nonlinear iteration", it)
                 it = it+1

                 # calculate current energy
                 E0 = a.Energy(sol.vec) - InnerProduct(f.vec, sol.vec)
                 print ("Energy old = ", E0)

                 # solve linearized problem
                 a.AssembleLinearization(sol.vec)

                 a.Apply (sol.vec, au)
                 r.data = f.vec - au

                 # inverse of a
                 inv = CGSolver (mat=a.mat, pre=c.mat)
                 w.data = inv * r

                 # calculate error
                 err = InnerProduct (w, r)
                 print ("err = ", err)


                 sol_new.data = sol.vec + w
                 # calculate new energy
                 E = a.Energy(sol_new) - InnerProduct(f.vec, sol_new)
                 print ("Enew = ", E)
                 tau = 1
                 while E > E0:
                     tau = 0.5*tau
                     sol_new.data = sol.vec + tau * w
                     E = a.Energy(sol_new) - InnerProduct(f.vec, sol_new)
                     print ("tau = ", tau, "Enew =", E)

                 sol.vec.data = sol_new
```

```
                Redraw()
        t_simulation = time.time() - t_simulation

        print("simulation time %.3lf seconds" % t_simulation)
```

```
nonlinear iteration 1
Energy old =  2.1995519996926786e-13
err =  0.11287987817845671
Enew =  -0.06556406373261799
nonlinear iteration 2
Energy old =  -0.06556406373261797
err =  0.06294947408964463
Enew =  -0.09655327847509289
nonlinear iteration 3
Energy old =  -0.09655327847509289
err =  0.0003346625162502495
Enew =  -0.09674258973388922
nonlinear iteration 4
Energy old =  -0.09674258973388924
err =  5.918883902927794e-05
Enew =  -0.09677707186727481
nonlinear iteration 5
Energy old =  -0.09677707186727484
err =  1.2064440360197632e-05
Enew =  -0.09678405704979519
nonlinear iteration 6
Energy old =  -0.09678405704979519
err =  1.4031774878884814e-06
Enew =  -0.0967848313229861
nonlinear iteration 7
Energy old =  -0.09678483132298611
err =  8.03895170779946e-08
Enew =  -0.09678487155881134
nonlinear iteration 8
Energy old =  -0.09678487155881134
err =  1.0828326756244667e-11
Enew =  -0.0967848715642249
simulation time 64.768 seconds
```

3.9 Final Step

To show the final solution on the surface of the iron the finite element space $H^1(\Omega)$ is defined on *iron* only. Further, the solution is projected in to that space.

In [13]:
```
        # draw Bnorm only on iron
        fesBnorm = H1(mesh, definedon=mesh.Materials("iron"))
```

```
B_norm = GridFunction(fesBnorm, "|B|")
B_norm.Set(B.Norm())
Draw(B_norm)

# some viewing options
ngsolve.internal.viewoptions.clipping.dist=-0.167
```

4 Results

The goal of the T.E.A.M. problem 13 is to simulate the problem and to compare some characteristic values of the solution with measured values provided.
Without details:

1. The resulting B gets evaluated at several evenly spread points.
2. These points are the base of a 2d integration to obtain the magnetic flux Φ.
3. With the known crosssection, the averaged flux density can be computed.
4. Finally, a comparison with the measured values is done.

```
In [14]: from measurement import measure1to25, measure26to36, measure37to40

         # results according to Team Problem 13 description page 13
         B_msm = [   1.33, 1.329, 1.286, 1.225, 1.129, 0.985, 0.655, \
                     0.259, 0.453, 0.554, 0.637, 0.698, 0.755, 0.809, 0.901, 0.945, 0.954, 0.956
                     0.960, 0.965, 0.970, 0.974, 0.981, 0.984, 0.985]

         # measure avarage magnetic flux density
         B_sim_fir = measure1to25(B, mesh, draw=False)
         B_sim_sec = measure26to36(B, mesh)
         B_sim_thi = measure37to40(B, mesh)

         B_sim = np.hstack([B_sim_fir, B_sim_sec, B_sim_thi])

         # output
         for i in range(len(B_msm)):
             print("measurement %d: sim\t %.3lf, msm\t %.3lf, rel. err\t %.3lf" % (i + 1, B_sim[

         for i in range(i + 1, len(B_sim)):
             print("measurement %d: sim\t %.3lf" % (i + 1, B_sim[i]))
```

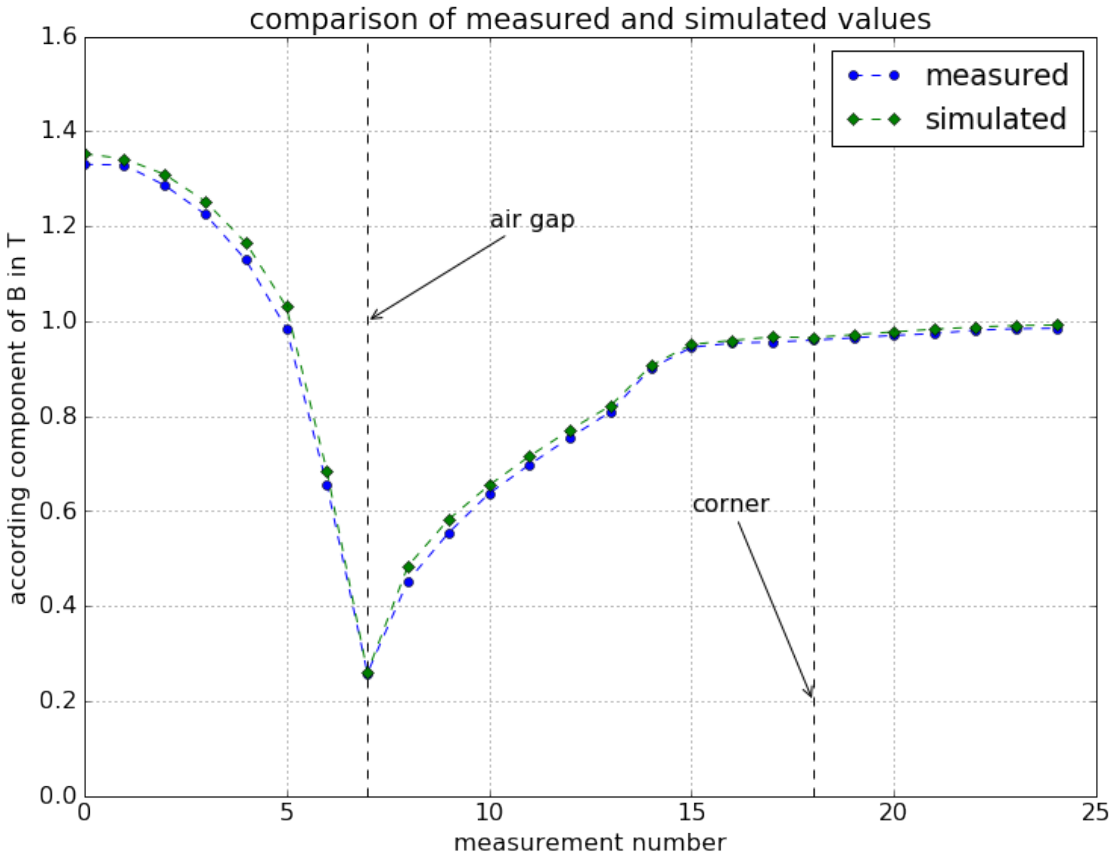
```
measurement: 25
measurement: 36
measurement: 40
measurement 1: sim         1.353, msm         1.330, rel. err         1.708
measurement 2: sim         1.342, msm         1.329, rel. err         0.969
measurement 3: sim         1.309, msm         1.286, rel. err         1.792
measurement 4: sim         1.252, msm         1.225, rel. err         2.228
```

```
measurement 5: sim         1.165, msm         1.129, rel. err         3.200
measurement 6: sim         1.031, msm         0.985, rel. err         4.645
measurement 7: sim         0.685, msm         0.655, rel. err         4.554
measurement 8: sim         0.261, msm         0.259, rel. err         0.645
measurement 9: sim         0.484, msm         0.453, rel. err         6.944
measurement 10: sim         0.582, msm         0.554, rel. err         5.093
measurement 11: sim         0.654, msm         0.637, rel. err         2.712
measurement 12: sim         0.716, msm         0.698, rel. err         2.520
measurement 13: sim         0.771, msm         0.755, rel. err         2.140
measurement 14: sim         0.822, msm         0.809, rel. err         1.600
measurement 15: sim         0.906, msm         0.901, rel. err         0.600
measurement 16: sim         0.951, msm         0.945, rel. err         0.643
measurement 17: sim         0.960, msm         0.954, rel. err         0.588
measurement 18: sim         0.967, msm         0.956, rel. err         1.145
measurement 19: sim         0.966, msm         0.960, rel. err         0.577
measurement 20: sim         0.972, msm         0.965, rel. err         0.695
measurement 21: sim         0.978, msm         0.970, rel. err         0.775
measurement 22: sim         0.983, msm         0.974, rel. err         0.938
measurement 23: sim         0.988, msm         0.981, rel. err         0.691
measurement 24: sim         0.991, msm         0.984, rel. err         0.689
measurement 25: sim         0.992, msm         0.985, rel. err         0.691
measurement 26: sim         0.035
measurement 27: sim         0.021
measurement 28: sim         0.016
measurement 29: sim         0.014
measurement 30: sim         0.014
measurement 31: sim         0.012
measurement 32: sim         0.010
measurement 33: sim         0.009
measurement 34: sim         0.005
measurement 35: sim         0.003
measurement 36: sim         0.002
measurement 37: sim         1.763
measurement 38: sim         0.030
measurement 39: sim         0.462
measurement 40: sim         1.283
```

A comparison of the real measured values and the simulated measured values is illustrated in the following figure.

```
In [15]: # plot magnetic flux density over position
         plt.figure(1, figsize=[12,9])
         plt.clf()
         xi = range(0, 25)
         plt.plot([7, 7], [0, 1.6], "--k")
         plt.plot([18, 18], [0, 1.6], "--k")
         plt.plot(xi, B_msm[0:25], '--o', label="measured")
```

```
plt.plot(xi, B_sim[0:25], '--D', label="simulated")
plt.title("comparison of measured and simulated values")
plt.ylabel("according component of B in T")
plt.xlabel("measurement number")
plt.ylim(0, 1.6)
plt.grid()

plt.annotate("air gap",xy=(7, 1), arrowprops=dict(arrowstyle='->'), xytext=(10, 1.2))
plt.annotate("corner",xy=(18, 0.2), arrowprops=dict(arrowstyle='->'), xytext=(15, 0.6))

plt.legend(loc=0)
font = {'size'   : 16}
matplotlib.rc('font', **font)
plt.show(block=False)
```



5 Related Work

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Valentin Hanser, TU Wien, Dec 2018, valentin . hanser @ student . tuwien . ac . at