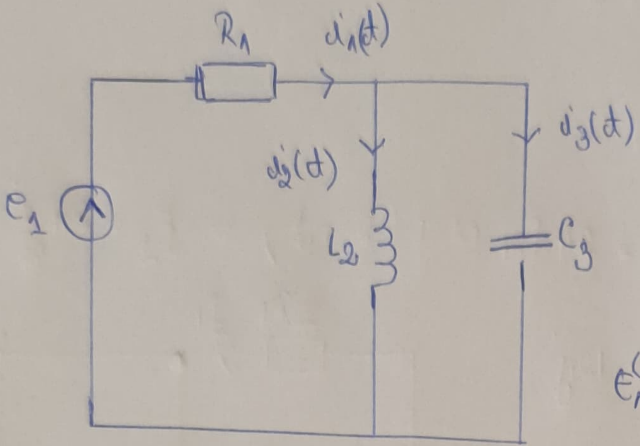
TEMĂ SEMINAR 11

(1.) $e_1(t) = 8 + 12\sqrt{2}\sin\left(\omega t + \frac{\pi}{2}\right) + 24\sqrt{2}\cos\left(2\omega t - \frac{\pi}{2}\right)$ [V]

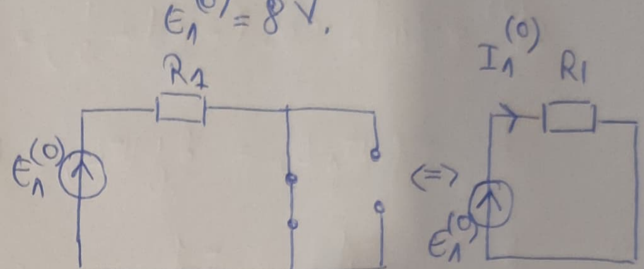
$R_1 = 4\,\Omega$, $\omega L_2 = 6\,\Omega$, $\frac{1}{\omega C_3} = 6\,\Omega$.



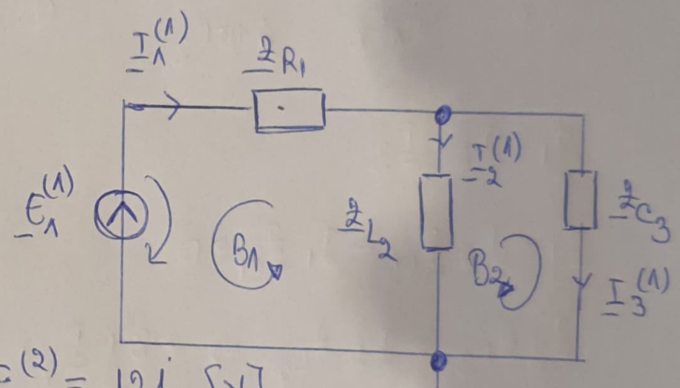
1. În curent continuu:

$e_1(t) = E_1^{(0)} + e_1^{(1)}(t) + e_2^{(2)}(t)$

$E_1^{(0)} = 8\,V.$



$$\begin{cases} I_1^{(0)} = \frac{E_1^{(0)}}{R_1} = \frac{8}{4} = 2\,A \\ \underline{I}_2^{(0)} = 2\,A \\ \underline{I}_3^{(0)} = 0\,A \end{cases}$$



2. Pentru armonica 1:

$e_1^{(2)} = 12\sqrt{2}\sin\left(\omega t + \frac{\pi}{2}\right) \Rightarrow \underline{E}_1^{(2)} = 12j$ [V]

$\underline{Z}_{R_1} = 4$, $\underline{Z}_{L_2} = j\omega L_2 = 6j$; $\underline{Z}_{C_3} = \frac{-j}{\omega C_3} = -6j$

$\underline{I}_1 = \underline{I}_2 + \underline{I}_3$ (TIK)

(TIIK) $$\begin{cases} \underline{E}_1 - \underline{I}_1^{(1)}\underline{Z}_{R_1} - \underline{I}_2\underline{Z}_{L_2} = 0 \Rightarrow 12j - 4\underline{I}_1 - 6j\underline{I}_2 = 0 \Rightarrow \underline{I}_1 = \frac{12j - 6j\underline{I}_2}{4} \Rightarrow \\ \underline{I}_3\underline{Z}_{C_3} - \underline{I}_2\underline{Z}_{L_2} = 0 \Rightarrow 6j\underline{I}_2 = -6j\underline{I}_3 \Rightarrow \boxed{\underline{I}_2 = -\underline{I}_3} \end{cases}$$

$\Rightarrow \underline{I}_1 = 3j - \frac{3}{2}j\underline{I}_2$

$3j - \frac{3}{2}j\underline{I}_2 = \underline{I}_2 - \underline{I}_2 \Rightarrow \underline{I}_2 = \frac{\cancel{3j}\cdot 2}{\cancel{3j}} \Rightarrow$

$\Rightarrow \underline{I}_2 = 2$ [A]

$\underline{I}_2^{(1)} = 2A$, $\underline{I}_3^{(1)} = -2A$, $\underline{I}_1^{(1)} = 0$ A

$\varphi_1 = \operatorname{arctg}\frac{0}{2} = 0$

$\varphi_2 = \operatorname{arctg}\frac{0}{-2} = 0$

$(-2,0)$ cadran II

$\Rightarrow \begin{cases} i_1(t) = 0 \\ i_2(t) = 2\sqrt{2}\sin(\omega t) \\ i_3(t) = +2\sqrt{2}\sin(\omega t + \pi) \end{cases}$

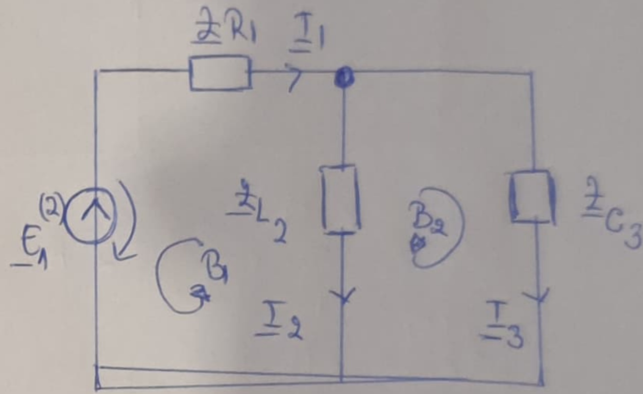
$\Rightarrow \varphi_2 = \operatorname{arctg}\frac{0}{-2} + \pi = \pi$

3. Pentru armonica 2:

$e_1^{(2)} = 24\sqrt{2}\cos\left(2\omega t + \frac{\pi}{2}\right) = 24\sqrt{2}\sin(2\omega t) \Rightarrow \underline{E}_2 = 24$ V

$\underline{Z}_{L_2} = j\omega_1 L_2 = 2j\omega L_2 = 12j$

$\underline{Z}_{C_3} = \frac{-j}{\omega_1 C_3} = \frac{-j}{2\omega C_3} = -3j$



$\begin{cases} \underline{I}_1 = \underline{I}_2 + \underline{I}_3 \\ \underline{E}_1 - \underline{I}_2\underline{Z}_{L_2} - \underline{I}_1\underline{Z}_{R_1} = 0 \\ \underline{I}_3\underline{Z}_{C_3} = \underline{I}_2\underline{Z}_{L_2} \Rightarrow -3j\underline{I}_3 = 12j\underline{I}_2 \Rightarrow \boxed{\underline{I}_3 = -4\underline{I}_2} \end{cases}$

$24 - 12j\underline{I}_2 - 4\underline{I}_1 = 0 \Rightarrow 24 - 12j\underline{I}_2 + 12\underline{I}_2 = 0 \Rightarrow$

$\underline{I}_1 = \underline{I}_2 - 4\underline{I}_2 = -3\underline{I}_2$

$\Rightarrow \underline{I}_2(1-j) + 2 = 0 \Rightarrow \underline{I}_2 = \frac{2}{j-1} = \frac{2(j+1)}{-2} = -1-j \Rightarrow \underline{I}_2 = -1-j$

$\underline{I}_1 = 3+3j$, $\underline{I}_3 = 4+4j$

$\underline{I}_1^{(2)} = 3+3j$, $\underline{I}_2^{(2)} = -1-j$, $\underline{I}_3^{(2)} = 4+4j$

$\varphi_1 = \operatorname{arctg} 1 = \frac{\pi}{4}$

$\varphi_2 = \operatorname{arctg} 1 + \pi = \frac{5\pi}{4} - \pi = -\frac{3\pi}{4}$

$\varphi_3 = \operatorname{arctg} 1 = \frac{\pi}{4}$ $(-1, 1)$ cadran 3.

$\Rightarrow \begin{cases} i_1^{(2)}(t) = 6\sin\left(2\omega t + \frac{\pi}{4}\right) \\ i_2^{(2)}(t) = 2\sin\left(2\omega t - \frac{3\pi}{4}\right) \\ i_3^{(2)}(t) = 8\sin\left(2\omega t + \frac{\pi}{4}\right) \end{cases}$

$i_1(t) = \underline{I}_1^{(0)} + i_1^{(1)}(t) + i_1^{(2)}(t) = 2 + 6\sin\left(2\omega t + \frac{\pi}{4}\right)$

$i_2(t) = \underline{I}_2^{(0)} + i_2^{(1)}(t) + i_2^{(2)}(t) = 2 + 2\sqrt{2}\sin\omega t + 2\sin\left(2\omega t - \frac{3\pi}{4}\right)$

$i_3(t) = I_3^{(0)} + i_3^{(1)}(t) + i_3^{(2)}(t) = +2\sqrt{2}\sin(\omega t + \pi) + 8\sin\left(2\omega t + \frac{\pi}{4}\right)$

[A]

b) valorile efective:

$I_{1ef} = \sqrt{2^2 + \left(\frac{6}{\sqrt{2}}\right)^2} = \sqrt{4 + \frac{36}{2}} = \sqrt{22}$

$I_{2ef} = \sqrt{4 + 4 + \frac{4}{2}} = \sqrt{10}$

$I_{3ef} = \sqrt{4 + \frac{64}{2}} = \sqrt{36} = 6$

! Rezonanta apare când părțile imaginare se anulează

$\boxed{X_L = |X_C|} \Rightarrow \omega L = \frac{1}{\omega C}$

c) factor de distorsiune pentru curentul i_2:

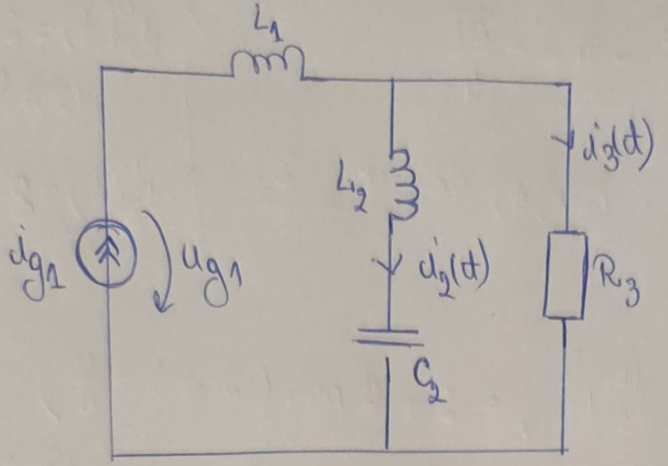
$k_{d(I_2)} = \frac{I_{d_2}}{I_{a_2}}$; $I_{a_2} = \sqrt{I_{ef_2}^2 - I_2^{(0)}} = \sqrt{10-4} = \sqrt{6}$ [A]

$I_{d_2} = \sqrt{I_{ef_2}^2 - I_2^{(0)2} - I_2^{(1)2}} = \sqrt{10-4-8} = \sqrt{4} = 2$

$k_d = \frac{\sqrt{6}}{2}$.

② $i_{g1}(t) = 2 + 2\sqrt{2}\sin(1000t + \frac{\pi}{2})$ [A] $+ \sqrt{2}\sin(2000t)$ [A],

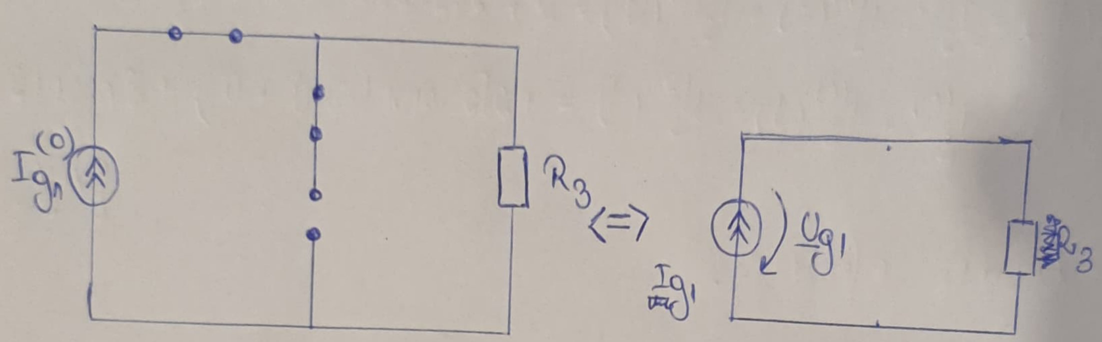
$L_1 = 3$ mH, $L_2 = 2$ mH, $C_2 = 0{,}125$ mF, $R_3 = 6\,\Omega$.



$i_{g1}(t) = I_{g1}^{(0)} + i_1^{(1)}(t) + i_1^{(2)}(t)$

1. Curent continuu.

$I_{g1}^{(0)} = 2$ A



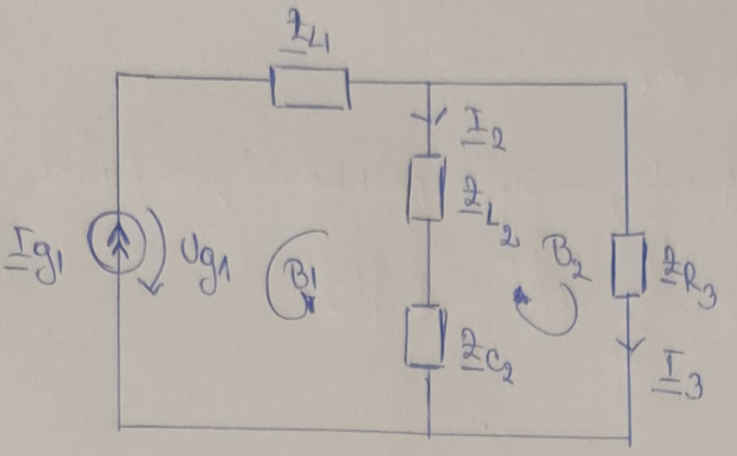
$$\begin{cases} U_{g1}^{(0)} = I_{g1} \cdot R_3 = 2 \cdot 6 = 12 \text{ V.} \\ I_{g1}^{(0)} = 2A = I_3^{(0)} \\ I_2^{(0)} = 0. \end{cases}$$

2. Pentru armonica 1: $i_{g1}^{(1)}(t) = 2\sqrt{2}\sin(1000t + \frac{\pi}{2})$

$\underline{I}_{g1}^{(1)} = 2e^{j\frac{\pi}{2}} = 2j$, $\omega = 1000$ rad/s $= 10^3$ rad/s.

$\underline{Z}_{L_1} = j\omega L_1 = j \cdot 10^3 \cdot 3 \cdot 10^{-3} = 3j$, $\underline{Z}_{L_2} = 2j \cdot \cancel{2} = 2j$

$\underline{Z}_{C_2} = \frac{-j}{\omega C_2} = -8j$, $\underline{Z}_{R_3} = 6$



$\underline{I}_{g_1} = \underline{I}_2 + \underline{I}_3$

$\underline{U}_{g_1} - \underline{I}_{g_1} \underline{Z}_{L_1} - \underline{I}_2(\underline{Z}_{L_2} + \underline{Z}_{C_2}) = 0$

$\underline{Z}_{R_3} \cdot \underline{I}_3 = \underline{I}_2(\underline{Z}_{L_2} + \underline{Z}_{C_2})$

$6\underline{I}_3 = -6j\,\underline{I}_2 \Rightarrow \boxed{\underline{I}_3 = -\underline{I}_2 j}$

$\underline{I}_{g_1} = \underline{I}_2 - j\cdot\underline{I}_2 \Rightarrow 2j = \underline{I}_2(1-j) \Rightarrow \underline{I}_2 = \dfrac{2j}{1-j} \Rightarrow \underline{I}_2 = \dfrac{2j(1+j)}{2} \Rightarrow$

$\Rightarrow \underline{I}_2 = (1+j)j = -1+j$

$\underline{I}_3 = j(-j+1) = 1+j$

$\underline{U}_{g_1} = \underline{I}_{g_1}\underline{Z}_{L_1} + \underline{I}_2(-6j) =$

$= 2j\cdot 3j + (j-1)(-6j) = -6+6+6j =$

$= 6j$

$\underline{I}_{g_1} = 2j$

$\underline{I}_2^{(1)} = -1+j, \quad \underline{I}_3^{(1)} = 1+j, \quad \underline{U}_{g_1}^{(1)} = 6j$

$i_2^{(1)}(t) = \sqrt{2}\cdot \sin(1000t + \frac{3\pi}{4})$

$\varphi_2 = \operatorname{arctg}\frac{1}{-1} = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$

$\varphi_3 = \operatorname{arctg}\frac{1}{1} = \frac{\pi}{4}$

$e_1 = \operatorname{arctg}\frac{6}{0} = \frac{\pi}{2}$

$$\begin{cases} i_2^{(1)}(t) = 2\sin(1000t + \frac{3\pi}{4}) \\ i_3^{(1)}(t) = 2\sin(1000t + \frac{\pi}{4}) \\ u_{g_1}^{(1)}(t) = 6\sin(1000t + \frac{\pi}{2}) \end{cases}$$

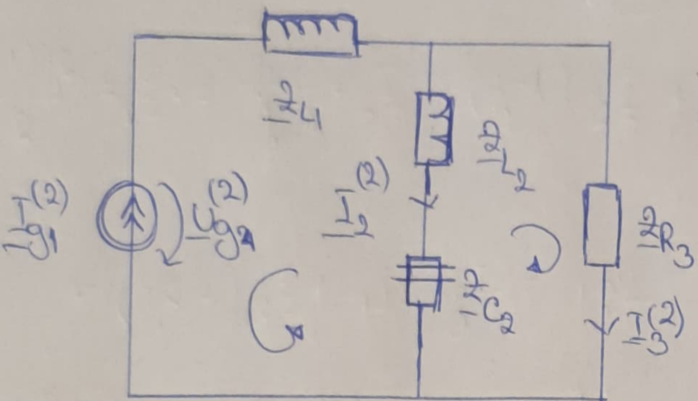
3. Pentru armonica 2.

$\cancel{i_{g_1}^{(2)} = 24\sqrt{2}\cos\left(2\omega t + \frac{\pi}{2}\right) = 24\sqrt{2}\sin(2\omega t) = 24\sqrt{2}}$

$i_{g_2}^{(2)} = \sqrt{2}\sin(2000t) = \frac{\sqrt{2}}{\sqrt{2}} \cdot e^{j\cdot 0} = 1\ [A]$

$\omega = 2\cdot 10^3$ rad/s.

$\underline{Z}_{L_1} = j\omega L_1 = 6j$, $\underline{Z}_{L_2} = 4j$, $\underline{Z}_{C_2} = -4j$, $\underline{Z}_{R_3} = 6$



T2K: $\underline{U}_{g_1}^{(2)} = \underline{I}_{g_1}\cdot\underline{Z}_{L_1} + \underline{I}_2(\underline{Z}_{L_2} + \underline{Z}_{C_2})$

$\underline{I}_3\underline{Z}_{R_3} = \underline{I}_2(\underline{Z}_{L_2} + \underline{Z}_{C_2})$

T1K: $\underline{I}_{g_1} = \underline{I}_2 + \underline{I}_3$

$\underline{I}_3 = 0 \Rightarrow \underline{I}_{g_1} = \underline{I}_2 = 1\ [A]$ $\qquad \varphi_2 = \varphi_1 = \text{arctg}\frac{0}{1} = 0$

$\underline{U}_{g_1} = 6j$ $\qquad \varphi_2 = \text{arctg}\frac{6}{0} = \frac{\pi}{2}$

$$\begin{cases} i_2^{(2)}(t) = \sqrt{2}\cdot\sin(2000t) \\ i_3^{(2)}(t) = 0 \\ u_{g_1}^{(2)}(t) = 6\sqrt{2}\sin\left(2000t + \frac{\pi}{2}\right) \end{cases}$$

$i_2(t) = \underline{I}_2^{(0)} + i_2^{(1)}(t) + i_2^{(2)}(t) = 2\sin\left(1000t + \frac{3\pi}{4}\right) + \sqrt{2}\sin(2000t)$

$i_3(t) = \underline{I}_3^{(0)} + i_3^{(1)}(t) + i_3^{(2)}(t) = 2 + 2\sin\left(1000t + \frac{\pi}{4}\right)$

$u_{g_1}(t) = \underline{U}_{g_1}^{(0)} + u_{g_1}^{(1)}(t) + u_{g_1}^{(2)}(t) = 12 + 6\sin\left(1000t + \frac{\pi}{2}\right) + 6\sqrt{2}\sin\left(2000 + \frac{\pi}{2}\right)$

b) Valorile efective:

$I_{2ef} = \sqrt{I_2^{(0)2} + I_2^{(1)2} + I_2^{(2)2}} = \sqrt{0+4+2} = \sqrt{6}$ [A]

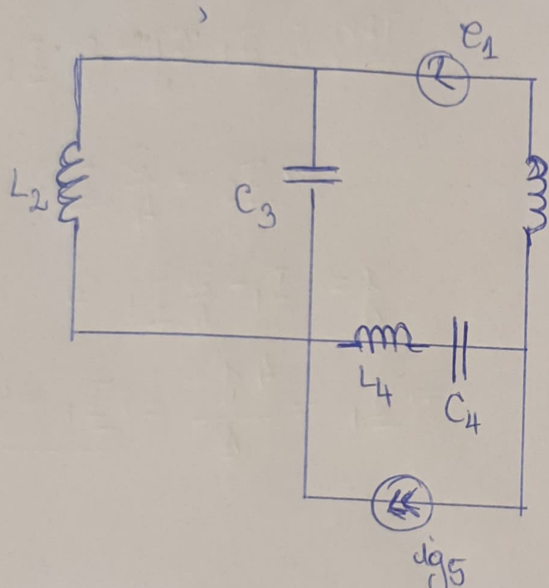
$I_{3ef} = \sqrt{I_3^{(0)2} + I_3^{(1)2} + I_3^{(2)2}} = \sqrt{4+4} = 2\sqrt{2}$ [A]

$U_{g_1ef} = \sqrt{U_{g_1}^{(0)2} + U_{g_1}^{(1)2} + U_{g_1}^{(2)2}} = \sqrt{12^2+6^2+2\cdot 6^2} = 6\sqrt{4+1+2} = 6\sqrt{7}$ [A]

3. $e_1(t) = 6\sqrt{2}\sin(3\omega t)$ [V], $i_{g_5}(t) = \sqrt{2}\sin\left(\omega t + \frac{\pi}{2}\right)$ [A]

$\omega L_2 = 2\Omega$, $\frac{1}{\omega C_3} = 6\Omega$, $\omega L_4 = 1\Omega$, $\frac{1}{\omega C_4} = 9\Omega$, $\omega L_1 = 3\Omega$.

Calculați curentul $i_1(t)$.



$e_1(t) = e_1^{(1)}(t) + e_1^{(3)}(t)$

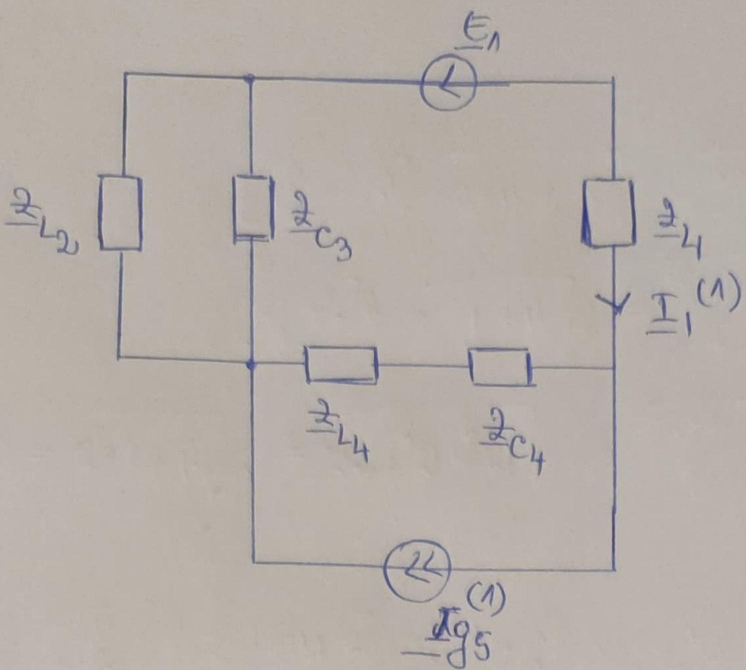
$i_{g_5}(t) = i_{g_5}^{(1)}(t) + i_{g_5}^{(3)}(t)$

1. Pentru armonica 1: $e_1^{(1)}(t) = 0$, $i_{g_5}(t) = \sqrt{2}\sin\left(\omega t + \frac{\pi}{2}\right)$

$\underline{Z}_{L_2} = j\omega L_2 = 2j$, $\underline{Z}_{C_3} = \frac{-j}{\omega C_3} = -6j$, $\underline{Z}_{L_4} = j\omega L_4 = j$

$\underline{Z}_{C_4} = \frac{-j}{\omega C_4} = -9j$, $\underline{Z}_{L_1} = j\omega L_1 = 3j$.

$\underline{E}_1^{(1)} = 0$, $\underline{I}_{g_5}^{(1)} = j$



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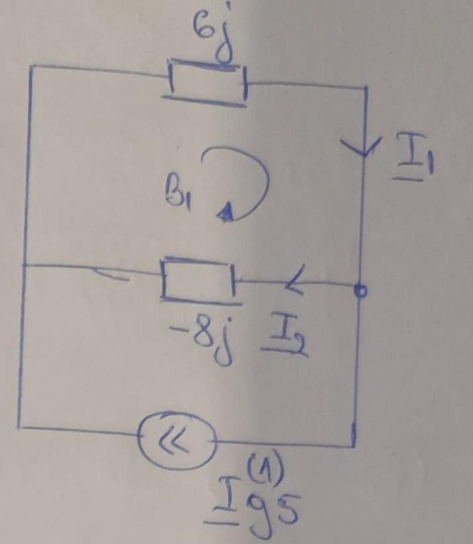


$$\underline{Z}_1 = \frac{\underline{Z}_{L_2} \cdot \underline{Z}_{C_3}}{\underline{Z}_{L_2} + \underline{Z}_{C_3}} = \frac{2j(-6j)}{-4j} = 3j$$

$$\underline{Z}_1' = \underline{Z}_1 + \underline{Z}_{L_1} = 3j + 3j = 6j$$

$$\underline{Z}_2 = \underline{Z}_{L_4} + \underline{Z}_{C_4} = -8j$$

T1K: $\underline{I}_1 = \underline{I}_2 + \underline{I}_{g5}$

T2K: $\underline{I}_1 \cdot 6j = 8j \cdot \underline{I}_2 \Rightarrow$

$$\Rightarrow \underline{I}_2 = \frac{3}{4}\underline{I}_1$$

$$j = \underline{I}_1 - \frac{3}{4}\underline{I}_1 \Rightarrow$$

$$\Rightarrow \underline{I}_1 = \frac{4j}{1} \Rightarrow \underline{I}_1 = 4j$$

$\underline{I}_1^{(1)} = 4j$, $\underline{E}_1^{(1)} = 0$.

$$\begin{cases} e_1^{(1)} = 0 \\ i_1^{(1)}(t) = 4\sqrt{2}\sin\left(\omega t + \frac{\pi}{2}\right) \end{cases}$$

Pentru armonica 3: $e_1^{(3)}(t) = 6\sqrt{2}\sin(3\omega t)$

$i_{g5}^{(3)}(t) = 0$.

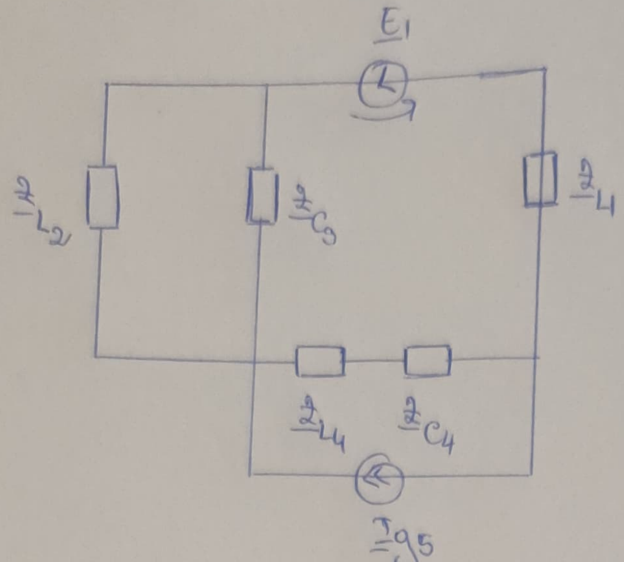
$\underline{E}_1^{(3)} = 6$ [V], $\underline{I}_{g5}^{(3)} = 0$ [A].

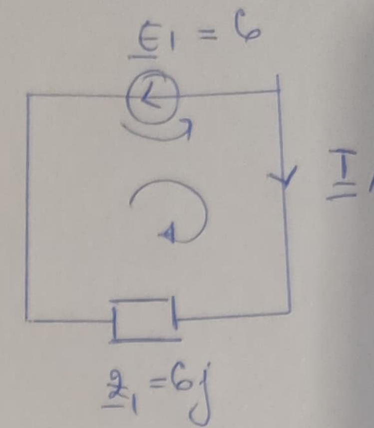
$\omega_1 = 3\omega$

$\underline{Z}_{L_1} = 3j\omega L_1 = 9j$ $\quad \underline{Z}_{C_3} = \frac{-j}{3\omega C_3} = -2j$

$\underline{Z}_{L_2} = 3j\omega L_2 = 6j$ $\quad \underline{Z}_{C_4} = \frac{-j}{3\omega C_4} = -3j$

$\underline{Z}_{L_4} = 3j\omega L_4 = 3j$





$$\underline{Z}_1 = \underline{Z}_{L_4} + \underline{Z}_{C_4} = 0$$

$$\underline{Z} = \frac{\underline{Z}_{L_2} \cdot \underline{Z}_{C_3}}{\underline{Z}_{L_2} + \underline{Z}_{C_3}} = \frac{6\cancel{j} \cdot (-2j)}{4\cancel{j}} = -3j$$

$$\underline{Z}_1 = -3j + 9j = 6j$$

$$\underline{E}_1 + \underline{I}_1 \underline{Z}_1 = 0 \Rightarrow \underline{I}_1 = \frac{-\underline{E}_1}{\underline{Z}_1} = -\frac{\cancel{6}}{\cancel{6}j} = -\frac{j}{-1} = 1 \text{ [A]}.$$

$\underline{I}_1^{(3)} = 1$ $\qquad i_1^{(3)}(t) = \sqrt{2} \cdot \sin(3\omega t)$

$\varphi = \operatorname{arctg} \frac{0}{1} = 0$

$$i_1(t) = i_1^{(1)}(t) + i_1^{(3)}(t) = 4\sqrt{2} \sin\left(\omega t + \frac{\pi}{2}\right) + \sqrt{2} \sin(3\omega t).$$