TEMĂ SEMINAR 9

① $e_1(t) = 8\sqrt{2}\sin\left(\omega t + \frac{\pi}{2}\right)$ [V]

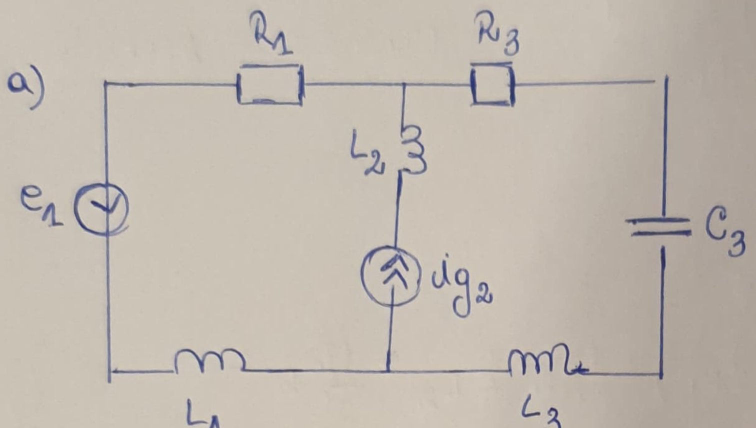
$i_{g2}(t) = 4\cos\left(\omega t - \frac{\pi}{4}\right)$ [A]

$X_C = -\frac{1}{\omega C}$ (reactanță capacitivă)

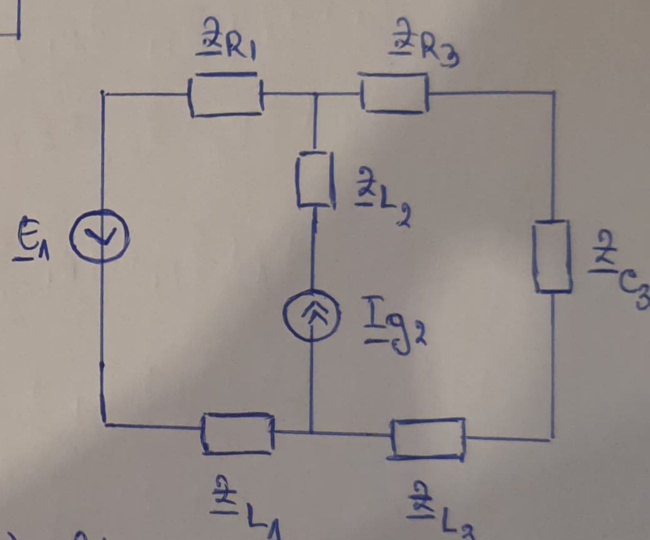
$R_1 = R_3 = 4\,\Omega$, $X_{L_1} = 8\,\Omega$, $X_{L_2} = X_{L_3} = 4\,\Omega$, $X_{C_3} = -8\,\Omega$.

$X_L = \omega L$ (reactanță inductivă)

a)



Desenați circuitul în complex.



$\underline{Z}_{R_1} = R_1 = \underline{Z}_{R_3} = R_3 = 4$

$\underline{Z}_{L_1} = j\omega L_1 = j \cdot X_{L_1} = 8j$

$\underline{Z}_{L_2} = \underline{Z}_{L_3} = j X_{L_2} = j X_{L_3} = 4j$

$\underline{Z}_{C_3} = \frac{-j}{\omega C} = j X_C = -8j$

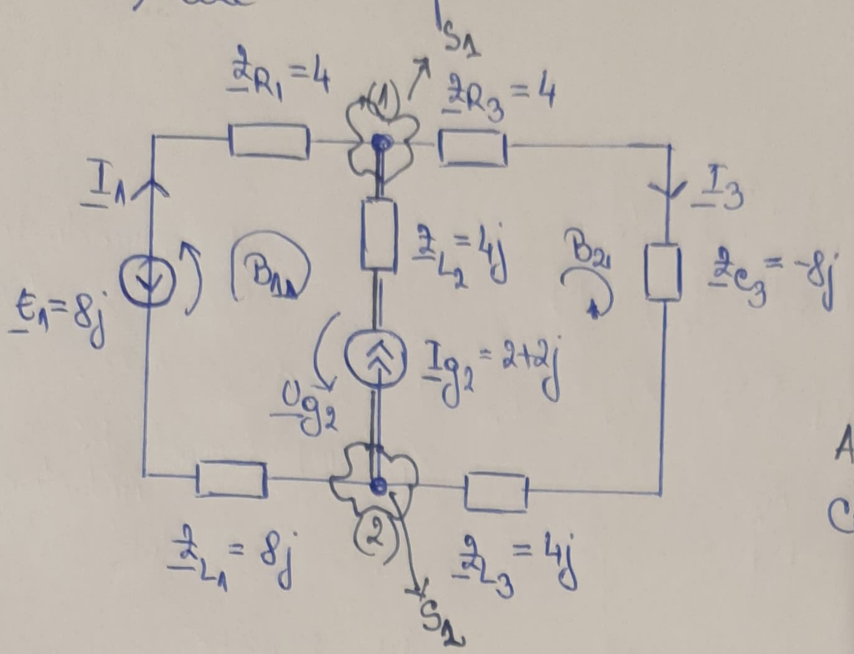
$\underline{E}_1 = 8 e^{j\frac{\pi}{2}} = 8\cdot\left(\cos\frac{\pi}{2} + j\cdot\sin\frac{\pi}{2}\right) = 8j$

$\underline{I}_{g2} = 2\sqrt{2}\, e^{j\frac{\pi}{4}} = 2\sqrt{2}\cdot\left(\frac{1}{\sqrt{2}} + j\cdot\frac{1}{\sqrt{2}}\right) = 2+2j = 2(1+j)$

$i_{g2}(t) = 4\cdot\sin\left(\omega t - \frac{\pi}{4} + \frac{\pi}{2}\right) = 4\sin\left(\omega t + \frac{\pi}{4}\right)$

$\underline{E}_1 = 8j$, $\underline{I}_{g2} = 2+2j$

b) Calc. rec. aplicând Teoremele lui Kirchhoff.



$N = 2$

$L = 3$

$N-1 = 1$ ec. (T1K)

$L-N+1 = 2$ ec (T2K)

Arbore – o ramură

Coarbore – 2 coarde

$(S_1|S_2)$: $\underline{I}_1 + \underline{I}_{g_2} = \underline{I}_3$

(B_1): $\underline{E}_1 + \underline{I}_1(\underline{Z}_{L_1} + \underline{Z}_{R_1}) - \underline{I}_{g_2} \cdot \underline{Z}_{L_2} + \underline{U}_{g_2} = 0$

(B_2): $\underline{I}_3(\underline{Z}_{R_3} + \underline{Z}_{C_3} + \underline{Z}_{L_3}) - \underline{U}_{g_2} + \underline{I}_{g_2} \cdot \underline{Z}_{L_2} = 0.$ $\Big\} \Rightarrow$

$$\overset{(+)}{\Longrightarrow} \underline{E}_1 + \underline{I}_1(\underline{Z}_{L_1} + \underline{Z}_{R_1}) + \underline{I}_3(\underline{Z}_{R_3} + \underline{Z}_{C_3} + \underline{Z}_{L_2}) = 0 \Rightarrow$$

$$\Rightarrow \begin{cases} 8j + (4+8j)\underline{I}_1 + (4-4j)\underline{I}_3 = 0 \\ \underline{I}_3 = \underline{I}_1 + 2+2j \end{cases}$$

$$\Rightarrow 8j + (4+8j)\underline{I}_1 + (4-4j)(\underline{I}_1 + 2+2j) = 0 \Rightarrow$$

$$\Rightarrow 8j + \underline{I}_1(4+8j) + \underline{I}_1(4-4j) + 2(2-2j)(2+2j) = 0$$

$$\Rightarrow 8j + 2(4+4) + \underline{I}_1(8+4j) = 0 \Rightarrow$$

$$\Rightarrow \underline{I}_1 = \cancel{\frac{-(8+8j)}{8+8j}} = \cancel{\frac{-(4+2j)}{2+j}} = -\frac{8j+16}{4j+8} = -2[A] \Rightarrow$$

$\Rightarrow \underline{I}_1 = 2[A]$

$\underline{I}_3 = -2+2+2j =$
$= 2j\ [A]$

$\underline{I}_1 = -2$ [A], $\underline{I}_3 = 2j$ [A], $\underline{I}g_2 = \underline{I}_3 - \underline{I}_1 = 2j + 2$

$\underline{U}g_2 = \underline{I}_3(\underline{Z}_{R_3} + \underline{Z}_{C_3} + \underline{Z}_{L_3}) + \underline{I}g_2 \cdot \underline{Z}_{L_2} =$

$\Rightarrow \underline{U}g_2 = 2j(4-4j) + (2+2j)\cdot 4j = 8j + 8 + 8j - 8 = 16j$ [V].

$\underline{I}_1 = 2$: $\varphi = \operatorname{arctg}\frac{0}{2} = \operatorname{arctg} 0 = 0$

$i_1(t) = 2\sqrt{2}\cdot e^{j\cdot 0} = 2\sqrt{2}\sin(\omega t)$

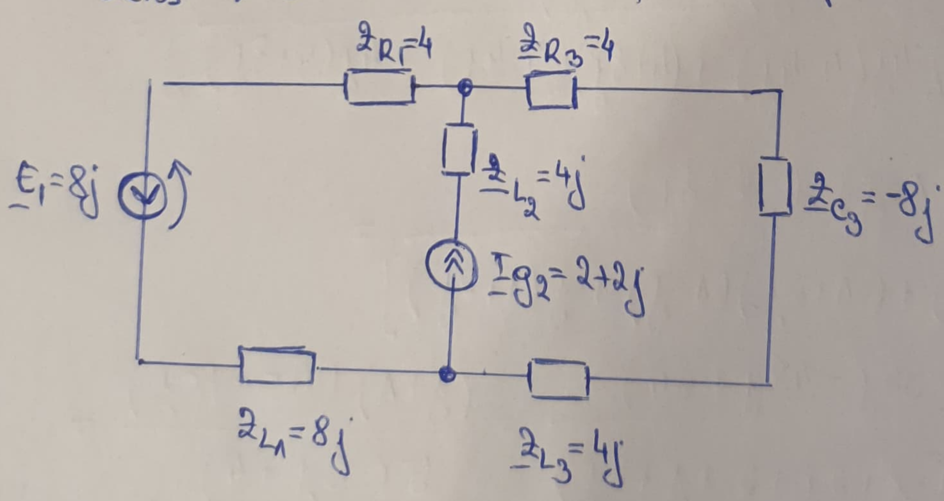
$\underline{I}_3 = 2j$: $\varphi = \operatorname{arctg}\frac{2}{0} = \frac{\pi}{2}$

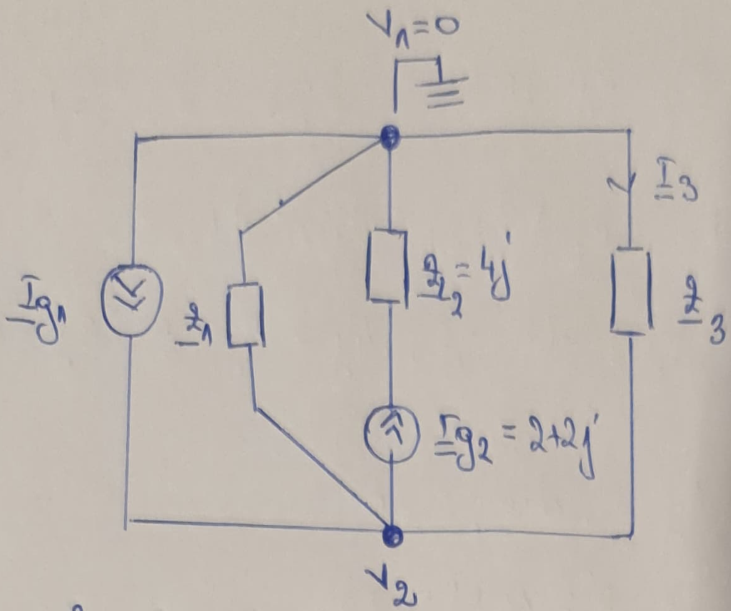
$i_3(t) = 2\sqrt{2}\sin(\omega t + \frac{\pi}{2})$

$u g_2(t) = 16\sqrt{2}\sin(\omega t + \frac{\pi}{2})$

$\underline{U}g_2 = 16j$, $\varphi = \operatorname{arctg}\frac{16}{0} = \frac{\pi}{2}$.

c) Calculați nec. folosind MNP ; calc. curenții lat. și tens. surselor de curent folosind potențialele.





Necunoscute: V_1, V_2.

$$\underline{Z}_1 = \underline{Z}_{R_1} + \underline{Z}_{L_1} = 4+8j$$

$$\underline{Z}_3 = \underline{Z}_{R_3} + \underline{Z}_{C_3} + \underline{Z}_{L_3} = 4-4j$$

$$\underline{I}_{g1} = \frac{E_1}{\underline{Z}_1} = \frac{8j}{4+8j} = \frac{2j}{1+2j} = \frac{2j(1-2j)}{5} = \frac{2j+4}{5}$$

$$V_2\left(\frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} + \frac{1}{\underline{Z}_3}\right) = \underline{I}_{g1} - \underline{I}_{g2}$$

$$V_2 \cdot \frac{\underline{Z}_{L_2}\cdot\underline{Z}_3 + \underline{Z}_1\cdot\underline{Z}_3 + \underline{Z}_1\cdot\underline{Z}_{L_2}}{\underline{Z}_1\cdot\underline{Z}_{L_2}\cdot\underline{Z}_3} = \frac{2j+4-10-10j}{5}$$

$$V_2 = \frac{(4+8j)\cdot 4j(4-4j)(-6-8j)}{5\cdot\left[4j(4-4j) + (4+8j)(4-4j) + 4j(4+8j)\right]} =$$

$$= \frac{(4+8j)\cdot j(1-j)\cdot(-6-8j)}{5\cdot\left[j(1-j) + (1+2j)(1-j) + j(1+2j)\right]} =$$

$$= \frac{(j+1)(-24-32j-48j+64)}{5\cdot(\cancel{j}+1+1-\cancel{j}+2j+2+\cancel{j}+\cancel{2j})} = \frac{(j+1)(40-80j)}{5(5j+4)} =$$

$$= \frac{(j+1)(8-16j)}{5j+4} = \frac{8(j+1)(1-2j)(5j-4)}{-41}$$ [illegible]

I_3 - Kirchhoff în buclă mică.

d) Bilanțul puterilor:

• Puteri absorbite (complexe): $\underline{S}_{a_1} = \underline{Z}_1 I_1^2$

$\underline{S}_{a_2} = \underline{Z}_2 I_2^2$

$\underline{S}_{a_3} = \underline{Z}_3 I_3^2$

$\underline{Z}_1 = \underline{Z}_{R_1} + \underline{Z}_{L_1} = 4+8j$

$\underline{Z}_2 = \underline{Z}_{L_2} = 4j$

$\underline{Z}_3 = \underline{Z}_{R_3} + \underline{Z}_{C_3} + \underline{Z}_{L_3} = 4-4j$

$\underline{I}_1 = 2\,[A]$, $\underline{I}_2 = \underline{I}_{g_2} = 2+2j$; $\underline{I}_3 = 2j$

$\underline{S}_{a_1} = (8+16j)\cdot 2 \rightarrow 16+32j$ $\quad \underline{S}_{a_2} = (8j-8)(2+2j) \rightarrow 32j$ $\quad \underline{S}_{a_3} = (8j+8)\cdot 2j \rightarrow 16+16j$

$\underline{S}_a = 32+48j = 16(2+3j)$

• Puteri complexe debitate: $\underline{S}_{d_1} = \underline{E}_1 \cdot \underline{I}_1^*$

$\underline{S}_{d_2} = \underline{U}_{g_2} \cdot \underline{I}_{g_2}^*$

$\underline{S}_{d_1} = 8j \cdot 2 = 16j$

$\underline{S}_{d_2} = 16j\cdot(2-2j) = 32j(1-j) = 32j+32$

$\underline{I}_{g_2} = 2+2j$

$\underline{I}_{g_2}^* = 2-2j$

$\underline{S}_d = 16(2+3j)$

$\boxed{\underline{S}_a = \underline{S}_d}$

e) Calculați puterile absorbite de dipolul ce formează latura 3: $\underline{S}_{a_3}$, P_{a_3}, Q_{a_3}, S_{a_3}; scrieți în dreptul fiecărei puteri denumirea acesteia;

Pe latura 3: $\underline{S}_{a_3} = 16 - 16j$ VA (din subpunctul d.)

$P_{a_3} = 16$ W

S_{a_3} - putere absorbită

$Q_{a_3} = -16$ var

$\boxed{\underline{S}_{a_3} = I_3^2 \cdot \underline{Z}_3}$

$S_{a_3} = \sqrt{16^2 + 16^2} = 16\sqrt{2}$ VA

f) Calculați mărimile: $\underline{Z}_3$, R_3, X_3, Z_3, $\underline{Y}_3$, G_3, B_3, Y_3.

$\underline{Z}_3 = 4 - 4j$ [Ω]; $R_3 = 4$ Ω

$Z_3 = \sqrt{32} = 4\sqrt{2}$ [Ω] $\underline{Y}_3 = \frac{1}{\underline{Z}_3} = \frac{1}{4-4j} = \frac{1}{4(1-j)} = \frac{1+j}{8}$ [S]

$X_3 = -4$ [Ω]

$G_3 = \frac{1}{8}$ [S], $B_3 = \frac{1}{8}$ [S], $Y_3 = \sqrt{\frac{2}{8^2}} = \frac{1}{8}\sqrt{2}$ [S]

$\underline{Z}_3$ impedanță complexă

$\underline{Y}_3$ admitanță complexă

Z_3 impedanță

Y_3 admitanța

R_3 rezistență

X_3 reactanță

B_3 - susceptanță

G_3 - conductanță

$G_3 = \frac{1}{R_3}$

$\frac{1}{8} \neq \frac{1}{4}$

(2.) $e_1(t) = 16 \sin\left(\omega t + \frac{5\pi}{4}\right)$ [V]

$e_2(t) = 4\, i_4(t)$ [V]

$e_3(t) = 8 \sin\left(\omega t - \frac{\pi}{4}\right)$ [V]

$i_{g6}(t) = 2\sqrt{2} \sin\left(\omega t + \frac{\pi}{2}\right)$ [A]

$\omega = 2$ rad/s, $L_3 = 1H$, $C_4 = 0{,}25$ F, $C_5 = 0{,}125$ F, $R_5 = 4\Omega$

$L_4 = 3H$

a) Rep. circuitul complex.

$\underline{E}_1 = 8\sqrt{2}\, e^{j\cdot\frac{5\pi}{4}} = 8\sqrt{2}\left(\cos\frac{5\pi}{4} + j\cdot\sin\frac{5\pi}{4}\right) = -8 - 8j$

$\sin\left(\pi + \frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = \frac{-1}{\sqrt{2}}, \quad \cos\left(\pi + \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$

$\underline{E}_2 = 4 \cdot \underline{I}_4$

$\underline{E}_3 = 4\sqrt{2} \cdot e^{-j\frac{\pi}{4}} = 4\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + j\sin\left(-\frac{\pi}{4}\right)\right] = 4\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j\right) =$

$= 4 - 4j.$

$\underline{I}_{g6} = 2\, e^{j\frac{\pi}{2}} = 2j$

$\underline{Z}_{L4} = 6j$

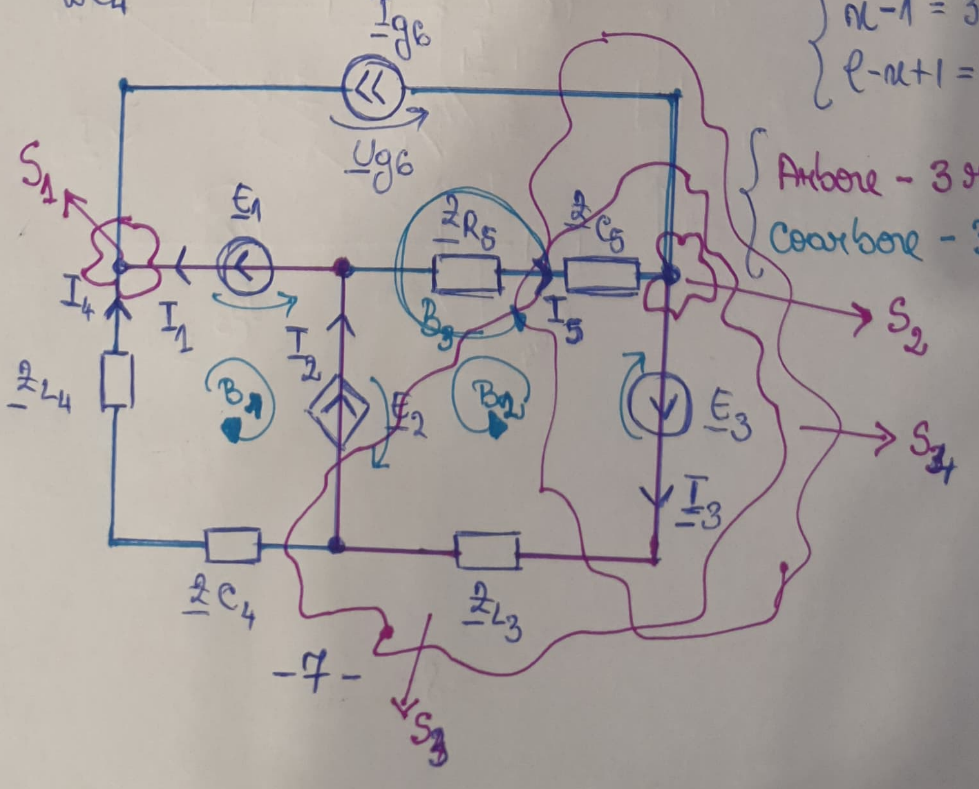
$\underline{Z}_{L3} = j\omega L_3 = 2j$

$\underline{Z}_{C5} = \frac{-j}{\omega C_5} = -8j$

$\underline{Z}_{C4} = \frac{-j}{\omega C_4} = -4j$

$\underline{Z}_{R5} = R_5 = 4.$

$\begin{cases} n-1 = 3 \text{ ec.} \\ l-n+1 = 4 \text{ ec.} \end{cases}$



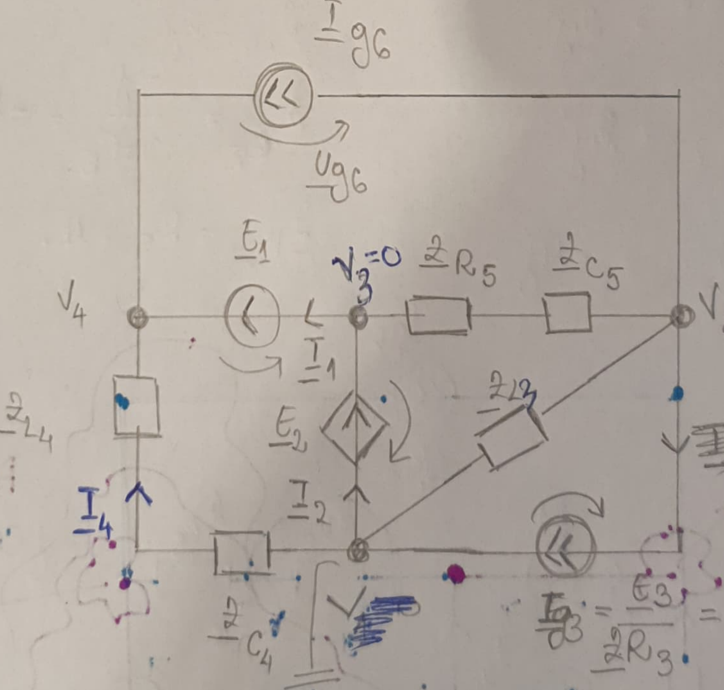
{ Arbore - 3 ramuri
Coarbore - 3 coarde

b) Scrieți ecuațiile corespunzătoare teoremelor lui Kirchhoff;

$$\begin{cases} S_1: \underline{I}_1 + \underline{I}_4 + \underline{I}_{g6} = 0 \\ S_2: \underline{I}_5 \pm \underline{I}_{g6} + \underline{I}_3 \\ S_3: \underline{I}_4 + \underline{I}_2 - \underline{I}_5 + \underline{I}_{g6} = 0 \\ S_4: \underline{I}_{g6} - \underline{I}_5 + \underline{I}_3 = 0 \end{cases}$$

$$\begin{cases} B_1: \underline{I}_4(\underline{Z}_{L4} + \underline{Z}_{C4}) + \underline{E}_1 + \underline{E}_2 = 0 \\ B_2: \underline{E}_2 + \underline{E}_3 - \underline{I}_3 \underline{Z}_{L3} - \underline{I}_5(\underline{Z}_{C5} + \underline{Z}_{L5}) = 0 \\ B_3: -\underline{U}_{g6} + \underline{E}_1 + \underline{E}_2 + \underline{E}_3 - \underline{I}_3 \underline{Z}_{L3} = 0. \end{cases}$$

c) Calc. nec. fol. MNP.



Necunoscute: $V_1, V_2, V_3, V_4,$ $\underline{I}_1, \underline{I}_2,$ $\underline{I}_4$.

$$\underline{I}_{g3} = \frac{\underline{E}_3}{\underline{Z}_{R3}} = \frac{\underline{E}_3}{\underline{Z}_{L3}} = \frac{4-4j}{2j} = -2+2j$$

$$= \frac{2-2j}{3j} = \frac{6j+6}{9} =$$

$\underline{E}_1 = V_4 - V_3 \Rightarrow V_4 = \underline{E}_1 \Rightarrow \boxed{V_4 = -8-8j}$

$\underline{E}_2 = V_3 - V_1 \Rightarrow V_1 = -\underline{E}_2 \Rightarrow \boxed{V_1 = -4\,\underline{I}_4}$

$$V_4\left(\frac{1}{\underline{Z}_{L4}} + \frac{1}{\underline{Z}_{C4}}\right) - V_1\left(\frac{1}{\underline{Z}_{L4}} + \frac{1}{\underline{Z}_{C4}}\right) = \underline{I}_1 + \underline{I}_{g6} + \underline{I}_4$$

$\underline{Z}_4 = \underline{Z}_{L4} + \underline{Z}_{C4} = 2j$

(1) $\frac{1}{2j}(V_4 - V_1) = \underline{I}_1 + \underline{I}_4 + 2j \Rightarrow \frac{-4-4j}{j} + \frac{4\,\underline{I}_4}{2j} =$

$= \underline{I}_1 + \underline{I}_4 + 2j \Rightarrow \frac{-4j+4}{-1} + \frac{2j}{-1}\underline{I}_4 = \underline{I}_1 + \underline{I}_4 + 2j \Rightarrow$

$\Rightarrow 4j - 4 - 2j\,\underline{I}_4 = \underline{I}_1 + \underline{I}_4 + 2j \Rightarrow \boxed{\underline{I}_1 = \underline{I}_4(-2j-1) + 2j - 4}$

(2) $\underline{Z}_{R5} + \underline{Z}_{C5} = \underline{Z}_5 = -8j + 4$

$$V_2\left(\frac{1}{\underline{Z}_5} + \frac{1}{\underline{Z}_{L4}}\right) - \frac{V_1}{\underline{Z}_{L4}} = -\underline{I}_{g3} - \underline{I}_{g6}$$

$$V_2\left(\frac{1}{-8j+4} + \frac{1}{6j}\right) + \frac{4\,\underline{I}_4}{6j} = 2+2j - 2j = 2$$

$$V_2 \cdot \frac{6j - 8j + 4}{48 + 24j} + \frac{4\,\underline{I}_4}{6j} = 2 \Rightarrow V_2 \cdot \frac{-j+2}{24+12j} + \frac{2\,\underline{I}_4}{3} = 2 \,\Big|\cdot 3$$

$$\Rightarrow \cancel{12}\, V_2 \cdot \frac{2-j}{8+4j} + 2\,\underline{I}_4 = 6 \,\Big|\cdot 4 \Rightarrow V_2 \cdot \frac{2-j}{2+j} + 8\,\underline{I}_4 = 24 \Rightarrow$$

$$\Rightarrow V_2 \cdot \frac{4-4j-1}{4+1} = 24 - 8\,\underline{I}_4 \Rightarrow \boxed{V_4 = \frac{5(24-8\,\underline{I}_4)}{3-4j}}$$

(3) $V_1\left(\frac{1}{\underline{Z}_4}+\frac{1}{\underline{Z}_{L4}}\right)-\frac{V_2}{\underline{Z}_{L3}}-\frac{V_4}{\underline{Z}_{L4}}=-\underline{I}_2+\underline{I}_{g3}-\underline{I}_4.$

înlocuim tot în funcție de I_4 și aflăm I_4.

d) $\underline{V}_1=8(1+j)$, $\underline{V}_2=-8$, $\underline{V}_3=8(-1+j)$, $V_4=0$.

$\underline{I}_1=-2(1+j)$, $\underline{I}_2=-2j$

Puteri absorbite : $\underline{S}_4=\underline{Z}_4\cdot\underline{I}_4^2$

$\underline{S}_3=\underline{Z}_{L3}\cdot\underline{I}_3^2$

$\underline{S}_5=\underline{Z}_5\cdot\underline{I}_5^2$

Puteri debitate : $\underline{S}_2=\underline{E}_2\cdot\underline{I}_2^*$

$\underline{S}_3=\underline{E}_3\cdot\underline{I}_3^*$

$\underline{S}_6=\underline{U}_{g6}\cdot\underline{I}_{g6}^*$

$\underline{S}_1=\underline{E}_1\cdot\underline{I}_1^*.$

e) $\underline{S}=\underline{S}_2+\underline{S}_3+\underline{S}_6+\underline{S}_1$

partea reală $\Rightarrow P$ [W]

parte imaginară $=Q$ [VAR]

$S=\sqrt{P^2+Q^2}.$