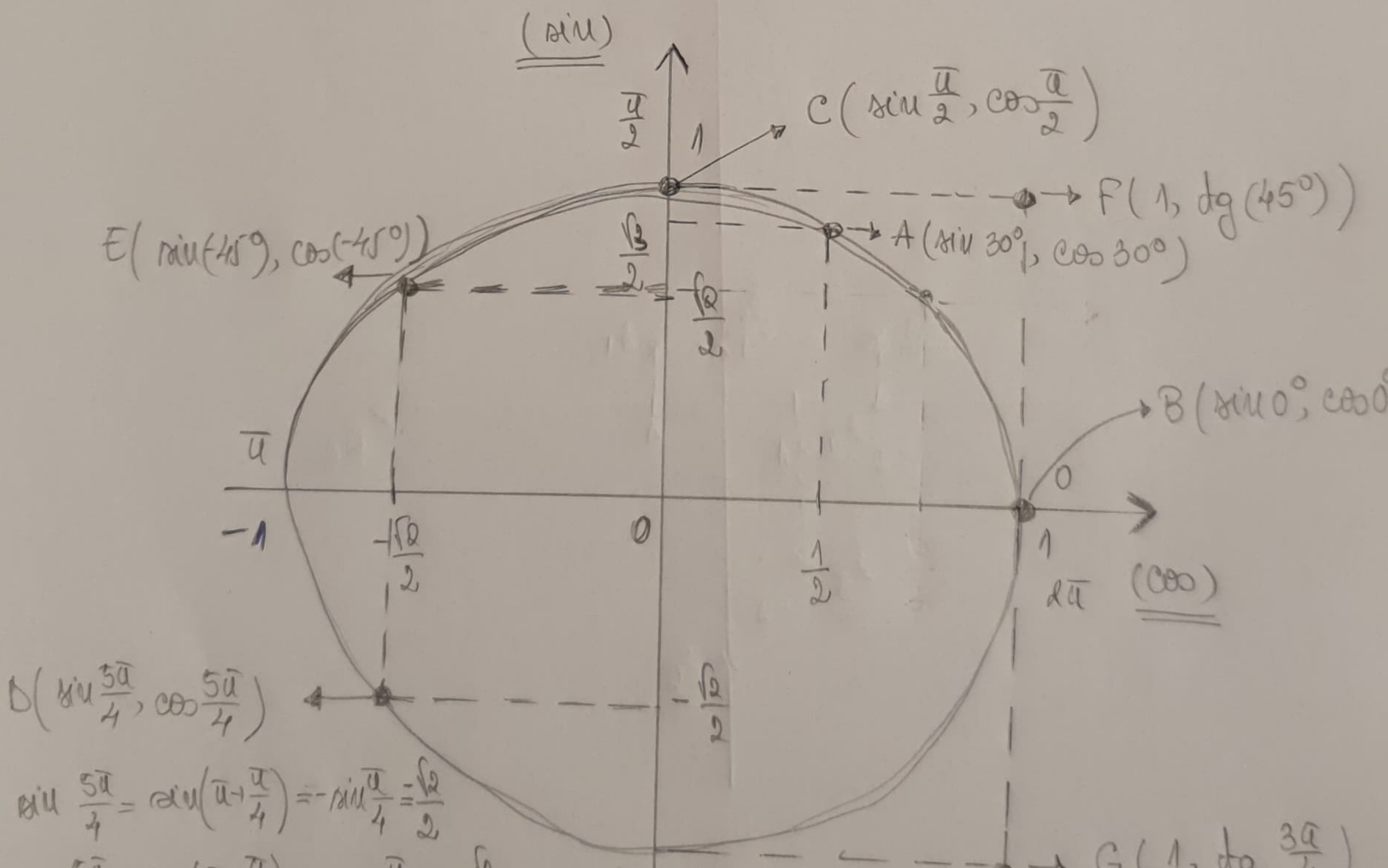
TEMĂ SEMINAR 7

① $R=1$. Desenați dreapta tangentă la cerc în punctul de coordonate $(1,0)$.

Punctele de coord: $A(\sin 30°, \cos 30°)$, $B(\sin 0°, \cos 0°)$, $C(\sin\frac{\pi}{2}, \cos\frac{\pi}{2})$, $D(\sin\frac{5\pi}{4}, \cos\frac{5\pi}{4})$, $E(\sin(-45°), \cos(-45°))$, $F(1, \operatorname{tg} 45°)$, $G(1, \operatorname{tg}\frac{3\pi}{4})$.



$\sin\frac{5\pi}{4} = \sin\left(\pi+\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} = -\frac{\sqrt2}{2}$

$\cos\frac{5\pi}{4} = \cos\left(\pi+\frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt2}{2}$

$\left.\begin{array}{l}\sin\frac{3\pi}{4} = \sin\left(\pi-\frac{\pi}{4}\right) = +\sin\frac{\pi}{4} = \frac{\sqrt2}{2} \\ \cos\frac{3\pi}{4} = \cos\left(\pi-\frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt2}{2}\end{array}\right\} \Rightarrow \operatorname{tg}\frac{3\pi}{4} = -1$

Dreapta tangentă la punctul $(1,0)$ este FG.

(2.) $z_1 = 1 + j$

$Re_{z_1} = 1, \quad Im_{z_1} = 1$

$z_2 = j$

$Re_{z_2} = 0, \quad Im_{z_2} = 1$

$z_3 = -2 + j$

$Re_{z_3} = -2, \quad Im_{z_3} = 1$

$z_4 = -2$

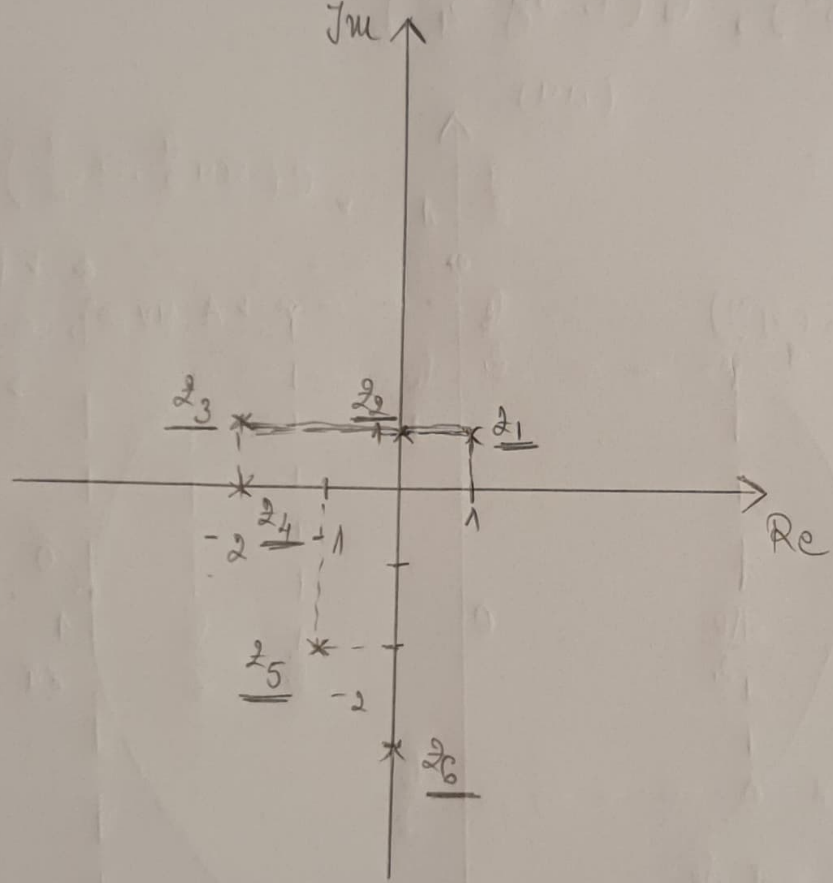
$Re_{z_4} = -2, \quad Im_{z_4} = 0$

$z_5 = -1 - 2j$

$Re_{z_5} = -1, \quad Im_{z_5} = -2$

$z_6 = -3j$

$Re_{z_6} = 0, \quad Im_{z_6} = -3$



③ Modulul și argumentul. Rep. grafic.

$z_1 = \sqrt{2} \cdot e^{j\frac{\pi}{4}}$

$|z_1| = \sqrt{2}$, $\varphi = \frac{\pi}{4}$ cadran I

$z_1 = \sqrt{2} \cdot \left(\cos\frac{\pi}{4} + j \sin\frac{\pi}{4}\right) = \sqrt{2} \cdot \left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) = 1 + j$

$\varphi = \operatorname{arctg}\frac{Im_{z_1}}{Re_{z_1}} = \frac{\pi}{4}$ $\boxed{z_1 = 1 + j}$

$z_2 = e^{j\frac{\pi}{4}}$

$|z_2| = 1$, $\varphi = \frac{\pi}{4}$.

$z_2 = \cos\frac{\pi}{4} + j \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2} \Rightarrow \boxed{z_2 = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}}$

$z_3 = 2 \cdot e^{j\frac{3\pi}{4}}$

$|z_3| = 2$, $\varphi = \frac{3\pi}{4}$

$z_3 = 2\left(\cos\frac{3\pi}{4} + j \sin\frac{3\pi}{4}\right) = 2 \cdot \left(\frac{-\sqrt{2}}{2} + j \cdot \frac{\sqrt{2}}{2}\right) = -1 + j \Rightarrow$

$\Rightarrow \boxed{z_3 = -1 + j}$

$z_4 = 3 \cdot e^{j\pi}$

$|z_4| = 3$, $\varphi = \pi$

$z_4 = 3(\cos\pi + j \cdot \sin\pi) = 3(-1 + j \cdot 0) = -3 \Rightarrow \boxed{z_4 = -3}$

$z_5 = e^{-j\frac{\pi}{4}}$

$|z_5| = 1$, $\varphi = -\frac{\pi}{4}$.

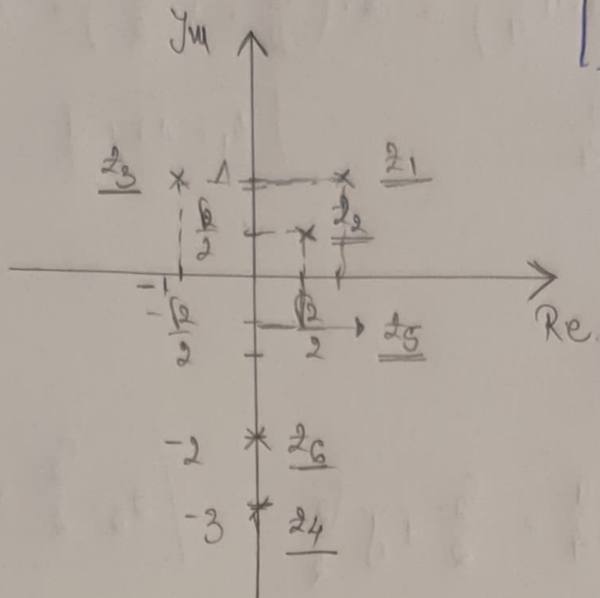
$z_5 = \cos\left(-\frac{\pi}{4}\right) + j \sin\left(\frac{-\pi}{4}\right) \Rightarrow \boxed{z_5 = +\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot j}$

$z_6 = 2\cdot e^{j3\pi}$

$|z_6| = 2, \quad \varphi = 3\pi$

$z_6 = 2(\cos 3\pi + j\sin 3\pi) = 2(-1 + j\cdot 0) = -2$

$\boxed{z_6 = -2}$



4. Forme polare. Modul şi argument pe desen.

$z_1 = 2$, $\quad Re\, z_1 = 2, \ Im\, z_1 = 0 \Rightarrow$ cadran I

$\varphi = \operatorname{arctg} \frac{0}{2} = 0 \Rightarrow \varphi = \frac{\pi}{\cancel{4}}$, $\quad |z_1| = 2$

$\varphi = 0$

$z_1 = 2\cdot e^{j\frac{\pi}{\cancel{4}}}$

$z_1 = 2\cdot e^{j\cdot 0}$

$z_2 = 2+2j$, $\quad Re\, z_2 = 2, \ Im\, z_2 = 2 \Rightarrow$ cadran I

$\varphi = \operatorname{arctg}\frac{2}{2} = 1 \Rightarrow \varphi = \frac{\pi}{4}$, $\quad |z_2| = 2\sqrt{2}$

$z_2 = 2\sqrt{2}\, e^{j\cdot\frac{\pi}{4}}$

$z_3 = j$, $Re_{z_3} = 0$, $Im_{z_3} = 1$ $\Rightarrow$ cadranul I

$\varphi = arctg \frac{1}{0} = arctg\ \infty = \frac{\pi}{2}$, $|z_3| = 1$

$z_3 = e^{j \cdot \frac{\pi}{2}}$

$z_4 = -1 + j$, $Re_{z_4} = -1$, $Im_{z_4} = 1$ $\Rightarrow$ cadran II

$\varphi = arctg \frac{1}{-1} + \pi = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$, $|z_4| = \sqrt{2}$

$z_4 = \sqrt{2} e^{j \cdot \frac{3\pi}{4}}$

$z_5 = -1 - j$, $Re_{z_5} = -1$, $Im\ z_5 = -1$ $\Rightarrow$ cadran III

$\varphi = arctg \frac{-1}{-1} + \pi = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$, $|z_5| = \sqrt{2}$

$z_5 = \sqrt{2} \cdot e^{j \cdot \frac{5\pi}{4}}$.

$z_6 = -1 - 2j$, $Re_{z_6} = -1$, $Im\ z_6 = -2$ $\Rightarrow$ cadran (III)

?

$\varphi = arctg \frac{-2}{-1} + \pi = arctg\ 2 + \pi = 1{,}107\ rad + \pi\ rad =$

$= (1{,}107 + \pi)\ rad$

π rad ... 180°

x rad 63

$|z_6| = \sqrt{1+4} = \sqrt{5}$

sau

$z_6 = \sqrt{5} \cdot e^{j(1{,}107 + \pi)}$

$arctg\ 2 \simeq 63°C$

$x \simeq \frac{\pi}{3}$

$z_6 = \sqrt{5} \cdot e^{j \cdot \frac{4\pi}{3}}$

$z_1 = 2\cos 0 + 2 j \sin 0$

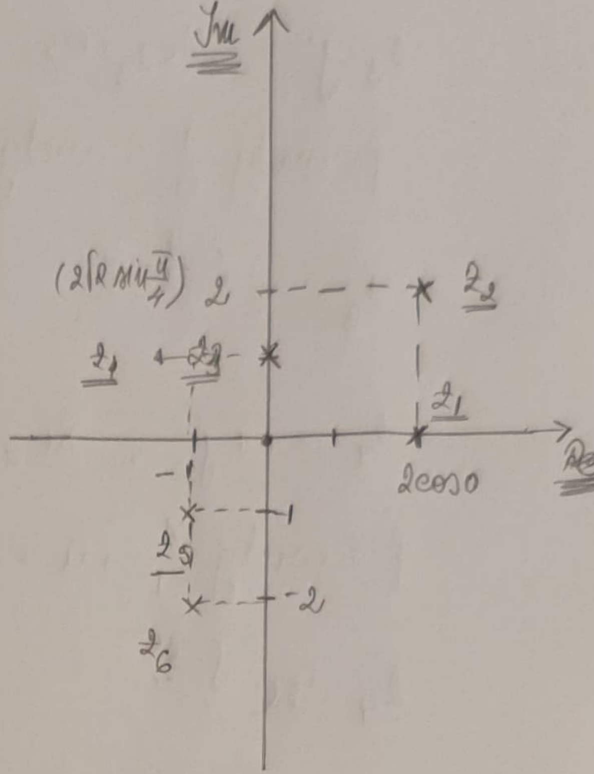
$z_2 = 2\sqrt{2} \cdot \cos\frac{\pi}{4} + 2\sqrt{2} \cdot j \cdot \sin\frac{\pi}{4}.$

$z_3 = j = e^{j\frac{\pi}{2}}$

$z_4 = -1 + j = \sqrt{2}\, e^{j \cdot \frac{3\pi}{4}}$

$z_5 = -1 - j = \sqrt{2} \cdot e^{j \cdot \frac{5\pi}{4}}$

$z_6 = -1 - 2j = \sqrt{5} \cdot e^{j(1{,}107 + \pi)}$



(5.) $z_1 = 4 \cdot e^{j\frac{\pi}{2}}$

$z_1 = 4 \cdot \left(\cos\frac{\pi}{2} + j \cdot \sin\frac{\pi}{2}\right) = 4\,(0 + j \cdot 1) = 4j.$

$z_2 = 2 \cdot e^{-j\pi}$

$z_2 = 2\left(\cos(-\pi) + j \cdot \sin(-\pi)\right) = 2(-1 + j \cdot 0) = -2$

$z_3 = 2 \cdot e^{-j \cdot \frac{3\pi}{4}}$

$z_3 = 2 \cdot \left(\cos\left(-\frac{3\pi}{4}\right) + j \cdot \sin\left(-\frac{3\pi}{4}\right)\right) = 2\left(-\frac{\sqrt{2}}{2} + j \cdot -\frac{\sqrt{2}}{2}\right) =$

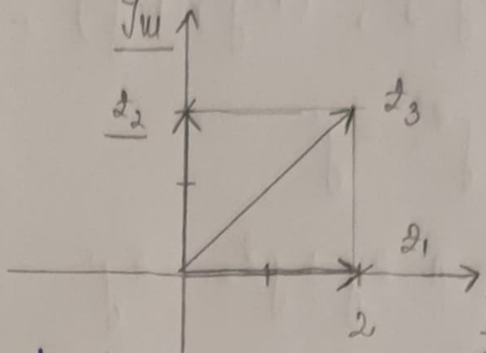
$= -\sqrt{2} - j\sqrt{2}$

$z_4 = 2 \cdot e^{j\pi}$

$z_4 = 2\left(\cos a + j \sin \pi\right) = 2(-1 + j \cdot 0) = -2.$

⑥ a) $z_1 = 2$, $z_2 = 2j$, $z_3 = z_1 + z_2$.

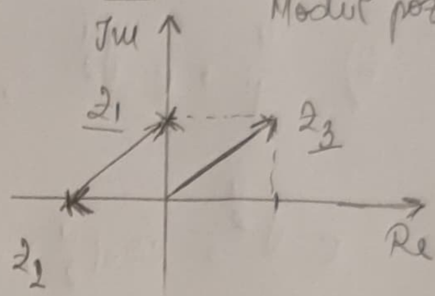
$z_3 = 2 + 2j$



Adunare vectori (reg ▱) = Ad. nr. complexe

b) $z_1 = e^{j\frac{\pi}{2}}$, $z_2 = -1$, $z_3 = z_1 - z_2$

Modul pozitiv

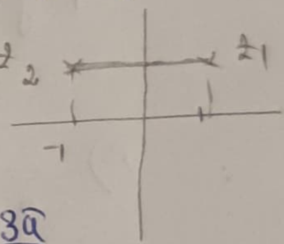


$z_1 = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = 0 + j = j$

$z_1 - z_2 = j - (-1) = j + 1 = z_3$.

c) $z_1 = 1 + j$, $z_2 = e^{j\frac{\pi}{2}} \cdot z_1$

$\cancel{z_2 = \left(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}\right)\cdot(1+j) = j(1+j) = j +} j^2 = j - 1$

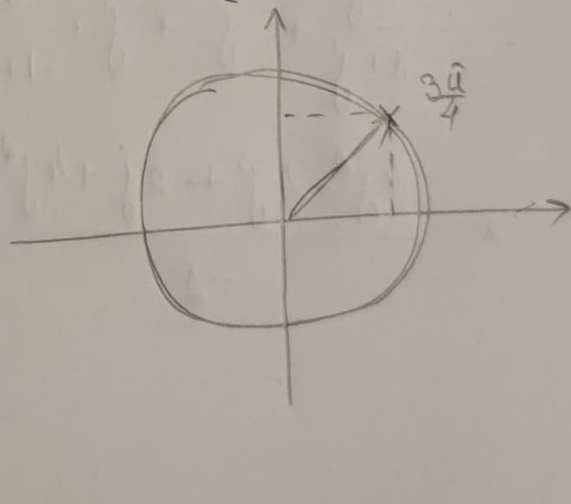


$|z_1| = 1$, $\varphi = \arctg\frac{1}{1} = \frac{\pi}{4}$

$z_1 = e^{j\frac{\pi}{4}}$

$z_2 = e^{j\frac{\pi}{2}} \cdot e^{j\frac{\pi}{4}} = e^{j\cdot\left(\frac{\pi}{2} + \frac{\pi}{4}\right)} = e^{j\cdot\frac{3\pi}{4}} =$

$= \cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}$.



(7.) Calculați:

a) j^3, $(1-j^2)^2$, $(1+j)(1-j)$, $j(1-j)$

$j^3 = j \cdot j^2 = -j$

$(1+j)(1-j) = 1-j^2 = 2$

$j(1-j) = j - j^2 = 1+j$.

$(1-j)^2 = 1-2j+j^2 = -2j$

b) $\frac{4}{j} + e^{-j\frac{\pi}{2}}$, $\frac{1+j}{1-j} + \frac{2+2j}{1+j}$.

$e^{-j\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) + j \cdot \sin\left(-\frac{\pi}{2}\right) = -j$

$\frac{4}{j} - j = \frac{4-j^2}{j} = \frac{4+1}{j} = \frac{5}{j} = -5j$

$\frac{1+j}{1-j} + \frac{2(1+j)}{1+j} = \frac{1(1+j)^2 + 2(1-j)(1+j)}{(1-j)(1+j)} = \frac{2j+4}{2} = j+2$

c) $\frac{j}{3+3j} \cdot 12 + e^{j \cdot 90^\circ} + j \cdot \frac{8}{-2+2j}$

$\frac{4j}{1+j} + j + j \cdot \frac{4}{-1+j} = \frac{4j(j-1) + j(j^2-1) + 4j(j+1)}{j^2-1} =$

$= \frac{4j^2 - \cancel{4j} + 2j + 4j^2 + \cancel{4j}}{-2} = \frac{-8-2j}{-2} = 4+j$.