2D to 3D Isontropic Oscillators with Drag - Shottlecock Services

Using the ODEs from JMA Danby:

$$\frac{d^2x}{dt^2} = -k_D * v * v_x$$

$$\downarrow$$

$$\ddot{x} = -k_D * \sqrt{\dot{x}^2 + \dot{z}^2} * \dot{x}$$

$$\frac{d^2z}{dt^2} = -k_D * v * v_z - g$$

$$\downarrow$$

$$\ddot{z} = -k_D * \sqrt{\dot{x}^2 + \dot{z}^2} * \dot{z} - g$$

Given $v = \sqrt{v_x^2 + v_z^2}$

$v_{term_{shottlecock}} = \sqrt{mg/k} \approx 22.3 \text{ ft/sec}$

and $k_D = 0.0643 \text{ ft}^{-1}$

Everything is measured in imperial units, as my primary source, JMA Danby, uses them for rescorded values.

Use Euler method to implement in a "for" loop in Python

Import Libraries

In [1]:
```
# prepares notebook for "inline" graphing, and imports libraries that we'll use later.
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
from numpy import * # imports all of numpy without "np" nickname
```

Start Code Exploration - Test Euler Method in 2D

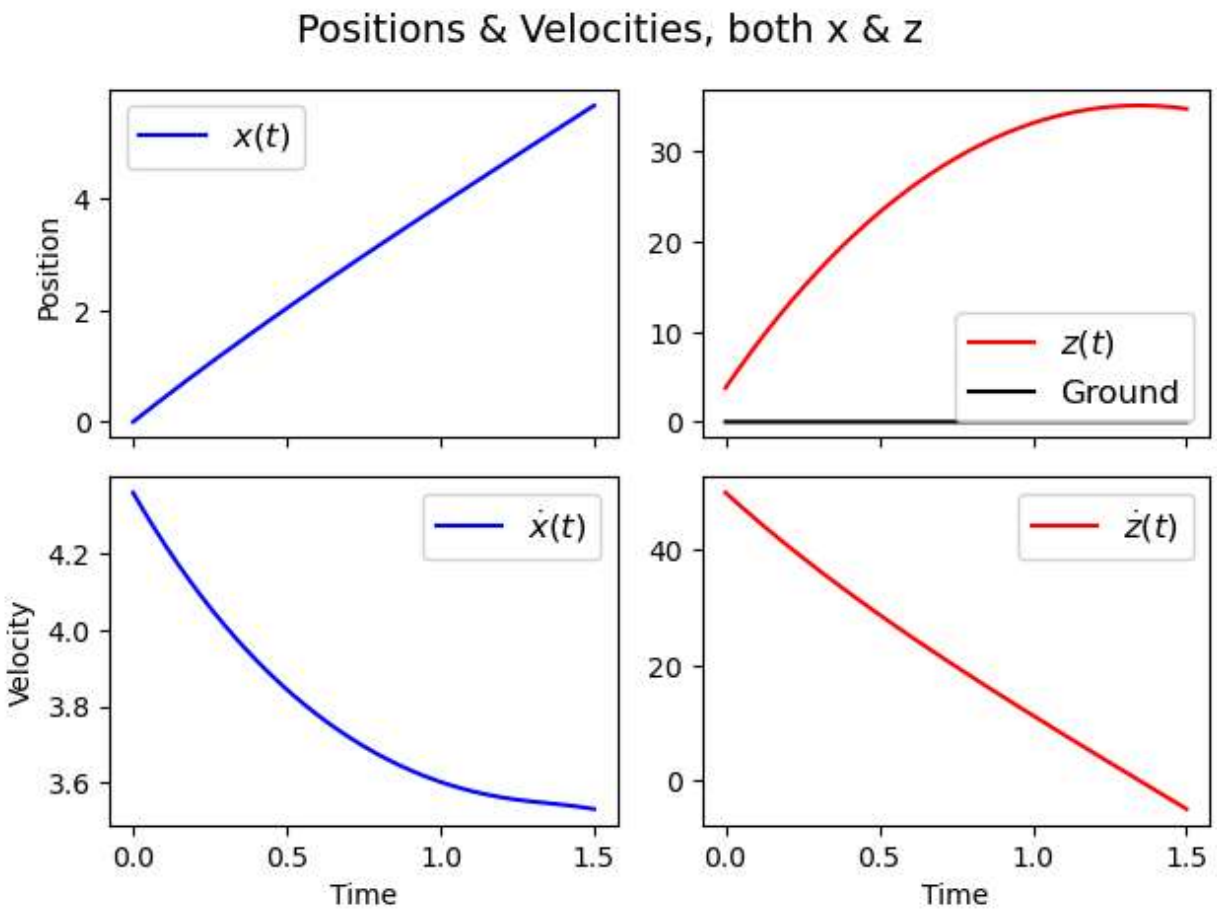
Initialize Variables and Init. Arrays; Intput to Array

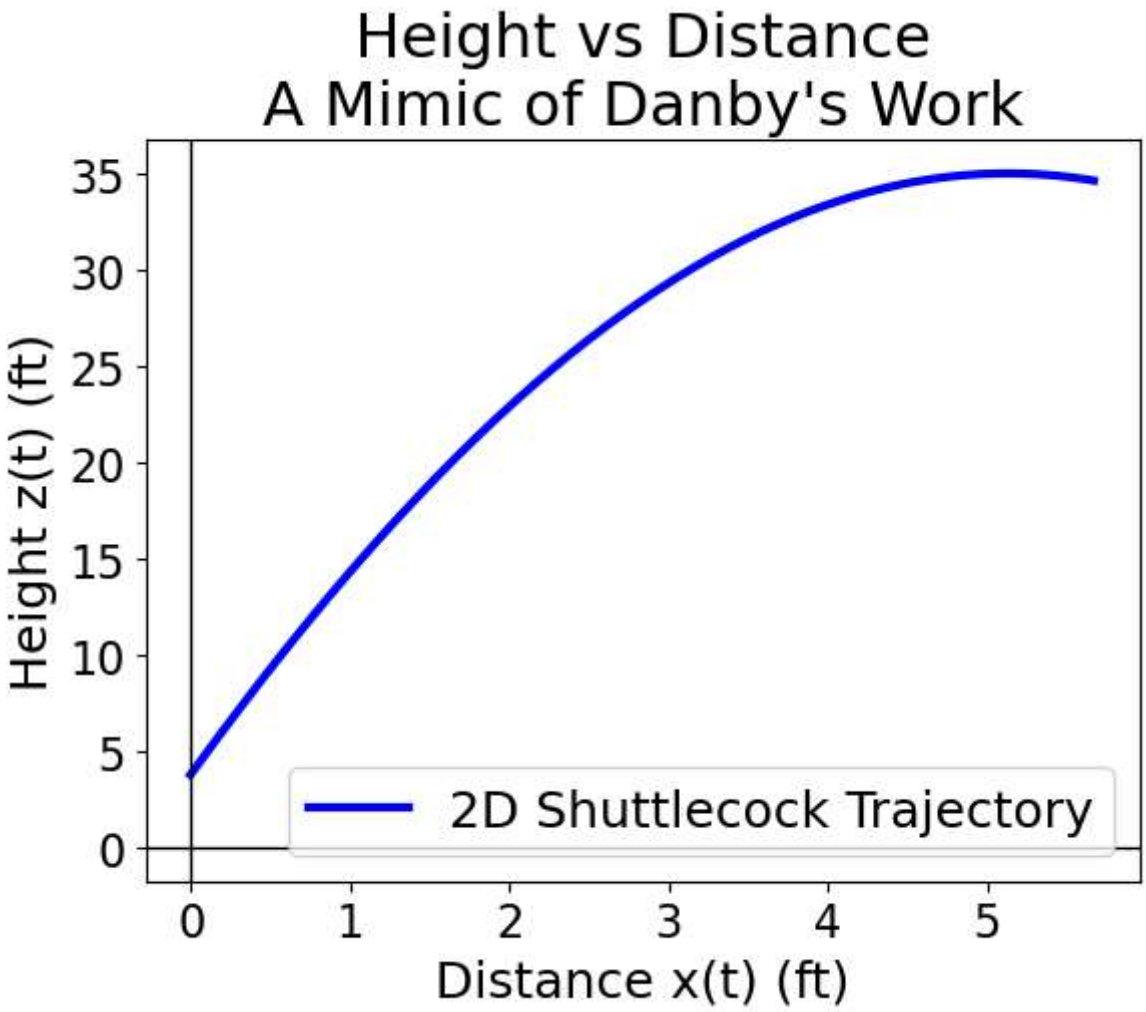
In [2]:
```
# Constants and Variables
m = (0.18 / 16) # lbs (since everything is in feet in J.M.A. Danby; 0.18 oz divided by
t_0 = 0
x_0 = 0
z_0 = (1.15 * 3.281) # ft (1.15 m times 3.281 ft/m) - https://rb.gy/lwa9pw
# v = 50 ft/s, and if v = sqrt(v_x**2 + v_z**2), we'll assume a serving angle of 20 de
v_x_0 = 50 * np.cos(np.pi * 85 / 180)
v_z_0 = 50 * np.sin(np.pi * 85 / 180)
k_D = 0.00643 # ft^-1
g = 32.17 # ft/s^2

# Start time variations of Position and Velocity
delta_t = 0.0001
Numerical_pos_x = np.array([])
Numerical_pos_z = np.array([])
Numerical_v_x = np.array([])
Numerical_v_z = np.array([])
Numerical_speed = np.array([])
time = np.array([])
t_min = 0
t_max = 1.5 # Analytical Guess from Changing t_max and viewing graphs
n = round((t_max-t_min)/ delta_t)

# Put initialized vars into first rows of arrays
Numerical_pos_x = np.append(Numerical_pos_x, x_0)
Numerical_pos_z = np.append(Numerical_pos_z, z_0)
Numerical_v_x = np.append(Numerical_v_x, v_x_0)
Numerical_v_z = np.append(Numerical_v_z, v_z_0)
time = np.append(time,t_0)


x = Numerical_pos_x[0] # initaial x value
z = Numerical_pos_z[0] # initial y value
x_dot = Numerical_v_x[0] # initial v_x value
z_dot = Numerical_v_z[0] # initial v_y value
t = time[0] # initial t value
```

Initial Euler ODE Calculations & Plots (Check if Code Works -> More Realistic)

In [3]:
```
for i in range(1, n): #For all the time steps
    x_ddot = -k_D * sqrt(x_dot**2 + z_dot**2) * x_dot
    z_ddot = -k_D * sqrt(x_dot**2 + z_dot**2) * z_dot - g

    x_dot += x_ddot*delta_t  #Euler method equation
    z_dot += z_ddot*delta_t

    x += x_dot*delta_t
    z += z_dot*delta_t

    t += delta_t #Update t to a new value based on Δt

    Numerical_pos_x = np.append(Numerical_pos_x, x)
    Numerical_pos_z = np.append(Numerical_pos_z, z) #Put your newest y value at the en
```

```
    Numerical_v_x = np.append(Numerical_v_x, x_dot)
    Numerical_v_z = np.append(Numerical_v_z, z_dot)

    time = np.append(time, t)  #Put your newest t value at the end of your table of t


# PLOT!
fig, ((ax1, ax2), (ax3, ax4)) = plt.subplots(2,2,sharex=True)
fig.suptitle('Positions & Velocities, both x & z', fontsize=14)

x_position = ax1.plot(time, Numerical_pos_x,'b', label=r'$x(t)$')
z_position = ax2.plot(time, Numerical_pos_z, 'r', label=r'$z(t)$')
ground_line = ax2.plot(time, np.linspace(0,0,n), 'black', label=r'Ground')

x_velocity = ax3.plot(time, Numerical_v_x, 'b', label=r'$\dot{x}(t)$')
z_velocity = ax4.plot(time, Numerical_v_z, 'r', label=r'$\dot{z}(t)$')

ax1.set_ylabel('Position')
ax3.set_ylabel('Velocity')
ax3.set_xlabel('Time')
ax4.set_xlabel('Time')


ax1.legend(loc='best', fontsize = 12)
ax2.legend(loc='best', fontsize = 12)
ax3.legend(loc='best', fontsize = 12)
ax4.legend(loc='best', fontsize = 12)
fig.tight_layout()
plt.show()


z_vs_x = plt.plot(Numerical_pos_x, Numerical_pos_z, 'b', label="2D Shuttlecock Traject
plt.title("Height vs Distance\nA Mimic of Danby's Work", fontsize=22)
plt.axhline(0, color='black', lw=1)
plt.axvline(0, color='black', lw=1)
plt.xlabel('Distance x(t) (ft)', fontsize=18)
plt.xticks(fontsize=16)
plt.ylabel('Height z(t) (ft)', fontsize=18)
plt.yticks(fontsize=16)
plt.legend(loc='best', fontsize=18)
plt.show()
```


Positions & Velocities, both x & z
x(t)
Position
z(t)
Ground
ẋ(t)
Velocity
ż(t)
Time
Time



Add Y-Component (Test 3D Models), Model Best Services From Right Court, Initial Velocity of 10 m/s

ϕ is xz angle, θ is yz angle

3D Variables and Arrays; Intput to Array

$$\ddot{x} = -k_D * \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} * \dot{x}$$

$$\ddot{y} = -k_D * \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} * \dot{y}$$

$$\ddot{z} = -k_D * \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} * \dot{z} - g$$

Prepping 3 Dimensions

In [4]:
```
# Constants and Variables
m = (0.18 / 16) # lbs (since everything is in feet in J.M.A. Danby; 0.18 oz divided by
```

```
t_0 = 0
x_0 = 8.5 # ft (if back court line is 0.0 ft, then service line is 13.5 ft, given the
y_0 = 13.5 # ft (if far-left court line is 0.0 ft, then divider is 8.5 ft, given the w
z_0 = (1.15 * 3.281) # ft (1.15 m times 3.281 ft/m) - https://rb.gy/lwa9pw
# v = 10 m/s, so (82*3.281) ft/s, and if v = sqrt(v_x**2 + v_y**2 + v_z**2)
v_0 = 10 * 3.281 # ft/s
theta = np.pi * 77 / 180
phi = np.pi * 8 / 180
v_x_0 = v_0 * np. sin(phi)
v_y_0 = v_0 * np.cos(theta) * np.cos(phi)
v_z_0 = v_0 * np.sin(theta) * np.cos(phi)
k_D = 0.00643 # ft^-1
g = 32.17 # ft/s^2

# Start time variations of Position and Velocity
delta_t = 0.001
Numerical_pos_x = np.array([])
Numerical_pos_y = np.array([])
Numerical_pos_z = np.array([])
Numerical_v_x = np.array([])
Numerical_v_y = np.array([])
Numerical_v_z = np.array([])
Numerical_speed = np.array([])
time = np.array([])
t_min = 0
t_max = 3 # Analytical Guess from Changing t_max and viewing graphs
n = round((t_max-t_min)/ delta_t)

# Put initialized vars into first rows of arrays
Numerical_pos_x = np.append(Numerical_pos_x, x_0)
Numerical_pos_y = np.append(Numerical_pos_y, y_0)
Numerical_pos_z = np.append(Numerical_pos_z, z_0)
Numerical_v_x = np.append(Numerical_v_x, v_x_0)
Numerical_v_y = np.append(Numerical_v_y, v_y_0)
Numerical_v_z = np.append(Numerical_v_z, v_z_0)
time = np.append(time,t_0)


x = Numerical_pos_x[0] # initaial x value
y = Numerical_pos_y[0] # initaial y value
z = Numerical_pos_z[0] # initial z value
x_dot = Numerical_v_x[0] # initial v_x value
y_dot = Numerical_v_y[0] # initial v_y value
z_dot = Numerical_v_z[0] # initial v_z value
t = time[0] # initial t value
```

In [5]:

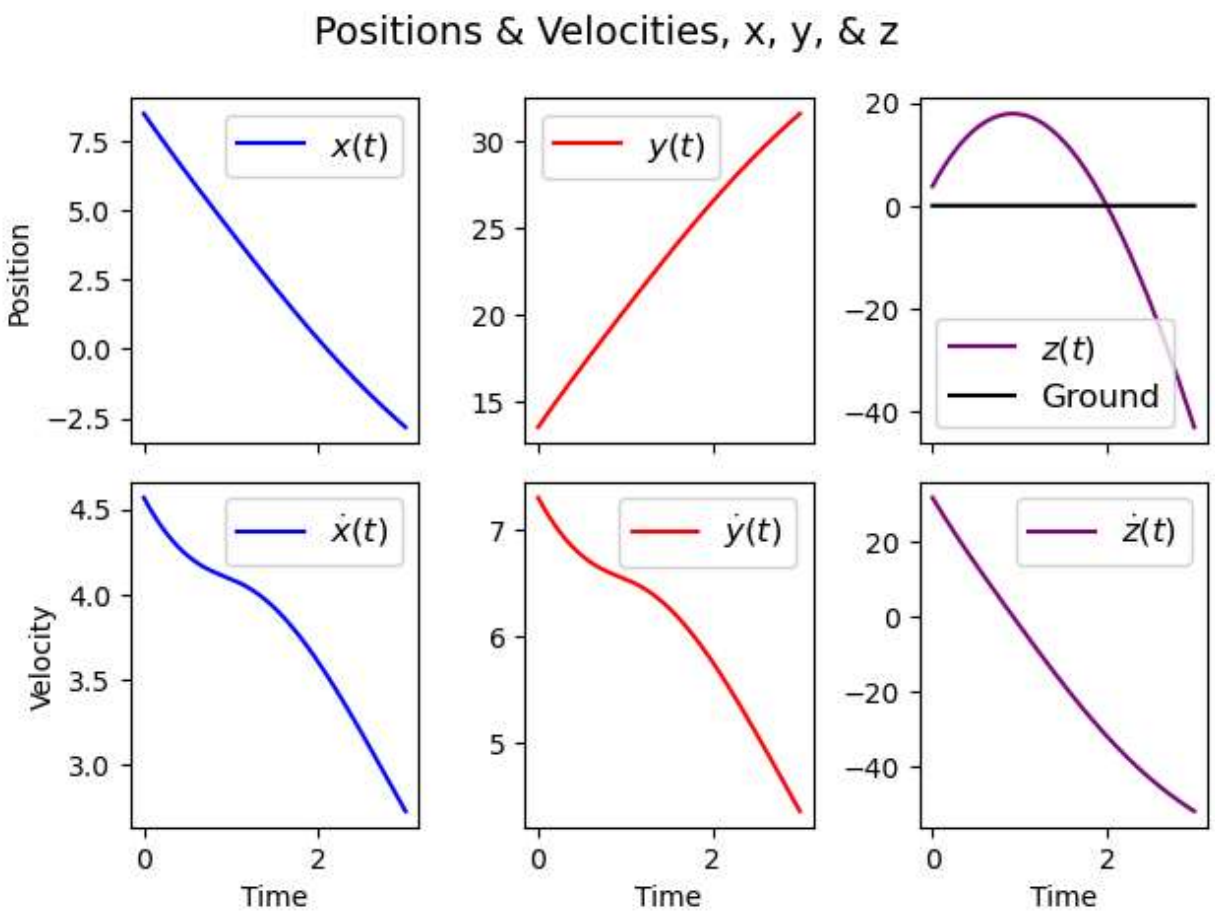
```
# Euler iterations
for i in range(1, n): #For all the time steps
    x_ddot = -k_D * sqrt(x_dot**2 + y_dot**2 + z_dot**2) * x_dot
    y_ddot = -k_D * sqrt(x_dot**2 + y_dot**2 + z_dot**2) * y_dot
    z_ddot = -k_D * sqrt(x_dot**2 + y_dot**2 + z_dot**2) * z_dot - g

    x_dot += x_ddot*delta_t  #Euler method equation
    y_dot += y_ddot*delta_t
    z_dot += z_ddot*delta_t

    x -= x_dot*delta_t # Starting from midline, serving to the left (-x direction)
    y += y_dot*delta_t
    z += z_dot*delta_t
```

```
    t += delta_t #Update t to a new value based on Δt

    Numerical_pos_x = np.append(Numerical_pos_x, x) #Put your newest x value at the en
    Numerical_pos_y = np.append(Numerical_pos_y, y)
    Numerical_pos_z = np.append(Numerical_pos_z, z)
    Numerical_v_x = np.append(Numerical_v_x, x_dot)
    Numerical_v_y = np.append(Numerical_v_y, y_dot)
    Numerical_v_z = np.append(Numerical_v_z, z_dot)

    time = np.append(time, t)  #Put your newest t value at the end of your table of t
```

2D Plots of a 3D System!

In [6]:

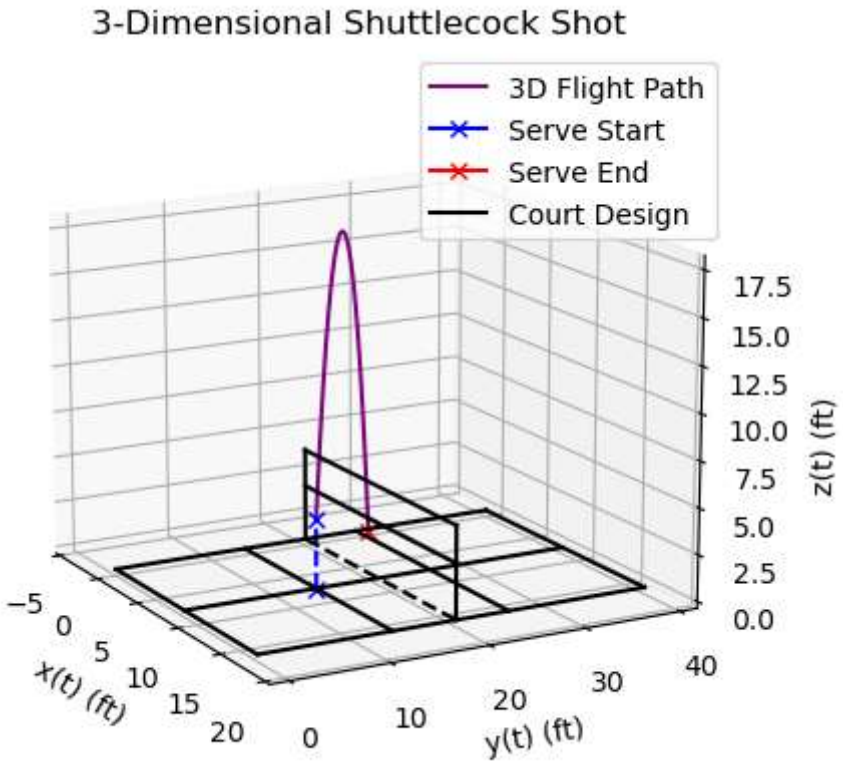
```
fig, ((ax1, ax2, ax3), (ax4, ax5, ax6)) = plt.subplots(2,3,sharex=True)
fig.suptitle('Positions & Velocities, x, y, & z', fontsize=14)

x_position = ax1.plot(time, Numerical_pos_x,'b', label=r'$x(t)$')
y_position = ax2.plot(time, Numerical_pos_y, 'r', label=r'$y(t)$')
z_position = ax3.plot(time, Numerical_pos_z, 'purple', label=r'$z(t)$')
ground_line = ax3.plot(time, np.linspace(0,0,n), 'black', label=r'Ground')

x_velocity = ax4.plot(time, Numerical_v_x, 'b', label=r'$\dot{x}(t)$')
y_velocity = ax5.plot(time, Numerical_v_y, 'r', label=r'$\dot{y}(t)$')
z_velocity = ax6.plot(time, Numerical_v_z, 'purple', label=r'$\dot{z}(t)$')

ax1.set_ylabel('Position')
ax4.set_ylabel('Velocity')
ax4.set_xlabel('Time')
ax5.set_xlabel('Time')
ax6.set_xlabel('Time')


ax1.legend(loc='best', fontsize = 12)
ax2.legend(loc='best', fontsize = 12)
ax3.legend(loc='best', fontsize = 12)
ax4.legend(loc='best', fontsize = 12)
ax5.legend(loc='best', fontsize = 12)
ax6.legend(loc='best', fontsize = 12)
fig.tight_layout()
plt.show()
```



Solving for 1 Ideal Service - 3D Plot

In [7]:
```
ax = plt.axes(projection='3d')
ax.set_title("3-Dimensional Shuttlecock Shot")
# ax.set_axhline(0, color='black', lw=1)
# ax.set_axvline(0, color='black', lw=1)
ax.set_xlabel('x(t) (ft)')
ax.set_ylabel('y(t) (ft)')
ax.set_zlabel('z(t) (ft)')


for hits_ground in range(len(time)):
    if Numerical_pos_z[hits_ground] <= 0:
        # print(hits_ground)
        z_vs_xy = ax.plot(Numerical_pos_x[:hits_ground], Numerical_pos_y[:hits_ground]

        start_point = ax.plot(x_0, y_0, z_0, 'blue', marker='x', label=r'Serve Start')
        start_point_ground = ax.plot(x_0, y_0, 0, 'blue', marker='x')
        start_drop_down = ax.plot(linspace(x_0, x_0, n), linspace(y_0, y_0, n), linspa
        end_point = ax.plot(Numerical_pos_x[hits_ground], Numerical_pos_y[hits_ground]

        # Plotting Court
        court_back = ax.plot(linspace(0,17,n), linspace(0,0,n), linspace(0,0,n), 'blac
        court_left = ax.plot(linspace(0,0,n), linspace(0,40,n), linspace(0,0,n), 'blac
        court_up = ax.plot(linspace(0,17,n), linspace(40,40,n), linspace(0,0,n), 'blac
        court_right = ax.plot(linspace(17,17,n), linspace(0,40,n), linspace(0,0,n), 'b
        long_mid = ax.plot(linspace(8.5,8.5,n), linspace(0,40,n), linspace(0,0,n), 'bl
        service_line_1 = ax.plot(linspace(0,17,n), linspace(13.5,13.5,n), linspace(0,0
        service_line_2 = ax.plot(linspace(0,17,n), linspace(25.5,25.5,n), linspace(0,0
```

```
        net_left = ax.plot(linspace(0,0,n), linspace(20,20,n), linspace(0,5,n), 'black
        net_right = ax.plot(linspace(17,17,n), linspace(20,20,n), linspace(0,5,n), 'bl
        net_bottom = ax.plot(linspace(0,17,n), linspace(20,20,n), linspace(3,3,n), 'bl
        net_top = ax.plot(linspace(0,17,n), linspace(20,20,n), linspace(5,5,n), 'black
        net_top = ax.plot(linspace(0,17,n), linspace(20,20,n), linspace(0,0,n), 'black

        # Viewing Angle
        # for azmith in range(0,30,15):
            # print(azmith)
        ax.view_init(azim=-29, elev=15)
        ax.set_xlim3d(-5, 20)
        ax.set_ylim3d(-1, 41)
        # ax.set_zlim3d(0, 6)

        ax.legend(loc='best', fontsize=10)
        ax.figsize=(100,100,100)
        plt.show()
        break
```



Function for 3D Modeling (Indiv. Graphs)!

In [8]:

```
# Given initial v is 10 m/s or ~32.81 ft/s
def Service_Deg(THETA, PHI, TITLE, AZM, ELEV):
    # Constants and Variables
    m = (0.18 / 16) # lbs (since everything is in feet in J.M.A. Danby; 0.18 oz divide
    t_0 = 0
    x_0 = 8.5 # ft (if back court line is 0.0 ft, then service line is 13.5 ft, given
    y_0 = 13.5 # ft (if far-left court line is 0.0 ft, then divider is 8.5 ft, given t
    z_0 = (1.15 * 3.281) # ft (1.15 m times 3.281 ft/m) - https://rb.gy/lwa9pw
    # v = 10 m/s, so (82*3.281) ft/s, and if v = sqrt(v_x**2 + v_y**2 + v_z**2)
    v_0 = 10 * 3.281 # ft/s
    theta = np.pi * THETA / 180
    phi = np.pi * PHI / 180
```

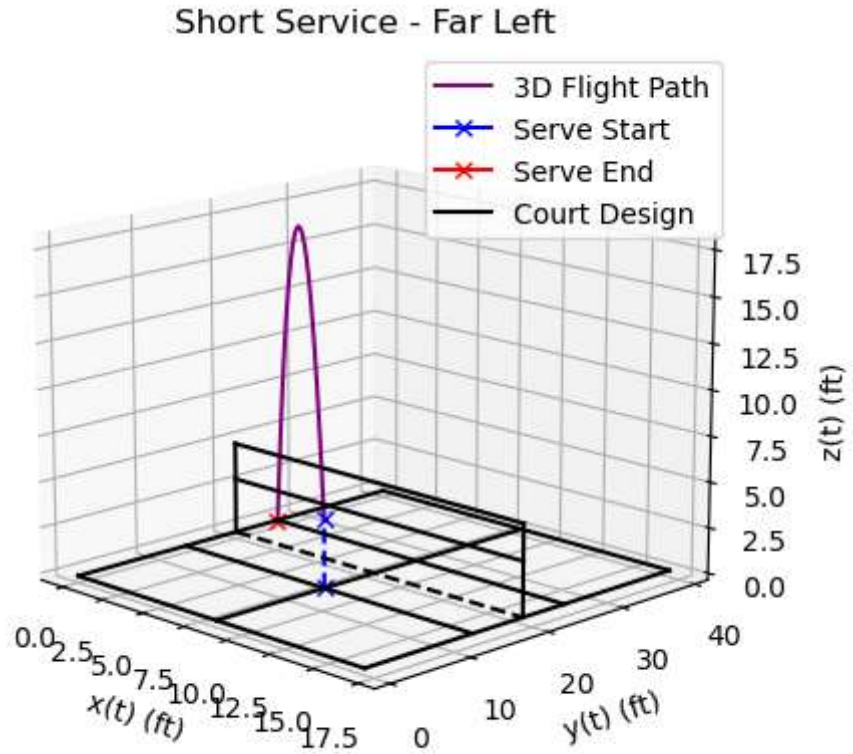
```
v_x_0 = v_0 * np. sin(phi)
v_y_0 = v_0 * np.cos(theta) * np.cos(phi)
v_z_0 = v_0 * np.sin(theta) * np.cos(phi)
k_D = 0.00643 # ft^-1
g = 32.17 # ft/s^2

# Start time variations of Position and Velocity
delta_t = 0.001
Numerical_pos_x = np.array([])
Numerical_pos_y = np.array([])
Numerical_pos_z = np.array([])
Numerical_v_x = np.array([])
Numerical_v_y = np.array([])
Numerical_v_z = np.array([])
Numerical_speed = np.array([])
time = np.array([])
t_min = 0
t_max = 3 # Analytical Guess from Changing t_max and viewing graphs
n = round((t_max-t_min)/ delta_t)

# Put initialized vars into first rows of arrays
Numerical_pos_x = np.append(Numerical_pos_x, x_0)
Numerical_pos_y = np.append(Numerical_pos_y, y_0)
Numerical_pos_z = np.append(Numerical_pos_z, z_0)
Numerical_v_x = np.append(Numerical_v_x, v_x_0)
Numerical_v_y = np.append(Numerical_v_y, v_y_0)
Numerical_v_z = np.append(Numerical_v_z, v_z_0)
time = np.append(time,t_0)


x = Numerical_pos_x[0] # initaial x value
y = Numerical_pos_y[0] # initaial y value
z = Numerical_pos_z[0] # initial z value
x_dot = Numerical_v_x[0] # initial v_x value
y_dot = Numerical_v_y[0] # initial v_y value
z_dot = Numerical_v_z[0] # initial v_z value
t = time[0] # initial t value

for i in range(1, n): #For all the time steps
    x_ddot = -k_D * sqrt(x_dot**2 + y_dot**2 + z_dot**2) * x_dot
    y_ddot = -k_D * sqrt(x_dot**2 + y_dot**2 + z_dot**2) * y_dot
    z_ddot = -k_D * sqrt(x_dot**2 + y_dot**2 + z_dot**2) * z_dot - g

    x_dot += x_ddot*delta_t  #Euler method equation
    y_dot += y_ddot*delta_t
    z_dot += z_ddot*delta_t

    x -= x_dot*delta_t # Starting from midline, serving to the left (-x direction)
    y += y_dot*delta_t
    z += z_dot*delta_t

    t += delta_t #Update t to a new value based on Δt

    Numerical_pos_x = np.append(Numerical_pos_x, x) #Put your newest x value at th
    Numerical_pos_y = np.append(Numerical_pos_y, y)
    Numerical_pos_z = np.append(Numerical_pos_z, z)
    Numerical_v_x = np.append(Numerical_v_x, x_dot)
    Numerical_v_y = np.append(Numerical_v_y, y_dot)
    Numerical_v_z = np.append(Numerical_v_z, z_dot)
```

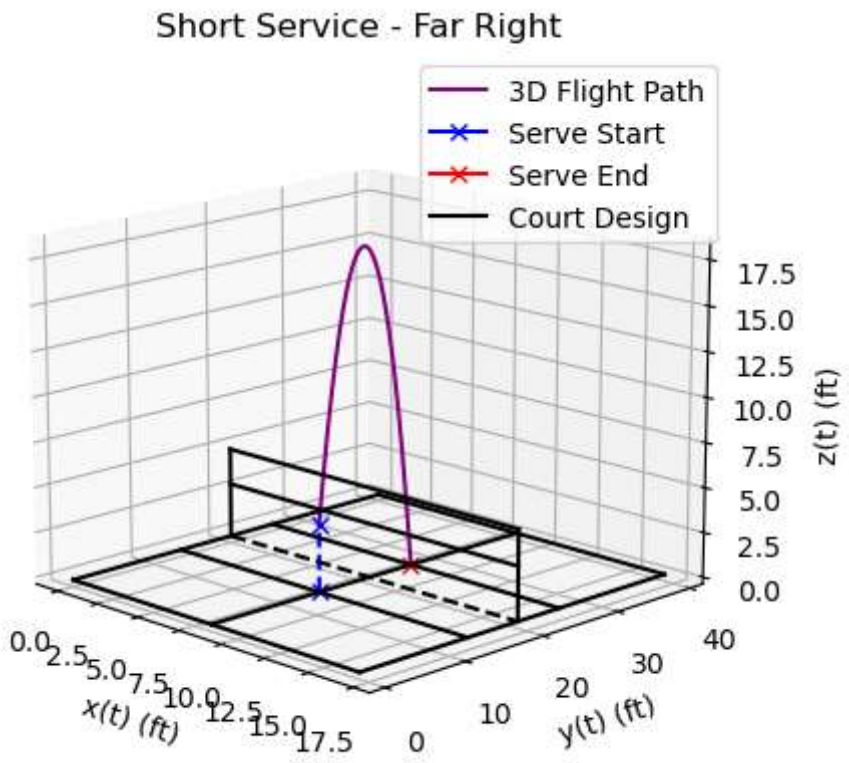
```
        time = np.append(time, t)  #Put your newest t value at the end of your table c

    ax = plt.axes(projection='3d')
    ax.set_title(TITLE)
    # ax.set_axhline(0, color='black', lw=1)
    # ax.set_axvline(0, color='black', lw=1)
    ax.set_xlabel('x(t) (ft)')
    ax.set_ylabel('y(t) (ft)')
    ax.set_zlabel('z(t) (ft)')


    for hits_ground in range(len(time)):
        if Numerical_pos_z[hits_ground] <= 0:
            # print(hits_ground)
            Flight_Path = ax.plot(Numerical_pos_x[:hits_ground], Numerical_pos_y[:hits

            start_point = ax.plot(x_0, y_0, z_0, 'blue', marker='x', label=r'Serve Sta
            start_point_ground = ax.plot(x_0, y_0, 0, 'blue', marker='x')
            start_drop_down = ax.plot(linspace(x_0, x_0, n), linspace(y_0, y_0, n), li
            end_point = ax.plot(Numerical_pos_x[hits_ground], Numerical_pos_y[hits_gro

            # Plotting Court
            court_back = ax.plot(linspace(0,17,n), linspace(0,0,n), linspace(0,0,n), '
            court_left = ax.plot(linspace(0,0,n), linspace(0,40,n), linspace(0,0,n), '
            court_up = ax.plot(linspace(0,17,n), linspace(40,40,n), linspace(0,0,n), '
            court_right = ax.plot(linspace(17,17,n), linspace(0,40,n), linspace(0,0,n)
            long_mid = ax.plot(linspace(8.5,8.5,n), linspace(0,40,n), linspace(0,0,n),
            service_line_1 = ax.plot(linspace(0,17,n), linspace(13.5,13.5,n), linspace
            service_line_2 = ax.plot(linspace(0,17,n), linspace(25.5,25.5,n), linspace
            net_left = ax.plot(linspace(0,0,n), linspace(20,20,n), linspace(0,5,n), 'b
            net_right = ax.plot(linspace(17,17,n), linspace(20,20,n), linspace(0,5,n),
            net_bottom = ax.plot(linspace(0,17,n), linspace(20,20,n), linspace(3,3,n),
            net_top = ax.plot(linspace(0,17,n), linspace(20,20,n), linspace(5,5,n), 'b
            net_top = ax.plot(linspace(0,17,n), linspace(20,20,n), linspace(0,0,n), 'b

            # Viewing Angle
            # for azmith in range(0,30,15):
                # print(azmith)
            ax.view_init(azim=AZM, elev=ELEV)
            ax.set_xlim3d(-1, 18)
            ax.set_ylim3d(-1, 41)
            # ax.set_zlim3d(0, 6)

            ax.legend(loc='best', fontsize=10)
            ax.figsize=(100,100,100)
            plt.show()
            return Flight_Path
            break
```

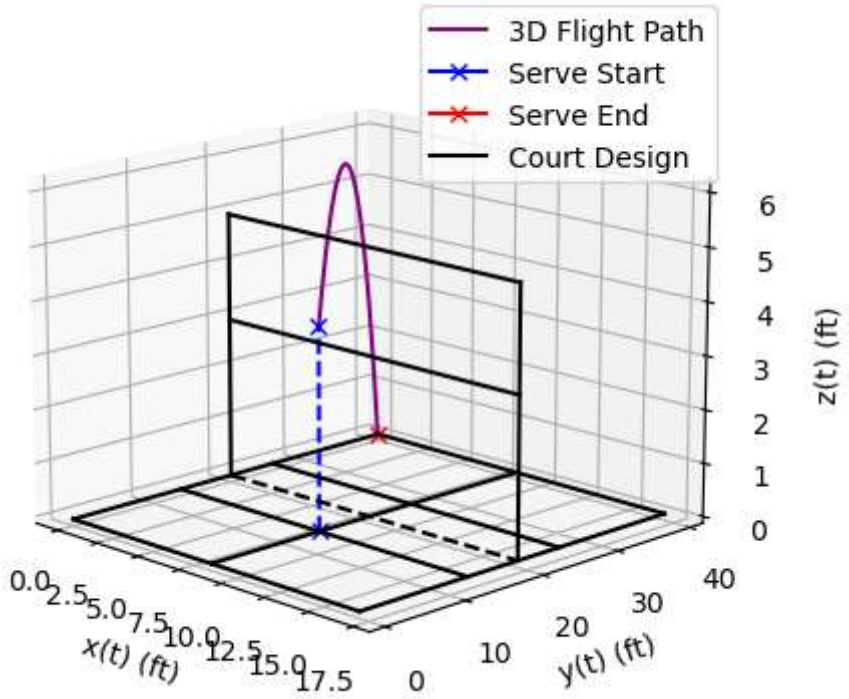
Individual Graphs!!!

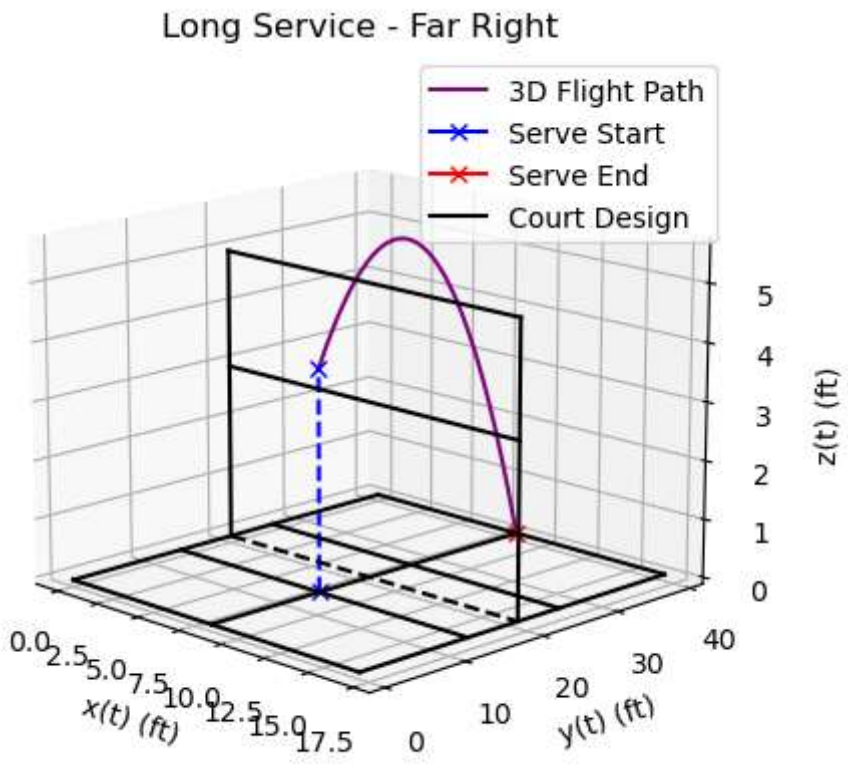
In [9]:
```
# Rights and Lefts are from Server's Perspective
theta_f_l = 78.07
phi_f_l = 8.32
title_f_l = f"Short Service - Far Left"

# ideal_title= f"Ideal Services"
```

```
theta_f_r = 78.35
phi_f_r = 0.01
title_f_r = f"Short Service - Far Right"

theta_b_l = 23.72
phi_b_l = 16.4
title_b_l = f"Long Service - Far Left"

theta_b_r = 19.96
phi_b_r = 0.01
title_b_r = f"Long Service - Far Right"

# Different Views
azm1 = -45
azm2 = -150
elev1 = 15
elev2 = 45

Front_Left = Service_Deg(theta_f_l, phi_f_l, title_f_l, azm1, elev1)
# Front_Left2 = Service_Deg(theta_f_l, phi_f_l, title_f_l, azm2, elev2)

Front_Right = Service_Deg(theta_f_r, phi_f_r, title_f_r, azm1, elev1)
# Front_Right2 = Service_Deg(theta_f_r, phi_f_r, title_f_r, azm2, elev2)

Back_Left = Service_Deg(theta_b_l, phi_b_l, title_b_l, azm1, elev1)
# Back_Left2 = Service_Deg(theta_b_l, phi_b_l, title_b_l, azm2, elev2)

Back_Right = Service_Deg(theta_b_r, phi_b_r, title_b_r, azm1, elev1)
# Back_Right2 = Service_Deg(theta_b_r, phi_b_r, title_b_r, azm2, elev2)
```




Short Service - Far Right
3D Flight Path
Serve Start
Serve End
Court Design
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
z(t) (ft)
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
x(t) (ft)
0
10
20
30
40
y(t) (ft)


Long Service - Far Left
3D Flight Path
Serve Start
Serve End
Court Design
6
5
4
3
2
1
0
z(t) (ft)
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
x(t) (ft)
0
10
20
30
40
y(t) (ft)



Code for Unified Graph

Function

In [10]:
```
def Service_Deg(THETA, PHI):
    # Constants and Variables
    m = (0.18 / 16) # lbs (since everything is in feet in J.M.A. Danby; 0.18 oz divide
    t_0 = 0
    x_0 = 8.5 # ft (if back court line is 0.0 ft, then service line is 13.5 ft, given
    y_0 = 13.5 # ft (if far-left court line is 0.0 ft, then divider is 8.5 ft, given t
    z_0 = (1.15 * 3.281) # ft (1.15 m times 3.281 ft/m) - https://rb.gy/lwa9pw
    # v = 10 m/s, so (82*3.281) ft/s, and if v = sqrt(v_x**2 + v_y**2 + v_z**2)
    v_0 = 10 * 3.281 # ft/s
    theta = np.pi * THETA / 180
    phi = np.pi * PHI / 180
    v_x_0 = v_0 * np. sin(phi)
    v_y_0 = v_0 * np.cos(theta) * np.cos(phi)
    v_z_0 = v_0 * np.sin(theta) * np.cos(phi)
    k_D = 0.00643 # ft^-1
    g = 32.17 # ft/s^2

    # Start time variations of Position and Velocity
    delta_t = 0.001
    Numerical_pos_x = np.array([])
    Numerical_pos_y = np.array([])
    Numerical_pos_z = np.array([])
    Numerical_v_x = np.array([])
    Numerical_v_y = np.array([])
    Numerical_v_z = np.array([])
    Numerical_speed = np.array([])
    time = np.array([])
```

```
    t_min = 0
    t_max = 3 # Analytical Guess from Changing t_max and viewing graphs
    n = round((t_max-t_min)/ delta_t)

    # Put initialized vars into first rows of arrays
    Numerical_pos_x = np.append(Numerical_pos_x, x_0)
    Numerical_pos_y = np.append(Numerical_pos_y, y_0)
    Numerical_pos_z = np.append(Numerical_pos_z, z_0)
    Numerical_v_x = np.append(Numerical_v_x, v_x_0)
    Numerical_v_y = np.append(Numerical_v_y, v_y_0)
    Numerical_v_z = np.append(Numerical_v_z, v_z_0)
    time = np.append(time,t_0)


    x = Numerical_pos_x[0] # initaial x value
    y = Numerical_pos_y[0] # initaial y value
    z = Numerical_pos_z[0] # initial z value
    x_dot = Numerical_v_x[0] # initial v_x value
    y_dot = Numerical_v_y[0] # initial v_y value
    z_dot = Numerical_v_z[0] # initial v_z value
    t = time[0] # initial t value

    for i in range(1, n): #For all the time steps
        x_ddot = -k_D * sqrt(x_dot**2 + y_dot**2 + z_dot**2) * x_dot
        y_ddot = -k_D * sqrt(x_dot**2 + y_dot**2 + z_dot**2) * y_dot
        z_ddot = -k_D * sqrt(x_dot**2 + y_dot**2 + z_dot**2) * z_dot - g

        x_dot += x_ddot*delta_t  #Euler method equation
        y_dot += y_ddot*delta_t
        z_dot += z_ddot*delta_t

        x -= x_dot*delta_t # Starting from midline, serving to the left (-x direction)
        y += y_dot*delta_t
        z += z_dot*delta_t

        t += delta_t #Update t to a new value based on Δt

        Numerical_pos_x = np.append(Numerical_pos_x, x) #Put your newest x value at th
        Numerical_pos_y = np.append(Numerical_pos_y, y)
        Numerical_pos_z = np.append(Numerical_pos_z, z)
        Numerical_v_x = np.append(Numerical_v_x, x_dot)
        Numerical_v_y = np.append(Numerical_v_y, y_dot)
        Numerical_v_z = np.append(Numerical_v_z, z_dot)

        time = np.append(time, t)  #Put your newest t value at the end of your table c

        zero = 0
        for hits_ground in range(len(time)):
            if Numerical_pos_z[hits_ground] <= 0:
                zero = hits_ground
                break

    return Numerical_pos_x, Numerical_pos_y, Numerical_pos_z, zero
```

Ideal Service Angles

Found via Guess & Check

In [11]:
```
theta_f_l = 78.07
phi_f_l = 8.32

theta_f_r = 78.35
phi_f_r = 0.01

theta_b_l = 23.72
phi_b_l = 16.4

theta_b_r = 19.96
phi_b_r = 0.01

n = round(3 / 0.001) # Bringing out of function for graphing the court over an linspac
```

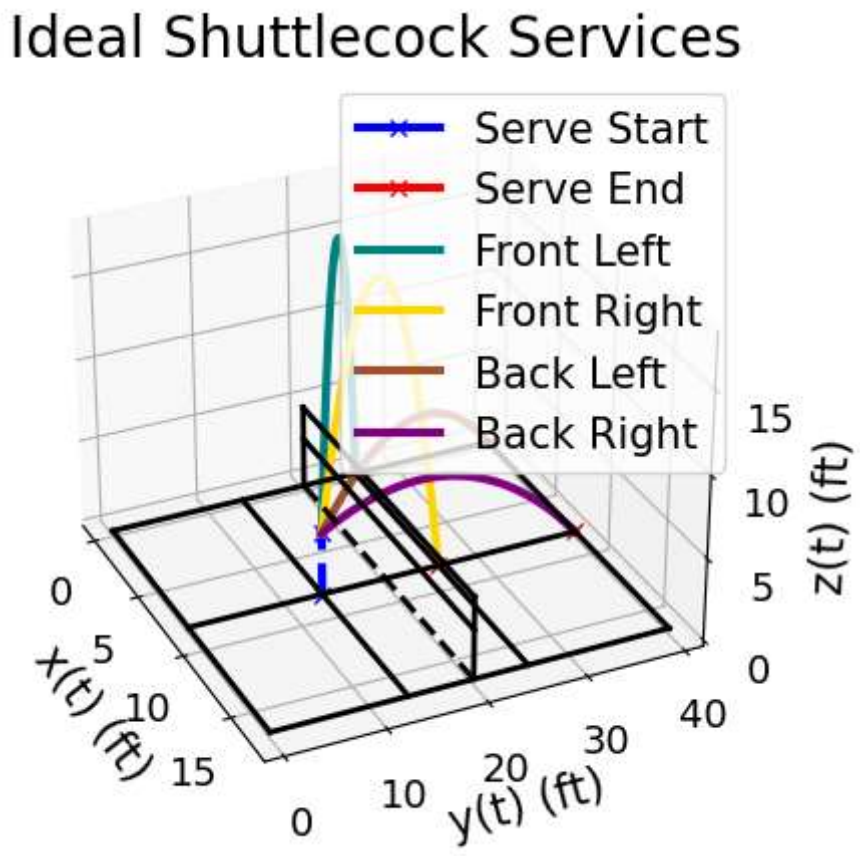
Ideal Graph!

I used Spyder for the images on the poster presentation because of its easy graph-manipulation

In [12]:
```
ax = plt.axes(projection='3d')
ax.set_title(f'Ideal Shuttlecock Services', fontsize=20)
ax.set_xlabel('x(t) (ft)', fontsize=16)
ax.set_ylabel('y(t) (ft)', fontsize=16)
ax.set_zlabel('z(t) (ft)', fontsize=16)

# Starting Point
x_0 = 8.5
y_0 = 13.5
z_0 = 1.15 * 3.281
start_point = ax.plot(x_0, y_0, z_0, 'blue', marker='x', label=r'Serve Start', linewic
start_point_ground = ax.plot(x_0, y_0, 0, 'blue', marker='x', linewidth=3)
start_drop_down = ax.plot(linspace(x_0, x_0, n), linspace(y_0, y_0, n), linspace(0, z_

# Plot trajectories for each type of serve
front_left = Service_Deg(theta_f_l, phi_f_l)
front_left_x = front_left[0]
front_left_y = front_left[1]
front_left_z = front_left[2]
zero_fl = front_left[3]
end_point = ax.plot(front_left_x[zero_fl], front_left_y[zero_fl], 0, 'red', marker='x'
ax.plot(front_left_x[:zero_fl], front_left_y[:zero_fl], front_left_z[:zero_fl], 'teal'

front_right = Service_Deg(theta_f_r, phi_f_r)
front_right_x = front_right[0]
front_right_y = front_right[1]
front_right_z = front_right[2]
zero_fr = front_right[3]
ax.plot(front_right_x[:zero_fr], front_right_y[:zero_fr], front_right_z[:zero_fr], 'gc
end_point = ax.plot(front_right_x[zero_fr], front_right_y[zero_fr], 0, 'red', marker='

back_left = Service_Deg(theta_b_l, phi_b_l)
back_left_x = back_left[0]
back_left_y = back_left[1]
back_left_z = back_left[2]
zero_bl = back_left[3]
ax.plot(back_left_x[:zero_bl], back_left_y[:zero_bl], back_left_z[:zero_bl], color='si
end_point = ax.plot(back_left_x[zero_bl], back_left_y[zero_bl], 0, 'red', marker='x',
```

```
back_right = Service_Deg(theta_b_r, phi_b_r)
back_right_x = back_right[0]
back_right_y = back_right[1]
back_right_z = back_right[2]
zero_br = back_right[3]
ax.plot(back_right_x[:zero_br], back_right_y[:zero_br], back_right_z[:zero_br], 'purpl
end_point = ax.plot(back_right_x[zero_br], back_right_y[zero_br], 0, 'red', marker='x'

# Court Code
court_back = ax.plot(linspace(0,17,n), linspace(0,0,n), linspace(0,0,n), 'black', line
#, label=r'Court Design'
court_left = ax.plot(linspace(0,0,n), linspace(0,40,n), linspace(0,0,n), 'black', line
court_up = ax.plot(linspace(0,17,n), linspace(40,40,n), linspace(0,0,n), 'black', line
court_right = ax.plot(linspace(17,17,n), linspace(0,40,n), linspace(0,0,n), 'black', l
long_mid = ax.plot(linspace(8.5,8.5,n), linspace(0,40,n), linspace(0,0,n), 'black', li
service_line_1 = ax.plot(linspace(0,17,n), linspace(13.5,13.5,n), linspace(0,0,n), 'bl
service_line_2 = ax.plot(linspace(0,17,n), linspace(25.5,25.5,n), linspace(0,0,n), 'bl
net_left = ax.plot(linspace(0,0,n), linspace(20,20,n), linspace(0,5,n), 'black', linew
net_right = ax.plot(linspace(17,17,n), linspace(20,20,n), linspace(0,5,n), 'black', li
net_bottom = ax.plot(linspace(0,17,n), linspace(20,20,n), linspace(3,3,n), 'black', li
net_top = ax.plot(linspace(0,17,n), linspace(20,20,n), linspace(5,5,n), 'black', linew
net_top = ax.plot(linspace(0,17,n), linspace(20,20,n), linspace(0,0,n), 'black', lines

ax.tick_params(labelsize=14)

ax.view_init(azim=-25, elev=30)
ax.set_xlim3d(-1, 18)
ax.set_ylim3d(-1, 41)
ax.set_zlim3d(0, 18)

ax.legend(loc='best', fontsize=15)
# ax.figsize=(100,100,100)
```

Out[12]: <matplotlib.legend.Legend at 0x1c4009675b0>



Initial Velocity Graph (For 1st Optimal Service)

In [13]:
```
# Initial Launch Conditions
x_0 = 8.5 # ft (if back court line is 0.0 ft, then service line is 13.5 ft, given the
y_0 = 13.5 # ft (if far-left court line is 0.0 ft, then divider is 8.5 ft, given the w
z_0 = (1.15 * 3.281) # ft (1.15 m times 3.281 ft/m) - https://rb.gy/lwa9pw
v_0 = 10 * 3.281 # ft/s
theta = np.pi * 78.07 / 180
phi = np.pi * 8.32 / 180
v_x_0 = v_0 * np.sin(phi)
v_y_0 = v_0 * np.cos(theta) * np.cos(phi)
v_z_0 = v_0 * np.sin(theta) * np.cos(phi)

# Set up graph and plot
v_ax = plt.axes(projection='3d')
v_ax.set_title(f'Initial Shuttlecock Service Velocity', fontsize=20)
v_ax.set_xlabel('x(t) (ft)', fontsize=16)
v_ax.set_ylabel('y(t) (ft)', fontsize=16)
v_ax.set_zlabel('z(t) (ft)', fontsize=16)

# Init. Vel
v_ax.plot(linspace(x_0, -v_x_0+x_0, 100), linspace(y_0, v_y_0+y_0, 100), linspace(z_0,
          'purple', label=r'$\mathbf{v_{o}}$', linewidth=3)
v_ax.plot(-v_x_0+x_0, v_y_0+y_0, (v_z_0+z_0), 'purple', marker='<')

# Init. Vel in xy plane
v_ax.plot(linspace(x_0, -v_x_0+x_0, 100), linspace(y_0, v_y_0+y_0, 100), linspace(z_0,
          'red', label=r'$\mathbf{v_{xy}}$ Projection', linestyle='--', linewidth=3)
v_ax.plot(-v_x_0+x_0, v_y_0+y_0, z_0, 'red', marker='v')
```

```
# Init. Vel in xz plane
v_ax.plot(linspace(x_0, -v_x_0+x_0, 100), linspace(y_0, y_0, 100), linspace(z_0, v_z_0
          'blue', label=r'$\mathbf{v_{xz}}$ Projection', linestyle='--', linewidth=3)
v_ax.plot(-v_x_0+x_0, y_0, v_z_0+z_0, 'blue', marker='^')

# Theta angle from xy up to Init. Vel ------------------------------------------------
v_ax.text((x_0*.75), (y_0*1.2), (z_0*2), r'$\mathbf{\theta}$', color='orange', fontsiz


# radius = v_0
# x_y_plane = linspace(v_0, 0, 100)
# xy_to_z = sqrt(radius**2 - x_y_plane**2)
# circ = np.array([])
# circ_count = 0
# for i in range(100):
#     if xy_to_z[i] <= v_0:
#         circ = np.append(circ, xy_to_z[i]/2)
#         if xy_to_z[i] == v_0:
#             circ_count = i-10
# v_ax.plot(linspace(v_x_0/1.5, v_x_0/3, circ_count), linspace(v_y_0/1.5, v_y_0/3, cir


# Theta angle from xz over to Init. Vel ----------------------------------------------
v_ax.text((x_0*.7), (y_0*1.1), (z_0*4), r'$\mathbf{\phi}$', color='green', fontsize=16


# radius = v_0
# x_z_plane = linspace(v_0, 0, 100)
# xz_to_y = sqrt(radius**2 - x_z_plane**2)
# circ = np.array([])
# circ_count = 0
# for i in range(100):
#     if xz_to_y[i] <= v_0:
#         circ = np.append(circ, xz_to_y[i]/8)
#     if xz_to_y[i] == v_0:
#         circ_count = i

# v_ax.plot(linspace(v_x_0/2, v_x_0/6, circ_count), circ[:circ_count], linspace(v_z_0/


# Plot xyz axis at origin
v_ax.plot(linspace(x_0, -v_x_0+x_0, 100), linspace(y_0, y_0, 100), linspace(0, 0, 100)
v_ax.plot(linspace(x_0, x_0, 100), linspace(y_0, v_y_0+y_0, 100), linspace(0, 0, 100),
v_ax.plot(linspace(x_0, x_0, 100), linspace(y_0, y_0, 100), linspace(0, v_z_0, 100), '

v_ax.tick_params(labelsize=14)

v_ax.view_init(azim=5, elev=15)
v_ax.legend(loc='best', fontsize=15)
plt.show()
```

Initial Shuttlecock Service Velocity