The difference between total and partial derivative

Partial derivative

Let we have the function of several parameters $f(x,y,z)$. According to the definition, the **partial derivative** is the derivative with respect to single variable considering all other variables as constants

$$\frac{\partial f(x,y,z)}{\partial x} = \frac{df(x)}{dx}|_{y=const,z=const}$$

Example:

$$f(x,y,z) = xyz + xy + z$$

$$\frac{\partial f(x,y,z)}{\partial x} = yz + y, \frac{\partial f(x,y,z)}{\partial y} = xz + x, \frac{\partial f(x,y,z)}{\partial z} = xy + 1$$

Total derivative

Now let's consider that the arguments x, y, z are the functions of variable t. This can be written as

$$f(x(t), y(t), z(t))$$

In contrast to a partial derivative, assumption that other variables are constant is not applied in the case of total derivative. Thus, in order to find the total change of the function f, we need to account all contributions to this change.

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

This definition uses the principle of the so-called **chain rule**. This rule is applied when a particular variable is a function of another variable, which is also the function of 3rd variable/variables.

When we need to use total/partial derivative

When we consider the change of the function f due to parameter t and the dependence is indirect, i.e.

$f(x(t), y(t), z(t))$

We need to account the changes

$$\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}$$

which introduce the contributions to the change of a function f.

If we have direct connection between the function f and the variable t, i.e.

$$f(t)$$

It is enough to use simply

$$\frac{\partial f}{\partial t}$$

Therefore, when constructing the differential equation, we need to analyze the independent variables and the connection between these variables.