DPS and survival as functions of several variables

We have considered **dps** and **survival** as a functions of a single variable only earlier. But of course we can increase all stats together. The main question in this case is the following. What is the optimal variant of allocation of gold between all parameters? In the case of **dsp** these parameters are the attack damage (*atk_dmg*), the attack speed (*atk_spd*) and the critical strike chance (*crt_ch*). In the case of **survival**, the corresponding parameters will be **health points** and **armor**.

1. DPS as a function of several variables

As was obtained at the previous stages, the **dps** formula is determined as follows

$$dps = atk\_dmg * atk\_spd * (1 + \frac{crt\_ch}{100\%}) \tag{1.1}$$

When an arbitrary champion achieves N^{th} level, **dps** can be written as (1.2):

$$dps_N = (atk\_dmg_N + atk\_dmg_{itm}) * atk\_spd_N * (1 + \frac{atk\_spd_{itm}}{100\%}) * (1 + \frac{crt\_ch_{itm}}{100\%}) \tag{1.2}$$

In this case there are two constants – atk_dmg_N and atk_spd_N – in (1.2). We can call these parameters constants because we can’t change them by gold or by any other until champion stays at the same level. Also there are three parameters – atk_dmg_{itm}, atk_spd_{itm} and crt_ch_{itm} that can be increased by items (by gold).

As we know

$$atk\_dmg_{itm} = \frac{AD_{gold}}{36}, atk\_spd_{itm} = \frac{AS_{gold}}{33.33}, crt\_ch_{itm} = \frac{CC_{gold}}{50} \tag{1.3}$$

Substitute (1.3) into (1.2) will give

$$dps_N = (atk\_dmg_N + \frac{AD_{gold}}{36}) * atk\_spd_N * (1 + \frac{AS_{gold}}{3333}) * (1 + \frac{CC_{gold}}{5000}) \quad (1.4)$$

Now we have to find the optimal allocation of gold between AD_{gold}, AS_{gold}, CC_{gold}.

Method from “lli03.pdf” is generally right, but result is not multipurpose because of atk_dmg_N=50 and atk_spd_N=1. In other words, in our consideration we will consider all parameter without initialization.

The gradient of the **dps** is a vector that consists of the components, each of components is determined as a partial derivative:

$$\nabla dsp = \{(dps_N)'_{AD_{gold}}, (dps_N)'_{AS_{gold}}, (dps_N)'_{CC_{gold}}\}$$

Taking the derivatives will give the following

$$\begin{cases} (dps_N)'_{AD_{gold}} = \frac{atk\_spd_N}{36} * (1 + \frac{AS_{gold}}{3333}) * (1 + \frac{CC_{gold}}{5000}) \\ (dps_N)'_{AS_{gold}} = (atk\_dmg_N + \frac{AD_{gold}}{36}) * \frac{atk\_spd_N}{3333} * (1 + \frac{CC_{gold}}{5000}) \\ (dps_N)'_{CC_{gold}} = (atk\_dmg_N + \frac{AD_{gold}}{36}) * \frac{atk\_spd_N}{5000} * (1 + \frac{AS_{gold}}{3333}) \end{cases}$$

One can observe that **dps** is a function that increasing monotonically, so it does not have any extrema.

Let’s express AS_{gold} and CC_{gold} as a function of AD_{gold}.

$$\frac{(dps_N)'_{AS_{gold}}}{(dps_N)'_{AD_{gold}}} \Rightarrow AS_{gold} = 36 * atk\_dmg_N - 3333 + AD_{gold}$$

$$\frac{(dps_N)'_{CC_{gold}}}{(dps_N)'_{AD_{gold}}} \Rightarrow CC_{gold} = 36 * atk\_dmg_N - 5000 + AD_{gold}$$

$$\begin{aligned} AS_{gold} &= 36 * atk\_dmg_N - 3333 + AD_{gold} \\ CC_{gold} &= 36 * atk\_dmg_N - 5000 + AD_{gold} \\ AS_{gold} &\geq 0, AD_{gold} \geq 0, CC_{gold} \geq 0 \end{aligned} \tag{1.5}$$

As so as the value of gold have been spent can't be negative, we have to find values of atk_dmg_N in (1.5) that $AS_{gold} = AD_{gold}$ and $CC_{gold} = AD_{gold}$. This condition ensures us that if we spent any quantity of gold all conditions in (1.5) will be fullfilled.

$$\begin{aligned} 36 * atk\_dmg_N - 3333 = 0 \\ 36 * atk\_dmg_N - 5000 = 0 \end{aligned} . \tag{1.6}$$

From equation (1.6) one can find two values of the attack damage

$$\begin{aligned} atk\_dmg_N^1 &= 92.58 \\ atk\_dmg_N^2 &= 138.89 \end{aligned} .$$

What is an important that we have already seen these values before!. But now we can make another conclusion.

If champion's attack damage is less than 92.58 all gold should be spent to increase attack damage ($gold_1^{sp}$). If champion's attack damage is more than 92.58 but less than 138.89, gold, that is spent over $gold_1^{sp}$, should be spent to increase attack damage and attack speed half-and-half ($gold_2^{sp}$). If champion's attack damage is more than 138.89, gold, that is spent over $gold_2^{sp}$, should be spent to increase all three stats in equal parts.

Survival as function of several variables

Let's remember formula for **survival** at N[th] level.

$$m\_dmg_N = \frac{HP_N + HP_{itm}}{1 - \dfrac{arm_N + arm_{itm}}{100 + arm_N + arm_{itm}}} \tag{2.1}$$

We can simplify (2.1).

$$m\_dmg_N = \frac{HP_N + HP_{itm}}{\dfrac{100 + arm_N + arm_{itm}}{100 + arm_N + arm_{itm}} - \dfrac{arm_N + arm_{itm}}{100 + arm_N + arm_{itm}}}$$

$$m\_dmg_N = \frac{HP_N + HP_{itm}}{\dfrac{100 + arm_N + arm_{itm} - arm_N - arm_{itm}}{100 + arm_N + arm_{itm}}}$$

$$m\_dmg_N = \frac{(HP_N + HP_{itm}) * (100 + arm_N + arm_{itm})}{100} \tag{2.2}$$

Similarly there are two constants – HP_N and arm_N – in (2.2). We can call these parameters constants because we can't change them by gold or by any other until champion stay at the same level. Also there are two parameters – HP_{itm} and arm_{itm} that can be increased by items (by gold).

As we know

$$HP_{itm} = \frac{HP_{gold}}{2.64}, arm_{itm} = \frac{ARM_{gold}}{20} \tag{2.3}$$

Substituting (2.3) into (2.2) will give

$$m\_dmg_N = \frac{(HP_N + \frac{HP_{gold}}{2.64}) * (100 + arm_N + \frac{ARM_{gold}}{20})}{100} \qquad (2.4)$$

Now we have to find the optimal allocation of gold between ARM_{gold} and HP_{gold}.

As in dps case, first of all find gradient m_dmg_N

$$\nabla m\_dmg_N = \{(m\_dmg_N)'_{HP_{gold}}, (m\_dmg_N)'_{ARM_{gold}}\}$$

$$(m\_dmg_N)'_{HP_{gold}} = \frac{100 + arm_N + \frac{ARM_{gold}}{20}}{264}$$

$$(m\_dmg_N)'_{ARM_{gold}} = \frac{HP_N + \frac{HP_{gold}}{2.64}}{2000}$$

Let's express ARM_{gold} through HP_{gold}

$$\frac{(m\_dmg_N)'_{AEM_{gold}}}{(m\_dmg_N)'_{HP_{gold}}} \Rightarrow 7.576 * (100 + arm_N + \frac{ARM_{gold}}{20}) = HP_N + \frac{HP_{gold}}{2.64}$$

After opening the brackets

$$757.6 + 7.576 * arm_N + 0.3788 * ARM_{gold} = HP_N + \frac{HP_{gold}}{2.64}$$

Multiply at 2.64.

$$\begin{aligned} ARM_{gold} &= 2.64 * HP_N - 2000 - 20 * arm_N + HP_{gold} \\ ARM_{gold} &\geq 0, HP_{gold} \geq 0 \end{aligned} \qquad (2.5)$$

Let's set as

$$check = 2.64 * HP_N - 2000 - 20 * arm_N$$

1. Check < 0, spent gold $\leq |check|$ - all gold should be spent to increase health points.

2. Check < 0, spent gold $> |check|$ - gold, that is spent over gold in first case, should be spent to increase armor and health points in equal parts.

3. Check > 0, spent gold $\leq$ check - all gold should be spent to increase armor.

4. Check ≥ 0, spent gold $>$ check - gold, that is spent over gold in third case, should be spent to increase armor and health points in equal parts.