Champ Analysis

Plan

1. Analysis of the attack damage and attack speed

From dps-formula

$$dps = (atk\_dmg_0 + (lvl-1)*atk\_dmg_{lvl} + atk\_dmg_{itm} + atk\_dmg_{db})* \\ ([atk\_spd_0 * (1 + \frac{(lvl-1)*atk\_spd_{lvl}}{100\%})]*[1 + \frac{atk\_spd_{itm} + atk\_spd_{bd}}{100\%}])* \\ (1 + \frac{crt\_ch_{itm} + ctr\_ch_{bd}}{100\%} \frac{crt\_dmg}{200\%}) \quad (1)$$

Let's simplify the consideration by assuming that critical chance and buff&debuff effects are absent

$$crt\_ch_{itm} = 0, atk\_dmg_{db} = 0, atk\_spd_{bd} = 0$$

In this case equation (1) can be rewritten as follows

$$dps = (atk\_dmg_0 + (lvl-1)*atk\_dmg_{lvl} + atk\_dmg_{itm})* \\ *([atk\_spd_0 * (1 + (lvl-1)*\frac{atk\_spd_{lvl}}{100\%}]*[1 + \frac{atk\_spd_{itm}}{100\%}]) \quad (2)$$

Let's consider **dps** when the champion has achieved a N^{th} level. Attack damage in this case will be equal to (without any items)

$$AD_N = atk\_dmg_0 + (N-1)*atk\_dmg_N \quad (3)$$

Attack speed on this level without any items is equal

$$AS_N = atk\_spd_0 * (1 + (N-1)*\frac{atk\_spd_N}{100\%}) \quad (4)$$

Substituting (3) and (4) into (2) one can derive the following

$$dps_N = (AD_N + atk\_dmg_{itm}) * [AS_N * (1 + \frac{atk\_spd_{itm}}{100\%})], \quad (5)$$

where AD_N is the attack damage when a champion has reached N^{th} level, AS_N is the attack speed when a champion has achieved N^{th} level, atk_dmg_{itm} is the value of attack damage which can increased by items, atk_spd_{itm} is the value of attack speed which can increased by items.

When $atk\_dmg_{itm} = 0$, **dps** is increased only by the attack speed

$$dps_N^{AS} = AD_N * AS_N * (1 + \frac{atk\_spd_{itm}}{100\%}) \quad (6)$$

Similarly, when $atk\_spd_{itm} = 0$, **dps** is increased only by the attack damage

$$dps_N^{AD} = (AD_N + atk\_dmg_{itm}) * AS_N \quad (7)$$

Let's find the condition when the increasing of the **dps** by the attack speed is more efficient than by the attack damage. This condition can be written as

$$dps_N^{AS} \geq dps_N^{AD} \quad (8)$$

Let's use equations (6), (7) in (8)

$$AD_N * AS_N * (1 + \frac{atk\_spd_{itm}}{100\%}) \geq (AD_N + atk\_dmg_{itm}) * AS_N$$

$$AD_N * AS_N + AD_N * AS_N \frac{atk\_spd_{itm}}{100\%} \geq AD_N * AS_N + atk\_dmg_{itm} * AS_N$$

After simplification, one can obtain

$$AD_N * AS_N \frac{atk\_spd_{itm}}{100\%} \geq atk\_dmg_{itm} * AS_N . \quad (9)$$

Since the attack speed at N^{th} level $AS_N \neq 0$, one can derive

$$AD_N \frac{atk\_spd_{itm}}{100\%} \geq atk\_dmg_{itm} \quad (10)$$

At this stage we can express the atk_spd_{itm} and atk_dmg_{itm} in gold:

$$atk\_spd_{itm} = \frac{AS_{gold}}{33.33}, atk\_dmg_{itm} = \frac{AD_{gold}}{36}, \quad (11)$$

where AS_{gold} is the gold which has been spent to increase the attack speed, AD_{gold} is the gold which has been spent to increase the attack damage.

After substituting (11) in (10) one can write

$$AD_N \frac{AS_{gold}}{3333} \geq \frac{AD_{gold}}{36}$$

If we spend the same quantity of gold for speed and attack $\left(AS_{gold} = AD_{gold}\right)$

$$AD_N \geq 3333/36 \Rightarrow AD_N \geq 92.58 \quad (12)$$

This result means that increase of **dps** by the attack speed **<u>is better</u>** than the increase of **dps** by the attack damage when champion's attack damage more than 92.58 regardless of its attack speed.

2. Analysis of the attack speed and critical strike chance

In this section the analysis of the **dps** equation with respect to the attack velocity and strike chance crt_ch_{itm} is performed.

Let's consider the **dps** equation (1) and assume that all buff&debuff effects are absent and the critical damage is equation 200%

$$crt\_dmg = 200\%, atk\_dmg_{db} = 0, atk\_spd_{bd} = 0$$

With above assumptions, the **dps** equation will be defined as follows

$$\begin{aligned} dps = &(atk\_dmg_0 + (lvl-1) * atk\_dmg_{lvl} + atk\_dmg_{itm}) * \\ &* ([atk\_spd_0 * (1 + (lvl-1) * \frac{atk\_spd_{lvl}}{100\%}] * [1 + \frac{atk\_spd_{itm}}{100\%}]) * (1 + \frac{crt\_ch_{itm}}{100\%}) \end{aligned} \quad (13)$$

Lets consider **dps** when the champion has achieved N^{th} level. Attack damage on this level without any items will be equal to

$$AD_N = atk\_dmg_0 + (N-1) * atk\_dmg_N \quad (14)$$

The attack speed on this level without any items will be equal to

$$AS_N = atk\_spd_0 * (1 + (N-1) * \frac{atk\_spd_N}{100\%}) \quad (15)$$

By substituting (14) and (15) into (13) one can derive:

$$dps_N = (AD_N + atk\_dmg_{itm}) * [AS_N * (1 + \frac{atk\_spd_{itm}}{100\%})] * (1 + \frac{crt\_ch_{itm}}{100\%}). \quad (16)$$

Here similarly to the previous section AD_N is the attack damage when a champion has achieved N^{th} level, AS_N is the corresponding attack speed, atk_dmg_{itm} is the value of attack damage which can increased by items, atk_spd_{itm} is the value of attack speed which can increased by items, crt_ch_{itm} is the value of critical strike chance which is increased by items.

In the case when $atk\_dmg_{itm} = 0$ and $crt\_ch_{itm} = 0$ (**dps** is increased only by the attack speed), one can write the following

$$dps_N^{AS} = AD_N * AS_N * (1 + \frac{atk\_spd_{itm}}{100\%}) \quad (17)$$

When $atk\_spd_{itm} = 0$ and $crt\_ch_{itm} = 0$ (**dps** is increased by attack damage only), similarly one can obtain

$$dps_N^{AD} = (AD_N + atk\_dmg_{itm}) * AS_N \quad (18)$$

When $atk\_dmg_{itm} = 0$ and $atk\_spd_{itm} = 0$ (**dps** is increased by the critical strike chance only)

$$dps_N^{CC} = AD_N * AS_N * (1 + \frac{crt\_ch_{itm}}{100\%}) \quad (19)$$

At this stage equations (17)-(19) can be used to derive an important results.

At first, one can find the condition when the increase of **dps** by the attack speed is more efficient that the increase by the critical strike chance. This condition can be written as

$$dps_N^{AS} \geq dps_N^{CC} \tag{20}$$

Let's use equations (17), (19) in (20):

$$AD_N * AS_N * (1 + \frac{atk\_spd_{itm}}{100\%}) \geq AD_N * AS_N * (1 + \frac{crt\_ch_{itm}}{100\%}) \tag{21}$$

Evidently that $AD_N \neq 0, AS_N \neq 0$, therefore one can obtain

$$\frac{atk\_spd_{itm}}{100\%} \geq \frac{crt\_ch_{itm}}{100\%} \tag{22}$$

Let's express the parameters in (22) in the gold

$$atk\_spd_{itm} = \frac{AS_{gold}}{33.33}, crt\_ch_{itm} = \frac{CC_{gold}}{50}, \tag{23}$$

where AS_{gold} is the gold that has been spent to increase the attack speed, CC_{gold} is the gold that has been spent to increase the critical strike chance.

Finally, by substituting (23) in (22) one can derive

$$\frac{AS_{gold}}{33.33} \geq \frac{CC_{gold}}{50}$$

If we spend the same quantity of gold for speed and critical chance $\left(AS_{gold} = CC_{gold}\right)$, one can obtain

$$\frac{1}{33.33} \geq \frac{1}{50}$$

This imparity is true. It means that increase **dps** by attack speed is more effective than increase **dps** by critical strike chance when $AS_{gold}=CC_{gold.}$

3. Analysis of the attack damage and critical strike chance

In this section similar analysis is performed with respect to attack damage and critical strike chance.

Similarly to the previous section, one can write the inequality

$$dps_N^{AD} \geq dps_N^{CC} \tag{24}$$

Using equations (18), (19) one can obtain

$$(AD_N + atk\_dmg_{itm}) * AS_N \geq AD_N * AS_N * (1 + \frac{crt\_ch_{itm}}{100\%})$$

$$AD_N * AS_N + atk\_dmg_{itm} * AS_N \geq AD_N * AS_N + AD_N * AS_N * \frac{crt\_ch_{itm}}{100\%})$$

$$atk\_dmg_{itm} * AS_N \geq AD_N * AS_N * \frac{crt\_ch_{itm}}{100\%})$$

Since $AS_N \neq 0$ one can derive

$$atk\_dmg_{itm} \geq AD_N * \frac{crt\_ch_{itm}}{100\%} \tag{25}$$

Similarly one can express the attack damage and the critical strike chance in gold. This gives us the following

$$crt\_ch_{itm} = \frac{CC_{gold}}{50}, atk\_dmg_{itm} = \frac{AD_{gold}}{36} \quad (26)$$

where CC_{gold} is the gold that gold has been spent to increase the critical strike chance, AD_{gold} is the gold that has been spent to increase the attack damage. Using (26) one can rewrite (25) as

$$\frac{AD_{gold}}{36} \geq ADN * \frac{CC_{gold}}{5000} \quad (27)$$

If we spend the same quantity of gold $\left(AD_{gold} = CC_{gold}\right)$

$$5000/36 \geq AD_N \Rightarrow AD_N \leq 138.89$$

This result means that the increase of **dps** by the attack damage is more effective than increase the **dps** by the critical strike chance when champion's attack damage less than 138.89.

4. Conclusions of the project phase

The above analysis has given answers on several important questions. The main results are as follows:

- When $AD_N < 92.58$ increasing of the **attack damage** is the **most effective** way to increase **dps**.
- The increase of the **attack speed** is **more effective** than the increase of the **critical strike chance** always.
- When $92.58 < AD_N < 138.89$ the increase of the **attack damage** is **more effective** than the increase the **critical strike chance** but **less effective** that the increase of the **attack speed**.

- When $AD_N > 138.89$ the increase of the **critical strike chance** is **more effective** than the increase of the **attack damage** but the increase of the **attack speed** is **more effective** than the increase of the **critical strike chance** always.
- When $AD_N < 92.58$ the increase of the **attack damage** is the **most effective** way to increase **dps**, when $AD_N > 92.58$ the increase of the **attack speed** is the **most effective** way to increase **dps**

Fig. 1 illustrates the obtained results in easy way.

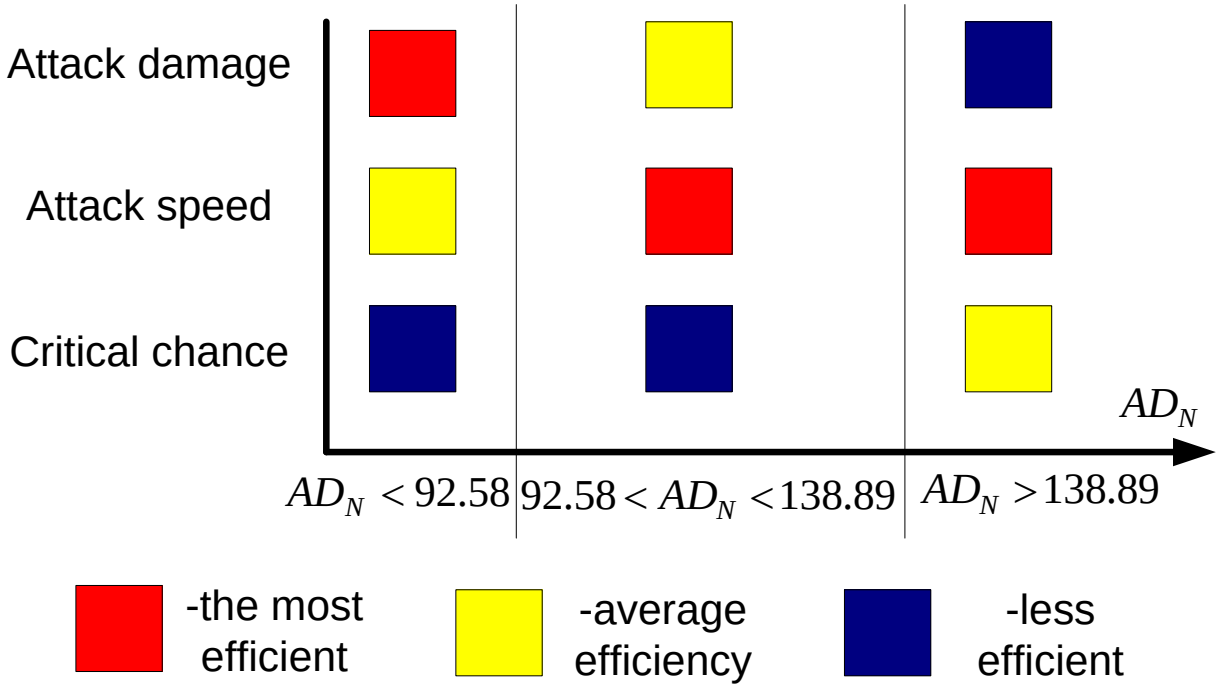

Fig. 1. Efficiency diagram

5. Testing of the models

Let's analyze the **dps** of some champions with different attack damage and speed. At the next step the **dps** will be calculated with particular items. The increase of the **dps** will be analyzed and gives a possibility to test the derived models.

The input item list is given in Table 1.

Item	Cost	Attack damage inc.	Attack speed inc.	Critical strike chance inc.
Long Sword+*	400	11.1	0	0
Dagger	400	0	12	0
Brawler's Gloves	400	0	0	8

Table 1. Input items

The reason we have chosen Long Sword+ instead of simple Long Sword is that the cost of all three items is the same. The simple Long Sword costs 360 gold and increases the attack damage at 10.

Also we have chosen the items which increase a single stat – the attack damage, the attack speed or the critical strike chance.

Let's consider several champions at two different levels with above described items.

Table 2 illustrates the **dps** at 15^{th} level for different champions.

Champion	**Attack damage at 15th level**	**Attack speed at 15th level**	**Dps at 15th level**	**Dps with Long Sword**	Δ_{AD}	**Dps with Dagger**	Δ_{AS}	**Dps with Braw. Gloves**	Δ_{CC}
Lulu	80.4	0.822	66,08	75,21	9,13	74,01	7,93	71,37	5,29
Kog'maw	88	0.912	80,23	90,36	10,13	89,86	9,63	86,65	6,42
Jinx	***92***	*0.713*	*65.55*	*73,47*	***7,92***	*73,42*	***7,87***	*70,79*	***5,24***
Anivia	***92.8***	*0.772*	*71.64*	*80,22*	***8,58***	*80,24*	***8,6***	*77,37*	***5,73***
Poppy	103.55	0.937	97,05	107,46	10,41	108,7	11,65	104,81	7,76
Alistar	105.71	0.811	85.72	94,73	9,01	96,01	10,29	92,58	6,86
Kassadin	106.9	0.972	103.86	114,65	10,79	116,32	12,46	112,16	8,31

Table 2. Champions table with **dps** increment (15^{th} level)

Δ_{AD} is the **dps** increment, caused by the attack damage only

Δ_{AS} is the **dps** increment, caused by the attack speed only

Δ_{CC} is the **dps** increment, caused by the critical chance only

The biggest value of increasing is in red color. The least value is in blue color. A medium value has a yellow color.

One can observe that is the attack damage level is smaller than the found threshold $AD_{15} < 92.58$, the largest **dps** increment corresponds to the item, which increases the attack damage. What is very important is that the found threshold is precise. For instance, let's consider Jinx, who has $AD_{15} = 92$ and Anivia with $AD_{15} = 92.8$. One can see that for the 2^{nd} hero the largest **dps** increment corresponds to the item, which increases the attack speed. While for Jinx $AD_{15} = 92 < 92.58$ and the most effective is the increment of the attack damage. These results totally matched with the developed models.

Now let's consider the same champions at 18th level (table 3).

Champion	**Attack damage at 18th level+25**	**Attack speed at 18th level**	**Dps at 18th level**	**Dps with Long Sword**	Δ_{ad}	**Dps with Dagger**	Δ_{as}	**Dps with Braw. Gloves**	Δ_{cc}
Lulu	113,2	0,86	97,81	107,41	9,6	109,55	11,74	105,64	7,82
Kog'maw	122	0,96	117,68	128,4	10,72	131,8	14,12	127,09	9,41
Jinx	126	0,73	92,14	100,26	8,12	103,19	11,06	99,51	7,37
Anivia	127,4	0,8	102,37	111,29	8,93	114,65	12,28	110,56	8,19
Poppy	**138,68**	1	138,86	149,99	**11,13**	155,52	**16,66**	149,97	**11,11**
Alistar	**141,57**	0,85	120,45	129,9	**9,45**	134,9	**14,45**	130,08	**9,64**
Kassadin	143,6	1,04	149,71	161,3	11,58	167,68	17,97	161,69	11,98

Table 2. Champions table with **dps** increment (18th level)

Let's consider two champions – Poppy and Alistar. The 2nd found threshold value in the analysis is $AD = 138.89$. One can see that for Poppy $AD_{18} < 138.89$ and for Alistar $AD_{18} > 138.89$. As we found from our models, the best way is to increase the attack speed for both heroes. Evidently, one can see that the largest **dps** increments in the table correspond to the attack speed, which exactly matches the efficiency diagram in Fig. 1. The average efficiency is increment of the attack speed for Poppy and the increment of the critical chance for Alistar, which again matches the derived model.

These tests prove the correctness of the developed models and obtained threshold values.