Updated champion's survival

In the previous analysis we consider the damage caused by the auto-attack only. Evidently, that all champions use additional abilities to make more damage. There are three existing types of the damage in league of legends:

physical damage, magic damage and true damage

Therefore, one can write for the total damage

$$tot\_dmg = ph\_dmg + mag\_dmg + tr\_dmg, \qquad (1)$$

where *ph_dmg* is a physical damage, *mag_dmg* is a magic damage and *tr_dmg*is a true damage respectively. Physical damage is caused by all auto-attacks and some champion's spells. Impact of this type of damage on health points (HP) can be weaken by the armor

$$ph\_dmg = \frac{HP}{1 - \dfrac{arm}{100 + arm}}, \qquad (2)$$

where *ph_dmg* is an existing physical damage, *HP* is the value at which target's heath points decreased when it achieved physical damage, *arm* is target's armor. Physical damage is the most popular type of damage.

The magic damage is caused by some items and the most champion's spells. Impaction of this type of damage on HP can be weakened by the magic resistance (*MR*)

$$mag\_dmg = \frac{HP}{1 - \frac{MR}{100 + MR}}, \qquad (3)$$

where *mag_dmg* is the magic damage, *HP* is the value at which target's heath points decreased when it achieved magic damage, *MR* is the magic resistance of a target.

True damage ignores armor and magic resistance. Therefore, it is is equal the value at which target's heath points decreased (4).

$$tr\_dmg = HP \qquad (4)$$

True damage is the least popular damage in the game.

Considering all types of damage and knowing the fact that their contributions into the total damage are different, one can write more precise equation for the total damage

$$tot\_dmg = w_{ph} * ph\_dmg + w_{mag} * mag\_dmg + w_{tr} * tr\_dmg, \quad (5)$$

where w_{ph}, w_{mag} and and w_{tr} are weight coefficients for the physical damage, magic damage and true damage respectively.

We have found the values of damage per second for all types for every champion. This has allowed us to calculate the averaged values of three types of damage. Therefore, knowing the average values of damage types, one can easily calculate the unknown weighting coefficients

$$w_{ph} = \frac{\langle ph\_dmg \rangle}{\langle ph\_total \rangle}, w_{mag} = \frac{\langle mag\_dmg \rangle}{\langle ph\_total \rangle}, w_{tr} = \frac{\langle ph\_true \rangle}{\langle ph\_total \rangle}. \qquad (6)$$
$$\langle ph\_total \rangle = \langle ph\_dmg \rangle + \langle mag\_dmg \rangle + \langle tr\_dmg \rangle$$

It was found that $w_{ph} = 0.63, w_{mag} = 0.36, w_{tr} = 0.01$. These averaged estimates can be used for further model analysis.

Using the calculated weighting coefficients, equation (5) can be written as

$$tot\_dmg = 0.63 * ph\_dmg + 0.36 * mag\_dmg + 0.01 * tr\_dmg . \quad (7)$$

Substituting (2)-(4) into (7) gives the following

$$tot\_dmg = 0.63 * \frac{HP}{1 - \frac{arm}{100 + arm}} + 0.36 * \frac{HP}{1 - \frac{MR}{100 + MR}} + 0.01 * HP$$

$$tot\_dmg = 0.63 * \frac{HP * (100 + arm)}{100} + 0.36 * \frac{HP * (100 + MR)}{100} + 0.01 * HP$$

$$tot\_dmg = HP * (0.63 * \frac{100 + arm}{100} + 0.36 * \frac{100 + MR}{100} + 0.01) \quad (8)$$

When champion achieves N^{th} level the maximum total damage that champion will be able to withstand:

$$tot\_dmg = (HP_N + HP_{itm}) * (0.63 * \frac{100 + arm_N + arm_{itm}}{100} +$$
$$+ 0.36 * \frac{100 + MR_N + MR_{itm}}{100} + 0.01) \quad , \quad (9)$$

where HP_N is the champion's health points at N^{th} level, arm_N is the champion's armor, MR_N is the champion's magic resistance, HP_{itm} is the quantity of health points that is added by items, arm_{tm} is the value of armor that is added by items, MR_{itm} is the value of magic resistance that is added by items.

Parameters HP_{itm}, arm_{itm} and MR_{itm} can be increased by gold.

$$HP_{itm} = \frac{HP_{gold}}{2.64}, arm_{itm} = \frac{ARM_{gold}}{20}, MR_{itm} = \frac{MR_{gold}}{20}, \qquad (10)$$

where HP_{gold} is the quantity of gold spent to buy items that increase maximum of health points, ARM_{gold} is the quantity of gold is spend to buy the items that increase armor, MR_{gold} is quantity of gold which is spent to buy items that increase the magic resistance.

After substitution (10) in (9) one can obtain

$$tot\_dmg = (HP_N + \frac{HP_{gold}}{2.64}) * (0.63 * \frac{100 + arm_N + \frac{ARM_{gold}}{20}}{100} +$$
$$+ 0.36 * \frac{100 + MR_N + \frac{MR_{gold}}{20}}{100} + 0.01) \qquad (11)$$

As we did before, first of all, let's find the gradient of tot_dmg

$$\nabla tot\_dmg = \left\{ (tot\_dmg)'_{HP_{gold}}, (tot\_dmg)'_{ARM_{gold}}, (tot\_dmg)'_{MR_{gold}} \right\}$$

$$(tot\_dmg)'_{HP_{gold}} = \frac{1}{2.64}(0.63 \frac{100 + arm_N + \frac{ARM_{gold}}{20}}{100} + 0.36 \frac{100 + MR_N + \frac{MR_{gold}}{20}}{100} + 0.01)$$

$$(tot\_dmg)'_{ARM_{gold}} = (HP_N + \frac{HP_{gold}}{2.64})(\frac{0.63}{2000}), (tot\_dmg)'_{MR_{gold}} = (HP_N + \frac{HP_{gold}}{2.64})(\frac{0.36}{2000})$$

Let's express ARM_{gold} through MR_{gold}

$$\frac{(tot\_dmg)'_{ARM_{gold}}}{(tot\_dmg)'_{MR_{gold}}} = \frac{(HP_N + \frac{HP_{gold}}{2.64})(\frac{0.63}{2000})}{(HP_N + \frac{HP_{gold}}{2.64})(\frac{0.36}{2000})} = 1.75 \Rightarrow ARM_{gold} = 1.75 * MR_{gold} \quad (12)$$

So, we have to spend gold to increase armor 1.75 times more than we spend to increase magic resistance. Therefore, the total gold that we need to spend to increase defense set as:

$$tot\_def_{gold} = ARM_{gold} + MR_{gold} = 2.75 * MR_{gold} \quad (13)$$

One can rewrite (11) using (12):

$$tot\_dmg = (HP_N + \frac{HP_{gold}}{2.64}) * (0.63 * \frac{100 + arm_N + \frac{1.75MR_{gold}}{20}}{100} +$$
$$+ 0.36 * \frac{100 + MR_N + \frac{MR_{gold}}{20}}{100} + 0.01)$$

After opening brackets one can obtain:

$$tot\_dmg = (HP_N + \frac{HP_{gold}}{2.64}) * (0.63 * (1 + \frac{arm_N}{100} + \frac{1.75MR_{gold}}{2000}) +$$
$$+ 0.36 * (1 + \frac{MR_N}{100} + \frac{MR_{gold}}{2000} + 0.01)$$

$$tot\_dmg = (HP_N + \frac{HP_{gold}}{2.64}) * (1 + \frac{0.63arm_N}{100} + \frac{1.1025MR_{gold}}{2000} + \frac{0.36MR_N}{100} + \frac{0.36MR_{gold}}{2000})$$

Further simplification gives

$$tot\_dmg = (HP_N + \frac{HP_{gold}}{2.64}) * (1 + \frac{0.63arm_N + 0.36MR_N}{100} + \frac{1.4625MR_{gold}}{2000}) \quad (14)$$

Now one can rewrite using (13)

$$tot\_dmg = (HP_N + \frac{HP_{gold}}{2.64}) * (1 + \frac{0.63arm_N + 0.36MR_N}{100} + \frac{tot\_def_{gold}}{3760}). \quad (15)$$

One can observe that the total damage is a function of two variables now. These variables are $HP_{gold}, tot\_dmg_{gold}$. Let's again find the gradient of tot_dmg:

$$\nabla tot\_dmg = \{(tot\_dmg)'_{HP_{gold}}, (tot\_dmg)'_{tot\_def_{gold}}\}$$

$$(tot\_dmg)'_{HP_{gold}} = \frac{1}{2.64}(1 + \frac{0.63arm_N + 0.36MR_N}{100} + \frac{tot\_def_{gold}}{3760})$$

$$(tot\_dmg)'_{tot\_dmg_{gold}} = \frac{1}{3760}(HP_N + \frac{HP_{gold}}{2.64})$$

$$\frac{(tot\_dmg)'_{HP_{gold}}}{(tot\_dmg)'_{tot\_dmg_{gold}}} \Rightarrow \frac{1}{2.64}(1 + \frac{0.63arm_N + 0.36MR_N}{100} + \frac{tot\_def_{gold}}{3760}) =$$

$$= \frac{1}{3760}(HP_N + \frac{HP_{gold}}{2.64})$$

After opening brackets and simplification one can obtain the following

$$3760 + 23.688arm_N + 13.536MR_N + tot\_def_{gold} = 2.64HP_N + HP_{gold}$$

$$HP_{gold} \geq 0, tot\_def_{gold} \geq 0$$

$$tot\_def_{gold} = 2.64HP_N - 3760 - 23.688arm_N - 13.536MR_N + HP_{gold}$$

One can introduce the expression and denote it as

$$check = 2.64HP_N - 3760 - 23{,}688arm_N - 13.536MR_N \qquad (16)$$

The value of this expression can be used to make a conclusion how to spend the gold in the way of maximum efficiency.

1. **Check < 0**, **spent gold ≤ |check|** - all gold should be spent to increase the health points (HP).

2. **Check < 0**, **spent gold > |check|** - gold, that is spent over gold in first case, should be spent to increase defense and health points in equal parts. All gold which is spent to increase defense should be spent in the next proportion: 36.36% - to increase the magic resistance, 63.64% - to increase the armor.

3. **Check > 0**, **spent gold ≤ check** - all gold should be spent to increase defense. All gold is spent to increase defense should be spent in the next proportion: 36.36% - to increase magic resistance, 63.64% - to increase armor.

4. **Check ≥ 0**, **spent gold > check** - gold, that is spent over gold in third case, should be spent to increase defense and health points in equal parts. All gold is spent to increase defense should be spent in the next proportion: 36.36% - to increase magic resistance, 63.64% - to increase armor.