Defensive stat analysis

Definitions

Incoming damage is classified according to three different types:

- TrueDmg (can't be mitigated)
- MagicDmg (reduced by magic resist)
- PhysicalDmg (reduced by armor)

Here we define {scaleAR, scaleMR} to be the percentages of total damage consisting of their respective damage types.
With [scaleAR + scaleMR <= 1.0] and the amount of TrueDmg being implicitly defined by the remainder of [1.0 - (scaleAR + scaleMR)]

```
hp = hpN + hpItem;
arm = armN + armItem;
mr = mrN + mrItem;

(* mdmg: defense/effective-HP rating *)
mdmg = FullSimplify[hp / ((100/(100+arm))scaleAR + (100/(100+mr))scaleMR + (1-(scaleAR+scaleMR)))]
```

$$\frac{hpItem + hpN}{1 - \frac{(armItem+armN)\ scaleAR}{100+armItem+armN} - \frac{(mrItem+mrN)\ scaleMR}{100+mrItem+mrN}}$$

Test

Assume:

- Incoming total amount of 100 mixed damage with the distribution of {75% physical, 25% magic, 0% TrueDmg}.
- 2000 base HP
- 100 armor
- 0 magic resist

```
admg = 100;
bhp = 2000;
scaleAR = 3/4;
scaleMR = 1/4;
dmgReduction = 1/2; (*100 armor*)

hpPercent = (bhp - (admg scaleAR dmgReduction) - (admg scaleMR))/bhp;

N@%*100
```

```
96.875
```

Find the corresponding effective-hp rating and make sure it matches mdmg results:

```
{ehp} = x /. Solve[(x - (admg scaleAR) - (admg scaleMR))/x == hpPercent]
ehp == mdmg /. {hp → bhp, arm → 100, mr → 0}
```

```
{3200}
```

```
True
```

Investigate the effect of scaleAR/MR

```
Clear[scaleAR,scaleMR]
mdmgHPN = mdmg /. {armItem → 0, mrItem → 0}
mdmgARMN = mdmg /. {hpItem → 0, mrItem → 0}
(*mdmgMRN = mdmg /. {hpItem → 0, armItem → 0} *)
```

$$\frac{\text{hpItem} + \text{hpN}}{1 - \frac{\text{armN scaleAR}}{100+\text{armN}} - \frac{\text{mrN scaleMR}}{100+\text{mrN}}}$$

$$\frac{\text{hpN}}{1 - \frac{(\text{armItem}+\text{armN})\ \text{scaleAR}}{100+\text{armItem}+\text{armN}} - \frac{\text{mrN scaleMR}}{100+\text{mrN}}}$$

```
$Assumptions = {armN≥0,mrN≥0,hpN>0,gold>0};
FullSimplify[(mdmgHPN > mdmgARMN) /.
    {hpItem → hpGold/(66/25), armItem → armGold/20, mrItem → mrGold/20}]
```

$$\frac{\frac{25\ \text{hpGold}}{66} + \text{hpN}}{1 - \frac{\text{armN scaleAR}}{100+\text{armN}} - \frac{\text{mrN scaleMR}}{100+\text{mrN}}} > \frac{\text{hpN}}{1 - \frac{(\text{armGold}+20\ \text{armN})\ \text{scaleAR}}{\text{armGold}+20\ (100+\text{armN})} - \frac{\text{mrN scaleMR}}{100+\text{mrN}}}$$

Assume the same amount of gold spent: hpGold = armGold = mrGold.
To simplify things we collapse scaleAR, scaleMR into one variable, ignoring TrueDamage.

```
% /. {hpGold → gold, armGold → gold, mrGold → gold}
f = FullSimplify[% /. {scaleMR → (1-scale), scaleAR → scale}]
```

$$\frac{\frac{25\,\text{gold}}{66}+\text{hpN}}{1-\frac{\text{armN scaleAR}}{100+\text{armN}}-\frac{\text{mrN scaleMR}}{100+\text{mrN}}}>\frac{\text{hpN}}{1-\frac{(20\,\text{armN}+\text{gold})\,\text{scaleAR}}{20\,(100+\text{armN})+\text{gold}}-\frac{\text{mrN scaleMR}}{100+\text{mrN}}}$$

$$\frac{(100+\text{armN})\left(\frac{25\,\text{gold}}{66}+\text{hpN}\right)(100+\text{mrN})}{100\,(100+\text{armN}-\text{armN scale}+\text{mrN scale})}>\frac{\text{hpN}}{1+\frac{\text{mrN}\,(-1+\text{scale})}{100+\text{mrN}}-\frac{(20\,\text{armN}+\text{gold})\,\text{scale}}{2000+20\,\text{armN}+\text{gold}}}$$

```
Reduce[FullSimplify[f //. {scale → 1, armN → 0, mrN → armN}], hpN]
N@%
```

$$\text{hpN}<\frac{25\,000}{33}$$

$$\text{hpN}<757.576$$

Display functions:

```
reduction[x_] := Expand@FullSimplify@Reduce[x];

show[t_,s_] := With[{expr=ToString[t,StandardForm] <> " / " <> ToString[s,StandardForm]

table[func_, threshold_, r_, ss_] :=
    Prepend[
        Map[x Ì {reduction[func /. Prepend[r,scale→x]], x, show[threshold, x]}, ss],
        {"Reduction: f /. " <> ToString[Prepend[r,scale→x], StandardForm],"x","Thresho
    ] // TableForm
```

```
t = 25000/33;
table[f, t, {armN → 0, mrN → 0}, {1, 2/3, 1/2, 1/3, 1/4}]
table[f, t/(1/2) (* because? : 100 armor = 1/2 dmg *), {armN → 100, mrN → 100}, {1, 2/3, 1/2, 1/3,
table[f, t/(1/3) (* because? : 200 armor = 1/3 dmg *), {armN → 200, mrN → 200}, {1, 2/3, 1/2, 1/3,
```

Reduction: f /. {scale → x, armN → 0, mrN → 0}	x	Threshold / scale
33 hpN < 25 000	1	$\frac{25000}{33} / 1 = \frac{25000}{33}$
150 000 + 25 gold > 132 hpN || 11 hpN ≤ 12 500	$\frac{2}{3}$	$\frac{25000}{33} / \frac{2}{3} = \frac{12500}{11}$
100 000 + 25 gold > 66 hpN || 33 hpN ≤ 50 000	$\frac{1}{2}$	$\frac{25000}{33} / \frac{1}{2} = \frac{50000}{33}$
3000 + gold > $\frac{33\ \text{hpN}}{25}$ || 11 hpN ≤ 25 000	$\frac{1}{3}$	$\frac{25000}{33} / \frac{1}{3} = \frac{25000}{11}$
200 000 + 75 gold > 66 hpN || 33 hpN ≤ 100 000	$\frac{1}{4}$	$\frac{25000}{33} / \frac{1}{4} = \frac{100000}{33}$

Reduction: f /. {scale → x, armN → 100, mrN → 100}	x	Threshold / scale
33 hpN < 50 000	1	$\frac{50000}{33} / 1 = \frac{50000}{33}$
12 000 + gold > $\frac{132\ \text{hpN}}{25}$ || 11 hpN ≤ 25 000	$\frac{2}{3}$	$\frac{50000}{33} / \frac{2}{3} = \frac{25000}{11}$
200 000 + 25 gold > 66 hpN || 33 hpN ≤ 100 000	$\frac{1}{2}$	$\frac{50000}{33} / \frac{1}{2} = \frac{100000}{33}$
150 000 + 25 gold > 33 hpN || 11 hpN ≤ 50 000	$\frac{1}{3}$	$\frac{50000}{33} / \frac{1}{3} = \frac{50000}{11}$
400 000 + 75 gold > 66 hpN || 33 hpN ≤ 200 000	$\frac{1}{4}$	$\frac{50000}{33} / \frac{1}{4} = \frac{200000}{33}$

Reduction: f /. {scale → x, armN → 200, mrN → 200}	x	Threshold / scale
11 hpN < 25 000	1	$\frac{25000}{11} / 1 = \frac{25000}{11}$
18 000 + gold > $\frac{132\ \text{hpN}}{25}$ || 11 hpN ≤ 37 500	$\frac{2}{3}$	$\frac{25000}{11} / \frac{2}{3} = \frac{37500}{11}$
12 000 + gold > $\frac{66\ \text{hpN}}{25}$ || 11 hpN ≤ 50 000	$\frac{1}{2}$	$\frac{25000}{11} / \frac{1}{2} = \frac{50000}{11}$
225 000 + 25 gold > 33 hpN || 11 hpN ≤ 75 000	$\frac{1}{3}$	$\frac{25000}{11} / \frac{1}{3} = \frac{75000}{11}$
200 000 + 25 gold > 22 hpN || 11 hpN ≤ 100 000	$\frac{1}{4}$	$\frac{25000}{11} / \frac{1}{4} = \frac{100000}{11}$

Graph

(Ignores magic resist)

```
effectiveHP[{gHP_,gAR_,gMR_}] := mdmg /.
    {hp → gHP/(66/25), arm → gAR/20, mr → gMR/20, scaleAR → scale, scaleMR → (1-scale)}

goldDistribution[h_,v_] := With[{f=Piecewise[{{h, 0<h<v}, {(h+v)/2, h>v}, {0, True}}]}, {f,Max[0,-v + f], 0 (*TODO*)}]

threshold = t * 66/25;

With[{x=goldDistribution[u,If[scale == 0, Infinity, threshold/scale]]},
    {x[[1]],x[[2]],effectiveHP[x]}];

region = Assuming[{scale≥0,scale≤1,Exists[scale,scale==0],scale ∈ Reals},
    FullSimplify[%]];
```

The purple surface is the union of all paths of steepest ascent from scale {0:1}.

```
graph
```