

Champ Analysis

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1. Analysis of the attack damage and attack speed

From dps-formula

$$\begin{aligned}
 dps = & (atk_dmg_0 + (lvl-1)*atk_dmg_{lvl} + atk_dmg_{itm} + atk_dmg_{db}) * \\
 & ([atk_spd_0 * (1 + \frac{(lvl-1)*atk_spd_{lvl}}{100\%})] * [1 + \frac{atk_spd_{itm} + atk_spd_{bd}}{100\%}]) * \\
 & (1 + \frac{crt_ch_{itm} + ctr_ch_{bd}}{100\%} \frac{crt_dmg}{200\%})
 \end{aligned} \tag{1}$$

Let's simplify the consideration by assuming that critical chance and buff&debuff effects are absent

$$crt_ch_{itm} = 0, atk_dmg_{db} = 0, atk_spd_{bd} = 0$$

In this case equation (1) can be rewritten as follows

$$\begin{aligned}
 dps = & (atk_dmg_0 + (lvl-1)*atk_dmg_{lvl} + atk_dmg_{itm}) * \\
 & * ([atk_spd_0 * (1 + (lvl-1)*\frac{atk_spd_{lvl}}{100\%})] * [1 + \frac{atk_spd_{itm}}{100\%}])
 \end{aligned} \tag{2}$$

Let's consider **dps** when the champion has achieved a N^{th} level. Attack damage in this case will be equal to (without any items)

$$AD_N = atk_dmg_0 + (N-1)*atk_dmg_N \tag{3}$$

Attack speed on this level without any items is equal

$$AS_N = atk_spd_0 * (1 + (N-1)*\frac{atk_spd_N}{100\%}) \tag{4}$$

Substituting (3) and (4) into (2) one can derive the following

$$dps_N = (AD_N + atk_dmg_{itm}) * [AS_N * (1 + \frac{atk_spd_{itm}}{100\%})], \quad (5)$$

where AD_N is the attack damage when a champion has reached N^{th} level, AS_N is the attack speed when a champion has achieved N^{th} level, atk_dmg_{itm} is the value of attack damage which can increased by items, atk_spd_{itm} is the value of attack speed which can increased by items.

When $atk_dmg_{itm} = 0$, **dps** is increased only by the attack speed

$$dps_N^{AS} = AD_N * AS_N * (1 + \frac{atk_spd_{itm}}{100\%}) \quad (6)$$

Similarly, when $atk_spd_{itm} = 0$, **dps** is increased only by the attack damage

$$dps_N^{AD} = (AD_N + atk_dmg_{itm}) * AS_N \quad (7)$$

Let's find the condition when the increasing of the **dps** by the attack speed is more efficient than by the attack damage. This condition can be written as

$$dps_N^{AS} \geq dps_N^{AD} \quad (8)$$

Let's use equations (6), (7) in (8)

$$AD_N * AS_N * (1 + \frac{atk_spd_{itm}}{100\%}) \geq (AD_N + atk_dmg_{itm}) * AS_N$$

$$AD_N * AS_N + AD_N * AS_N \frac{atk_spd_{itm}}{100\%} \geq AD_N * AS_N + atk_dmg_{itm} * AS_N$$

After simplification, one can obtain

$$AD_N * AS_N \frac{atk_spd_{im}}{100\%} \geq atk_dmg_{im} * AS_N. \quad (9)$$

Since the attack speed at N^{th} level $AS_N \neq 0$, one can derive

$$AD_N \frac{atk_spd_{im}}{100\%} \geq atk_dmg_{im} \quad (10)$$

At this stage we can express the atk_spd_{im} and atk_dmg_{im} in gold:

$$atk_spd_{im} = \frac{AS_{gold}}{33.33}, atk_dmg_{im} = \frac{AD_{gold}}{36}, \quad (11)$$

where AS_{gold} is the gold which has been spent to increase the attack speed, AD_{gold} is the gold which has been spent to increase the attack damage.

After substituting (11) in (10) one can write

$$AD_N \frac{AS_{gold}}{3333} \geq \frac{AD_{gold}}{36}$$

If we spend the same quantity of gold for speed and attack ($AS_{gold} = AD_{gold}$)

$$AD_N \geq 3333/36 \Rightarrow AD_N \geq 92.58 \quad (12)$$

This result means that increase of **dps** by the attack speed **is better** than the increase of **dps** by the attack damage when champion's attack damage more than 92.58 regardless of its attack speed.

2. Analysis of the attack speed and critical strike chance

In this section the analysis of the **dps** equation with respect to the attack velocity and strike chance crt_ch_{im} is performed.

Let's consider the **dps** equation (1) and assume that all buff&debuff effects are absent and the critical damage is equation 200%

$$crt_dmg = 200\%, atk_dmg_{db} = 0, atk_spd_{bd} = 0$$

With above assumptions, the **dps** equation will be defined as follows

$$dps = (atk_dmg_0 + (lvl - 1) * atk_dmg_{lvl} + atk_dmg_{im}) * ([atk_spd_0 * (1 + (lvl - 1) * \frac{atk_spd_{lvl}}{100\%}) * [1 + \frac{atk_spd_{im}}{100\%}]] * (1 + \frac{crt_ch_{im}}{100\%})) \quad (13)$$

Lets consider **dps** when the champion has achieved N^{th} level. Attack damage on this level without any items will be equal to

$$AD_N = atk_dmg_0 + (N - 1) * atk_dmg_N \quad (14)$$

The attack speed on this level without any items will be equal to

$$AS_N = atk_spd_0 * (1 + (N - 1) * \frac{atk_spd_N}{100\%}) \quad (15)$$

By substituting (14) and (15) into (13) one can derive:

$$dps_N = (AD_N + atk_dmg_{im}) * [AS_N * (1 + \frac{atk_spd_{im}}{100\%})] * (1 + \frac{crt_ch_{im}}{100\%}). \quad (16)$$

Here similarly to the previous section AD_N is the attack damage when a champion has achieved N^{th} level, AS_N is the corresponding attack speed, atk_dmg_{itm} is the value of attack damage which can be increased by items, atk_spd_{itm} is the value of attack speed which can be increased by items, crt_ch_{itm} is the value of critical strike chance which is increased by items.

In the case when $atk_dmg_{itm} = 0$ and $crt_ch_{itm} = 0$ (**dps** is increased only by the attack speed), one can write the following

$$dps_N^{AS} = AD_N * AS_N * (1 + \frac{atk_spd_{itm}}{100\%}) \quad (17)$$

When $atk_spd_{itm} = 0$ and $crt_ch_{itm} = 0$ (**dps** is increased by attack damage only), similarly one can obtain

$$dps_N^{AD} = (AD_N + atk_dmg_{itm}) * AS_N \quad (18)$$

When $atk_dmg_{itm} = 0$ and $atk_spd_{itm} = 0$ (**dps** is increased by the critical strike chance only)

$$dps_N^{CC} = AD_N * AS_N * (1 + \frac{crt_ch_{itm}}{100\%}) \quad (19)$$

At this stage equations (17)-(19) can be used to derive an important result.

At first, one can find the condition when the increase of **dps** by the attack speed is more efficient than the increase by the critical strike chance. This condition can be written as

$$dps_N^{AS} \geq dps_N^{CC} \quad (20)$$

Let's use equations (17), (19) in (20):

$$AD_N * AS_N * (1 + \frac{atk - spd_{itm}}{100\%}) \geq AD_N * AS_N * (1 + \frac{crt - ch_{itm}}{100\%}) \quad (21)$$

Evidently that $AD_N \neq 0, AS_N \neq 0$, therefore one can obtain

$$\frac{atk - spd_{itm}}{100\%} \geq \frac{crt - ch_{itm}}{100\%} \quad (22)$$

Let's express the parameters in (22) in the gold

$$atk - spd_{itm} = \frac{AS_{gold}}{33.33}, crt - ch_{itm} = \frac{CC_{gold}}{50}, \quad (23)$$

where AS_{gold} is the gold that has been spent to increase the attack speed, CC_{gold} is the gold that has been spent to increase the critical strike chance.

Finally, by substituting (23) in (22) one can derive

$$\frac{AS_{gold}}{33.33} \geq \frac{CC_{gold}}{50}$$

If we spend the same quantity of gold for speed and critical chance ($AS_{gold} = CC_{gold}$), one can obtain

$$\frac{1}{33.33} \geq \frac{1}{50}$$

This imparity is true. It means that increase **dps** by attack speed is more effective than increase **dps** by critical strike chance when $AS_{\text{gold}} = CC_{\text{gold}}$.

3. Analysis of the attack damage and critical strike chance

In this section similar analysis is performed with respect to attack damage and critical strike chance.

Similarly to the previous section, one can write the inequality

$$dps_N^{AD} \geq dps_N^{CC} \quad (24)$$

Using equations (18), (19) one can obtain

$$\begin{aligned} (AD_N + atk_dmg_{itm}) * AS_N &\geq AD_N * AS_N * (1 + \frac{crt_ch_{itm}}{100\%}) \\ AD_N * AS_N + atk_dmg_{itm} * AS_N &\geq AD_N * AS_N + AD_N * AS_N * \frac{crt_ch_{itm}}{100\%} \\ atk_dmg_{itm} * AS_N &\geq AD_N * AS_N * \frac{crt_ch_{itm}}{100\%} \end{aligned}$$

Since $AS_N \neq 0$ one can derive

$$atk_dmg_{itm} \geq AD_N * \frac{crt_ch_{itm}}{100\%} \quad (25)$$

Similarly one can express the attack damage and the critical strike chance in gold.

This gives us the following

$$crt_ch_{im} = \frac{CC_{gold}}{50}, atk_dmg_{im} = \frac{AD_{gold}}{36} \quad (26)$$

where CC_{gold} is the gold that has been spent to increase the critical strike chance, AD_{gold} is the gold that has been spent to increase the attack damage. Using (26) one can rewrite (25) as

$$\frac{AD_{gold}}{36} \geq ADN * \frac{CC_{gold}}{5000} \quad (27)$$

If we spend the same quantity of gold ($AD_{gold} = CC_{gold}$)

$$5000/36 \geq ADN \Rightarrow ADN \leq 138.89$$

This result means that the increase of **dps** by the attack damage is more effective than increase the **dps** by the critical strike chance when champion's attack damage less than 138.89.

4. Conclusions of the project phase

The above analysis has given answers on several important questions. The main results are as follows:

- When $AD_N < 92.58$ increasing of the attack damage is the **most effective** way to increase **dps**.
- The increase of the attack speed is **more effective** than the increase of the critical strike chance always.
- When $92.58 < AD_N < 138.89$ the increase of the attack damage is **more effective** than the increase the critical strike chance but **less effective** that the increase of the attack speed.
- When $AD_N > 138.89$ the increase of the critical strike chance is **more effective** than the increase of the attack damage but the increase of the attack speed is **more effective** than the increase of the critical strike chance always.
- When $AD_N < 92.58$ the increase of the attack damage is the **most effective** way to increase **dps**, when $AD_N > 92.58$ the increase of the attack speed is the **most effective** way to increase **dps**

Fig. 1 illustrates the obtained results in easy way.

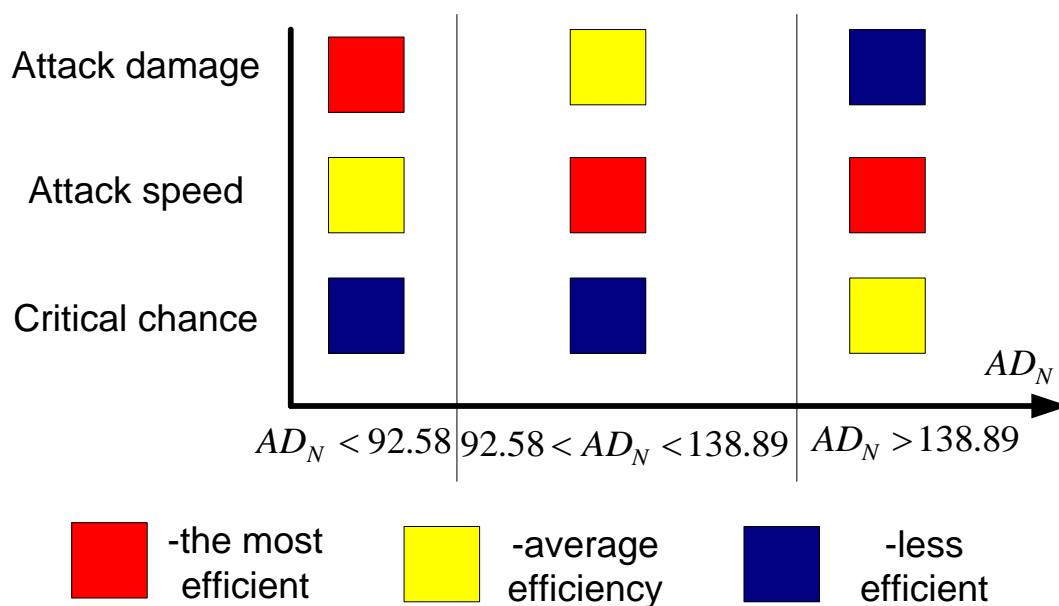


Fig. 1. Efficiency diagram