Champion's survival

Champion dies when value of achieved damage (*ach_dmg*) exceeds the value of him health points (*HP*) at the moment. If champion doesn't have any armor (*arm*) all of damage (made damage) transfers into the achieved damage (*ach_dmg*). If the value of champion's value armor is not equal zero ($arm \neq 0$), some part of the made damage is blocked (*bl_dmg*) by the armor. All these things can be related via the following equation

$$(1 - bl\_dmg) = \frac{ach\_dmg}{m\_dmg}$$

Blocked damage depends on the armor. This dependence can be written as follows

$$bl\_dmg = \frac{arm}{100 + arm} \tag{1}$$

So if a particular champion has 100 points of armor it achieves only half of made damage.

Champion's survival is defined by *m_dmg* that champion is able to withstand.

$$m\_dmg = \frac{HP}{1 - bl\_dmg} \tag{2}$$

After using (1) in (2) one can obtain

$$m\_dmg = \frac{HP}{1 - \frac{arm}{100 + arm}} \tag{3}$$

Thus, the maximization of champ survival implies the maximization of *m_dmg*.

Health points

Health points are determined by the next equation

$$HP = HP_0 + (lvl - 1) * HP_{lvl} + HP_{itm}, \quad (4)$$

where HP_0 is the champion's health points at first level(minimum – 290 (LeBlanc), maximum – 495 (Gangplank)),

lvl – champion's level(maximum - 18),

HP_{lvl} is a quantity of the health points, that is added when the champion gets level up (minimum – 70 (Anivia, Sona), maximum – 104 (Sion))

HP_{itm} is a quantity of health points is added by items.

Armor

Champion's armor is determined by the next formula

$$arm = arm_0 + (lvl - 1) * arm_{lvl} + arm_{itm} \quad (5)$$

where arm_0 is the champion's armor at first level(minimum – 6.75 (Zilean), maximum – 21 (Rammus)),

lvl is the champion's level(maximum - 18),

arm_{lvl} is the value of armor, that is added when the champion gets level up (minimum – 0 (Thresh), maximum – 4)

arm_{itm} is the value of armor is added by items.

Substituting equations (4) and (5) into (3) gives the following:

$$m\_dmg = \frac{HP_0 + (lvl-1) * HP_{lvl} + HP_{itm}}{1 - \frac{arm_0 + (lvl-1) * arm_{lvl} + arm_{itm}}{100 + arm_0 + (lvl-1) * arm_{lvl} + arm_{itm}}} \quad (6)$$

When champion achieves Nth level it's HP and armor will be equal:

$$HP = HP_N + HP_{itm} \quad (7)$$

$$arm = arm_N + arm_{itm} \quad (8)$$

So, the value of damage that champion is able to withstand at Nth level

$$m\_dmg_N = \frac{HP_N + HP_{itm}}{1 - \frac{arm_N + arm_{itm}}{100 + arm_N + arm_{itm}}} \quad (9)$$

Compare what stat is more effectively increases the champion's survival. When *m_dmg* is increased by health points only $arm_{itm} = 0$ (9) becomes

$$m\_dmg_N^{HP} = \frac{HP_N + HP_{itm}}{1 - \frac{arm_N}{100 + arm_N}} \quad (10)$$

When *m_dmg* is increased by armor only $HP_{itm} = 0$ (9) becomes

$$m\_dmg_N^{arm} = \frac{HP_N}{1 - \frac{arm_N + arm_{itm}}{100 + arm_N + arm_{itm}}} \quad (11)$$

Let's find the conditions when $m\_dmg_N^{HP}$ more than $m\_dmg_N^{arm}$ if it is spent the same value of gold for both stats arm_{itm} and HP_{itm}

$$m\_dmg_N^{HP} > m\_dmg_N^{arm} \tag{12}$$

Substituting (10) and (11) into (12) gives

$$\frac{HP_N + HP_{itm}}{1 - \frac{arm_N}{100 + arm_N}} > \frac{HP_N}{1 - \frac{arm_N + arm_{itm}}{100 + arm_N + arm_{itm}}}$$

$$(HP_N + HP_{itm}) * (1 - \frac{arm_N + arm_{itm}}{100 + arm_N + arm_{itm}}) > HP_N * (1 - \frac{arm_N}{100 + arm_N})$$

After opening the brackets:

$$HP_N + HP_{itm} - HP_N \frac{arm_N + arm_{itm}}{100 + arm_N + arm_{itm}} - HP_{itm} \frac{arm_N + arm_{itm}}{100 + arm_N + arm_{itm}} >$$

$$> HP_N - HP_N * \frac{arm_N}{100 + arm_N}$$

$$HP_{itm} - HP_N \frac{arm_N + arm_{itm}}{100 + arm_N + arm_{itm}} - HP_{itm} \frac{arm_N + arm_{itm}}{100 + arm_N + arm_{itm}} >$$

$$> -HP_N * \frac{arm_N}{100 + arm_N}$$

Further simplification gives

$$HP_{itm} * (100 + arm_N + arm_{itm}) - HP_N * (arm_N + arm_{itm}) - HP_{itm} * (arm_N + arm_{itm}) >$$

$$> -HP_N * \frac{arm_N}{100 + arm_N} * (100 + arm_N + arm_{itm})$$

$$100*HP_{itm}+arm_N*HP_{itm}+arm_{itm}*HP_{itm}-HP_N*arm_N-HP_N*arm_{itm}-$$
$$-HP_{itm}*arm_N-HP_{itm}*arm_{itm}>-HP_N*arm_N*\frac{100+arm_N+arm_{itm}}{100+arm_N}$$

$$100*HP_{itm}-HP_N*arm_N-HP_N*arm_{itm}>-HP_N*arm_N*(1+\frac{arm_{itm}}{100+arm_N})$$

Open brackets:

$$100*HP_{itm}-HP_N*arm_N-HP_N*arm_{itm}>$$
$$>-HP_N*arm_N-HP_N*arm_N*\frac{arm_{itm}}{100+arm_N}$$

$$100*HP_{itm}-HP_N*arm_{itm}>-HP_N*arm_N*\frac{arm_{itm}}{100+arm_N}$$

$$100*HP_{itm}*(100+arm_N)-HP_N*arm_{itm}*(100+arm_N)>-HP_N*arm_N*arm_{itm}$$

Open brackets:

$$10000*HP_{itm}+100*HP_{itm}*arm_N-100*HP_N*arm_{itm}-HP_N*arm_{itm}*arm_N>$$
$$>-HP_N*arm_N*arm_{itm}$$

$$10000*HP_{itm}+100*HP_{itm}*arm_N-100HP_N*arm_{itm}>1$$
$$10000*HP_{itm}+100*HP_{itm}arm_N>1+100*HP_N*arm_{itm} \quad (13)$$

At this stage we can express the HP_{itm} and arm_{itm} in gold:

$$HP_{itm}=\frac{HP_{gold}}{2.64},\ arm_{itm}=\frac{arm_{gold}}{20} \quad (14)$$

where HP_{gold} is the gold which has been spent to increase the health points, arm_{gold} is the gold which has been spent to increase the armor.

After substituting (14) into (13) one can write

$$10000*\frac{HP_{gold}}{2.64}+100*\frac{HP_{gold}}{2.64}arm_N > 1+100HP_N*\frac{arm_{gold}}{20}$$

$$100*(100+arm_N)*HP_{gold} > 2.64+13.2HP_N*arm_{gold}$$

$$100*(100+arm_N)*HP_{gold}-2.64 > 13.2HP_N*arm_{gold} \qquad (15)$$

As so as $100*(100+arm_N)*HP_{gold} >> 2.64$ (15) can be rewritten as

$$100*(100+arm_N)*HP_{gold} > 13.2HP_N*arm_{gold}$$

$$7.576*(100+arm_N)*HP_{gold} > HP_N*arm_{gold}$$

If we spend the same quantity of gold for health points and armor ($HP_{gold}=arm_{gold}$) one can obtain the following

$$7.576*(100+arm_N) > HP_N$$

$$757.6+7.576*arm_N-HP_N > 0 \qquad (16)$$

So, if imparity (16) is true increase health points (HP) is more effective way than the increase of armor to improve the champion's survival.

Let's estimate the limits of values of arm_N and HP_N in (16). The minimum value of arm_N is 6.75 (Zilean at first level) and we have $757.6+7.576*6.75 \approx 809$.

Thus if $HP_N < 809$, the imparity (16) is true anyway and therefore increase the health points is better than increase the armor. The maximum value of arm_N is 86 (Poppy and Maokai at 18^{th} level) and we have $757.6 + 7.576 * 86 \approx 1409$. Thus, if $HP_N > 1409$, the imparity (16) is false anyway and therefore increase the armor is better than increase the health points (HP).

So, the conclusions are strict and coincide with the intuitive assumptions.
The champions have a low value of health points at lower levels and it is better increase the HP at the beginning. But when champions get up levels, basic health points increase and priority shifts to the armor.

Checking the results

Let's analyze and check the obtained results. This can be done by the calculation of *m_dmg* of some champions with different health points and armor. After that calculate the *m_dmg* of same champions with items, that increase one of considered stats and have equal cost in gold.

Let's consider the champions at 7^{th} level with the following items:

Item	**Cost**	**Health points increase**	**Armor increase**
Giant's Belt (GB)	1000	380	0
Chain Vest + Cloth Armor (CV+CA)	720+300	0	40+15

One can see that each item increases single stat, this means that so we can check the influence of every stat on *m_dmg* separately.

Table 1 illustrates the obtained results.

Champion	HP at 7^{th} level	Armor at 7^{th} level	*m_dmg* at 7^{th} level	Switch7	*m_dmg* with GB	Δ_{HP}	*m_dmg* CV+CA	Δ_{arm}
Zyra	799	29	1031	178	1521	490	1470	439
Heimerdinger	800	28	1024	170	1510	486	1464	440
Vayne	857	29.7	1112	126	1604	492	1583	471
Nocturne	950	38	1311	95	1835	525	1834	523
Trundle	1031	35.2	1394	-6.7	1908	514	1961	567
Alistar	1054	35.5	1428	-27	1943	515	2008	580
Tryndamere	1049	33.5	1400	-38	1908	508	1977	577

Table 1. Made damages at 7^{th} level

Switch7 – the left part of imparity (16) value at 7th level

Δ_{HP} – the value of *m_dmg* is increased by item, that increases the HP only

Δ_{arm} - the value of *m_dmg* is increased by items, that increase the armor only

Larger values of increasing are green, fewer values are red.

Let's consider the same champions at 12th level with items. Table 2 illustrates the calculated made damage for this level.

Champion	HP at 12th level	Armor at 12th level	*m_dmg* at 12th level	Switch12	*m_dmg* with GB	Δ_{HP}	*m_dmg* CV+CA	Δ_{arm}
Zyra	1169	44	1683	-78	2231	548	2326	643
Heimerdinger	1175	43	1680	-92	2224	544	2327	647
Vayne	1272	46.7	1866	-161	2423	557	2566	700
Nocturne	1375	55.5	2138	-197	2729	591	2894	756
Trundle	1511	48.7	2247	-384	2812	565	3078	831
Alistar	1564	53	2393	-405	2974	581	3253	860
Tryndamere	1539	49	2293	-410	2859	566	3139	846

Table 2. Made damages at 12th level

In the table 2 the column "Switch12" is the left part of imparity (16).

One can see that the obtained model is correct.

-When switch>0 increasing quantity of health points is more effective to increase survival.

-When switch<0 increasing quantity of health points become a bad idea and priority shifts to the armor increament.