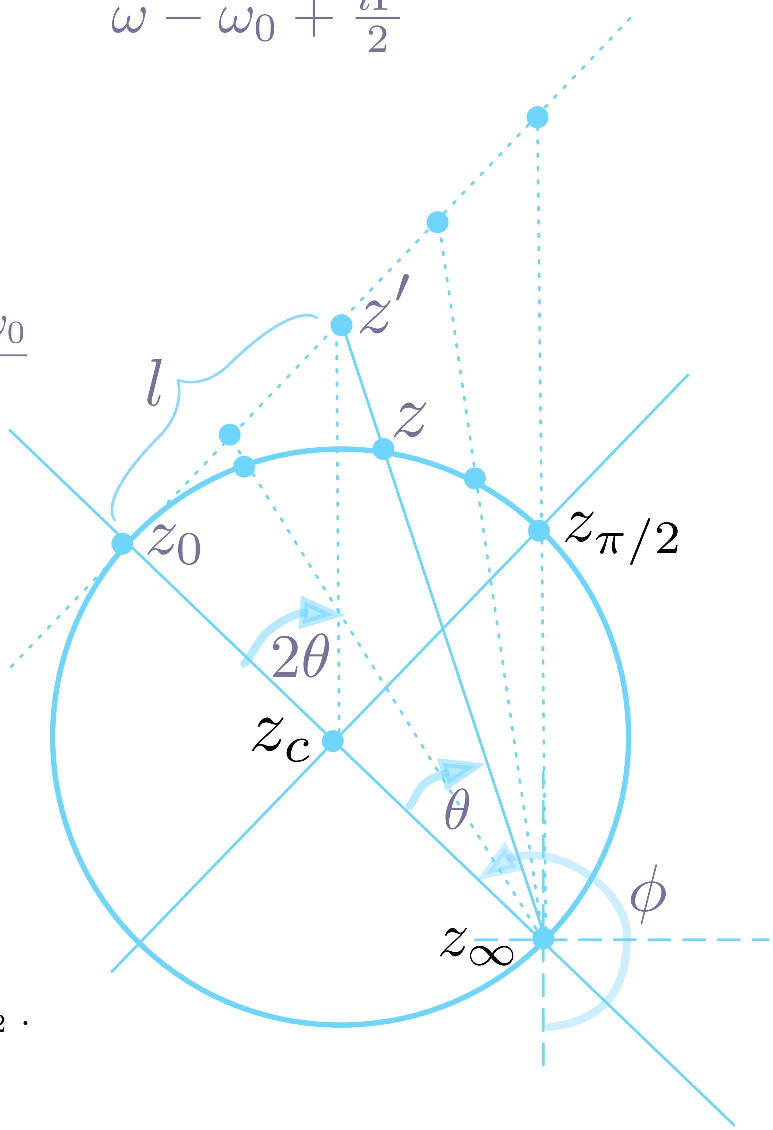
Lorentzian in the complex plane

$$z = z_\infty + \frac{Ae^{i\phi}}{\omega - \omega_0 + \frac{i\Gamma}{2}}$$

$$z' - z_\infty = \frac{|z_0 - z_\infty|^2}{(z - z_\infty)^*}$$

$$l = |z' - z_0| = |z_0 - z_\infty| \frac{\omega - \omega_0}{\Gamma/2}$$



$\omega = \infty$ maps into point z_∞ .

$\omega = \omega_0$ maps into point z_0 .

$\omega = \omega_0 + \Gamma/2$ maps into point $z_{\pi/2}$.

$$\theta = \arctan \frac{\omega - \omega_0}{\Gamma/2}$$

$$z_0 - z_\infty = 2\frac{A}{\Gamma} e^{i(\phi - \pi/2)}$$

$$\begin{aligned} z - z_c &= \frac{1}{\omega - \omega_0 + i\Gamma/2} - \frac{1}{i\Gamma} \\ &= \left(\frac{i}{\Gamma}\right) \frac{\omega - \omega_0 - i\Gamma/2}{\omega - \omega_0 + i\Gamma/2} \\ &= \frac{i}{\Gamma} e^{-2i\theta} \end{aligned}$$