Bài tập 1) Tính các giới hạn sau: (VCB + L'Hospital)

a) $\lim_{x\to 0} \frac{\sin 5x - \sin 4x}{\sin 3x}$ b) $\lim_{x\to 1} \frac{\ln x}{\ln(2-x)}$

c) $\lim_{x\to 1} \frac{\sin \pi x}{x-1}$ d) $\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}} \to 1^\infty$

e) $\lim_{x\to 0} \frac{e^x - 1}{5^x - 1}$ f) $\lim_{x\to a} \frac{\ln x - \ln a}{x-a}$

$\frac{0}{0}$

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a) $I = \lim_{x\to 0} \frac{\sin 5x - \sin 4x}{\sin 3x}$

Sử dụng VCB, ta có $\begin{cases} \sin 5x \sim 5x \\ \sin 4x \sim 4x \\ \sin 3x \sim 3x \end{cases}$

$\Rightarrow I = \lim_{x\to 0} \frac{5x-4x}{3x} = \lim_{x\to 0} \frac{x}{3x} = \frac{1}{3}.$

$\boxed{(\ln u)' = \frac{u'}{u} \quad (u = 2-x)}$

b) $\lim_{x\to 1} \frac{\ln(x)}{\ln(2-x)} \overset{L}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{-\frac{1}{2-x}} = \lim_{x\to 1}\left(-\frac{2-x}{x}\right) = -1.$

$\boxed{(\sin ax)' = a\cos ax}$

c) $\lim_{x\to 1} \frac{\sin \pi x}{x-1} \overset{L}{=} \lim_{x\to 1} \frac{\pi \cos \pi x}{1} = \pi \cos \pi = -\pi.$

$\boxed{(a^x)' = a^x \ln a}$

e) $\lim_{x\to 0} \frac{e^x - 1}{5^x - 1} \overset{L}{=} \lim_{x\to 0} \frac{e^x}{5^x \ln 5} = \frac{e^0}{5^0 \ln 5} = \frac{1}{\ln 5}$

C2: $e^x - 1 \sim x$ $\Rightarrow \lim_{x\to 0} \frac{e^x - 1}{5^x - 1} = \lim_{x\to 0} \frac{x}{x\ln 5} = \frac{1}{\ln 5}$

$5^x - 1 = e^{x\ln 5} - 1 \sim x\ln 5$

$a^{\log_a x} = x$ $(a = e)$: $e^{\ln x} = x \Rightarrow 5^x = \left(e^{\ln 5}\right)^x$

f) $\lim_{x\to a} \frac{\ln x - \ln a}{x-a} \overset{L}{=} \lim_{x\to a} \frac{\frac{1}{x}}{1} = \frac{1}{a}.$

d) $\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}} = e^{\lim_{x\to 0} \frac{\ln \cos x}{x^2}}$

$\boxed{\cos x = e^{\ln \cos x} \quad (\ln u)' = \frac{u'}{u} \quad u = \cos x}$

Xét $\lim_{x\to 0} \frac{\ln \cos x}{x^2} \overset{L}{=} \lim_{x\to 0} \frac{-\frac{\sin x}{\cos x}}{2x} \overset{VCB}{=} \lim_{x\to 0} \frac{-\frac{x}{\cos x}}{2x} = -\frac{1}{2}$

$\Rightarrow \lim_{x\to 0} (\cos x)^{1/x^2} = e^{\lim_{x\to 0} \frac{\ln \cos x}{x^2}} = e^{-\frac{1}{2}}$

$\cos x = 1 + (\cos x - 1)$

$\ln \cos x = \ln(1 + (\cos x - 1)) \sim \cos x - 1 = -(1-\cos x) = -\frac{x^2}{2}$