Computational Methods

RELATIONS AND MAPPINGS

Exercises

Describe the rule associated with the following functions, sketch their graphs and state their domains and ranges:

(a) $f(x) = 2x^2$

(b) $f(x) = x^2 - 1$ $\quad 0 \leqslant x$

(c) $g(t) = 3t - 4$ $\quad 0 \leqslant t$

(d) $y(x) = x^3$

(e) $f(t) = 0.5t + 2$ $\quad -2 \leqslant t \leqslant 10$

(f) $z(x) = 3x - 2$ $\quad 3 \leqslant x \leqslant 8$

Exercises

If $f(x) = 5x + 4$, find

(a) $f(3)$

(b) $f(-3)$

(c) $f(\alpha)$

(d) $f(x+1)$

(e) $f(3\alpha)$

(f) $f(x^2)$

Exercises

If $g(t) = 5t^2 - 4$, find

(a) $g(0)$

(b) $g(2)$

(c) $g(-3)$

(d) $g(x)$

(e) $g(2t - 1)$

Exercises

Find the inverse of the following functions:

(a) $f(x) = x + 4$

(b) $g(t) = 3t + 1$

(c) $y(x) = x^3$

(d) $h(t) = \dfrac{t - 8}{3}$

(e) $f(t) = \dfrac{t - 1}{3}$

(f) $h(x) = x^3 - 1$

(g) $k(v) = 7 - v$

(h) $m(n) = \frac{1}{3}(1 - 2n)$

Exercises

Given $f(t) = 2t$, $g(t) = t - 1$ and $h(t) = t^2$ write
expressions for

(a) $f(g(t))$ (b) $f(h(t))$

(c) $g(h(t))$ (d) $g(f(t))$

(e) $h(g(t))$ (f) $h(f(t))$

(g) $f(f(t))$ (h) $g(g(t))$

(i) $h(h(t))$ (j) $f(g(h(t)))$

(k) $g(f(h(t)))$ (l) $h(g(f(t)))$

Exercises

Simplify

(a) $\dfrac{e^{2x}}{3e^{3x}}$

(b) $\dfrac{e^{2t-3}}{e^2}$

(c) $\dfrac{e^x(e^x + e^{2x})}{e^{2x}}$

(d) $\dfrac{e^{-3}e^{-7}}{e^6e^{-2}}$

(e) $\dfrac{(e^{2t})^3(e^{3t})^4}{e^{10t}}$

Exercises

Evaluate

(a) $\log_2 8$ (b) $\log_2 15$

(c) $\log_{16} 50$ (d) $\log_{16} 123$

(e) $\log_8 23$ (f) $\log_8 47$

Simplify each of the following to a single log term:

(a) $\log 7 + \log x$

(b) $\log x + \log y + \log z$

(c) $\ln y - \ln 3$

(d) $2 \log y + \log x$

(e) $\ln(xy) + \ln(y^2)$

Exercises

Solve the following equations:

(a) $e^x = 70$

(b) $e^x = \dfrac{1}{3}$

(c) $e^{-x} = 1$

(d) $3e^x = 50$

(e) $e^{3x} = 50$

(f) $e^{2x+3} = 300$

(g) $e^{-x+1} = 0.75$

(h) $2ee^{2x} = 50$

(i) $\dfrac{3}{e^x + 1} = 0.6$

(j) $\dfrac{3}{e^{x+1}} = 0.6$

(k) $(e^x)^3 = 200$

(l) $\sqrt{e^{2x}} = 2$

(m) $\sqrt{e^{2x} + 4} = 6$

(n) $\dfrac{e^x}{e^x + 2} = 0.7$

(o) $e^{2x} = 7e^x$

(p) $2e^{-x} = 9$

(q) $(e^x + 3)^2 = 25$

(r) $(3e^{-x} - 6)^3 = 8$

(s) $e^{2x} - 3e^x + 2 = 0$

(t) $2e^{2x} - 7e^x + 3 = 0$

(u) $e^x(5 - e^x) = 6$

(v) $e^x - 7 + \dfrac{12}{e^x} = 0$

Exercises

Solve
(a) $\log x = 1.6$ (b) $\log 2x = 1.6$
(c) $\log(2 + x) = 1.6$ (d) $2\log(x^2) = 2.4$
(e) $\log(2x - 3) = 0.7$

Solve
(a) $\ln x = 2.4$ (b) $\ln 3x = 4$
(c) $2\ln(2x - 1) = 5$ (d) $\ln(2x^2) = 4.5$
(e) $\ln\left(\dfrac{x+1}{3}\right) = 0.9$

Solve
(a) $e^{3x} = 21$ (b) $10^{-2x} = 6.7$
(c) $\dfrac{1}{e^{-x} + 2} = 0.3$ (d) $2e^{(x/2)} - 1 = 0$
(e) $3(10^{(-4x+6)}) = 17$
(f) $(e^{x-1})^3 + e^{3x} = 500$
(g) $\sqrt{10^{2x} + 100} = 3(10^x)$

Exercises

Evaluate the following:

(a) $\sinh 3$ 　　　　　　　(b) $\cosh 1.6$
(c) $\tanh 0.95$ 　　　　　(d) $\operatorname{sech} 1$
(e) $\operatorname{cosech} 2$ 　　　　　(f) $\coth 1.5$
(g) $\cosh(-3)$ 　　　　　(h) $\operatorname{cosech}(-1.6)$
(i) $\sinh(-2)$ 　　　　　(j) $\coth(-2.7)$
(k) $\tanh(-1.4)$ 　　　　(l) $\operatorname{sech}(-0.5)$

Evaluate

(a) $\sinh^{-1} 3$

(b) $\cosh^{-1} 2$

(c) $\tanh^{-1}(-0.25)$

Exercises

Express

(a) $6e^x + 5e^{-x}$ in terms of $\sinh x$ and $\cosh x$,

(b) $4e^{2x} - 3e^{-2x}$ in terms of $\sinh 2x$ and $\cosh 2x$,

(c) $2e^{-3x} - 5e^{3x}$ in terms of $\sinh 3x$ and $\cosh 3x$.

Express

(a) $4\sinh x + 3\cosh x$ in terms of e^x and e^{-x},

(b) $3\sinh 2x - \cosh 2x$ in terms of e^{2x} and e^{-2x},

(c) $3\cosh 3x - 0.5\sinh 3x$ in terms of e^{3x} and e^{-3x}.

Exercises

Use the identities for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ to simplify the following:

(a) $\sin\left(\theta - \dfrac{\pi}{2}\right)$

(b) $\cos\left(\theta - \dfrac{\pi}{2}\right)$

(c) $\tan(\theta + \pi)$

(d) $\sin(\theta - \pi)$

(e) $\cos(\theta - \pi)$

(f) $\tan(\theta - 3\pi)$

(g) $\sin(\theta + \pi)$

(h) $\cos\left(\theta + \dfrac{3\pi}{2}\right)$

(i) $\sin\left(2\theta + \dfrac{3\pi}{2}\right)$

(j) $\cos\left(\theta - \dfrac{3\pi}{2}\right)$

(k) $\cos\left(\dfrac{\pi}{2} + \theta\right)$