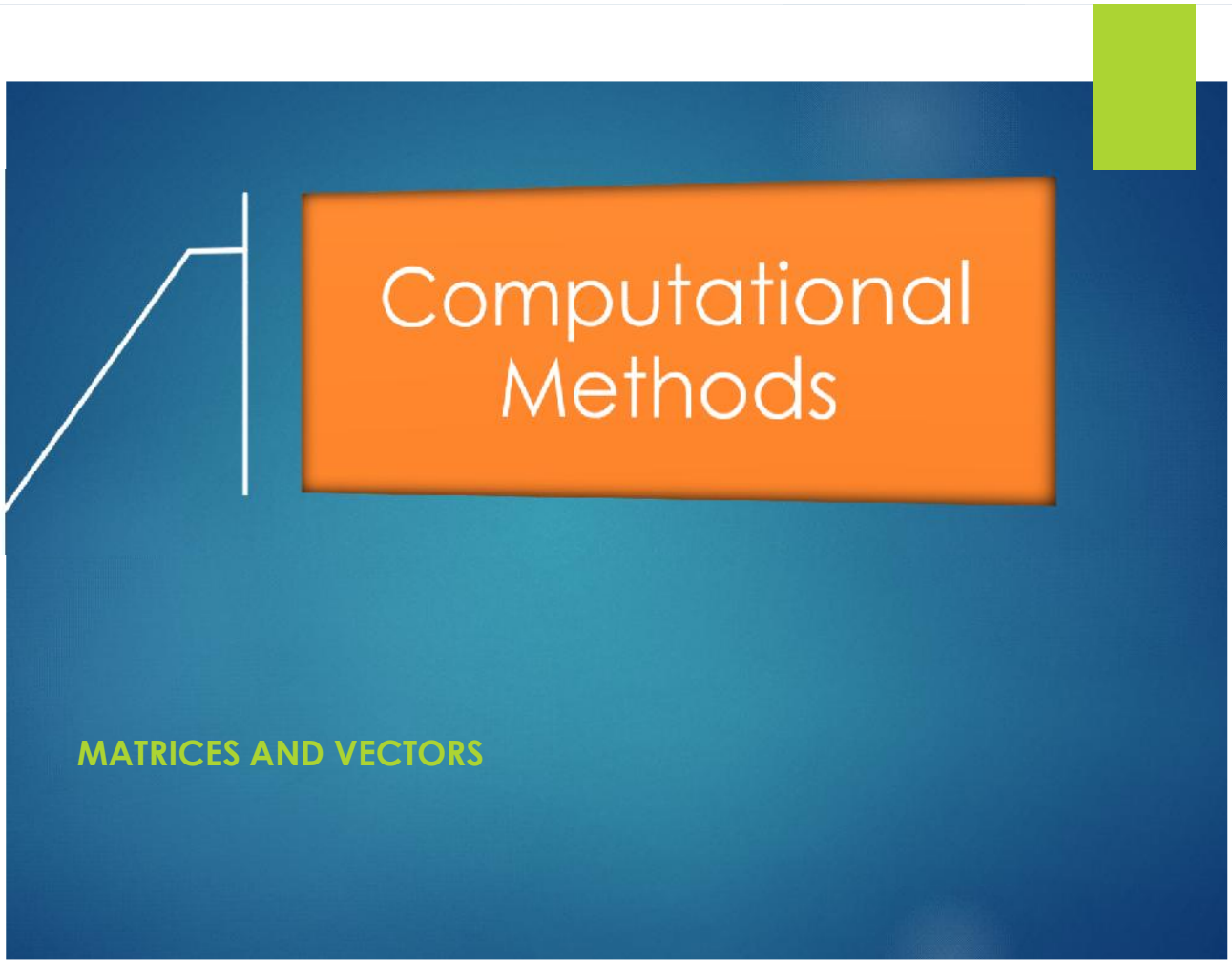
Computational Methods
MATRICES AND VECTORS

Exercises

Find AB and BA where

$$A = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 0 & 4 \\ 5 & 1 & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 5 & 2 & 1 \\ 0 & 3 & 4 \\ 1 & 3 & 5 \end{pmatrix}$$

Given that A^2 means the product of a matrix A with itself, find A^2 when $A = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$. Find A^3.

Exercises

If $A = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & 5 \end{pmatrix}$ find AB, BA, $A + B$ and $(A + B)^2$. Show that

$$(A + B)^2 = A^2 + AB + BA + B^2$$

Why is $(A + B)^2$ not equal to $A^2 + 2AB + B^2$?

Exercises

If $A = \begin{pmatrix} 3 & 1 \\ 2 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 4 \\ 3 & 8 \end{pmatrix}$

(a) find A^T,

(b) find B^T,

(c) find AB,

(d) find $(AB)^T$,

(e) deduce that $(AB)^T = B^T A^T$.

Exercises

Find the inverse, if it exists, of each of the following matrices:

(a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} -1 & 0 \\ -1 & 7 \end{pmatrix}$ (e) $\begin{pmatrix} 6 & 2 \\ 9 & 3 \end{pmatrix}$ (f) $\begin{pmatrix} -6 & 2 \\ 9 & 3 \end{pmatrix}$

(g) $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} \end{pmatrix}$

Exercises

Given that the matrix

$$M = \begin{pmatrix} \cos \omega t & -\sin \omega t & 0 \\ \sin \omega t & \cos \omega t & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is orthogonal, find M^{-1}.

Exercises

Given

$$A = \begin{pmatrix} 3 & 7 & 6 \\ -2 & 1 & 0 \\ 4 & 2 & -5 \end{pmatrix}$$

(a) find $|A|$

(b) find the cofactors of the elements of row 2, that is $-2, 1, 0$

(c) calculate

$-2 \times$ (cofactor of -2)

$+1 \times$ (cofactor of 1)

$+0 \times$ (cofactor of 0).

What do you deduce?

Exercises

Find adj(A), $|A|$ and, if it exists, A^{-1}, if

(a) $A = \begin{pmatrix} 2 & -3 & 1 \\ 5 & 4 & 1 \\ 2 & -2 & -1 \end{pmatrix}$

(b) $A = \begin{pmatrix} 3 & 1 & 0 \\ 2 & -1 & 1 \\ 5 & 5 & -7 \end{pmatrix}$

(c) $A = \begin{pmatrix} 2 & -1 & 4 \\ 5 & -2 & 9 \\ 3 & 2 & -1 \end{pmatrix}$

Exercises

If $\mathbf{a} = 3\mathbf{i} - 7\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} + 4\mathbf{j}$ find $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{b} \cdot \mathbf{a}$, $\mathbf{a} \cdot \mathbf{a}$ and $\mathbf{b} \cdot \mathbf{b}$.

If $\mathbf{a} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$ and $\mathbf{c} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$, find

(a) $\mathbf{a} \cdot \mathbf{a}$ (b) $\mathbf{a} \cdot \mathbf{b}$

(c) $\mathbf{a} \cdot \mathbf{c}$ (d) $\mathbf{b} \cdot \mathbf{c}$

Evaluate $(-13\mathbf{i} - 5\mathbf{j}) \cdot (-3\mathbf{i} + 4\mathbf{j})$.

Find the angle between the vectors $\mathbf{p} = 7\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{q} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$.

Find the angle between the vectors $7\mathbf{i} + \mathbf{j}$ and $4\mathbf{j} - \mathbf{k}$.

Find the angle between the vectors $4\mathbf{i} - 2\mathbf{j}$ and $3\mathbf{i} - 3\mathbf{j}$.

If $\mathbf{a} = 7\mathbf{i} + 8\mathbf{j}$ and $\mathbf{b} = 5\mathbf{i}$ find $\mathbf{a} \cdot \hat{\mathbf{b}}$.

If $\mathbf{r}_1 = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}$ find $\mathbf{r}_1 \cdot \mathbf{r}_1$, $\mathbf{r}_1 \cdot \mathbf{r}_2$ and $\mathbf{r}_2 \cdot \mathbf{r}_2$.

Given that $\mathbf{p} = 2\mathbf{q}$ simplify $\mathbf{p} \cdot \mathbf{q}$, $(\mathbf{p} + 5\mathbf{q}) \cdot \mathbf{q}$ and $(\mathbf{q} - \mathbf{p}) \cdot \mathbf{p}$.

Exercises

Evaluate

(a) $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 2 \\ 2 & 1 & 4 \end{vmatrix}$ (b) $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & -3 \\ -4 & 0 & 1 \end{vmatrix}$

(c) $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 1 & 0 & 4 \end{vmatrix}$ (d) $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 5 & 2 \\ -3 & -1 & 4 \end{vmatrix}$

If $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$, find

(a) $\mathbf{a} \times \mathbf{b}$

(b) $\mathbf{b} \times \mathbf{a}$

If $\mathbf{a} = \mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = 5\mathbf{i} + 5\mathbf{k}$ find $\mathbf{a} \times \mathbf{b}$.

If $\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j}$ and $\mathbf{c} = 2\mathbf{i} + \mathbf{k}$ find

(a) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$

(b) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

If $\mathbf{p} = 6\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$ and $\mathbf{q} = 3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ find $|\mathbf{p}|$, $|\mathbf{q}|$ and $|\mathbf{p} \times \mathbf{q}|$. Deduce the sine of the angle between $\mathbf{p}$ and $\mathbf{q}$.