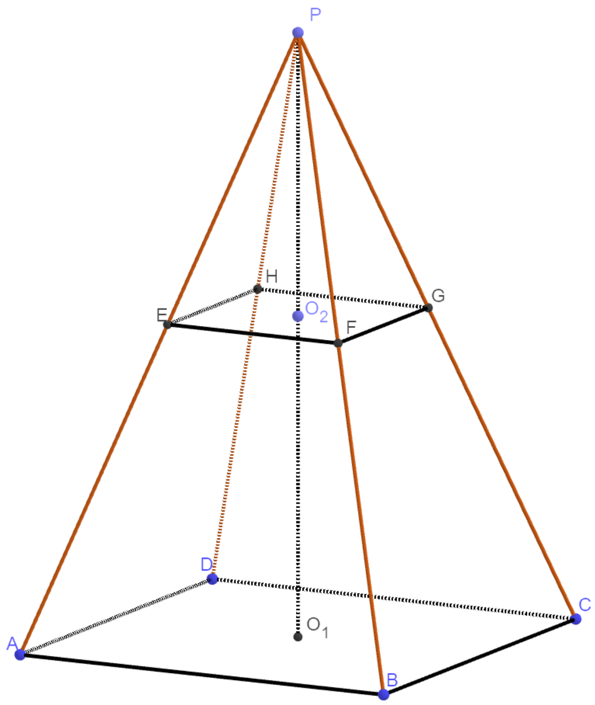
锥体（台体）的体积公式推导

已知：S_0(即$S_{四边形ABCD}$),h_0(即PO_1),h_1(即O_2O_1)



$V_{P-ABCD}=\int_0^{h_0} S(h)\mathrm{d}h$

$\because \left(\frac{h_0-h}{h_0}\right)^2=\frac{s}{s_0} \therefore S(h)=S_0+\frac{s_0}{h_0^2}h^2-\frac{2s_0}{h_0}h$

记$S(h)$原函数为$F(h)$

$\therefore F(h)=\frac{s_0}{3h_0^2}h^3-\frac{s_0}{h_0}h^2+s_0h$

$\therefore V_{P-ABCD}=\int_0^{h_0} S(h)\mathrm{d}h=F(h_0)-F(0)=\frac{1}{3}s_0h_0$

$V_{EFGH-ABCD}=\int_0^{h_1} S(h)\mathrm{d}h=F(h_1)-F(0)=\frac{s_0h_1^3}{3h_0^2}-\frac{s_0h_1^2}{h_0}+s_0h_1$

注：原函数的求法

$\because f(x)=ax^3+bx^2+cx+d$

$\therefore f'(x)=\lim\limits_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$

$=\lim\limits_{\Delta x\to 0} a\Delta x^2+3ax_0^2+3ax_0^2+3ax_0\Delta x+b\Delta x+2bx_0+c$

$=3ax^2+2bx+c$