The Stable Marriage Problem

這個算法也是iteration

men’s preferences

	1st	2nd	3rd
Bob	Lea	Ann	Sue
Jim	Lea	Sue	Ann
Tom	Sue	Lea	Ann

women’s preferences

	1st	2nd	3rd
Ann	Jim	Tom	Bob
Lea	Tom	Bob	Jim
Sue	Jim	Tom	Bob

men's preferences

	1st	2nd	3rd
Bob	Lea	Ann	Sue
Jim	Lea	Sue	Ann
Tom	Sue	Lea	Ann

women's preferences

	1st	2nd	3rd
Ann	Jim	Tom	Bob
Lea	Tom	Bob	Jim
Sue	Jim	Tom	Bob

輸入進來之後就轉換成二維陣列

ranking matrix

	Ann	Lea	Sue
Bob	2, 3	1, 2	3, 3
Jim	3, 1	1, 3	2, 1
Tom	3, 2	2, 1	1, 2

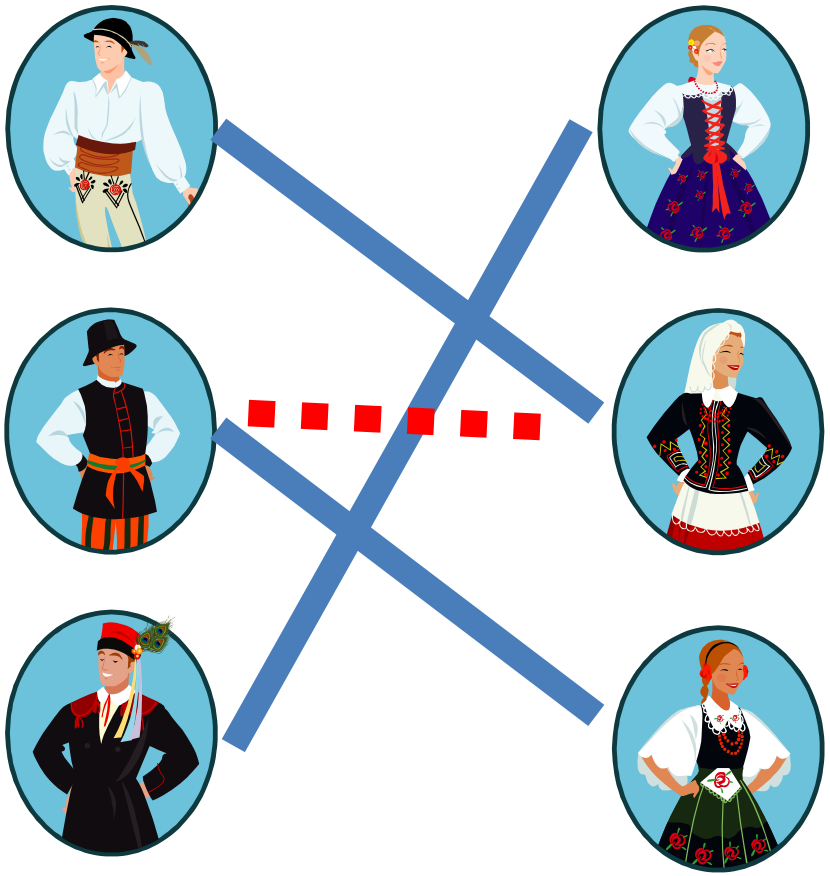
對女生的排序，來自男生

對男生的排序

blocking pair就是互相看對眼但沒配在一起（沒有配出最好的結果）

Stable Marriage Problem

要根據喜好表配對，希望配對的結果是stale matching



- Find a *stable* marriage matching for men's and women's preferences.

只要不存在blocking pair，就是stable matching

- Stable: no *blocking pair* in the matching
- Blocking pair: man *m* and woman *w* are not matched in the matching *M* but they prefer each other to their mates in *M*

Blocking pair

- M = {(Bob, Ann),
  (Jim, Lea),
  (Tom, Sue)}
- (Bob, Lea) is a block pair!
  - Bob: Lea (1) >> Ann (2)
  - Lea: Bob (2) >> Jim (3)

ranking matrix

	Ann	Lea	Sue
Bob	2, 3	1, 2	3, 3
Jim	3, 1	1, 3	2, 1
Tom	3, 2	2, 1	1, 2

這樣亂配就會產生blocking pair

- Does the problem always have a solution?
- **Yes!**
- How to solve the problem?

Free men:
Bob, Jim, Tom

	Ann	Lea	Sue
Bob	2, 3	1, 2	3, 3
Jim	3, 1	1, 3	2, 1
Tom	3, 2	2, 1	1, 2

Bob proposed to Lea

Lea accepted

Free men:
Jim, Tom

	Ann	Lea	Sue
Bob	2, 3	1, 2	3, 3
Jim	3, 1	1, 3	2, 1
Tom	3, 2	2, 1	1, 2

Jim proposed to Lea

Lea rejected

Free men:
Jim, Tom

	Ann	Lea	Sue
Bob	2, 3	1, 2	3, 3
Jim	3, 1	1, 3	2, 1
Tom	3, 2	2, 1	1, 2

Jim proposed to Sue

Sue accepted

Free men:
Tom

	Ann	Lea	Sue
Bob	2, 3	1, 2	3, 3
Jim	3, 1	1, 3	2, 1
Tom	3, 2	2, 1	1, 2

Tom proposed to Sue

Sue rejected

Free men: ☹
Tom

	Ann	Lea	Sue
Bob	2, 3	1, 2	3, 3
Jim	3, 1	1, 3	2, 1
Tom	3, 2	2, 1	1, 2

Tom proposed to Lea

Lea accepted

Free men:
Bob

	Ann	Lea	Sue
Bob	2, 3	1, 2	3, 3
Jim	3, 1	1, 3	2, 1
Tom	3, 2	2, 1	1, 2

Bob proposed to Ann

Ann accepted

	Ann	Lea	Sue
Bob	2, 3	1, 2	3, 3
Jim	3, 1	1, 3	2, 1
Tom	3, 2	2, 1	1, 2

這樣就跑完了，queue沒東西了，這就是stable matching的結果

Algorithm

1. Start with all the men and women being free.
2. While there are free men, arbitrarily select one and do the following:

   Proposal: The selected free man *m* proposes to the next woman *w* on his reference list.

   Response: If *w* is free, accepted. Otherwise, compare *m* with her current mate. Replace her mate with *m* if she prefers *m* better.

   如果有更好的就要更新，替代目前的

3. Return matched pairs.

D. Gale and L. S. Shapley, "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, vol. 69, pp. 9-14, 1962.

Theorem

會在n^2次以內結束，答案也不會存在blocking pair存在

- The stable marriage algorithm terminates in n^2 iterations with a stable marriage output.

n^2剛好是matrix的格子數，每一格都match一次，所以是n^2

先鎖定男生的喜好順序，女生的部分是看有沒有更好的，有就替換，所以可以跑出最好的結果

Shortcomings?

- The algorithm is not “gender neutral.”
- Man-optimal: it assigns to each man the highest-ranked woman possible under any stable marriage.

如果今天只有兩組，喜好順序剛好是完全相反，但還是會按照男生的喜好順序排出結果，雖然沒有blocking pari但不中立

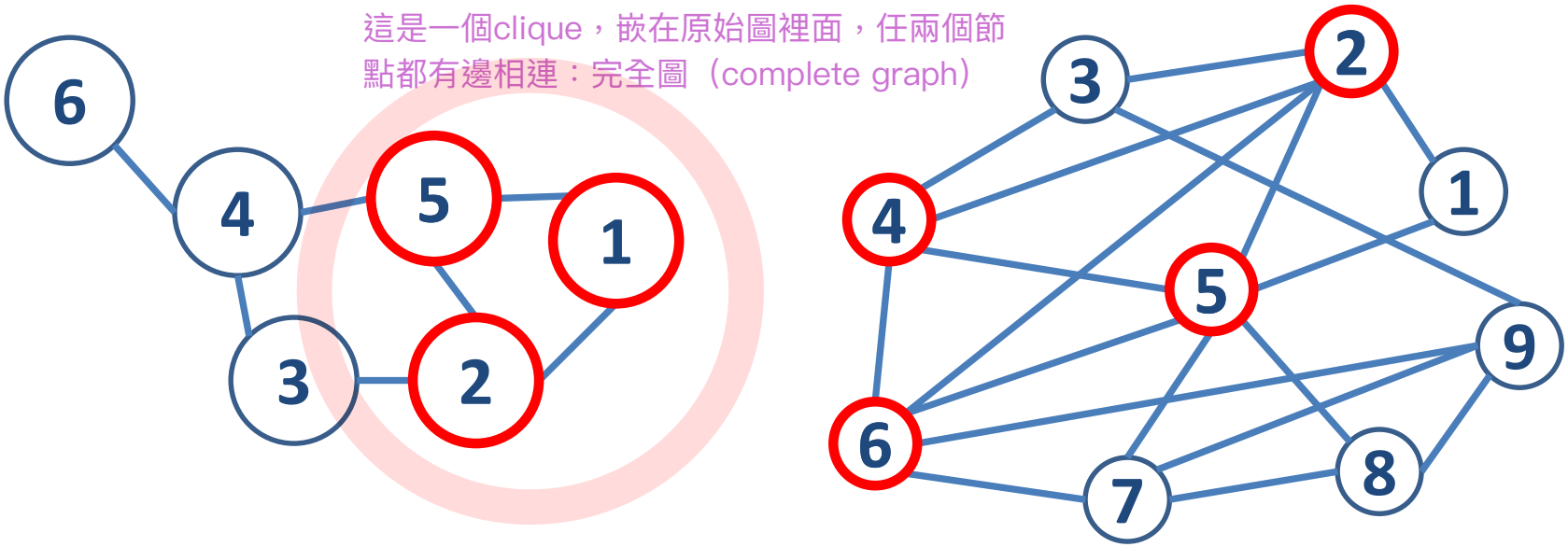
	woman 1	woman 2
man 1	1, 2	2, 1
man 2	2, 1	1, 2

Other graph problems

- The ***clique*** problem

a subset *V'* of vertices, each pair of which is connected by an edge in *E*

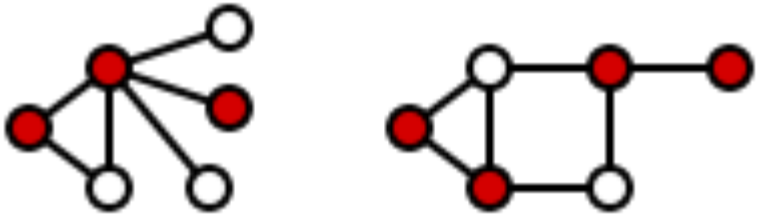
Find a clique of ***maximum*** size in a graph



Other graph problems

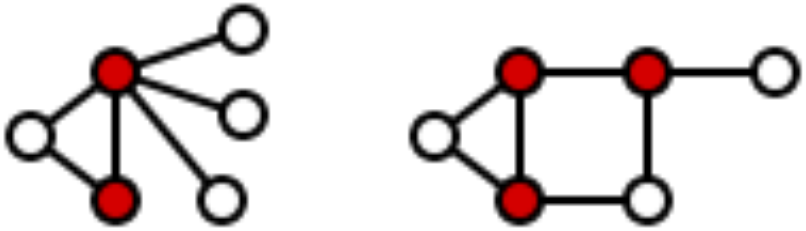
- The vertex-cover problem

a subset *V'* of vertices such that each edge of *G* is incident to at least one vertex in *V'*



幫最少的節點塗上顏色，並確保每個邊都會碰到一個塗色的

Find a vertex cover of ***minimum*** size in a graph

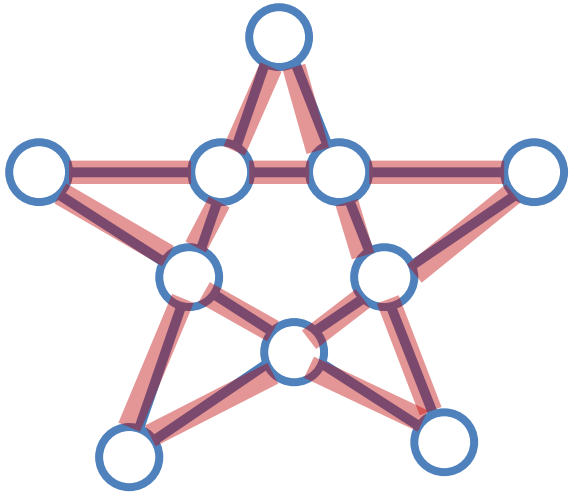


Other graph problems

- The Euler tour problem

一筆劃路徑

a simple cycle that contains each edge in *E*

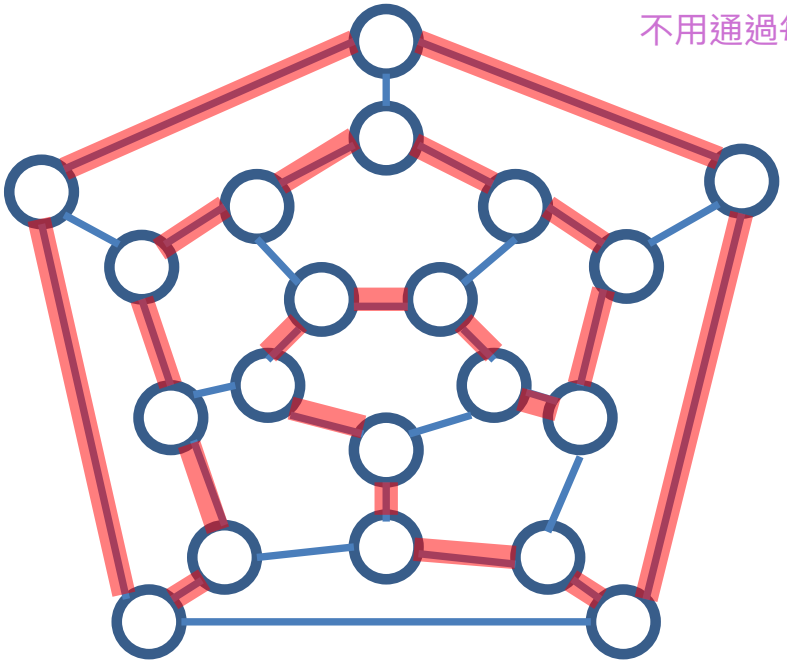


Other graph problems

- The hamiltonian-cycle problem

a simple cycle that contains each vertex in V

不用通過每個邊，但一次就要通過每個節點



*Can you figure out any **polynomial-time** algorithm for above problems?*

5/25 (목) End