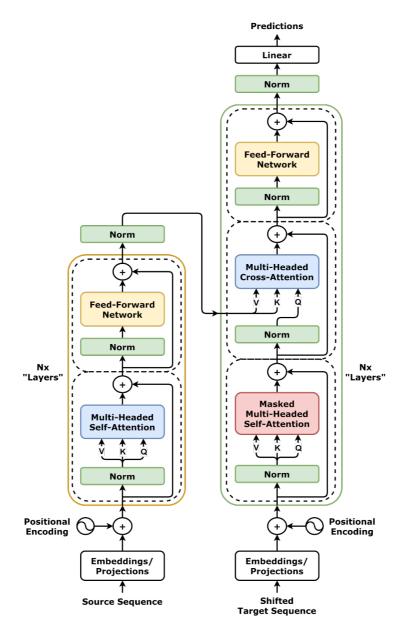
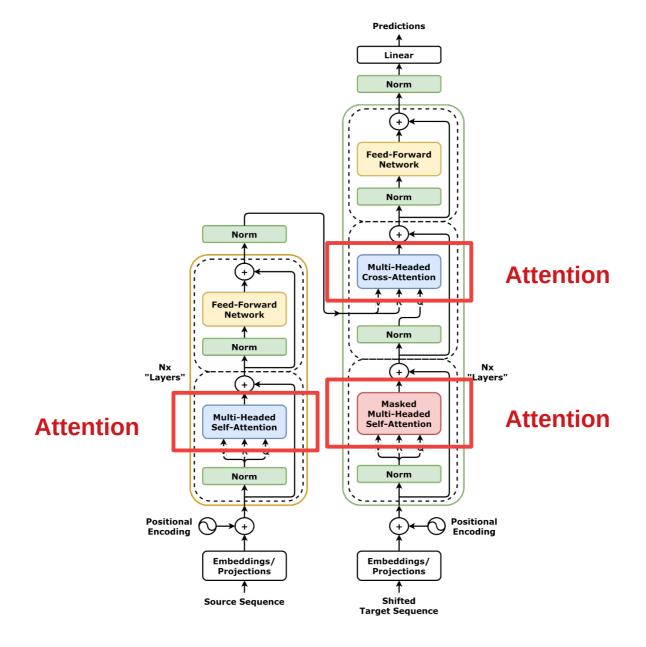
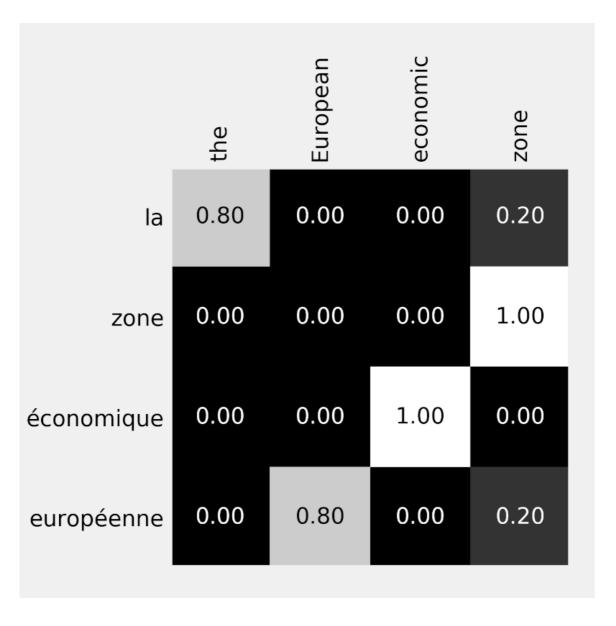
by Daniel Voigt Godoy Data Science Retreat – October 2024

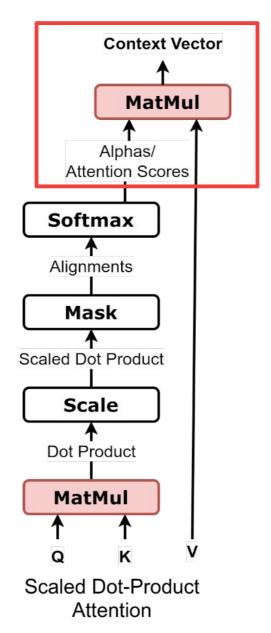
Why Quantize?

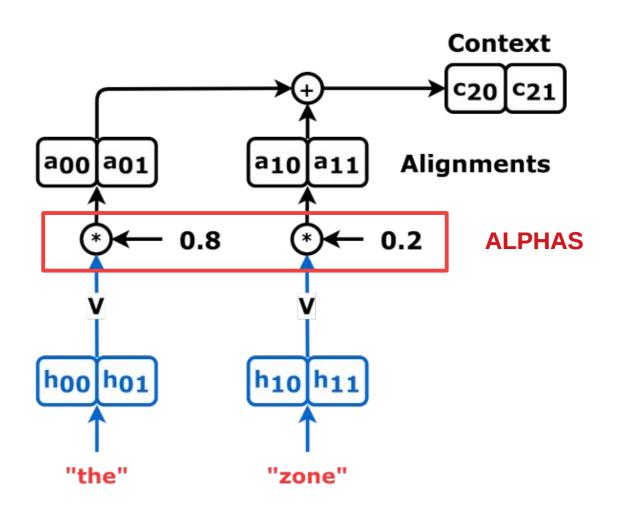
- To lower memory usage
 - Transformers are HUGE
 - Models do not fit into GPU RAM
 - FP32 is wasteful
- For faster inference
 - Loads weights faster
 - Integer arithmetics runs faster

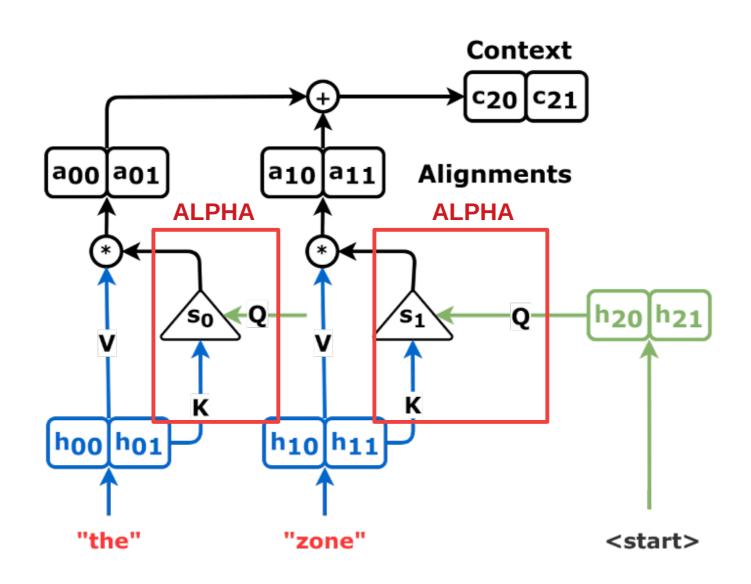


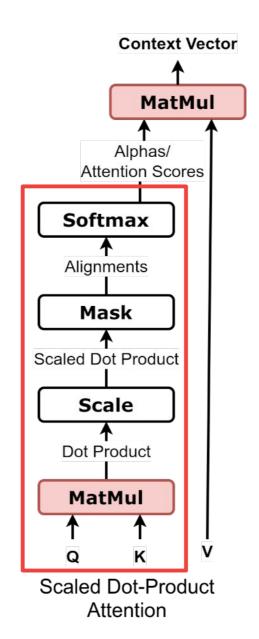








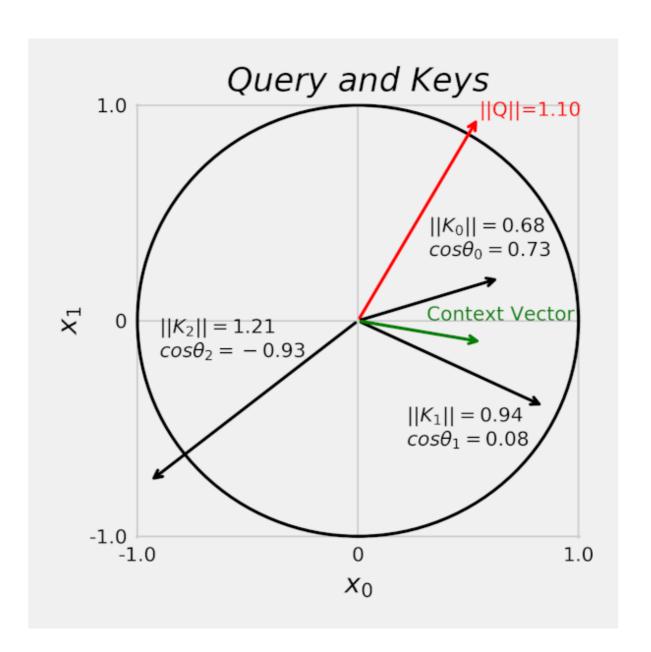




scaled dot product =
$$\frac{Q \cdot K}{\sqrt{d_k}}$$

n_dims = 10
vector1 = torch.randn(10000, 1, n_dims)
vector2 = torch.randn(10000, 1, n_dims).permute(0, 2, 1)
torch.bmm(vector1, vector2).squeeze().var()

$$\cos \theta ||Q|| ||K|| = Q \cdot K$$



$$\alpha_{00}, \alpha_{01} = \operatorname{softmax}(\frac{Q_0 \cdot K_0}{\sqrt{2}}, \frac{Q_0 \cdot K_1}{\sqrt{2}})$$

$$\operatorname{context\ vector}_0 = \alpha_{00} V_0 + \alpha_{01} V_1$$

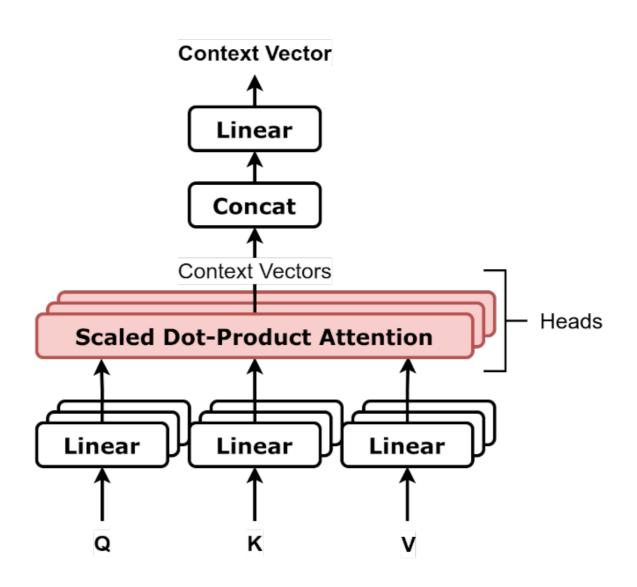
$$\alpha_{10}, \alpha_{11} = \operatorname{softmax}(\frac{Q_1 \cdot K_0}{\sqrt{2}}, \frac{Q_1 \cdot K_1}{\sqrt{2}})$$

$$\operatorname{context\ vector}_1 = \alpha_{10}V_0 + \alpha_{11}V_1$$

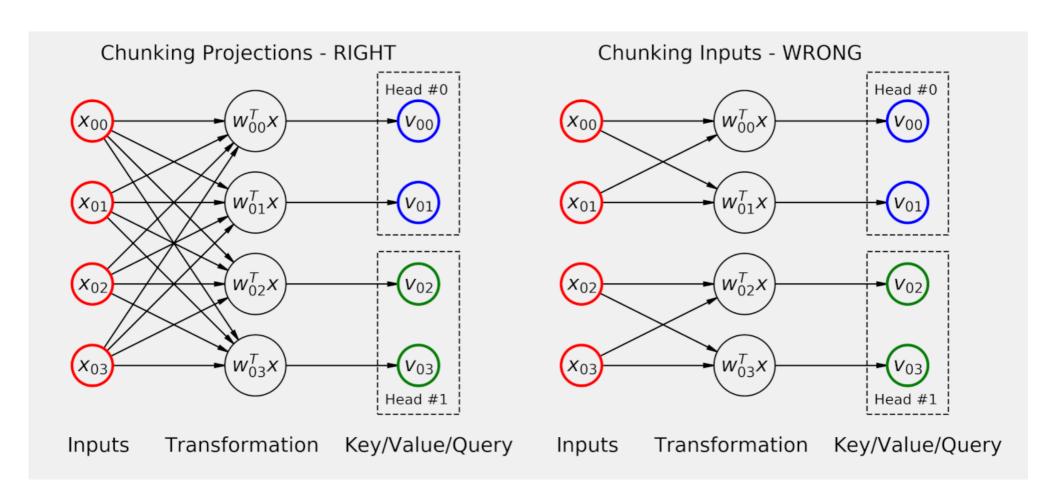
```
1 class Attention(nn.Module):
      def __init__(self, hidden_dim, input_dim=None,
                   proj_values=False):
          super().__init__()
          self.d_k = hidden_dim
          self.input_dim = hidden_dim if input_dim is None \
                           else input_dim
          self.proj_values = proj_values
 8
 9
          # Affine transformations for O. K. and V
10
          self.linear_query = nn.Linear(self.input_dim, hidden_dim)
                                                                        PROJECTION
          self.linear_key = nn.Linear(self.input_dim, hidden_dim)
11
                                                                        LAYERS
          self.linear_value = nn.Linear(self.input_dim, hidden_dim)
12
13
          self.alphas = None
14
15
      def init_keys(self, keys):
16
          self.kevs = kevs
          self.proj_keys = self.linear_key(self.keys)
17
                                                                Projections of Ks and Vs
          self.values = self.linear_value(self.keys) \
18
                                                                or simply Ks and Vs
19
                        if self.proj_values else self.keys
```

```
20
21
      def score_function(self, query):
           proj_query = self.linear_query(query)
22
23
          # scaled dot product
24
           # N, 1, H x N, H, L -> N, 1, L
25
           dot_products = torch.bmm(proj_query,
26
                                    self.proj_keys.permute(0, 2, 1))
27
           scores = dot_products / np.sqrt(self.d_k)
28
          return scores
29
30
      def forward(self, query, mask=None):
31
           # Query is batch-first N, 1, H
           scores = self.score_function(query) # N, 1, L
32
          if mask is not None:
33
               scores = scores.masked_fill(mask == 0, -1e9)
34
35
           alphas = F.softmax(scores, dim=-1) # N, 1, L 2
36
           self.alphas = alphas.detach()
37
38
           # N, 1, L x N, L, H -> N, 1, H
           context = torch.bmm(alphas, self.values)
                                                           (3)
39
40
          return context
```

Projections of Qs



```
1 class MultiHeadAttention(nn.Module):
       def __init__(self, n_heads, d_model,
                    input_dim=None, proj_values=True):
           super().__init__()
           self.linear_out = nn.Linear(n_heads * d_model, d_model)
           self.attn heads = nn.ModuleList(
               [Attention(d_model,
8
                          input_dim=input_dim,
                                                         Multiple heads!
9
                          proj_values=proj_values)
                for _ in range(n_heads)]
10
11
12
      def init_keys(self, key):
13
14
           for attn in self.attn_heads:
15
               attn.init keys(key)
16
17
       @property
       def alphas(self):
           # Shape: n_heads, N, 1, L (source)
19
20
           return torch.stack(
21
               [attn.alphas for attn in self.attn heads], dim=0
22
23
24
      def output_function(self, contexts):
25
           # N, 1, n heads * D
26
           concatenated = torch.cat(contexts, axis=-1)
27
           # Linear transf. to go back to original dimension
           out = self.linear_out(concatenated) # N, 1, D
28
29
           return out
30
      def forward(self, query, mask=None):
31
           contexts = [attn(query, mask=mask)
32
33
                      for attn in self.attn_heads]
34
           out = self.output_function(contexts)
35
           return out
```



```
1 class MultiHeadedAttention(nn.Module):
      def __init__(self, n_heads, d_model, dropout=0.1):
          super(MultiHeadedAttention, self). init ()
          self.n_heads = n_heads
          self.d model = d_model
          self.d k = int(d model / n heads)
          self.linear query = nn.Linear(d model, d model)
                                                                          PROJECTION
          self.linear_key = nn.Linear(d model, d model)
                                                                          LAYERS
          self.linear value = nn.Linear(d model, d model)
10
          self.linear out = nn.Linear(d model, d model)
          self.dropout = nn.Dropout(p=dropout)
11
                                                                (4)
          self.alphas = None
14
      def make chunks(self, x):
                                                                (1)
15
          batch_size, seq_len = x.size(0), x.size(1)
          # N, L, D -> N, L, n_heads * d_k
          x = x.view(batch size, seq len, self.n heads, self.d k)
                                                                             CHUNKING
          # N, n heads, L, d k
19
          x = x.transpose(1, 2)
20
          return x
21
22
      def init_keys(self, key):
          # N, n_heads, L, d_k
          self.proj_key = self.make_chunks(self.linear_key(key)) ①
25
          self.proi value = \
                                                                            CHUNKING
                  self.make_chunks(self.linear_value(kev))
26
```

```
27
28
       def score function(self, query):
           # Scaled dot product
30
           proj_query = self.make_chunks(self.linear_query(query))
           # N, n_heads, L, d_k x N, n_heads, d_k, L ->
31
32
           # N, n_heads, L, L
           dot_products = torch.matmul(
                                                                   (2)
33
               proj_query, self.proj_key.transpose(-2, -1)
34
35
           scores = dot_products / np.sqrt(self.d_k)
36
37
           return scores
38
       def attn(self, query, mask=None):
39
           # Query is batch-first: N, L, D
40
           # Score function will generate scores for each head
41
42
           scores = self.score_function(query) # N, n_heads, L, L
43
           if mask is not None:
44
               scores = scores.masked_fill(mask == 0, -1e9)
           alphas = F.softmax(scores, dim=-1) # N, n_heads, L, L
45
46
           alphas = self.dropout(alphas)
           self.alphas = alphas.detach()
49
           # N, n_heads, L, L x N, n_heads, L, d_k ->
50
           # N, n_heads, L, d_k
51
52
           context = torch.matmul(alphas, self.proj_value)
53
           return context
54
```

CHUNKING

```
def output_function(self, contexts):
           # N, L, D
56
           out = self.linear_out(contexts) # N, L, D
           return out
      def forward(self, query, mask=None):
           if mask is not None:
61
               # N, 1, L, L - every head uses the same mask
               mask = mask.unsqueeze(1)
63
64
65
           # N, n heads, L, d k
           context = self.attn(query, mask=mask)
           # N, L, n_heads, d_k
           context = context.transpose(1, 2).contiguous()
                                                                   (5)
           \# N, L, n_heads * d_k = N, L, d_model
           context = context.view(query.size(0), -1, self.d_model)(5)
           # N, L, d_model
           out = self.output_function(context)
           return out
```

CONCAT CHUNKS

HEAD #0

ı	q							
	0.23	0.03	0.1	0.3	0.87	0.84	0.3	0.3
	0.27	0.61	0.7	0.02	0.83	0.94	0.12	0.21
	0.79	0.23	0.03	0.28	0.02	0.47	0.97	0.61
	0.11	0.1	0.3	1.0	0.08	0.88	0.83	0.69
	0.07	0.01	0.16	0.05	0.51	0.54	0.23	0.47
_								
	k							
	0.08	0.41	0.36	0.1	0.15	0.03	0.95	0.16
	0.7	0.77	0.57	0.9	0.65	0.36	0.58	0.32
	0.77	0.29	0.42	0.58	0.16	0.49	0.17	0.73
	0.94	0.36	0.16	0.03	0.31	0.67	0.81	0.94
	0.76	1.0	0.45	0.94	0.6	0.49	0.68	0.54
	v							
	0.54	8.0	0.73	0.35	0.97	0.05	0.07	0.45
	0.81	0.42	0.13	0.33	0.6	0.75	0.41	0.36
	0.81	0.47	0.2	0.05	0.63	0.75	0.58	0.66
	0.69	0.89	0.09	0.49	0.49	0.63	0.91	0.88
	0.19	0.39	0.22	0.36	1.0	0.17	0.66	0.02

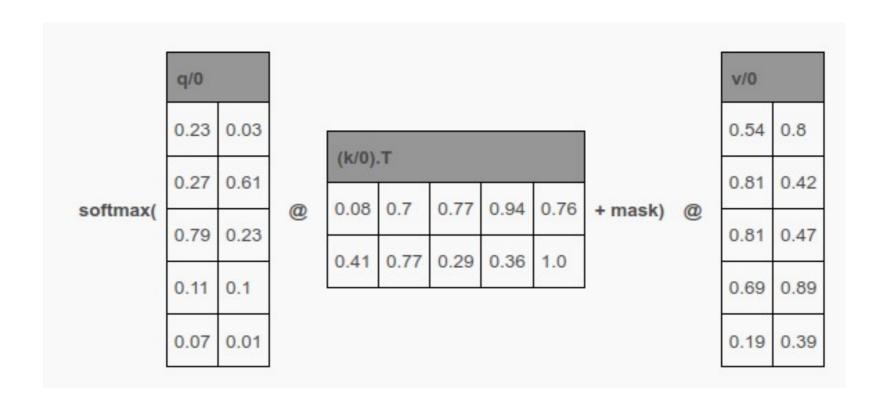
FOUR

PROJECTIONS

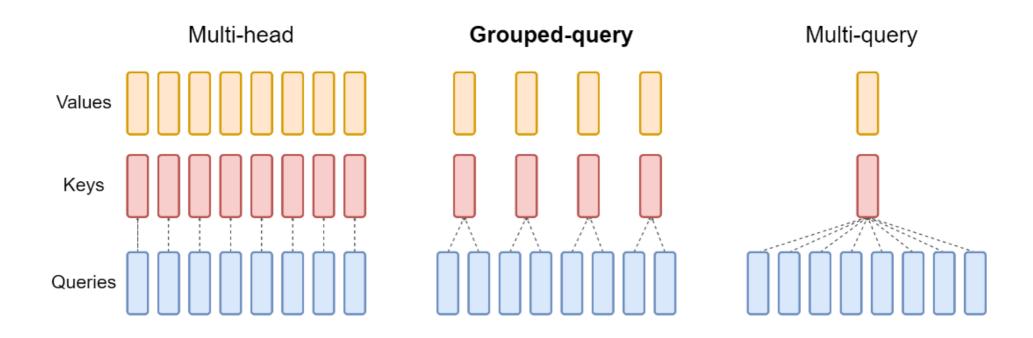
FOR Q, K, V

	q/0		q/1			q/2			q/3		
	0.23	0.03	0.1	0.3		0.87	0.84		0.3	0.3	
	0.27	0.61	0.7	0.02		0.83	0.94		0.12	0.21	
	0.79	0.23	0.03	0.28		0.02	0.47		0.97	0.61	
	0.11	0.1	0.3	1.0		0.08	0.88		0.83	0.69	
	0.07	0.01	0.16	0.05		0.51	0.54		0.23	0.47	
	k/0		k/1			k/2			k/3		
	0.08	0.41	0.36	0.1		0.15	0.03		0.95	0.16	
	0.7	0.77	0.57	0.9		0.65	0.36		0.58	0.32	
	0.77	0.29	0.42	0.58		0.16	0.49		0.17	0.73	
	0.94	0.36	0.16	0.03		0.31	0.67		0.81	0.94	
	0.76	1.0	0.45	0.94		0.6	0.49		0.68	0.54	
	v/0		v/1			v/2			v/3		
•	0.54	0.8	0.73	0.35		0.97	0.05		0.07	0.45	
	0.81	0.42	0.13	0.33		0.6	0.75		0.41	0.36	
	0.81	0.47	0.2	0.05		0.63	0.75		0.58	0.66	
	0.69	0.89	0.09	0.49		0.49	0.63	0.91		0.88	
	0.19	0.39	0.22	0.36		1.0	0.17		0.66	0.02	

Source: https://vgel.me/posts/faster-inference

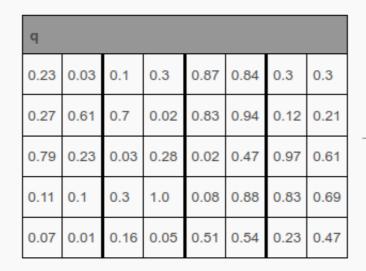


MHA, GQA, MQA



Multi-Query Attention(MQA)

FOUR PROJEC-TIONS FOR Q



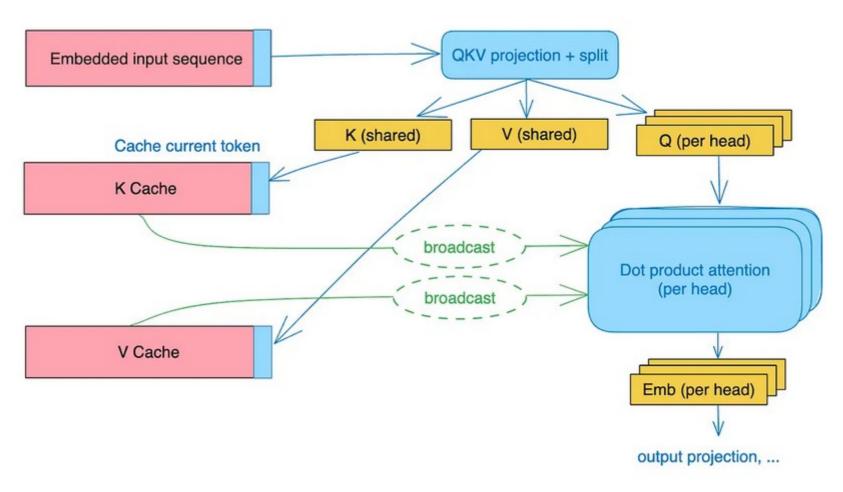
q/0		q/1	q/1		q/2			q/3		
0.23	0.03	0.1	0.3		0.87	0.84		0.3	0.3	
0.27	0.61	0.7	0.02		0.83	0.94		0.12	0.21	
0.79	0.23	0.03	0.28		0.02	0.47		0.97	0.61	
0.11	0.1	0.3	1.0		0.08	0.88		0.83	0.69	
0.07	0.01	0.16	0.05		0.51	0.54		0.23	0.47	

SINGLE PROJECTIONS FOR K AND V (SHARED WITH EACH PROJECTION OF Q)

k							
0.08	0.41						
0.7	0.77						
0.77	0.29						
0.94	0.36						
0.76	1.0						

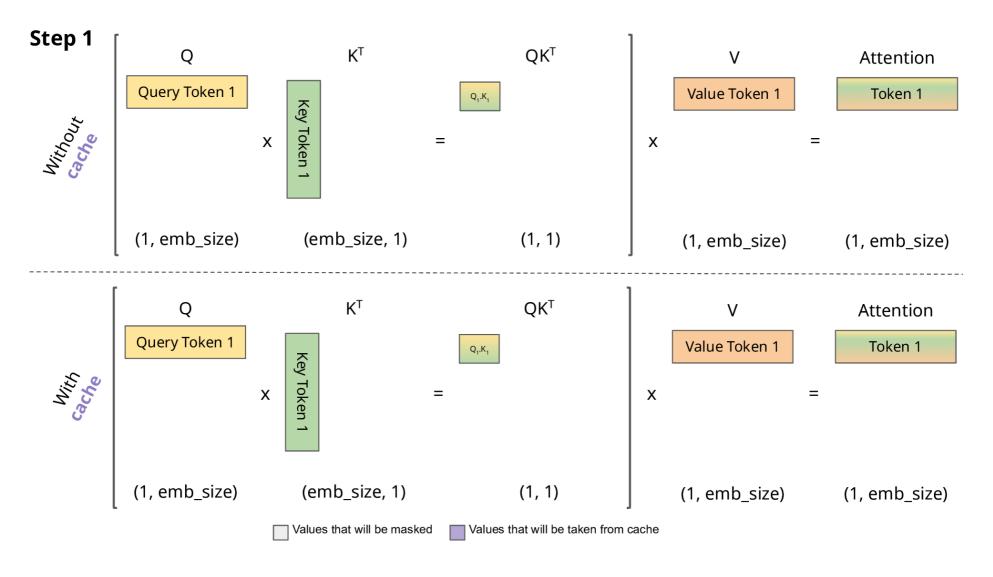
v	
0.54	0.8
0.81	0.42
0.81	0.47
0.69	0.89
0.19	0.39

Multi-Query Attention(MQA)



Multi-Query Attention

KV Caching



Hands-On

Notebook 1 - MHA vs MQA

Sparse Attention

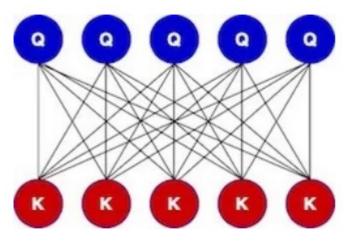


Figure 1. Attention Layer Representation

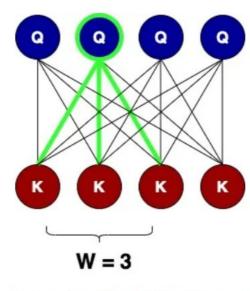


Figure 4. Sliding Attention Window of Size 3.

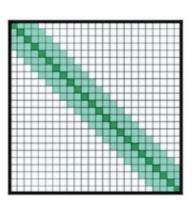
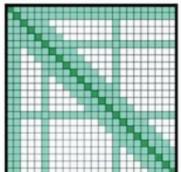
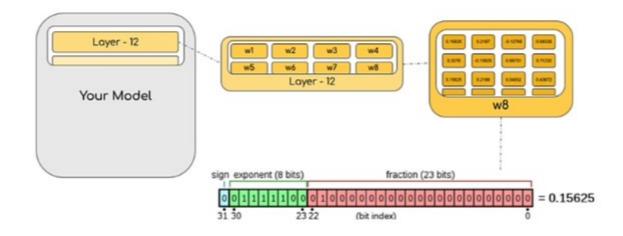


Figure 7. Dilated Sliding Window Attention Layer



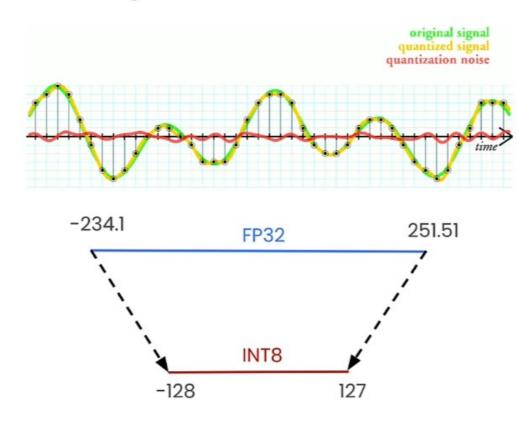
What is a model parameter?



It is possible to inspect each parameter's data type!

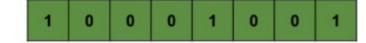
Quantization - Concept

Quantization refers to the process of mapping a large set to a smaller set of values.



Data representation in ML Dtypes

- Integer (int8)



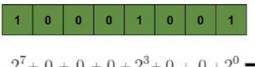
- Floating Point (FP32, F16, BF16)

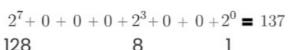
- Sign: 1 bit
- Exponent (range): 8 bit Fraction (precision): 23 bit
- Total: 32 bit

Integer

- Unsigned integer:
 - Range for n-bits: $[0, 2^n 1]$

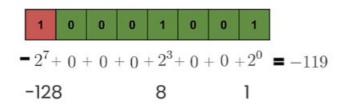
Example with 8-bit (torch.uint8): [0, 255]





- Signed integer two's complement representation
 - Range for n-bits: $[-2^{n-1}, 2^{n-1} 1]$

Example with 8-bit (torch.int8): [-128, 127]



Integer - PyTorch - Ranges

- Unsigned integer: $[0, 2^n - 1]$

iinfo(min=0, max=255, dtype=uint8)



Floating Point

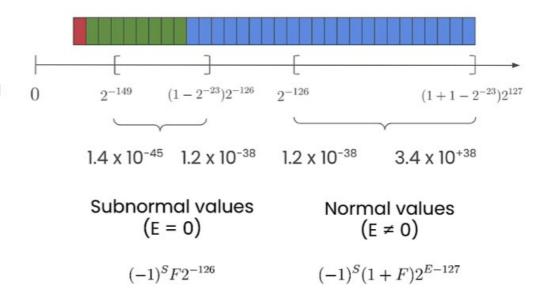
3 components in floating point:

- **Sign**: positive/negative (always 1 bit)
- Exponent (range): impact the representable range of the number
- Fraction (precision): impact on the precision of the number

FP32, BF16, FP16, FP8 are floating point format with a specific number of bits for **exponent** and the **fraction.**

FP32

- Sign: 1 bit
- Exponent (range): 8 bit
- Fraction (precision): 23 bit
- Total: 32 bit



FP16

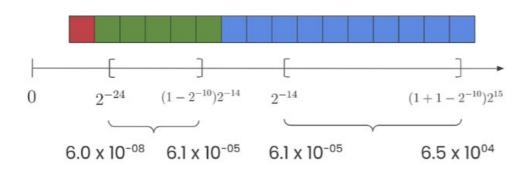
BF16

Sign: 1 bit

- Exponent (range): 5 bit

- Fraction (precision): 10 bit

- Total: 16 bit

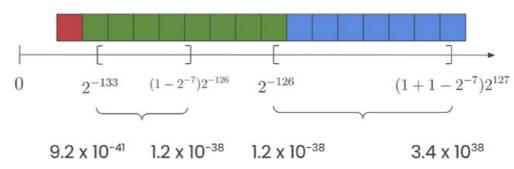


- Sign: 1 bit

- Exponent (range): 8 bit

- Fraction (precision): 7 bit

Total: 16 bit



Subnormal values (E = 0)

 $(-1)^S F 2^{-14}$

Normal values (E ≠ 0)

$$(-1)^S(1+F)2^{E-15}$$

Subnormal values (E = 0)

$$(-1)^S F 2^{-126}$$

Normal values (E ≠ 0)

$$(-1)^{S}(1+F)2^{E-127}$$

Comparison

Data Type	Precision	maximum				
FP32	best	~10+38				
FP16	better	~10 ⁰⁴				
BF16	good	~10 ³⁸				

Floating Point - PyTorch Downcasting

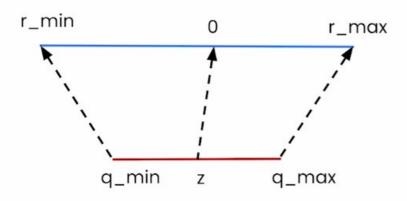
Advantages:

- Reduced memory footprint.
 - More efficient use of GPU memory.
 - Enables the training of larger models
 - Enables larger batch sizes
- Increased compute and speed
 - Computation using low precision (fp16, bf16) can be faster than fp32 since it require less memory.
 - Depends on the hardware (e.g. Google TPU, NVIDIA A100)

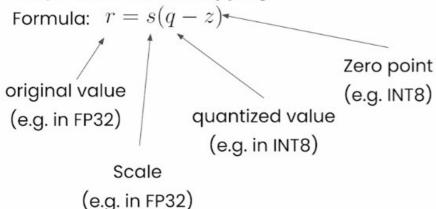
Disadvantages:

 Less precise: We are using less memory, hence the computation is less precise.

(Optional) Linear Quantization



Simple idea: linear mapping



(Optional) scale and zero point

Linear quantization maps the floating point range $[r_{\min}, r_{\max}]$ to the quantized range $[q_{\min}, q_{\max}]$

If we look that the **extreme values**, we should get:

$$\begin{cases} r_{\min} = s(q_{\min} - z) & r_{-\min} & 0 & r_{-\max} \\ r_{\max} = s(q_{\max} - z) & & & & \\ q_{-\min} & z & q_{-\max} \end{cases}$$

If we subtract the first equation from the second one, we get the **scale s**:

$$s = (r_{\text{max}} - r_{\text{min}})/(q_{\text{max}} - q_{\text{min}})$$

For the **zero point z**, we need to round the value since it is a n-bit integer:

$$z = \operatorname{int}(\operatorname{round}(q_{\min} - r_{\min}/s))$$

Notebook 2 - Quantization

bitsandbytes



The bitsandbytes library is a lightweight Python wrapper around CUDA custom functions, in particular 8-bit optimizers, matrix multiplication (LLM.int8()), and 8 & 4-bit quantization functions.

The library includes quantization primitives for 8-bit & 4-bit operations, through bitsandbytes.nn.Linear8bitLt and bitsandbytes.nn.Linear4bit and 8-bit optimizers through bitsandbytes.optim module.

There are ongoing efforts to support further hardware backends, i.e. Intel CPU + GPU, AMD GPU, Apple Silicon. Windows support is quite far along and is on its way as well.

```
class transformers.BitsAndBytesConfig

( load_in_8bit = False, load_in_4bit = False, llm_int8_threshold = 6.0, llm_int8_skip_modules = None,
llm_int8_enable_fp32_cpu_offload = False, llm_int8_has_fp16_weight = False, bnb_4bit_compute_dtype =
None, bnb_4bit_quant_type = 'fp4', bnb_4bit_use_double_quant = False, bnb_4bit_quant_storage = None,
**kwargs )
```

Quickstart

The basic way to load a model in 4bit is to pass the argument load_in_4bit=True when calling the from_pretrained method by providing a device map (pass "auto" to get a device map that will be automatically inferred).

```
from transformers import AutoModelForCausalLM

model = AutoModelForCausalLM.from_pretrained("facebook/opt-350m", load_in_4bit=True, device_map=
...
```

That's all you need!

As a general rule, we recommend users to not manually set a device once the model has been loaded with device_map. So any device assignment call to the model, or to any model's submodules should be avoided after that line - unless you know what you are doing.

Keep in mind that loading a quantized model will automatically cast other model's submodules into float16 dtype. You can change this behavior, (if for example you want to have the layer norms in float32), by passing torch_dtype=dtype to the from_pretrained method.

```
import torch
from transformers import BitsAndBytesConfig

quantization_config = BitsAndBytesConfig(
    load_in_4bit=True,
    bnb_4bit_compute_dtype=torch.bfloat16
)
```

Advanced usage

You can play with different variants of 4bit quantization such as NF4 (normalized float 4 (default)) or pure FP4 quantization. Based on theoretical considerations and empirical results from the paper, we recommend using NF4 quantization for better performance.

Other options include bnb_4bit_use_double_quant which uses a second quantization after the first one to save an additional 0.4 bits per parameter. And finally, the compute type. While 4-bit bitsandbytes stores weights in 4-bits, the computation still happens in 16 or 32-bit and here any combination can be chosen (float16, bfloat32 etc).

The matrix multiplication and training will be faster if one uses a 16-bit compute dtype default torch.float32). One should leverage the recent BitsAndBytesConfig from transformers to change these parameters. An example to load a model in 4bit using NF4 quantization below with double quantization with the compute dtype bfloat16 for faster training:

```
from transformers import BitsAndBytesConfig

nf4_config = BitsAndBytesConfig(
    load_in_4bit=True,
    bnb_4bit_quant_type="nf4",
    bnb_4bit_use_double_quant=True,
    bnb_4bit_compute_dtype=torch.bfloat16
)
```

bfloat16 is the ideal compute_dtype if your hardware supports it. While the default compute_dtype, float32, ensures backward compatibility (due to wide-ranging hardware support) and numerical stability, it is large and slows down computations. In contrast, float16 is smaller and faster but can lead to numerical instabilities. bfloat16 combines the best aspects of both; it offers the numerical stability of float32 and the reduced memory footprint and speed of a 16-bit data type. Check if your hardware supports bfloat16 and configure it using the bnb_4bit_compute_dtype parameter in BitsAndBytesConfig!

Notebook 3 - BitsAndBytes

llama.cpp



Description

The main goal of liama.cpp is to enable LLM inference with minimal setup and state-of-the-art performance on a wide variety of hardware - locally and in the cloud.

Release Madness!



Releases 2,185

b3531 Latest
2 hours ago
+ 2,184 releases



8 hours ago

ighthub-actions

ighthub-a

Compare *

b3527

[CANN]: Fix ggml_backend_cann_buffer_get_tensor (#8871)

* cann: fix ggml_backend_cann_buffer_get_tensor

1. fix data ptr offset
2. enable the acquisition of incomplete tensors

GGUF

GGUF

GGUF is a file format for storing models for inference with GGML and executors based on GGML. GGUF is a binary format that is designed for fast loading and saving of models, and for ease of reading. Models are traditionally developed using PyTorch or another framework, and then converted to GGUF for use in GGML.

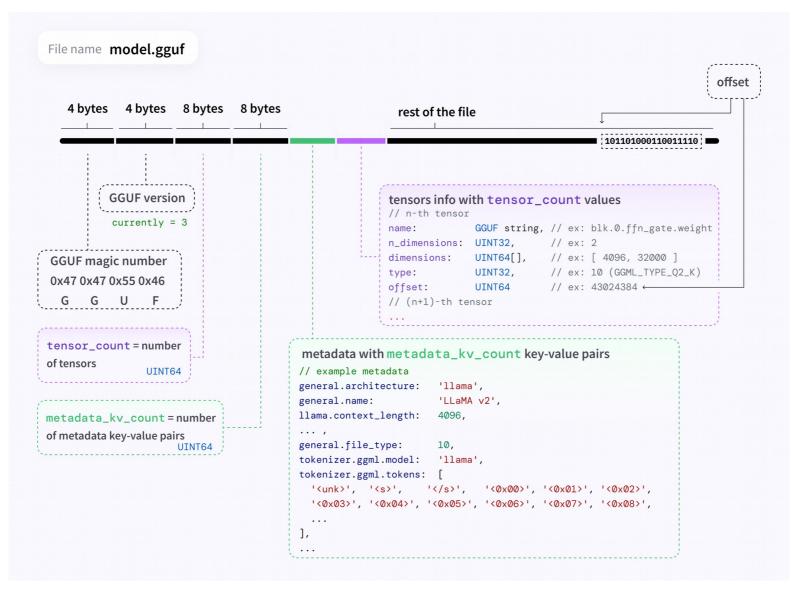
It is a successor file format to GGML, GGMF and GGJT, and is designed to be unambiguous by containing all the information needed to load a model. It is also designed to be extensible, so that new information can be added to models without breaking compatibility.

For more information about the motivation behind GGUF, see Historical State of Affairs.

GGUF

Hugging Face Hub supports all file formats, but has built-in features for <u>GGUF format</u>, a binary format that is optimized for quick loading and saving of models, making it highly efficient for inference purposes. GGUF is designed for use with GGML and other executors. GGUF was developed by <u>@ggerganov</u> who is also the developer of <u>llama.cpp</u>, a popular C/C++ LLM inference framework. Models initially developed in frameworks like PyTorch can be converted to GGUF format for use with those engines.

GGUF - File Format



Notebook 4 - Converting to GGUF

Ollama



Get up and running with large language models.

Run <u>Llama 3</u>, <u>Phi 3</u>, <u>Mistral</u>, <u>Gemma</u>, and other models. Customize and create your own.

Download ↓

Available for macOS, Linux, and Windows (preview)

Ollama - Model Library

Model library

Ollama supports a list of models available on ollama.com/library

Here are some example models that can be downloaded:

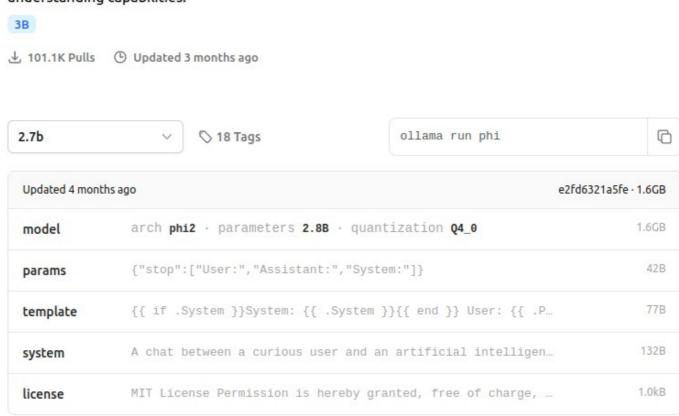
Model	Parameters	Size	Download
Llama 3	8B	4.7GB	ollama run llama3
Llama 3	70B	40GB	ollama run llama3:70b
Phi-3	3.8B	2.3GB	ollama run phi3
Mistral	7B	4.1GB	ollama run mistral
Neural Chat	7B	4.1GB	ollama run neural-chat
Starling	7B	4.1GB	ollama run starling-lm
Code Llama	7B	3.8GB	ollama run codellama
Llama 2 Uncensored	7B	3.8GB	ollama run llama2-uncensored
LLaVA	7B	4.5GB	ollama run llava
Gemma	2B	1.4GB	ollama run gemma:2b
Gemma	7B	4.8GB	ollama run gemma:7b
Solar	10.7B	6.1GB	ollama run solar

Note: You should have at least 8 GB of RAM available to run the 7B models, 16 GB to run the 13B models, and 32 GB to run the 33B models.

Ollama - Model Details

phi

Phi-2: a 2.7B language model by Microsoft Research that demonstrates outstanding reasoning and language understanding capabilities.



Ollama - Custom Model

Customize a model

Import from GGUF

Ollama supports importing GGUF models in the Modelfile:

1. Create a file named Modelfile, with a FROM instruction with the local filepath to the model you want to import.

FROM ./vicuna-33b.Q4_0.gguf

9

2. Create the model in Ollama

ollama create example -f Modelfile

O

3. Run the model

ollama run example

0

Ollama - Model File

```
> ollama show --modelfile llama3
# Modelfile generated by "ollama show"
# To build a new Modelfile based on this one, replace the FROM line with:
# FROM llama3:latest
FROM /Users/pdevine/.ollama/models/blobs/sha256-00e1317cbf74d901080d7100f57580ba8dd8de57203072dc6f668324ba545f29
TEMPLATE """{{ if .System }}<|start_header_id|>system<|end_header_id|>
{{ .System }}<|eot_id|>{{ end }}{{ if .Prompt }}<|start_header_id|>
{{ .Prompt }}<|eot_id|>{{ end }}<|start_header_id|>
## To build a new Modelfile based on this one, replace the FROM line with:
## FROM llama3:latest
## FROM llama5:latest
## FROM llama6:latest
```

Ollama

REST API

Ollama has a REST API for running and managing models.

Generate a response

```
curl http://localhost:11434/api/generate -d '{
   "model": "llama3",
   "prompt":"Why is the sky blue?"
}'
```

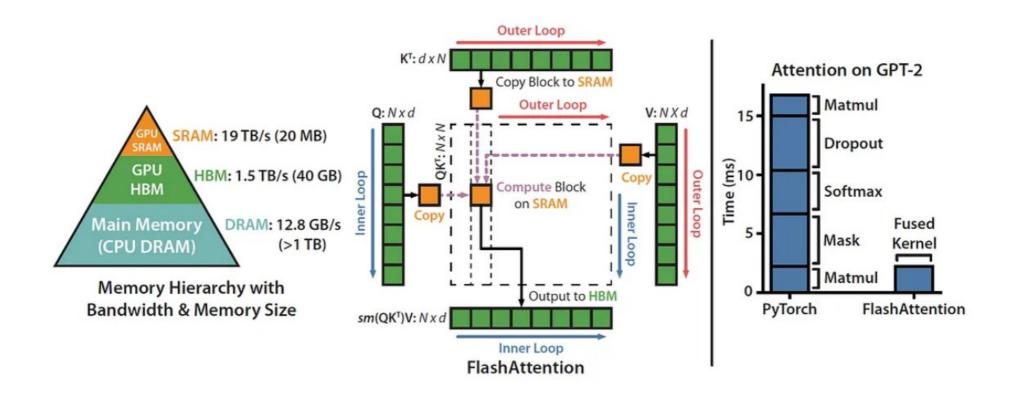
Chat with a model

Notebook 5 - Running Ollama

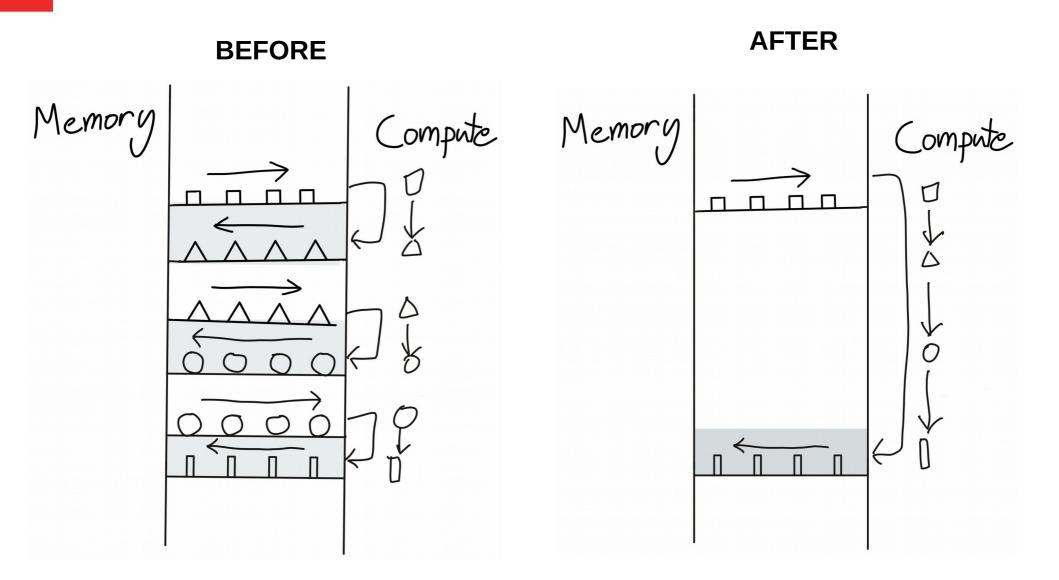
Exercise

Exercise – Quantizing and Serving a Model

BONUS: Flash Attention



Flash Attention



Flash Attention

Algorithm 0 Standard Attention Implementation

Require: Matrices $Q, K, V \in \mathbb{R}^{N \times d}$ in HBM.

- 1: Load \mathbf{Q}, \mathbf{K} by blocks from HBM, compute $\mathbf{S} = \mathbf{Q}\mathbf{K}^{\mathsf{T}}$, write \mathbf{S} to HBM.
- Read S from HBM, compute P = softmax(S), write P to HBM.
- Load P and V by blocks from HBM, compute O = PV, write O to HBM.
- 4: Return O.

Algorithm 1 FlashAttention

```
Require: Matrices \mathbf{Q}, \mathbf{K}, \mathbf{V} \in \mathbb{R}^{N \times d} in HBM, on-chip SRAM of size M.
```

- 1: Set block sizes $B_c = \left[\frac{M}{4d}\right], B_r = \min\left(\left[\frac{M}{4d}\right], d\right)$.
- 2: Initialize $\mathbf{O} = (0)_{N \times d} \in \mathbb{R}^{N \times d}, \ell = (0)_N \in \mathbb{R}^N, m = (-\infty)_N \in \mathbb{R}^N$ in HBM.
- 3: Divide **Q** into $T_r = \begin{bmatrix} N \\ B_r \end{bmatrix}$ blocks $\mathbf{Q}_1, \dots, \mathbf{Q}_{T_r}$ of size $B_r \times d$ each, and divide \mathbf{K}, \mathbf{V} in to $T_c = \begin{bmatrix} N \\ B_c \end{bmatrix}$ blocks $\mathbf{K}_1, \dots, \mathbf{K}_{T_c}$ and $\mathbf{V}_1, \dots, \mathbf{V}_{T_c}$, of size $B_c \times d$ each.
- 4: Divide \mathbf{O} into T_r blocks $\mathbf{O}_i, \ldots, \mathbf{O}_{T_r}$ of size $B_r \times d$ each, divide ℓ into T_r blocks $\ell_i, \ldots, \ell_{T_r}$ of size B_r each, divide m into T_r blocks m_1, \ldots, m_{T_r} of size B_r each.
- 5: for $1 \le j \le T_c$ do
- Load K_i, V_i from HBM to on-chip SRAM.
- 7: for $1 \le i \le T_r$ do
- 8: Load $\mathbf{Q}_i, \mathbf{O}_i, \ell_i, m_i$ from HBM to on-chip SRAM.
- 9: On chip, compute $S_{ij} = \mathbf{Q}_i \mathbf{K}_j^T \in \mathbb{R}^{B_r \times B_c}$.
- 10: On chip, compute $\tilde{m}_{ij} = \operatorname{rowmax}(\mathbf{S}_{ij}) \in \mathbb{R}^{B_r}$, $\tilde{\mathbf{P}}_{ij} = \exp(\mathbf{S}_{ij} \tilde{m}_{ij}) \in \mathbb{R}^{B_r \times B_c}$ (pointwise), $\tilde{\ell}_{ij} = \operatorname{rowsum}(\tilde{\mathbf{P}}_{ij}) \in \mathbb{R}^{B_r}$.
- 11: On chip, compute $m_i^{\text{new}} = \max(m_i, \tilde{m}_{ij}) \in \mathbb{R}^{B_r}$, $\ell_i^{\text{new}} = e^{m_i m_i^{\text{new}}} \ell_i + e^{\tilde{m}_{ij} m_i^{\text{new}}} \tilde{\ell}_{ij} \in \mathbb{R}^{B_r}$.
- 12: Write $\mathbf{O}_i \leftarrow \operatorname{diag}(\ell_i^{\text{new}})^{-1}(\operatorname{diag}(\ell_i)e^{m_i m_i^{\text{new}}}\mathbf{O}_i + e^{\tilde{m}_{ij} m_i^{\text{new}}}\tilde{\mathbf{P}}_{ij}\mathbf{V}_i)$ to HBM.
- 13: Write $\ell_i \leftarrow \ell_i^{\text{new}}$, $m_i \leftarrow m_i^{\text{new}}$ to HBM.
- 14: end for
- 15: end for
- 16: Return O.

Tiling Softmax

$$\sigma(\mathbf{z})_i = rac{e^{z_i}}{\sum_{j=1}^K e^{z_j}}$$

$$m(x) := \max_{i} x_{i}, \quad f(x) := \left[e^{x_{1} - m(x)} \dots e^{x_{B} - m(x)} \right]$$
$$\ell(x) := \sum_{i} f(x)_{i}, \quad \text{softmax}(x) := \frac{f(x)}{\ell(x)}$$

For vectors $x^{(1)}, x^{(2)} \in \mathbb{R}^B$, we can decompose the softmax of the concatenated $x = \begin{bmatrix} x^{(1)} & x^{(2)} \end{bmatrix} \in \mathbb{R}^{2B}$ as: $m(x) = m(\begin{bmatrix} x^{(1)} & x^{(2)} \end{bmatrix}) = \max(m(x^{(1)}), m(x^{(2)})), \quad f(x) = \begin{bmatrix} e^{m(x^{(1)}) - m(x)} f(x^{(1)}) & e^{m(x^{(2)}) - m(x)} f(x^{(2)}) \end{bmatrix},$ $\ell(x) = \ell(\begin{bmatrix} x^{(1)} & x^{(2)} \end{bmatrix}) = e^{m(x^{(1)}) - m(x)} \ell(x^{(1)}) + e^{m(x^{(2)}) - m(x)} \ell(x^{(2)}), \quad \text{softmax}(x) = \frac{f(x)}{\ell(x)}.$

BONUS - Flash Attention

The End

THANK YOU!