This script is a part of the supplementary material for the following paper:

- Jamshidi Idaji et al, "Nonlinear Interaction Decomposition (NID): A Method for Separation of Cross-frequency Coupled Sources in Human Brain". doi: https://doi.org/10.1101/680397

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https://github.com/minajamshidi/NID

Supplementary Code (2)

Linear Mixture of Phase-Coupled Oscillations

In this script, we will provide a simple simulation regarding the distribution of a linear mixture of two phase-coupled signals. For this purpose, we filter a white Gaussian noise in alpha (10Hz) and beta (20Hz) frequency bands and make phase-coupled signals out of them, and then investigate the distribution of their linear mixtures.

We want to filter a 10-min white Gaussian noise around 10Hz and 20Hz:

```
fs = 256;
order = 10*fs+1; % FIR filter order
n_samples = 10*60*fs;
phase_lag = pi/8;
times = (0:n_samples-1)/fs;
sig = randn(1, n_samples);

% build filter
Wn1 =  [9,11]/fs*2;
Wn2 =  [19,21]/fs*2;
b1 = fir1(order, Wn1);
b2 = fir1(order, Wn2);

% filter sig
u1 = filter(b1,1, sig);
x2 = filter(b2,1, sig);
```

We have extracted the phase and amplitude of the filtered signals. Now we make a faster oscillation phase-coupled to the slow oscillation and apply a time-shift to the faster oscillation:

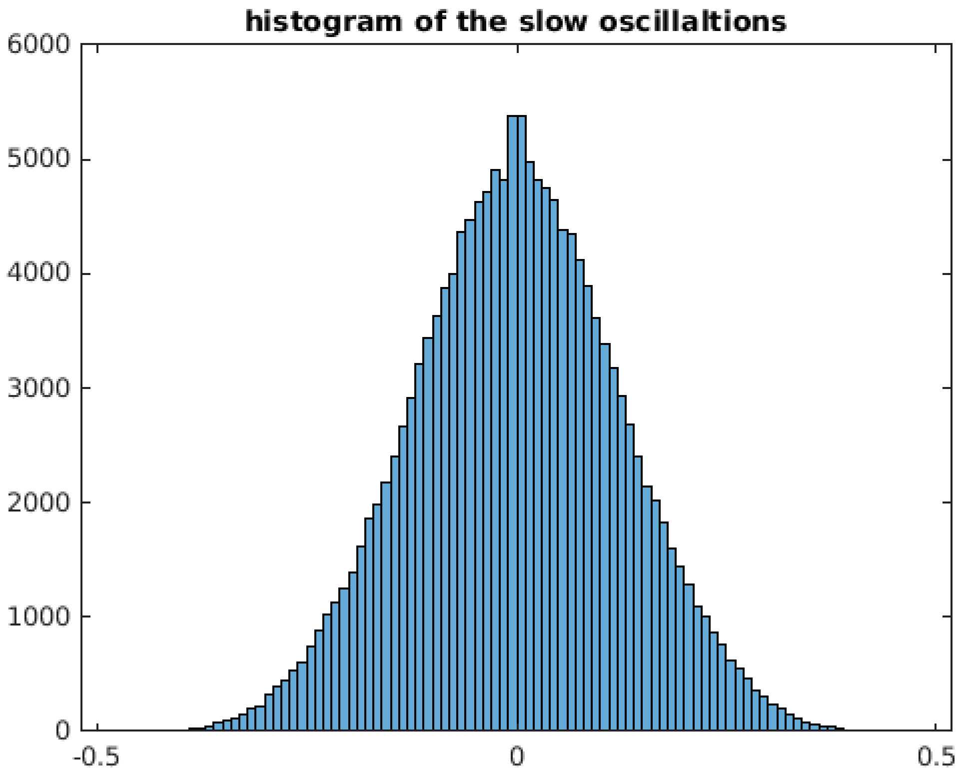
```
x2_amp = abs(hilbert(x2));
u1_angle = angle(hilbert(u1));
u2 = real(x2_amp .* exp(1i*2*u1_angle + 1i*phase_lag));

cPLV = mean(exp(1i*2*angle(hilbert(u1)))...
```

```
    .*exp(-1i*angle(hilbert(u2))),2)
```

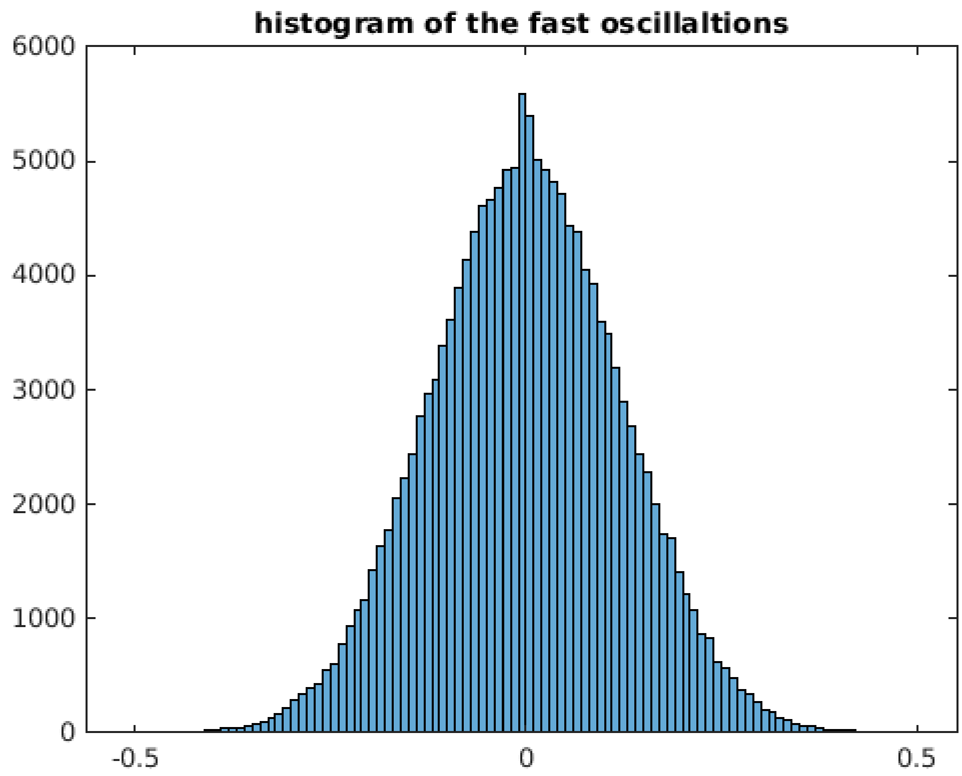
```
cPLV = 0.9228 - 0.3820i
```

Now, lets look at the distrubutions of u1 and u2:

```
figure, histogram(u1),
title('histogram of the slow oscillaltions')
```




```
figure, histogram(u2),
title('histogram of the fast oscillaltions')
```




```
fprintf('kurtosis of u1 = %.2f\n', kurtosis(u1)-3)
```

```
kurtosis of u1 = -0.11
```

```
fprintf('kurtosis of u2 = %.2f\n', kurtosis(u2)-3)
```

```
kurtosis of u2 = 0.02
```

```
fprintf('skewness of u1 = %.2f\n', skewness(u1))
```

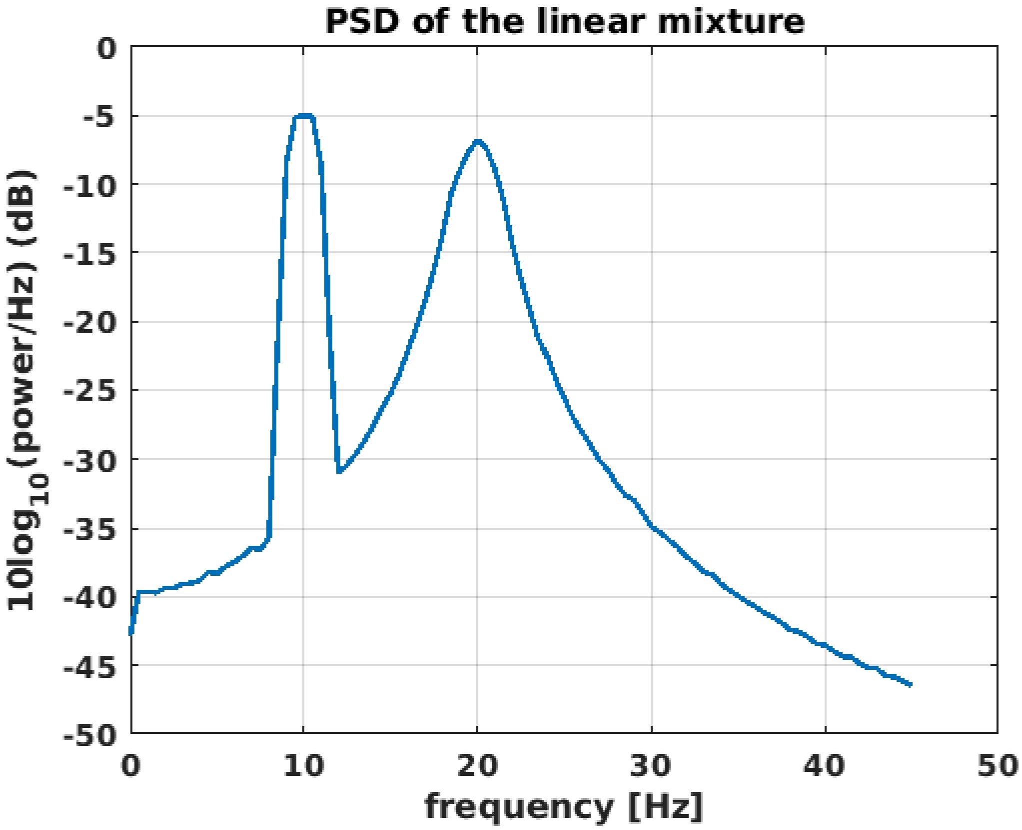
```
skewness of u1 = -0.00
```

```
fprintf('skewness of u2 = %.2f\n', skewness(u2))
```

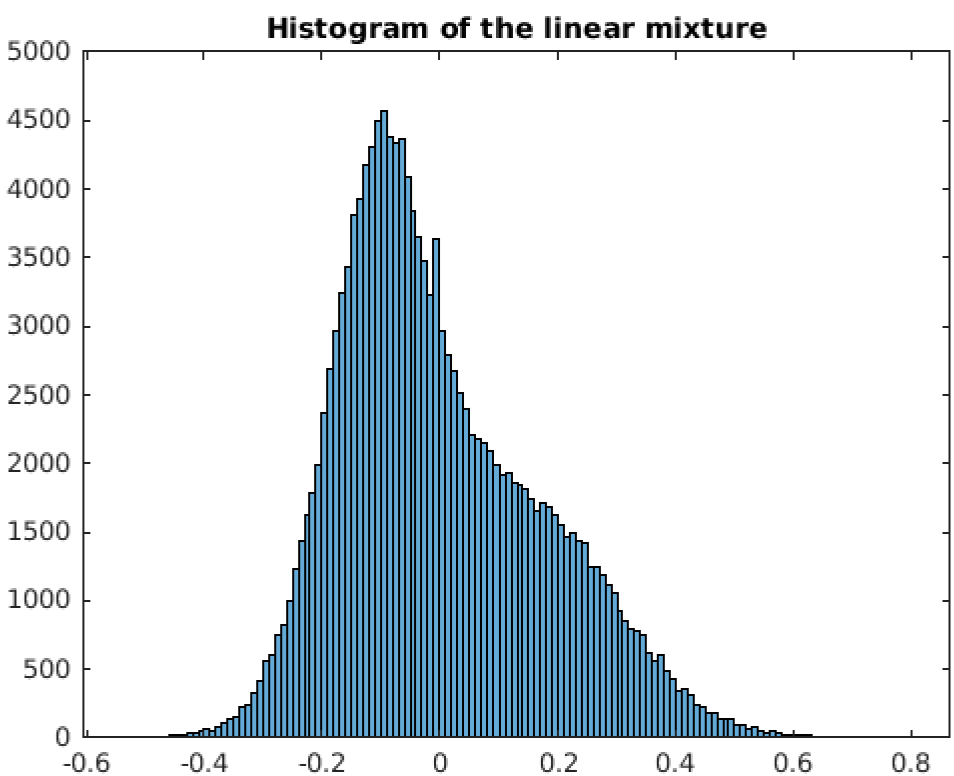
```
skewness of u2 = 0.00
```

One can see they are quite Gaussian. Now we mix them linearly and check the distribution of the mixture:

```
z = u1 + u2;
figure, plot_psd(z, fs, 45)
title('PSD of the linear mixture')
```




```
figure, histogram(z)
title('Histogram of the linear mixture')
```



```
fprintf('kurtosis of z = %.2f\n', kurtosis(z)-3)
```

```
kurtosis of z = -0.01
```

```
fprintf('skewness of z = %.2f\n', skewness(z))
```

```
skewness of z = 0.62
```

```
fprintf('5th moment of z = %.2f\n', mean((z/std(z)).^5))
```

```
5th moment of z = 4.38
```

```
fprintf('7th moment of z = %.2f\n', mean((z/std(z)).^7))
```

```
7th moment of z = 34.46
```

The distribution of the linear mixture apparently deviates from Gaussian distribution. Please take a look at the odd higher order moments.

Conclusion

In this script, we provided an exemplar simulation pipeline, where one can see that phase-coupled oscillations can be linearly mixed in the way that the distribution of the mixture is non-Gaussian. Note that here we did not use any non-Gaussinity maximization algorithm to find the optimum coefficients, and simply added the two narrow-band oscillations to each other.

Functions

```
function plot_psd(x, fs, max_freq)
% a function to plot the PSD of the input signal
    [pxx,f] = pwelch(x, 2*fs, fs);
    f = f/pi * fs/2;
    pxx = pxx(f<=max_freq, :);
    f = f(f<=max_freq);
    plot(f, 10*log10(pxx), 'linewidth', 2);
    ylabel('10log_{10}(power/Hz) (dB)','fontsize', 12)
    xlabel('frequency [Hz]','fontsize', 12);
    set(gca, 'fontweight', 'bold', 'fontsize', 12)
    grid on,
end
```

Notification: