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https://github.com/minajamshidi/NID

Supplementary Code (1)

Filtering a Gaussian-distributed signal: the distribution of the filtered signal

In this script, we will filter a white Gaussian noise with different bandwidths and investigate the impact of bandwidth of the filter on the distribution of the filtered signal.

We want to filter a 10-min white Gaussian noise:

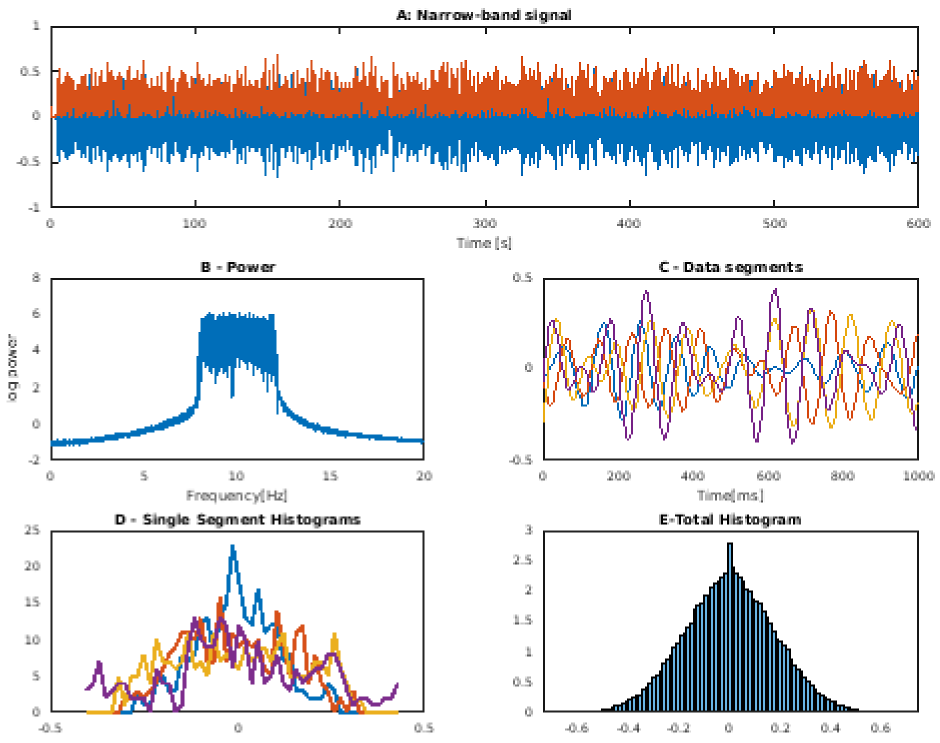
```
fs = 256;
n_samples = 10*60*fs;
times = (0:n_samples-1)/fs;
sig = randn(1, n_samples);
```

Let's start with a bandwidth of 4Hz around central frequency of 10Hz:

```
filt_order = 10*fs + 1;
Wn1 = [8, 12];
[b1, b1_spec] = build_fir1(filt_order, Wn1, fs);
[y1, y1_spec] = filter_signal(b1, sig, fs);
```

Now, we can look at the distribution of the filtered signal and its time segments:

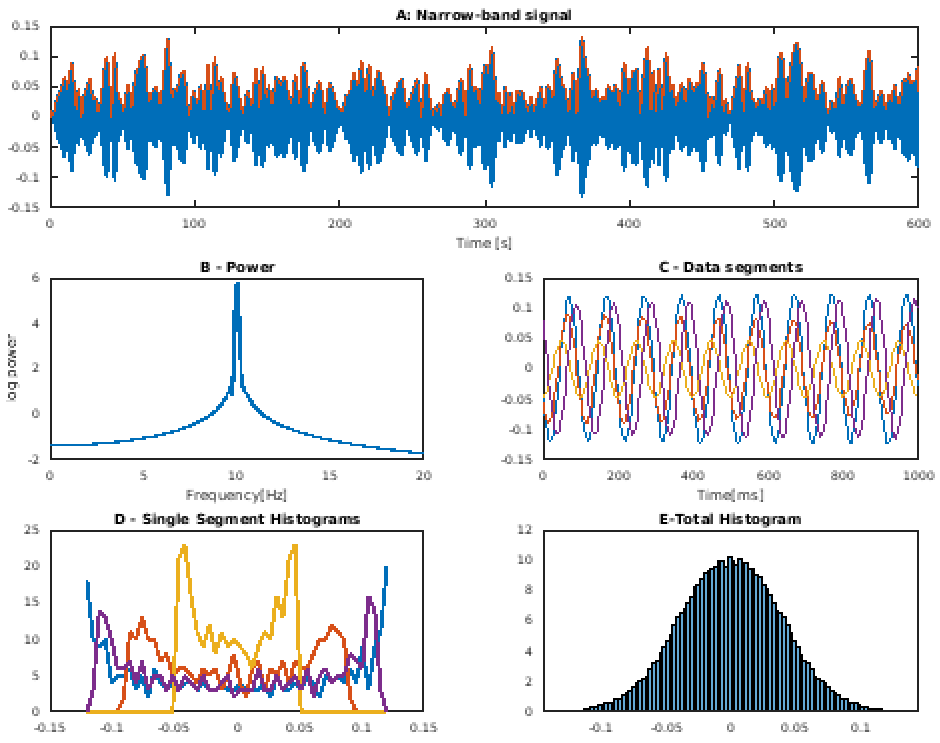
```
plot_fig(times, y1, y1_spec, fs)
```



- As it can be seen, the distribution of the whole time-series (panel E) can be approximated well with Gaussian distribution. Even the very-short data segments in panel C have bell-shaped histograms. This is actually because of the bandwidth of the narrow-band signal (panel B), which lets faster fluctuations appear in the amplitude-envelope.

Now, we set the bandwidth of the filter *very* narrow (0.2Hz) and plot the figure again:

```
filt_order = 10*fs + 1;
Wn2 = [9.9, 10.1];
[b2, b2_spec] = build_fir1(filt_order, Wn2, fs);
[y2, y2_spec] = filter_signal(b2, sig, fs);
plot_fig(times, y2, y2_spec, fs)
```



- The first important observation is that because of very narrow bandwidth of the signal (panel B) the fluctuations of the amplitude envelope are much slower than the previous case. This results in the observation that short data segments look like pure sinusoids with no fluctuation in their amplitude-envelope and their histograms (panel D) look *very* non-Gaussian, with peaks at the extremes. However, the whole-signal has a Gaussian distribution (pannel E).

Conclusion

- The two simulations above illustrate the fact that filtering of a Gaussian signal, as a linear transformation, results in a Gaussian signal. However, based on the width of the filter, more or less data points are required so that the histograms of the data segments are eligible to be assumed as approximations of data distribution.

Functions

```
function plot_fig(times, y, y_spec, fs)
    FS = 5;
    LW = 1.5;
    figure;
    subplot(311) %A
    plot(times,y)
    hold on,
    plot(times, abs(hilbert(y)), 'linewidth',LW)
    xlabel('Time [s]')
    title('A: Narrow-band signal')
    set(gca,'fontsize',FS)

    subplot(323); %B
    plot(y_spec.f,log10(y_spec.pow),'linewidth', LW);
    xlim([0,20]);
    xlabel('Frequency[Hz]','fontsize',FS);
    ylabel('log power')
    title('B - Power','fontsize',FS);
    set(gca,'fontsize',FS)

    n_seg = 4;
    seg_len = 1*fs;
    y_segments = zeros(seg_len, n_seg);
    % select random segments
    m1 = randi(length(y) - seg_len, n_seg);
    for k = 1:n_seg
        y_segments(:, k) = y(m1(k):m1(k)+seg_len-1)';
    end

    subplot(324); %C
    xt = (0:seg_len-1)/fs*1000;
    plot(xt, y_segments,'linewidth', LW);
    xlabel('Time[ms]','fontsize', FS);
    title('C - Data segments','fontsize',FS);
    set(gca,'fontsize',FS)


    subplot(325); %D
    [r,c] = hist(y_segments, 50);
    plot(c,r, 'linewidth', LW)
    title('D - Single Segment Histograms','fontsize',FS);
    set(gca,'fontsize',FS)

    subplot(326) %E
    hist(y, 100,'Normalization','pdf');
    title('E-Total Histogram','fontsize',FS);
    set(gca,'fontsize',FS)
    [r,c] = hist(y, 100);
    [fitresult, gof] = createFit(c, r);
    gof
    hold on,
    gauss = fitresult.a1*exp(-((c-fitresult.b1)/fitresult.c1).^2);
    plot(c, gauss, 'r', 'linewidth', 4)
end
```

A function for building the filter and returning its spectrum:

```
function  [b, spec] = build_fir1(order, Wn, fs)
    % INPUTS:
    % order: filter order
    % Wn [2x1]: pass-band frequencies
    % fs: sampling frequency
    % OUTPUTS:
    % b: filter coefficients
    % spec: spectrum of the filter
    Wn = Wn/(fs/2); % normalize the frequencies
    b = fir1(order, Wn);
    nfft = 2^ceil(2*log2(order));
    b_fft = fft(b ,nfft);
    f = (0:nfft-1)/nfft*fs;
    b_fft = b_fft(f<=20);
    f = f(f<=20);
    b_pow = abs(b_fft).^2;
    spec.f = f;
    spec.pow = b_pow;
end
```

A function for filtering a given signal:

```
function [y, spec] = filter_signal(b, sig, fs)
    % INPUTS:
    % b: filter coefficients
    % sig [time x channel]: the signal to be filtered
    % fs: sampling frequency
    % OUTPUTS:
    % y: filered signal
    % spec [structure content: f [frequecy bins], pow[power of the signal]]:
    % spectrum of y
    nfft = 2^ceil(log2(length(sig)));

    y = filter(b, 1, sig);
    y_fft = fft(y ,nfft);
    f = (0:nfft-1)/nfft*fs;
    y_fft = y_fft(f<=20);
    f = f(f<=20);
    y_pow = abs(y_fft).^2;
    spec.f = f;
    spec.pow = y_pow;
end
```

Function for fitting gaussian curve:

```
function [fitresult, gof] = createFit(X, N)
%  Auto-generated by MATLAB on 28-Aug-2019 11:57:20


%% Fit: 'untitled fit 1'.
[xData, yData] = prepareCurveData( X, N );

% Set up fittype and options.
ft = fittype( 'gauss1' );
opts = fitoptions( 'Method', 'NonlinearLeastSquares' );
opts.Display = 'Off';
opts.Lower = [-Inf -Inf 0];
opts.StartPoint = [66.8664281549551 -0.000277078636324379 0.00441470769173704];

% Fit model to data.
[fitresult, gof] = fit( xData, yData, ft, opts );

% Plot fit with data.
% figure( 'Name', 'untitled fit 1' );
% h = plot( fitresult, xData, yData );
% legend( h, 'N vs. X', 'untitled fit 1', 'Location', 'NorthEast' );
% Label axes
xlabel X
ylabel N
grid on
end
```

Notification: