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1 Background and Motivation

Bipedal locomotion is an area of biomechanics and robotics that requires more research. While there have been many studies trying to model motions like bipedal walking and hopping, many of these models tend to be simple due to assumptions. An area of unipedal locomotion that is interesting is unipedal skateboarding or snowboarding. The author in [2] describes the mathematical model for the equations of motion of a skateboard with no rod. The first paper describes more specifically a skateboard with one degree of freedom. Another author in [1] describes human control of a skateboard, treating the human as a rigid body attached to the board by a pin. Both studies provide input into how the equations of motion may look for a skateboard that applies ankle torques to the skateboard. We will extend this work (with some simplification) to a three-link rider, where the human applies torques to the knee, ankle, and hip.

2 Our Approach

We will model the system as a three-linked inverted pendulum mounted to the center of a simplified skateboard rolling on an inclined plane. The rider will have concentrated masses at the hip and head (on either side of the topmost link in the chain) and the board will have a small inertia about its long axis. We will constrain the three-link rider to the plane orthogonal to the long axis of the skateboard. The skateboard wheels will roll without slip, and the instantaneous radius of curvature of the path along which it rolls will be inversely proportional to the lean angle of the board, which will be subject to an elastic restoring torque. As well, the board will be assumed to be of negligible length, so the whole system will rotate about an axis passing through center of the board, and the velocity constraint of the board will apply to the point at the center of the board (unlike for a real skateboard, where the constraints apply at the wheels). The board will also be subject to a damping force proportional to its velocity. The board lean angle and the joint angles will be zero when all links are parallel to the normal vector of the inclined surface and the board is parallel to the inclined surface. We will seek periodic motions that minimize metabolic cost as approximated by the model we learned in class. For the optimization, constraints will be placed on average downhill velocity and range of motion of the joints (including a constraint encouraging the snowboarder not to fall over and through the floor). The rider will actuate its motion via torques at the ankle, knee, and hip.

BIBLIOGRAPHY

[1] Mont Hubbard. "Human control of the skateboard". In: *Journal of Biomechanics* 13.9 (1980), pp. 745–754. ISSN: 0021-9290. DOI: `https://doi.org/10.1016/0021-9290(80)90236-5`. URL: `https://www.sciencedirect.com/science/article/pii/0021929080902365`.

[2] Alexander S. Kuleshov. "Mathematical model of a skateboard with one degree of freedom". In: *Doklady Physics* 52 (2007), pp. 283–286.