# Selective DF Protocol for MIMO STBC Based Single/Multiple Relay Cooperative Communication: End-to-End Performance and Optimal Power Allocation

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***Abstract*—In this paper, we consider the performance of a selective decode-and-forward (DF) relaying based multiple-input multiple-output (MIMO) space-time block coded (STBC) cooperative communication system with single and multiple relays. We begin with a single relay based MIMO STBC system and derive the closed form expression for the end-to-end PEP of coded block detection at the destination node. It is also demonstrated that the MIMO STBC cooperative communication system achieves the full diversity order of the system. We also derive the optimal source relay power allocation, which minimizes the end-to-end decoding error of the cooperative system for a given power budget. Subsequently, for the multiple relay scenario, we consider two different relaying protocols based on two-phase and multi-phase communication. For each of these multi-relay protocols, we derive the closed form expressions for the end-to-end error rate, diversity order, and optimal power allocation. Simulation results are presented to validate the performance of the proposed single and multiple relay based cooperative communication schemes and the derived analytical results. Further, these schemes can also be seen to lead to a performance improvement compared to several other relaying schemes in existing literature.**

***Index Terms*—Cooperative communication, decode-and-forward, MIMO, STBC, optimal power allocation, diversity order.**

## I. INTRODUCTION

THE paradigm of cooperative communication [1]–[5], in which one or more relays are employed to enhance the signal quality at the destination node, has generated a significant research interest in recent times. Such systems have been shown to achieve a significantly high diversity gain due to their ability to form virtual multiple-input multiple-output (MIMO) transmit and receive arrays, by coordinating the signals relayed between the source and the destination nodes. Towards this purpose, the amplify-and-forward (AF) and decode-and-forward (DF) protocols [1] have emerged as the most popular choices for cooperative communication due to their robust performance and simplicity of implementation. AF relaying, as the name suggests, is based on amplification and re-transmission of the received signal at the relay. However, it suffers from drawbacks such as noise amplification [6], [7]. In contrast, the DF protocol [1], [2] which employs a simple decoding followed by retransmission operation at the relay is suited for a cost effective implementation of cooperative communication. Moreover, the conventional fixed DF relaying scheme, in which the relay retransmits the decoded symbol irrespective of the decoding accuracy, leads to a degradation of the overall end-to-end error rate performance of the system, which arises from the retransmission of erroneously decoded symbols at the relay. Further, its performance progressively worsens with deteriorating source-relay link quality of the cooperative communication system. A robust scheme to overcome this drawback is selective DF based cooperative communication [1], [8] in which the relay forwards the symbol to the destination only if the instantaneous signal to noise ratio (SNR) at the relay is greater than a threshold. Further, it has been shown that MIMO technology can be employed at each of the nodes [2] to additionally enhance the data rates and reliability in such cooperative wireless systems through spatial multiplexing and diversity gains. This multi-antenna relay based cooperation can also increase the capacity of DF based cooperative systems. Moreover, DF based cooperation can be readily employed in conjunction with MIMO transmission schemes, such as space-time block coding (STBC) and beamforming, also known as maximum ratio transmission (MRT), to further enhance the end-to-end performance of the cooperative system by exploiting spatial diversity along with cooperative diversity. For MRT based cooperative systems, the source requires knowledge of the complete MIMO channel matrices of the source-relay and source-destination links, in order to jointly beamform the data to the relay and destination nodes. Further, it requires the computation of the optimal beamforming vector at the source, which leads to additional complexity. Selective DF MIMO cooperative systems with STBC based transmission do not require any channel state information (CSI) at the transmitter while achieving the full diversity of the system and thus overcome the drawbacks of beamforming based MIMO cooperative schemes. However, a comprehensive analysis of STBC based MIMO cooperative schemes in the presence of both single and multiple relays is lacking in existing works. Towards this end, this paper presents

Manuscript received January 14, 2015; revised April 11, 2015; accepted May 13, 2015. Date of publication May 22, 2015; date of current version July 13, 2015. The associate editor coordinating the review of this paper and approving it for publication was G. Bauch.

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Digital Object Identifier 10.1109/TCOMM.2015.2436912

a framework based on the end-to-end pairwise error probability (PEP) analysis of a general STBC MIMO cooperative system. Further, in the literature, it has been demonstrated that equal power allocation at each node is optimal in the cooperative scenarios only when the source-relay link is stronger than the relay-destination link [8]. However, for the other scenarios, i.e., either when the relay-destination link is stronger than the source-relay link or when both the links have same strength, equal power allocation at each node is not the optimal solution in order to improve the error performance at the destination. For these scenarios, therefore it is necessary to employ optimal power factors at the transmitting nodes, which can significantly improve the end-to-end performance of the cooperative system. In this context, the presented PEP analysis is subsequently used to derive the optimal power factors, thus leading to optimal performance. The dependence of the optimal power factors on the structure of the STBC used can be clearly seen through its dependence on the singular values of the codeword difference matrices. Results are also developed for the end-to-end PEP performance of a multi-relay system considering various protocols with and without inter-relay communication. Moreover, in the multiple relay scenario, using the derived PEP results, we also demonstrate the impact of inter relay communication on the optimal power allocation compared to the case when there is no communication amongst the relays. Next, we present a brief overview and comparative survey of related works in existing literature and also clarify our contribution.

### A. Comparative Survey of Related Work

Several works in existing literature [9]–[11] have analyzed the performance of DF based MIMO-STBC cooperative communication systems. A closed form expression for the SNR outage probability has been derived in [9] for a MIMO orthogonal-STBC (OSTBC) based cooperative communication system. However, the work does not present an analysis for the end-to-end error rate or diversity order of the system. The authors in [10] present an analysis for the diversity of an adaptive DF based cooperative communication protocol [1] employing a PEP framework for various power control scenarios with specific relay locations. The study in [10] does not consider a general multiple relay scenario and cannot be readily extended to include multiple relays due to the location and power restrictions. The authors of [12] have derived the PEP bounds and diversity order of a cooperative MIMO STBC system employing the conventional fixed DF protocol. In the scheme considered therein, the symbol decoded at the relay is transmitted to the destination irrespective of the SNR conditions at the relay and the quality of the source-relay link, which leads to suboptimal end-to-end performance in comparison to the selective DF protocol. In [11], an exact outage probability analysis has been presented for the MIMO relay channel with OSTBC employing selective DF based cooperative communication over spatially uncorrelated Rayleigh flat fading channels. This work is also limited to OSTBCs and is not generalized for arbitrary STBCs. Due to the orthogonal nature of the effective channel matrix for an OSTBC, it is relatively straightforward to analyze the performance of the optimal ML decoder. However, this is not possible for a general STBC, which renders the analysis challenging. Towards this end we develop a novel end-to-end analysis of the PEP for a single relay scenario with a general STBC and extend the results to various multiple relay protocols. Moreover, to the best of our knowledge, none of the studies in existing literature consider the problem of optimal source-relay power allocation in MIMO-STBC based cooperative wireless systems.

In order to overcome the shortcomings of the schemes presented in the other related works described above, we consider a MIMO STBC based selective DF cooperative relaying system in this work and comprehensively analyze the various performance aspects of this system. Also, importantly, the framework developed and results presented in this paper are in the context of any general STBC and not restricted to an OSTBC such as [9], [11]. We begin by deriving the closed form expression for the PEP of end-to-end decoding in a single relay system. However, unlike the work in [10], since we do not consider any specific relay locations or power control mechanisms, the derived expressions are applicable for a general single relay scenario. Further, we also derive the diversity order of this system and demonstrate that it achieves the full decoding diversity at the receiver. Moreover, an optimization framework is developed to compute the optimal source-relay power allocation for end-to-end error rate minimization. However, it can be noted that this does not imply transmit power sharing between the source and the relays. Rather, it derives optimal power usage by the source and relays for a given total power budget. This optimal power allocation paradigm is similar to works such as [8], [13] and enhances the performance of the cooperative system for a given total transmit power of the net system comprising of the source and the relays.

Next we consider MIMO-STBC cooperative communication for multiple-relay scenarios. In [8], [14], a class of multi-relay DF cooperative protocols $C(m)$; $1 \leq m < N$, where $N$ is the number of relays, have been analyzed in which each relay combines the signals received from the source and $m$ previous relays. In the simplest $C(1)$ version of the cooperative protocol, each relay receives the signal from only one previous relay along with the signal transmitted by the source. Therefore, this protocol requires a total of $N + 1$ transmission phases for communication between the source and destination. However, the analysis therein has been presented only for single antenna nodes based multiple relay cooperative wireless scenarios. In our work, we consider the general version of the $C(m)$ protocol proposed in [8], [14], which allows inter-relay communication and derive the results for the PEP, diversity order and optimal power allocation to comprehensively characterize the performance of a multi-relay MIMO-STBC cooperative system. Further, since the work in [8], [14] is based on a single-input single-output (SISO) cooperative communication scenario, the optimal power allocation depends only on the corresponding average channel gains of the source-relay and relay-destination links. However, interestingly, the MIMO nature of the cooperative communication system considered in this work introduces a new paradigm for optimal power allocation, where the optimal power factors depend not only on the average source-relay and relay-destination channel gains but also on the number of antennas at the relay and destination nodes. Further, we have explicitly characterized the impact of the diversity properties and number of antennas of the individual MIMO source-destination, source-relay and relay-destination channel links on the optimal power allocation. Moreover, a new two-phase MIMO-STBC based DF cooperative protocol $C(0)$ is also proposed, in which each relay receives the STBC coded block from the source in the first phase and selectively re-transmits the STBC coded block to the destination during the second, i.e., the cooperative relaying phase.

We now summarize the various novel contributions of this work below.

1) A closed form expression is derived for the end-to-end PEP of STBC decoding in a selective DF based MIMO-STBC cooperative system.
2) An asymptotic approximation is derived for the end-to-end PEP of the proposed MIMO STBC cooperative relaying scheme.
3) The diversity order of this cooperative MIMO-STBC system is characterized and it is demonstrated that the proposed scheme achieves the full diversity order for STBC decoding at the destination node.
4) Subsequently, results are also presented for optimal source-relay power allocation towards end-to-end error rate minimization.
5) For a multi-relay scenario, considering a two phase multi-relay MIMO-STBC based selective DF cooperative system termed $C(0)$, closed form expressions are derived for the PEP and diversity order followed by the results for optimal source-relay power allocation.
6) Further, considering the general $C(m)$ multi-relay protocol, which allows inter relay communication, the PEP, diversity order and optimal power allocation results are presented to comprehensively characterize the performance of the system.

The following notation is employed for the mathematical expressions presented in this work. The Hermitian transpose of the matrix $\mathbf{A}$ is denoted by $\mathbf{A}^H$, $|a|$ denotes the magnitude of the complex number $a$, $\|\mathbf{A}\|_F$ denotes the matrix Frobenius norm of $\mathbf{A}$ and $\mathrm{Tr}(\mathbf{A})$ denotes the trace of $\mathbf{A}$. Also, $\mathbb{E}\{\cdot\}$ denotes the expectation operator, and $Q(x)$ denotes the Gaussian $Q$ function which characterizes the tail probability $P(X \geq x)$ of the standard Gaussian random variable $X$ with zero mean and unit variance, also represented as $\mathcal{N}(0, 1)$. The notation $\mathbb{C}^{N\times M}$ denotes the space of $N \times M$ matrices over the complex field $\mathbb{C}$.

The organization of the paper is as follows. The selective DF single/multiple relay MIMO STBC cooperative system model is described in Section II, followed by a detailed derivation of the end-to-end PEP, diversity order and optimal power allocation in Section III for the single relay scenario. The general multiple relay $C(0)$ protocol based cooperative system is introduced in Section IV along with the corresponding end-to-end performance and optimal power allocation. The multi-phase multiple relay $C(m)$ protocol is analyzed in Section V. Simulation results are presented in Section VI, followed by the conclusion in Section VII.

## II. Selective DF Based MIMO STBC Cooperative System Model

Let $\mathbf{X} \in \mathbb{C}^{N\times T}$ denote the transmitted codeword matrix in the MIMO STBC system, with $N$ transmit antennas and block length $T$. Let the received symbol blocks $\mathbf{Y}_{SD} \in \mathbb{C}^{N_d\times T}$, $\mathbf{Y}_{SR}^{(k)} \in \mathbb{C}^{N\times T}$ denote the output codewords at the destination and the $k^{th}$ relay respectively, corresponding to the transmission of the STBC codeword $\mathbf{X}$ by the source, where $N_s = N_r = N$ denotes the number of antennas at the source and relay, since the source and relay employ the same STBC and $N_d$ denotes the number of antennas at the destination. The received codewords $\mathbf{Y}_{SD}$, $\mathbf{Y}_{SR}^{(k)}$ can be modeled as,

$$\mathbf{Y}_{SD} = \sqrt{\frac{P_0}{N}}\mathbf{H}_{SD}\mathbf{X} + \mathbf{W}_{SD},$$
$$\mathbf{Y}_{SR}^{(k)} = \sqrt{\frac{P_0}{N}}\mathbf{H}_{SR}^{(k)}\mathbf{X} + \mathbf{W}_{SR}^{(k)},$$

where $\mathbf{H}_{SD} \in \mathbb{C}^{N_d\times N}$, $\mathbf{H}_{SR}^{(k)} \in \mathbb{C}^{N\times N}$ denote the Rayleigh fading MIMO channel matrices between the source and destination and the source and $k^{th}$ relay respectively, where $1 \leq k \leq K$, and $K$ denotes the number of relays. Also, the average gains of the channel coefficients corresponding to the individual links between the source-destination and source-relay are given as $\delta_{sd}^2$, $\left(\delta_{sr}^{(k)}\right)^2$. The quantity $P_0$ denotes the source transmit power. The noise samples at the relay and destination, which are the entries of the matrices $\mathbf{W}_{SR}^{(k)}$, $\mathbf{W}_{SD}$ respectively, are complex circularly symmetric additive white Gaussian of power $\eta_0/2$ per complex dimension, i.e., with distribution given by $\mathcal{CN}(0, \eta_0)$. In the subsequent cooperative phase(s), a single relay or a group of relays employ selective decode-and-forward based transmission, thus forwarding only if the decoding SNR exceeds a threshold similar to the one considered in works such as [8], [13]. The received codeword $\mathbf{Y}_{RD}^{(k)}$, corresponding to the forwarding by the $k^{th}$ relay is given as,

$$\mathbf{Y}_{RD}^{(k)} = \sqrt{\frac{P_k}{N}}\mathbf{H}_{RD}^{(k)}\mathbf{X} + \mathbf{W}_{RD}^{(k)},$$

where $\mathbf{H}_{RD}^{(k)} \in \mathbb{C}^{N_d\times N}$ is the fading MIMO channel matrix between the $k^{th}$ relay and destination node of average power gain $\left(\delta_{rd}^{(k)}\right)^2$ per link. The quantity $P_k$ denotes the transmit power of the $k^{th}$ relay. The additive Gaussian noise matrix $\mathbf{W}_{RD}^{(k)}$ at the destination is statistically similar to the noise matrices $\mathbf{W}_{SR}^{(k)}$ and $\mathbf{W}_{SD}$ described above with variance $\eta_0/2$ per dimension.

## III. Single Relay Based MIMO-STBC Selective DF Cooperative Communication

### A. PEP Analysis

We begin by deriving the end-to-end PEP for STBC decoding in a single, i.e., $K = 1$ relay based MIMO cooperative wireless system with the selective DF communication protocol as shown in Fig. 1. Let $\mathcal{C} = \{\mathbf{X}_j\}$ denote the MIMO STBC codeword set, where each $\mathbf{X}_j \in \mathbb{C}^{N\times T}$ and $1 \leq j \leq |\mathcal{C}|$, where $|\mathcal{C}|$, which denotes the cardinality of the codeword set $\mathcal{C}$, is the number of codewords in the MIMO STBC. The PEP for the error event

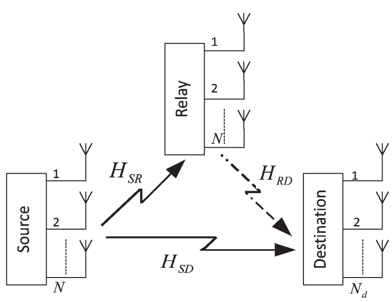


Fig. 1. Schematic representation of a single-relay based MIMO-STBC selective DF cooperative system.

corresponding to the transmitted codeword $\mathbf{X}_0 \in \mathbb{C}^{N\times T}$ being confused for the codeword $\mathbf{X}_j \in \mathbb{C}^{N\times T}$ at the relay where $j \neq 0$, conditioned on the fading channel matrix $\mathbf{H}_{SR}$ is given from [15] as,

$$P_{S\to R}(\mathbf{X}_0 \to \mathbf{X}_j|\mathbf{H}_{SR}) = Q\left(\sqrt{\frac{P_0\left\|\mathbf{H}_{SR}(\mathbf{X}_0-\mathbf{X}_j)\right\|_F^2}{2N\eta_0}}\right),$$

where $\mathbf{X}_0 \to \mathbf{X}_j$ denotes the error event and $P_{S\to R}$ denotes the error probability for the source to relay transmission. Let the singular value decomposition (SVD) of the codeword difference matrix $\mathbf{X}_0-\mathbf{X}_j$ be given as $\mathbf{X}_0-\mathbf{X}_j=\mathbf{U}_j\mathbf{\Lambda}_j\mathbf{V}_j^H$, where $\mathbf{U}_j \in \mathbb{C}^{N\times N}$ is a unitary matrix, i.e., $\mathbf{U}_j\mathbf{U}_j^H=\mathbf{U}_j^H\mathbf{U}_j=\mathbf{I}_N$ and the diagonal matrix $\mathbf{\Lambda}_j \in \mathbb{R}^{N\times N}$ given as,

$$\mathbf{\Lambda}_j = \begin{bmatrix} \lambda_{j,1} & 0 & \cdots & 0 \\ 0 & \lambda_{j,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_{j,N} \end{bmatrix}$$

contains the non-negative singular values $\lambda_{j,1}, \lambda_{j,2}, \cdots \lambda_{j,N}$ of the codeword difference matrix. The matrix $\mathbf{V}_j^H \in \mathbb{C}^{T\times N}$ contains orthogonal row vectors which form a basis for the row space of the difference matrix $\mathbf{X}_0-\mathbf{X}_j$, and satisfies the property $\mathbf{V}_j^H\mathbf{V}_j=I_N$. The Frobenius norm $\|\mathbf{H}_{SR}(\mathbf{X}_0-\mathbf{X}_j)\|_F^2$ can therefore be simplified as,

$$\begin{aligned}\left\|\mathbf{H}_{SR}(\mathbf{X}_0-\mathbf{X}_j)\right\|_F^2 &= \mathrm{Tr}\left\{\mathbf{H}_{SR}(\mathbf{X}_0-\mathbf{X}_j)(\mathbf{X}_0-\mathbf{X}_j)^H\mathbf{H}_{SR}^H\right\} \\ &= \mathrm{Tr}\left\{\mathbf{H}_{SR}\mathbf{U}_j\mathbf{\Lambda}_j\mathbf{V}_j^H\mathbf{V}_j\mathbf{\Lambda}_j\mathbf{U}_j^H\mathbf{H}_{SR}^H\right\} \\ &= \mathrm{Tr}\left\{\mathbf{H}_{SR}\mathbf{U}_j{\mathbf{\Lambda}_j}^2\mathbf{U}_j^H\mathbf{H}_{SR}^H\right\} \\ &= \mathrm{Tr}\left\{{\mathbf{\Lambda}_j}^2\tilde{\mathbf{H}}_{SR}\tilde{\mathbf{H}}_{SR}^H\right\} \\ &= \sum_{n=1}^{N}\lambda_{jn}^2\sum_{\tilde{n}=1}^{N}|\tilde{h}_{\tilde{n},n}|^2, \end{aligned} \tag{1}$$

where the coefficient $\tilde{h}_{\tilde{n},n}$ is the $(\tilde{n}, n)$ entry of the matrix $\tilde{\mathbf{H}}_{SR}=\mathbf{H}_{SR}\mathbf{U}_j$ for $1 \le \tilde{n}, n \le N$. Therefore, the above expression for the PEP conditioned on $\mathbf{H}_{SR}$ can be simplified by substituting this expression for $\|\mathbf{H}_{SR}(\mathbf{X}_0-\mathbf{X}_j)\|_F^2$ as,

$$P_{S\to R}(\mathbf{X}_0 \to \mathbf{X}_j|\tilde{\mathbf{H}}_{SR}) = Q\left(\sqrt{\frac{P_0\sum_{n=1}^N\lambda_{jn}^2\sum_{\tilde{n}=1}^N|\tilde{h}_{\tilde{n},n}|^2}{2N\eta_0}}\right).$$

It follows from the result in [15] that the coefficients $\tilde{h}_{\tilde{n},n}$ of the effective channel matrix $\tilde{\mathbf{H}}_{SR}$ are Rayleigh distributed, identical to $h_{\tilde{n},n}$ with average power gain $\delta_{sr}^2$. Further, it can be readily seen that the gain $\tilde{g}_{\tilde{n},n}=|\tilde{h}_{\tilde{n},n}|^2$ is exponentially distributed with parameter $\frac{1}{\delta_{sr}^2}$ as,

$$f(\tilde{g}_{\tilde{n},n}) = \frac{1}{\delta_{sr}^2}\exp\left\{-\frac{\tilde{g}_{\tilde{n},n}}{\delta_{sr}^2}\right\}. \tag{2}$$

The average PEP at the relay can be obtained by taking the expectation of the above PEP with respect to the exponential distribution of the gains $\tilde{g}_{\tilde{n},n}$ as,

$$P_{S\to R}(\mathbf{X}_0 \to \mathbf{X}_j) = \mathbb{E}_{\tilde{\mathbf{H}}_{SR}}\left\{Q\left(\sqrt{\frac{P_0\sum_{n=1}^N\lambda_{jn}^2\sum_{\tilde{n}=1}^N|\tilde{h}_{\tilde{n},n}|^2}{2N\eta_0}}\right)\right\}.$$

We now employ the identity $Q(x)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta$ for the Gaussian $Q(\cdot)$ function, to simplify the average PEP as given in Appendix A,

$$P_{S\to R}(\mathbf{X}_0 \to \mathbf{X}_j) = G\left(\prod_{n=1}^{N}\left(1+\frac{P_0\lambda_{jn}^2\delta_{sr}^2}{4\eta_0 N\sin^2\theta}\right)^N\right), \tag{3}$$

where the function $G(\nu(\theta))$ is defined as $G(\nu(\theta)) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{1}{\nu(\theta)}d\theta$ [8]. Therefore, the total PEP for the decoding at the relay can be upper bounded using the union bound which is very tight at high SNR as the sum of the PEP over codewords $\mathbf{X}_j \in \mathcal{C}$ as,

$$\begin{aligned} P_{S\to R} &\le \sum_{\mathbf{X}_j\in\mathcal{C},\ \mathbf{X}_j\neq\mathbf{X}_0} P_{S\to R}(\mathbf{X}_0 \to \mathbf{X}_j) \\ &= \sum_{j=1}^{|\mathcal{C}|} G\left(\prod_{n=1}^{N}\left(1+\frac{P_0\lambda_{jn}^2\delta_{sr}^2}{4\eta_0 N\sin^2\theta}\right)^N\right). \end{aligned} \tag{4}$$

Following a similar approach, the average PEP of the confusion event $\mathbf{X}_0 \to \mathbf{X}_i$, corresponding to the direct source-destination link, averaged over the fading channel gain matrix $\tilde{\mathbf{H}}_{SD}$ with Rayleigh fading coefficients of average power $\delta_{sd}^2$ is,

$$P_{S\to D}(\mathbf{X}_0 \to \mathbf{X}_i) = G\left(\prod_{n=1}^{N}\left(1+\frac{P_0\lambda_{in}^2\delta_{sd}^2}{4\eta_0 N\sin^2\theta}\right)^{N_d}\right). \tag{5}$$

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$$P_{S\to D,R\to D}(\mathbf{X}_0 \to \mathbf{X}_i|\tilde{\mathbf{H}}_{SD},\tilde{\mathbf{H}}_{RD}) = Q\left(\sqrt{\frac{P_0\sum_{n=1}^N\lambda_{in}^2\sum_{\tilde{l}=1}^{N_d}\left|\tilde{h}_{\tilde{l},n}^{(SD)}\right|^2+P_1\sum_{n=1}^N\lambda_{in}^2\sum_{l=1}^{N_d}\left|\tilde{h}_{l,n}^{(RD)}\right|^2}{2N\eta_0}}\right). \tag{6}$$

Let $\tilde{h}_{\tilde{l}n}^{(SD)}$, $\tilde{h}_{ln}^{(RD)}$ denote the Rayleigh distributed coefficients corresponding to the effective source-destination and relay-destination matrices $\tilde{\mathbf{H}}_{SD} = \mathbf{H}_{SD}\mathbf{U}_i$ and $\tilde{\mathbf{H}}_{RD} = \mathbf{H}_{RD}\mathbf{U}_i$ respectively. The expression for the PEP of the error event $\mathbf{X}_0 \to \mathbf{X}_i$ at the destination, when the relay decodes all the symbols transmitted by the source during the first phase correctly, conditioned on the source-destination and relay-destination matrices $\tilde{\mathbf{H}}_{SD}, \tilde{\mathbf{H}}_{RD}$ is given in (6), shown at the bottom of the previous page. Averaging now over the Rayleigh distribution of the fading channel coefficients $\tilde{h}_{\tilde{l},n}^{(SD)}$, $\tilde{h}_{l,n}^{(RD)}$, the average PEP for the error event $\mathbf{X}_0 \to \mathbf{X}_i$ is given in (7), shown at the bottom of the page, which can be simplified as,

$$P_{S\to D,R\to D}(\mathbf{X}_0 \to \mathbf{X}_i) = G\left(\prod_{n=1}^{N}\left[\left(1+\frac{P_0\lambda_{in}^2\delta_{sd}^2}{4\eta_0 N\sin^2\theta}\right)^{N_d}\left(1+\frac{P_1\lambda_{in}^2\delta_{rd}^2}{4\eta_0 N\sin^2\theta}\right)^{N_d}\right]\right). \quad (8)$$

Therefore, the end-to-end PEP for the error event $\mathbf{X}_0 \to \mathbf{X}_i$ denoted by, $P_e(\mathbf{X}_0 \to \mathbf{X}_i)$, is given as,

$$P_e(\mathbf{X}_0 \to \mathbf{X}_i) = P_{S\to D}(\mathbf{X}_0 \to \mathbf{X}_i) \times P_{S\to R} + P_{S\to D,R\to D}(\mathbf{X}_0 \to \mathbf{X}_i) \times (1 - P_{S\to R}),$$

where $P_{S\to D}(\mathbf{X}_0 \to \mathbf{X}_i) \times P_{S\to R}$ and $P_{S\to D,R\to D}(\mathbf{X}_0 \to \mathbf{X}_i) \times (1 - P_{S\to R})$ are the end-to-end PEPs at the destination corresponding to the events when the relay decodes the transmitted codeword matrix erroneously and correctly respectively. Substituting the expressions for $P_{S\to R}$, $P_{S\to D}(\mathbf{X}_0 \to \mathbf{X}_i)$ and $P_{S\to D,R\to D}(\mathbf{X}_0 \to \mathbf{X}_i)$ derived in (4), (5) and (8) above respectively and employing the fact that $1 - P_{S\to R} \approx 1$ at high SNR in above expression, yields the expression for the average end-to-end PEP $P_e(\mathbf{X}_0 \to \mathbf{X}_i)$ corresponding to the error event $\mathbf{X}_0 \to \mathbf{X}_i$ as,

$$\begin{aligned} P_e(\mathbf{X}_0 \to \mathbf{X}_i) &\le G\left(\prod_{n=1}^{N}\left(1+\frac{P_0\lambda_{in}^2\delta_{sd}^2}{4\eta_0 N\sin^2\theta}\right)^{N_d}\right) \\ &\times \sum_{j=1}^{|\mathcal{C}|} G\left(\prod_{n=1}^{N}\left(1+\frac{P_0\lambda_{jn}^2\delta_{sr}^2}{4\eta_0 N\sin^2\theta}\right)^{N}\right) \\ &+ G\left(\prod_{n=1}^{N}\left(1+\frac{P_0\lambda_{in}^2\delta_{sd}^2}{4\eta_0 N\sin^2\theta}\right)^{N_d}\left(1+\frac{P_1\lambda_{in}^2\delta_{rd}^2}{4\eta_0 N\sin^2\theta}\right)^{N_d}\right). \end{aligned}$$

Finally, the PEP bound for the maximum likelihood (ML) decoding at the destination is now readily obtained by considering the sum of all the PEP terms corresponding to $\mathbf{X}_0 \to \mathbf{X}_i$ over all the possible codewords $\mathbf{X}_i \in \mathcal{C}$ as,

$$P_e \le \sum_{i=1}^{|\mathcal{C}|} P_e(\mathbf{X}_0 \to \mathbf{X}_i). \quad (9)$$

Next, we present the diversity analysis and the framework for optimal source relay power allocation in the above cooperative communication system.

### B. Diversity Order Analysis and Optimal Source Relay Power Allocation

At high SNR, i.e., with the source and relay signal to noise power ratios $\frac{P_0}{\eta_0}, \frac{P_1}{\eta_0} \to \infty$, since the union bound is tight, the PEP expression $P_e(\mathbf{X}_0 \to \mathbf{X}_i)$ given above can be tightly approximated using the relations $1 + \frac{P_0\lambda_{in}^2\delta_{sd}^2}{4\eta_0 N\sin^2\theta} \approx \frac{P_0\lambda_{in}^2\delta_{sd}^2}{4\eta_0 N\sin^2\theta}$, $1 + \frac{P_0\lambda_{jn}^2\delta_{sr}^2}{4\eta_0 N\sin^2\theta} \approx \frac{P_0\lambda_{jn}^2\delta_{sr}^2}{4\eta_0 N\sin^2\theta}$ and $1 + \frac{P_1\lambda_{in}^2\delta_{rd}^2}{4\eta_0 N\sin^2\theta} \approx \frac{P_1\lambda_{in}^2\delta_{rd}^2}{4\eta_0 N\sin^2\theta}$, to yield the following asymptotic expression for the end-to-end PEP $\mathbf{X}_0 \to \mathbf{X}_i$ as,

$$\begin{aligned} P_e(\mathbf{X}_0 \to \mathbf{X}_i) \approx &\frac{(4N)^{NN_d+N^2}\zeta(NN_d)\zeta(N^2)\kappa}{\left(\sigma_i^2\right)^{N_d}\left(\delta_{sd}^2\right)^{NN_d}\left(\delta_{sr}^2\right)^{N^2}\beta_0^{NN_d+N^2}}\left(\frac{\eta_0}{P}\right)^{NN_d+N^2} \\ &+ \frac{(4N)^{2NN_d}\zeta(2NN_d)}{\left(\sigma_i^2\right)^{2N_d}\left(\delta_{sd}^2\right)^{NN_d}\left(\delta_{rd}^2\right)^{NN_d}\beta_0^{NN_d}\beta_1^{NN_d}}\left(\frac{\eta_0}{P}\right)^{2NN_d}, \end{aligned} \quad (10)$$

where $\beta_i = P_i/P$ for $i = 0,1$ denote the ratio of the powers $P_0$, $P_1$, allocated to the source and relay respectively, to the total power. Further, since $P_0 + P_1 = P$, it naturally follows that the power factors $\beta_0$, $\beta_1$ at the source and relay satisfy $\beta_0 + \beta_1 = 1$ with $\beta_0, \beta_1 \ge 0$. Also, let the quantity $\sigma_i^2$ be defined in terms of the singular values $\lambda_{in}$ of the codeword difference matrix as $\sigma_i^2 = \prod_{n=1}^{N}\lambda_{in}^2$ and $\kappa = \sum_{j=1}^{|\mathcal{C}|}\sigma_j^{-2N}$. The constant $\zeta(z)$ is defined as $\zeta(z) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}(\sin^2\theta)^z d\theta$. Using the above relation for the PEP of the event $\mathbf{X}_0 \to \mathbf{X}_i$, the asymptotic approximation for the net end-to-end PEP of the above system can be obtained at high SNR employing the union bound as,

$$\begin{aligned} P_e &\approx \sum_{i=1}^{|\mathcal{C}|} P_e(\mathbf{X}_0 \to \mathbf{X}_i) \\ &\approx \sum_{i=1}^{|\mathcal{C}|}\left[\frac{(4N)^{NN_d+N^2}\zeta(NN_d)\zeta(N^2)\kappa}{\left(\sigma_i^2\right)^{N_d}\left(\delta_{sd}^2\right)^{NN_d}\left(\delta_{sr}^2\right)^{N^2}\beta_0^{NN_d+N^2}}\left(\frac{\eta_0}{P}\right)^{NN_d+N^2}\right. \\ &\qquad\left. + \frac{(4N)^{2NN_d}\zeta(2NN_d)}{\left(\sigma_i^2\right)^{2N_d}\left(\delta_{sd}^2\right)^{NN_d}\left(\delta_{rd}^2\right)^{NN_d}\beta_0^{NN_d}\beta_1^{NN_d}}\left(\frac{\eta_0}{P}\right)^{2NN_d}\right]. \end{aligned} \quad (11)$$

---

$$P_{S\to D,R\to D}(\mathbf{X}_0 \to \mathbf{X}_i) = \mathbb{E}_{\tilde{\mathbf{H}}_{SD},\tilde{\mathbf{H}}_{RD}}\left\{Q\left(\sqrt{\frac{P_0\sum_{n=1}^{N}\lambda_{in}^2\sum_{\tilde{l}=1}^{N_d}\left|\tilde{h}_{\tilde{l},n}^{(SD)}\right|^2 + P_1\sum_{n=1}^{N}\lambda_{in}^2\sum_{l=1}^{N_d}\left|\tilde{h}_{l,n}^{(RD)}\right|^2}{2N\eta_0}}\right)\right\} =$$

$$\frac{1}{\pi}\int_0^{\pi/2}\left[\prod_{n=1}^{N}\prod_{\tilde{l}=1}^{N_d}\int_0^{\infty}\exp\left(-\frac{P_0\lambda_{in}^2\left|\tilde{h}_{\tilde{l}n}^{(SD)}\right|^2}{4N\eta_0\sin^2\theta}\right)f\left(\tilde{h}_{\tilde{l},n}^{(SD)}\right)d\tilde{h}_{\tilde{l},n}^{(SD)}\prod_{n=1}^{N}\prod_{l=1}^{N_d}\int_0^{\infty}\exp\left(-\frac{P_1\lambda_{in}^2\left|\tilde{h}_{ln}^{(RD)}\right|^2}{4N\eta_0\sin^2\theta}\right)f\left(\tilde{h}_{l,n}^{(RD)}\right)d\tilde{h}_{l,n}^{(RD)}\right]d\theta \quad (7)$$

It is demonstrated in Appendix B that the diversity order for the above system is, $-\lim_{\frac{P}{\eta_0}\to\infty}\frac{\log(P_e)}{\log\left(\frac{P}{\eta_0}\right)} = NN_d + N\min\{N, N_d\}$.

Let the quantity $\sigma_{\min}$ be defined as $\sigma_{\min} = \min\{\sigma_i\}_{i=1}^{|\mathcal{C}|}$. The asymptotic PEP expression calculated above can be upper bounded by including only the dominant term in the expression corresponding to $\sigma_{\min}$ as,

$$P_e \le |\mathcal{C}| \left[ \frac{(4N)^{NN_d+N^2}\zeta(NN_d)\zeta(N^2)\kappa}{\left(\sigma^2_{\min}\right)^{N_d}\left(\delta^2_{sd}\right)^{NN_d}\left(\delta^2_{sr}\right)^{N^2}\beta_0^{NN_d+N^2}}\left(\frac{\eta_0}{P}\right)^{NN_d+N^2} + \frac{(4N)^{2NN_d}\zeta(2NN_d)}{\left(\sigma^2_{\min}\right)^{2N_d}\left(\delta^2_{sd}\right)^{NN_d}\left(\delta^2_{rd}\right)^{NN_d}\beta_0^{NN_d}\beta_1^{NN_d}}\left(\frac{\eta_0}{P}\right)^{2NN_d}\right]. \quad (12)$$

Hence, it is now relatively straightforward to see that the convex optimization framework for computing the optimal source-relay power factors $\beta_0, \beta_1$ with the constraint $\beta_0 + \beta_1 = 1$, towards PEP minimization can be formulated as,

$$\min_{\beta_0,\beta_1} \left\{ \frac{(4N)^{N^2}\zeta(NN_d)\zeta(N^2)\kappa}{\left(\delta^2_{sr}\right)^{N^2}\beta_0^{NN_d+N^2}}\left(\frac{\eta_0}{P}\right)^{NN_d+N^2} + \frac{(4N)^{NN_d}\zeta(2NN_d)}{\left(\sigma^2_{\min}\right)^{N_d}\left(\delta^2_{rd}\right)^{NN_d}\beta_0^{NN_d}\beta_1^{NN_d}}\left(\frac{\eta_0}{P}\right)^{2NN_d}\right\},$$
$$\text{s.t.} \quad \beta_0 + \beta_1 = 1. \quad (13)$$

The standard Karush-Kuhn-Tucker (KKT) based Lagrangian framework can be employed to compute the optimal power factor $\beta_0$. It can be shown by differentiating the above cost function and setting equal to zero that the optimal power factor $\beta_0$ is given as the non-negative zero of the polynomial equation,

$$(1-\beta_0)^{NN_d+1}\chi(N, N_d) - \beta_0^{N^2+1} + \beta_0^{N^2}(1-\beta_0) = 0, \quad (14)$$

where the factor $\chi(N, N_d)$ is defined in (15), shown at the bottom of the page. This yields the optimal source-relay power allocation for a given total cooperative power budget $P$. We now extend this framework to a multi-relay MIMO space time block coded cooperative communication system with $K$ relaying nodes.

## IV. Multiple Relay $C(0)$ Protocol Based Selective DF Relaying

### A. PEP Analysis

Consider now a selective DF cooperative wireless system with $K$ relays as shown in Fig. 2. This section presents the PEP analysis for this multiple relay system with the $C(0)$ protocol, i.e., one in which each relay receives the signal directly from the source. Therefore, this is a two phase cooperative protocol, similar to the one considered previously for the single relay scenario. The general scenario of $C(m)$ based cooperative communication, described in [14], [8], in which each relay receives the signal from $m$ previous relays is considered in the next section. The $K$ relays $R_1, R_2, \cdots, R_K$ together with the destination node receive the STBC coded symbol block transmitted by the source during the first phase. In the second phase, a subset of relays which are able to successfully decode the coded block, based on the SNR threshold criteria, retransmit the coded block to the destination. Therefore, unlike the $C(m)$ protocol, this scheme in general has a shorter overall time duration since it involves only two phases. Further, since there is no inter relay communication, it is easier to implement. Each of the $K$ relays can be in either of 2 states depending on correct/incorrect decoding of the space time coded block, leading to a total of $2^K$ possible states for the cooperating relays in the selective DF system. Let $S(k)$ denote the state of the relay $k$ for $1 \le k \le K$, with $S(k) = 1$ if relay $k$ decodes correctly and 0 otherwise. Let the set $\mathcal{A}$ denote the set of all the possible $2^K$ binary vector states, with $\mathbf{a}_j \in \mathcal{A}$ for $0 \le j \le 2^K - 1$ representing one such possible state. Employing the above framework, the total end-to-end PEP for the error event $\mathbf{X}_0 \to \mathbf{X}_i$ at the destination, where $\mathbf{X}_0$ is the coded block transmitted by the source, can be written as [14], [16],

$$P_e(\mathbf{X}_0 \to \mathbf{X}_i|\mathcal{H}) = \sum_{\mathbf{a}_j\in\mathcal{A}} P_e(\mathbf{X}_0 \to \mathbf{X}_i|\mathbf{S} = \mathbf{a}_j)P(\mathbf{S} = \mathbf{a}_j),$$

where $\mathbf{S} = [S(1), S(2), \cdots, S(K)]^T$ denotes the network state vector and $\mathcal{H} = \{\mathbf{H}_{SD}, \mathbf{H}_{SR}^{(k)}, \mathbf{H}_{RD}^{(k)}, 1 \le k \le K\}$ denotes the total

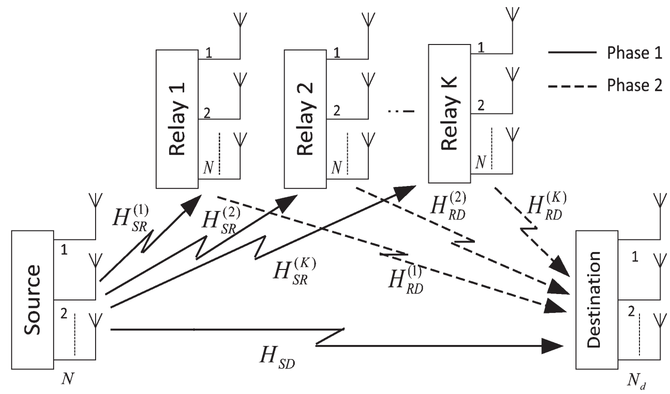


Fig. 2. Schematic representation of the two phase $C(0)$ protocol based multi-relay selective DF cooperative system.

$$\chi(N, N_d) = \begin{cases} \dfrac{2\left(\zeta(N^2)\right)^2\kappa\left(\sigma^2_{\min}\right)^N\left(\delta^2_{rd}\right)^{N^2}}{\zeta(2N^2)\left(\delta^2_{sr}\right)^{N^2}} & \text{for } N_d = N, \\ \dfrac{(NN_d+N^2)\zeta(NN_d)\zeta(N^2)\kappa\left(\sigma^2_{\min}\right)^{N_d}\left(\delta^2_{rd}\right)^{NN_d}(P/\eta_0)^{NN_d-N^2}}{NN_d(4N)^{NN_d-N^2}\zeta(2NN_d)\left(\delta^2_{sr}\right)^{N^2}} & \text{for } N_d > N, \\ \dfrac{(NN_d+N^2)(4N)^{N^2-NN_d}\zeta(NN_d)\zeta(N^2)\kappa\left(\sigma^2_{\min}\right)^{N_d}\left(\delta^2_{rd}\right)^{NN_d}}{NN_d\zeta(2NN_d)\left(\delta^2_{sr}\right)^{N^2}(P/\eta_0)^{N^2-NN_d}} & \text{for } N_d < N. \end{cases} \quad (15)$$

channel state information (CSI). Also $P_e(\mathbf{X}_0 \to \mathbf{X}_i|\mathbf{S} = \mathbf{a}_j)$ is the end-to-end PEP when the cooperative system is in state $\mathbf{a}_j$, while $P(\mathbf{S} = \mathbf{a}_j)$ denotes the corresponding probability of the system being in state $\mathbf{a}_j \in \mathcal{A}$. Since the various links of the cooperative system fade independently, due to the nodes being spatially distributed, both the probability terms in the above equation are independent. Hence, averaging over the CSI $\mathcal{H}$, the above expression can be simplified as,

$$P_e(\mathbf{X}_0 \to \mathbf{X}_i) = \sum_{\mathbf{a}_j \in \mathcal{A}} \mathbb{E}_{\mathcal{H}}\left\{P_e(\mathbf{X}_0 \to \mathbf{X}_i|\mathbf{S} = \mathbf{a}_j)\right\} \mathbb{E}_{\mathcal{H}}\left\{P(\mathbf{S} = \mathbf{a}_j)\right\}. \tag{16}$$

The probability of the system being in state $\mathbf{S} = \mathbf{a}_j \in \mathcal{A}$ can be calculated as,

$$P(\mathbf{S} = \mathbf{a}_j) = \prod_{k=1}^{K} P\left(S(k) = a_j(k)\right),$$

where each of the components $P(S(k) = a_j(k))$, corresponding to the net PEP at the relay $k$, can be expressed as,

$$P\left(S(k) = a_j(k)\right) = \begin{cases} \sum_{l=1}^{|\mathcal{C}|} P_{S \to R^{(k)}}(\mathbf{X}_0 \to \mathbf{X}_l) & \text{if } a_j(k) = 0 \\ 1 - \sum_{l=1}^{|\mathcal{C}|} P_{S \to R^{(k)}}(\mathbf{X}_0 \to \mathbf{X}_l) & \text{if } a_j(k) = 1. \end{cases}$$

From expression (3) for the source to relay PEP derived in the previous section, the average PEP for the error event $\mathbf{X}_0 \to \mathbf{X}_l$, when the block $\mathbf{X}_0$ is transmitted by the source, is given as,

$$\mathbb{E}_{\mathcal{H}}\left\{P_{S \to R^{(k)}}(\mathbf{X}_0 \to \mathbf{X}_l)\right\} = G\left(\prod_{n=1}^{N}\left(1 + \frac{P_0 \lambda_{ln}^2 \left(\delta_{sr}^{(k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^{N}\right). \tag{17}$$

Let the average PEP at the destination for the error event $\mathbf{X}_0 \to \mathbf{X}_i$, with the network in state $\mathbf{S} = \mathbf{a}_j \in \mathcal{A}$, given as $\mathbb{E}_{\mathcal{H}}\{P_e(\mathbf{X}_0 \to \mathbf{X}_i|\mathbf{S} = \mathbf{a}_j)\}$, be denoted by $P_D(i, j)$. Let the set $\Psi_j$ defined as $\Psi_j = \{k|a_j(k) = 1, 1 \le k \le K\}$ include all the relays which decode the coded block $\mathbf{X}_0$ correctly. Using the expression in (8), the above equation for $P_D(i, j)$ can be simplified as (18), shown at the bottom of the page. Employing this expression for $P_D(i, j)$ and the equation for $\mathbb{E}_{\mathcal{H}}\{P_{S \to R^{(k)}}(\mathbf{X}_0 \to \mathbf{X}_l)\}$ derived in (17), it follows from (16) that the net PEP for end-to-end decoding of the $C(0)$ protocol based cooperative system can be written as (19), shown at the bottom of the page, where we have employed the approximation that $1 - \sum_{l=1}^{|\mathcal{C}|} P_{S \to R^{(k)}}(\mathbf{X}_0 \to \mathbf{X}_l) \approx 1$ if $a_j(k) = 1$ at high SNR and $\bar{\Psi}_j$ denotes the compliment of the set $\Psi_j$. Next, we present the diversity order analysis for the $C(0)$ protocol based multiple relay cooperative system.

### B. Diversity Order Analysis and Optimal Source Relay Power Allocation

An asymptotic approximation of the end-to-end PEP for the multi-relay system can be obtained as follows. Ignoring the additive unity factors in each of the PEP components similar to the single relay case in Section III-B along with the approximation $1 - \sum_{l=1}^{|\mathcal{C}|} P_{S \to R^{(k)}}(\mathbf{X}_0 \to \mathbf{X}_l) \approx 1$ used above for the PEP, the average probability of state $\mathbf{S} = \mathbf{a}_j$ can be approximated at high SNR as,

$$\begin{aligned} \mathbb{E}_{\mathcal{H}}\left\{P(\mathbf{S} = \mathbf{a}_j)\right\} &\approx \prod_{k \in \bar{\Psi}_j}\left\{\sum_{l=1}^{|\mathcal{C}|} G\left(\prod_{n=1}^{N}\left(1 + \frac{P_0 \lambda_{ln}^2 \left(\delta_{sr}^{(k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^{N}\right)\right\} \\ &\approx \prod_{k \in \bar{\Psi}_j}\left\{\sum_{l=1}^{|\mathcal{C}|} G\left(\prod_{n=1}^{N}\left(\frac{P_0 \lambda_{ln}^2 \left(\delta_{sr}^{(k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^{N}\right)\right\} \\ &= \prod_{k \in \bar{\Psi}_j}\left[\left(\frac{4N}{\left(\delta_{sr}^{(k)}\right)^2}\frac{\eta_0}{P_0}\right)^{N^2} \zeta(N^2)\kappa\right] \\ &= \left(4N\frac{\eta_0}{P_0}\right)^{N^2|\bar{\Psi}_j|}\left(\kappa\zeta(N^2)\right)^{|\bar{\Psi}_j|} \prod_{k \in \bar{\Psi}_j}\left(\frac{1}{\left(\delta_{sr}^{(k)}\right)^2}\right)^{N^2}, \end{aligned} \tag{20}$$

$$P_D(i, j) = G\left(\prod_{n=1}^{N}\left[\left(1 + \frac{P_0 \lambda_{in}^2 \delta_{sd}^2}{4\eta_0 N \sin^2\theta}\right)^{N_d} \prod_{k \in \Psi_j}\left(1 + \frac{P_k \lambda_{in}^2 \left(\delta_{rd}^{(k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^{N_d}\right]\right). \tag{18}$$

$$\begin{aligned} P_e \le \sum_{i=1}^{|\mathcal{C}|} P_e(\mathbf{X}_0 \to \mathbf{X}_i) = \sum_{i=1}^{|\mathcal{C}|} \sum_{\mathbf{a}_j \in \mathcal{A}} &\left[\prod_{k \in \bar{\Psi}_j}\left\{\sum_{l=1}^{|\mathcal{C}|} G\left(\prod_{n=1}^{N}\left(1 + \frac{P_0 \lambda_{ln}^2 \left(\delta_{sr}^{(k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^{N}\right)\right\}\right. \\ &\left.\times G\left(\prod_{n=1}^{N}\left[\left(1 + \frac{P_0 \lambda_{in}^2 \delta_{sd}^2}{4\eta_0 N \sin^2\theta}\right)^{N_d} \prod_{k \in \Psi_j}\left(1 + \frac{P_k \lambda_{in}^2 \left(\delta_{rd}^{(k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^{N_d}\right]\right)\right], \end{aligned} \tag{19}$$

where $\zeta(\cdot)$ and $\kappa$ are as defined in Section III. Let the power factors for the source and $k$th relay, $\beta_0, \beta_k$ respectively be defined as $\beta_0 = P_0/P$, $\beta_k = P_k/P$, $1 \le k \le K$. Substituting $P_0 = \beta_0 P$ and $P_k = \beta_k P$, the above approximation can be further simplified as,

$$\mathbb{E}_{\mathcal{H}}\left\{P(\mathbf{S}=\mathbf{a}_j)\right\} \approx \left(\frac{4N}{\beta_0}\right)^{N^2|\bar{\Psi}_j|}\left(\kappa\zeta(N^2)\right)^{|\bar{\Psi}_j|} \times \left(\prod_{k\in\bar{\Psi}_j}\left(\frac{1}{\left(\delta_{sr}^{(k)}\right)^2}\right)^{N^2}\right)\left(\frac{\eta_0}{P}\right)^{N^2|\bar{\Psi}_j|}. \quad (21)$$

The quantity $P_D(i,j)$, which denotes the average PEP for the error event $\mathbf{X}_0 \to \mathbf{X}_i$ conditioned on the relay state $\mathbf{a}_j$ in (18) can be tightly approximated at high SNR similar to (20) by ignoring the additive unity factors in each of the PEP components as,

$$\begin{aligned} P_D(i,j) &\approx G\left(\prod_{n=1}^{N}\left[\left(\frac{P_0\lambda_{in}^2\delta_{sd}^2}{4\eta_0 N\sin^2\theta}\right)^{N_d}\prod_{k\in\Psi_j}\left(\frac{P_k\lambda_{in}^2\left(\delta_{rd}^{(k)}\right)^2}{4\eta_0 N\sin^2\theta}\right)^{N_d}\right]\right)\\ &= \frac{\left(\frac{4N\eta_0}{\delta_{sd}^2P_0}\right)^{NN_d}\zeta\left(NN_d+NN_d|\Psi_j|\right)}{\left(\sigma_i^2\right)^{N_d+N_d|\Psi_j|}}\prod_{k\in\Psi_j}\left(\frac{4N\frac{\eta_0}{P_k}}{\left(\delta_{rd}^{(k)}\right)^2}\right)^{NN_d}\\ &= \frac{(4N)^{NN_d+NN_d|\Psi_j|}\zeta\left(NN_d+NN_d|\Psi_j|\right)}{\left(\sigma_i^2\right)^{N_d+N_d|\Psi_j|}\left(\delta_{sd}^2\beta_0\right)^{NN_d}}\\ &\quad\times\left(\prod_{k\in\Psi_j}\left(\frac{1}{\left(\delta_{rd}^{(k)}\right)^2\beta_k}\right)^{NN_d}\right)\left(\frac{\eta_0}{P}\right)^{NN_d+NN_d|\Psi_j|}. \end{aligned} \quad (22)$$

Substituting this expression for $P_D(i,j)$ and the relation for $\mathbb{E}_{\mathcal{H}}\{P(\mathbf{S}=\mathbf{a}_j)\}$ from (21), in the equation in (16), the net end-to-end PEP for the error event $\mathbf{X}_0 \to \mathbf{X}_i$ can be tightly approximated at high SNR as (23), shown at the bottom of the page. The PEP bound for the end-to-end decoding at the destination node is now readily derived as,

$$\begin{aligned} P_e &\le \sum_{i=1}^{|\mathcal{C}|}P_e(\mathbf{X}_0\to\mathbf{X}_i)\\ &= \sum_{i=1}^{|\mathcal{C}|}\sum_{\mathbf{a}_j\in\mathcal{A}}\rho_{i,j}\left(\frac{\eta_0}{P}\right)^{d_j}, \end{aligned} \quad (24)$$

where $d_j = NN_d + NN_d|\Psi_j| + N^2|\bar{\Psi}_j| = NN_d + N^2K + N\{(N_d - N)|\Psi_j|\}$, and $\rho_{i,j}$ is defined in (25), shown at the bottom of the page. Therefore, it is now easy to see that the diversity order $d$ for this multi-relay cooperative $C(0)$ protocol is given as $d = \min_j d_j = NN_d + N^2K + N\min_j\{(N_d - N)|\Psi_j|\}$. Further, the following observations yield a deeper insight into the diversity order of this system. When $N_d > N$, i.e., with more antennas at the destination compared to the source and relays, $\min_j\{(N_d-N)|\Psi_j|\}$ occurs for $|\Psi_j| = 0$. Retaining only the dominant term corresponding to $j = 0$ and neglecting the other terms in the sum in (24), the net end-to-end PEP can be further approximated at high SNR as,

$$P_e \approx \sum_{i=1}^{|\mathcal{C}|}\left[\frac{(4N)^{NN_d+N^2K}\zeta(NN_d)\left(\kappa\zeta(N^2)\right)^K}{\left(\sigma_i^2\right)^{N_d}\left(\delta_{sd}^2\right)^{NN_d}\beta_0^{NN_d+N^2K}} \times\left(\prod_{k=1}^{K}\frac{1}{\left(\left(\delta_{sr}^{(k)}\right)^2\right)^{N^2}}\right)\right]\left(\frac{\eta_0}{P}\right)^{NN_d+N^2K}. \quad (26)$$

$$P_e(\mathbf{X}_0\to\mathbf{X}_i) \approx \sum_{\mathbf{a}_j\in\mathcal{A}}\left[\frac{(4N)^{NN_d+N^2|\bar{\Psi}_j|+NN_d|\Psi_j|}\zeta\left(NN_d+NN_d|\Psi_j|\right)\left(\kappa\zeta(N^2)\right)^{|\bar{\Psi}_j|}}{\left(\sigma_i^2\right)^{N_d+N_d|\Psi_j|}\left(\delta_{sd}^2\right)^{NN_d}\beta_0^{NN_d+N^2|\bar{\Psi}_j|}} \times\left(\prod_{k\in\bar{\Psi}_j}\frac{1}{\left(\left(\delta_{sr}^{(k)}\right)^2\right)^{N^2}}\right)\left(\prod_{k\in\Psi_j}\frac{1}{\left(\beta_k\left(\delta_{rd}^{(k)}\right)^2\right)^{NN_d}}\right)\right]\left(\frac{\eta_0}{P}\right)^{NN_d+NN_d|\Psi_j|+N^2|\bar{\Psi}_j|} \quad (23)$$

$$\rho_{i,j} = \frac{(4N)^{NN_d+N^2|\bar{\Psi}_j|+NN_d|\Psi_j|}\zeta\left(NN_d+NN_d|\Psi_j|\right)\left(\kappa\zeta(N^2)\right)^{|\bar{\Psi}_j|}}{\left(\sigma_i^2\right)^{N_d+N_d|\Psi_j|}\left(\delta_{sd}^2\right)^{NN_d}\beta_0^{NN_d+N^2|\bar{\Psi}_j|}}\left(\prod_{k\in\bar{\Psi}_j}\frac{1}{\left(\left(\delta_{sr}^{(k)}\right)^2\right)^{N^2}}\right)\left(\prod_{k\in\Psi_j}\frac{1}{\left(\beta_k\left(\delta_{rd}^{(k)}\right)^2\right)^{NN_d}}\right) \quad (25)$$

On the other hand, when $N_d < N$, $\min_j\{(N_d - N)|\Psi_j|\}$ occurs for $|\Psi_j| = K$. Hence, the term corresponding to $j = 2^K - 1$ is the dominant term in the PEP expression, which is well approximated at high SNR by,

$$P_e \approx \sum_{i=1}^{|\mathcal{C}|}\left[\frac{(4N)^{NN_d+NN_dK}\zeta(NN_d+NN_dK)}{\left(\sigma_i^2\right)^{N_d+N_dK}\left(\delta_{sd}^2\right)^{NN_d}\beta_0^{NN_d}}\right.$$
$$\left.\times\left(\prod_{k=1}^{K}\frac{1}{\left(\beta_k\left(\delta_{rd}^{(k)}\right)^2\right)^{NN_d}}\right)\right]\left(\frac{\eta_0}{P}\right)^{NN_d+NN_dK}. \quad (27)$$

For the case $N_d = N$, all the terms $0 \le j \le 2^K - 1$ will contribute to $P_e$ in (24). From the relations in (24), (25), (26), one can readily conclude that the achieved diversity order is $N^2 + N^2K$ for $N_d = N$, $NN_d + N^2K$ for $N_d > N$ and $NN_d + NN_dK$ for $N_d < N$. The different expressions above corresponding to the various cases can be combined to yield the succinct expression $NN_d + NK\min\{N, N_d\}$ for the final diversity order of the $C(0)$ system.

The optimal source relay power allocation can be derived as follows. For the case $N_d > N$, interestingly, as can be seen from the expression in (26), the term corresponding to $\beta_0$ dominates the end-to-end performance. Therefore, it naturally follows that the error rate is minimized by maximizing $\beta_0$, i.e., setting $\beta_0 = 1$, implying that all power is allocated to the source. Similarly, for $N_d < N$, the product term $\beta_0\prod_{k=1}^{K}\beta_k$ dominates, from which it can be seen that $\beta_0 = \beta_k = \frac{1}{K+1}$, $1 \le k \le K$ are the optimal power factors, i.e., equal power allocation at the source and relays. For the scenario $N_d = N$, $P_e$ can be upper bounded as,

$$P_e \le |\mathcal{C}|\left[\sum_{\mathbf{a}_j\in\mathcal{A}} D_j\left(\frac{\eta_0}{P}\right)^{NN_d+NN_d|\Psi_j|+N^2|\bar{\Psi}_j|}\right], \quad (28)$$

where $D_j$ is defined as,

$$D_j = \left[\frac{(4N)^{NN_d+N^2|\bar{\Psi}_j|+NN_d|\Psi_j|}\zeta\left(NN_d+NN_d|\Psi_j|\right)\left(\kappa\zeta(N^2)\right)^{|\bar{\Psi}_j|}}{\left(\sigma_{\min}^2\right)^{N_d+N_d|\Psi_j|}\left(\delta_{sd}^2\right)^{NN_d}\beta_0^{NN_d+N^2|\bar{\Psi}_j|}}\right.$$
$$\left.\times\left(\prod_{k\in\bar{\Psi}_j}\frac{1}{\left(\left(\delta_{sr}^{(k)}\right)^2\right)^{N^2}}\right)\left(\prod_{k\in\Psi_j}\frac{1}{\left(\beta_k\left(\delta_{rd}^{(k)}\right)^2\right)^{NN_d}}\right)\right]. \quad (29)$$

The optimization problem for source relay power allocation to compute the optimal power factors $\beta_0, \beta_k, 1 \le k \le K$, which can in turn be used to compute the optimal powers $P_0, P_k, 1 \le k \le K$, towards minimizing the end-to-end PEP, can be formulated as,

$$\min_{\beta_0,\beta_1,\ldots,\beta_K}\left[\sum_{\mathbf{a}_j\in\mathcal{A}} D_j\left(\frac{\eta_0}{P}\right)^{NN_d+NN_d|\Psi_j|+N^2|\bar{\Psi}_j|}\right],$$
$$\text{s.t.}\quad \beta_0 + \sum_{k=1}^{K}\beta_k = 1,$$
$$\beta_0 \ge 0,\ \beta_k \ge 0,\ 1 \le k \le K. \quad (30)$$

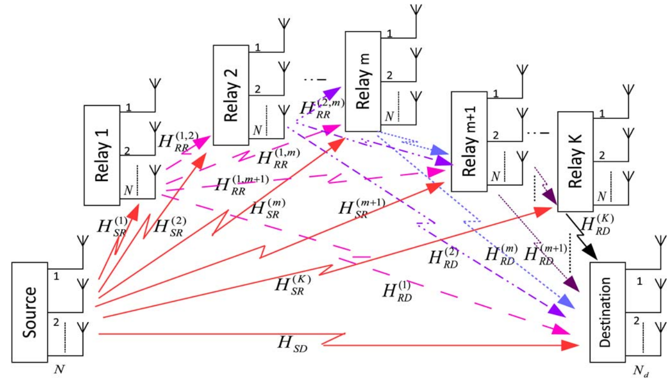

Fig. 3. Schematic representation of a multiple phase $C(m)$ protocol based multi-relay selective DF cooperative system.

The above optimization problem can be seen to be a geometric program (GP) [17] which can be readily solved using a convex solver such as CVX [18]. Next we analyze the performance of a $C(m)$ protocol based multi-relay cooperative system.

## V. Multiple Phase $C(m)$ Protocol Based Multi-Relay System

### A. PEP Analysis

We now consider the $C(m)$ protocol for a multi-relay cooperative system, originally described in [8], in which the cooperative relaying occurs in a total of $K + 1$ phases. Similar to the single relay and $C(0)$ scenarios, the source transmits the STBC coded symbol block in the first phase, while each relay $k$ transmits in phase $k + 1$ employing the selective DF protocol and combining the transmission of the source along with the signals received from $\min\{m, k-1\}$ previous relay transmissions to perform ML decoding. Finally, the destination decodes the symbol block after $K + 1$ communication phases using the signal received from the source and the $K$ relays as shown in Fig. 3. Let the state $S(k)$ and state space $\mathcal{A}$ of the multiple relay system be defined similar to the $C(0)$ protocol based on the decoding at the individual relays. Hence, the probability of the system being in state $\mathbf{a}_j \in \mathcal{A}$ is given as $P(\mathbf{S} = \mathbf{a}_j) = \prod_{k=1}^{K} P(k, j)$, where the quantity $P(k, j)$ is the average conditional probability of the relay being in state $S(k) = a_j(k)$, defined as,

$$P(k,j) = \mathbb{E}_{\mathcal{H}}\left\{\Pr\left(S(k) = a_j(k)\middle|\left\{S(k-1) = a_j(k-1), \cdots,\right.\right.\right.$$
$$\left.\left.\left.S(k-m) = a_j(k-m)\right\}\right)\right\}$$
$$= \begin{cases}\sum_{l=1}^{|\mathcal{C}|}\mathbb{E}_{\mathcal{H}}\left\{P_{S\to R^{(k)}}(\mathbf{X}_0 \to \mathbf{X}_l)\right\}, & \text{if } a_j(k) = 0,\\ 1-\sum_{l=1}^{|\mathcal{C}|}\mathbb{E}_{\mathcal{H}}\left\{P_{S\to R^{(k)}}(\mathbf{X}_0 \to \mathbf{X}_l)\right\}, & \text{if } a_j(k) = 1,\end{cases} \quad (31)$$

where $\mathcal{H} = \{\mathbf{H}_{SD}, \mathbf{H}_{SR}^{(k)}, \mathbf{H}_{RD}^{(k)}, \mathbf{H}_{RR}^{(\tilde{k},k)}, 1 \le k \le K, \max\{1, k-m\} \le \tilde{k} \le k-1\}$ and $\mathbf{H}_{RR}^{(\tilde{k},k)}$ denotes the channel matrix between the relay $\tilde{k}$ and the relay $k$ with a power of $\left(\delta_{rr}^{(\tilde{k},k)}\right)^2$. Let $\Psi_j(k)$ be defined as the set of previous nodes corresponding to node $k$ which decode the transmitted coded block correctly, i.e., $\Psi_j(k) = \{q \| a_j(q) = 1; \quad \max\{1, k-m\} \le q \le k-1\}$. The average error probability corresponding to the source-relay transmission can be expressed as (32), shown at the bottom of the page. Therefore, the expression for the average probability of the network being in state $\mathbf{a}_j$, denoted by $\mathbb{E}_{\mathcal{H}}\{P(\mathbf{S} = \mathbf{a}_j)\}$ can be obtained by substituting this equation for $\mathbb{E}_{\mathcal{H}}\{P_{S\to R^{(k)}}(\mathbf{X}_0 \to \mathbf{X}_l)\}$ in $P(\mathbf{S} = \mathbf{a}_j) = \prod_{k=1}^{K} P(k, j)$ and using the property (31). The expression for end-to-end PEP of the event $\mathbf{X}_0 \to \mathbf{X}_i$ is now given as,

$$P_e(\mathbf{X}_0 \to \mathbf{X}_i) = \sum_{\mathbf{a}_j \in \mathcal{A}} \mathbb{E}_{\mathcal{H}}\left\{P_e(\mathbf{X}_0 \to \mathbf{X}_i | \mathbf{S} = \mathbf{a}_j)\right\} \mathbb{E}_{\mathcal{H}}\left\{P(\mathbf{S} = \mathbf{a}_j)\right\}$$

The above given expression can be simplified as (33), shown at the bottom of the page, where $P_D(i, j)$ is the average PEP for the event $\mathbf{X}_0 \to \mathbf{X}_i$ conditioned on the network state $\mathbf{a}_j$ defined as $P_D(i, j) = \mathbb{E}_{\mathcal{H}}\{P_e(\mathbf{X}_0 \to \mathbf{X}_i | \mathbf{S} = \mathbf{a}_j)\}$. The union bound for the total PEP at the destination can be now obtained by the addition of the above PEP over all possible codewords $\mathbf{X}_i$. Further, at high SNR, employing the approximation $1 - \mathbb{E}\{\cdot\} \approx 1$ in (31) and ignoring the additive factor of unity in the SNR components, the quantity $\mathbb{E}_{\mathcal{H}}\{P(\mathbf{S} = \mathbf{a}_j)\}$ can be approximated as (34), shown at the bottom of the page, where $\tilde{\kappa} = \left(\sum_{l=1}^{|\mathcal{C}|} \frac{1}{(\prod_{n=1}^{N} \lambda_{ln}^2)^{N+N|\Psi_j(k)|}}\right)$. Once again, using the power factors $\beta_0 = \frac{P_0}{P}$, $\beta_q = \frac{P_q}{P}$, the approximation in (34) for the PEP can be simplified as,

$$\mathbb{E}_{\mathcal{H}}\left\{P(\mathbf{S} = \mathbf{a}_j)\right\} \approx \left(\frac{\eta_0}{P}\right)^{N^2|\bar{\Psi}_j| + N^2 \sum_{k \in \bar{\Psi}_j} |\Psi_j(k)|} \times \underbrace{\prod_{k \in \bar{\Psi}_j} \left[\frac{(4N)^{N^2+N^2|\Psi_j(k)|} \zeta\left(N^2 + N^2\left|\Psi_j(k)\right|\right)}{\left(\beta_0\left(\delta_{sr}^{(k)}\right)^2\right)^{N^2} \prod_{q \in \Psi_j(k)}\left(\beta_q\left(\delta_{rr}^{(q,k)}\right)^2\right)^{N^2}} \tilde{\kappa}\right]}_{X_j}. \quad (35)$$

---

$$\begin{aligned}\mathbb{E}_{\mathcal{H}}\left\{P_{S\to R^{(k)}}(\mathbf{X}_0 \to \mathbf{X}_l)\right\} &= G\left(\prod_{n=1}^{N}\left[\left(1 + \frac{P_0\lambda_{ln}^2\left(\delta_{sr}^{(k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^N \prod_{q=\max\{1,k-m\}}^{k-1}\left(1 + \frac{P_q\lambda_{ln}^2\left(\delta_{rr}^{(q,k)}\right)^2 a_j(q)}{4\eta_0 N \sin^2\theta}\right)^N\right]\right)\\ &= G\left(\prod_{n=1}^{N}\left[\left(1 + \frac{P_0\lambda_{ln}^2\left(\delta_{sr}^{(k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^N \prod_{q\in\Psi_j(k)}\left(1 + \frac{P_q\lambda_{ln}^2\left(\delta_{rr}^{(q,k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^N\right]\right) \quad (32)\end{aligned}$$

---

$$P_e(\mathbf{X}_0 \to \mathbf{X}_i) = \sum_{\mathbf{a}_j\in\mathcal{A}} \underbrace{G\left(\prod_{n=1}^{N}\left[\left(1 + \frac{P_0\lambda_{in}^2\delta_{sd}^2}{4\eta_0 N \sin^2\theta}\right)^{N_d} \prod_{k'\in\Psi_j}\left(1 + \frac{P'_k\lambda_{in}^2\left(\delta_{rd}^{(k')}\right)^2}{4\eta_0 N \sin^2\theta}\right)^{N_d}\right]\right)}_{P_D(i,j)} \prod_{k=1}^{K}\mathbb{E}_{\mathcal{H}}\left\{P(k,j)\right\} \quad (33)$$

---

$$\begin{aligned}\mathbb{E}_{\mathcal{H}}\left\{P(\mathbf{S} = \mathbf{a}_j)\right\} &\approx \prod_{k\in\bar{\Psi}_j}\underbrace{\left[\sum_{l=1}^{|\mathcal{C}|} G\left(\prod_{n=1}^{N}\left[\left(1 + \frac{P_0\lambda_{ln}^2\left(\delta_{sr}^{(k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^N \prod_{q\in\Psi_j(k)}\left(1 + \frac{P_q\lambda_{ln}^2\left(\delta_{rr}^{(q,k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^N\right]\right)\right]}_{\mathbb{E}_{\mathcal{H}}\{P(S(k)=a_j(k))\}}\\ &\approx \prod_{k\in\bar{\Psi}_j}\left[\sum_{l=1}^{|\mathcal{C}|} G\left(\prod_{n=1}^{N}\left[\left(\frac{P_0\lambda_{ln}^2\left(\delta_{sr}^{(k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^N \prod_{q\in\Psi_j(k,m)}\left(\frac{P_q\lambda_{ln}^2\left(\delta_{rr}^{(q,k)}\right)^2}{4\eta_0 N \sin^2\theta}\right)^N\right]\right)\right]\\ &= \prod_{k\in\bar{\Psi}_j}\left[\frac{(4N\eta_0)^{N^2+N^2|\Psi_j(k)|}\zeta\left(N^2 + N^2\left|\Psi_j(k)\right|\right)}{\left(P_0\left(\delta_{sr}^{(k)}\right)^2\right)^{N^2}\prod_{q\in\Psi_j(k)}\left(P_q\left(\delta_{rr}^{(q,k)}\right)^2\right)^{N^2}}\tilde{\kappa}\right] \quad (34)\end{aligned}$$

Similarly, the expression for $P_D(i,j)$ defined in (33) can be approximated at high SNR by ignoring the additive unity factors added to the individual SNR terms to obtain (36), shown at the bottom of the page. From the relations in (35), (36), the end-to-end PEP for the error event $\mathbf{X}_0 \to \mathbf{X}_i$ can be approximated at high SNR as,

$$P_e(\mathbf{X}_0 \to \mathbf{X}_i) \approx \sum_{\mathbf{a}_j \in \mathcal{A}} X_j Y_{i,j} \left(\frac{\eta_0}{P}\right)^{d_j}, \tag{37}$$

where $d_j = NN_d + NN_d|\Psi_j| + N^2|\bar{\Psi}_j| + N^2 \sum_{k\in\bar{\Psi}_j} |\Psi_j(k)|$ and the expressions for $X_{i,j}, Y_{i,j}$ are defined in (35), (36) respectively. The exponent $d_j$ can be lower bounded as $d_j \geq NN_d + NN_d|\Psi_j| + N^2|\bar{\Psi}_j|$, with equality if $\sum_{k\in\bar{\Psi}_j} |\Psi_j(k)| = 0$, i.e., $|\Psi_j(k)| = 0 \ \forall \ k \in \bar{\Psi}_j$, which corresponds to the network states such as $111\cdots1, 011\cdots1, 001\cdots1, \cdots, 000\cdots0$ where for every relay $k$ which decodes incorrectly, the previous $\max\{k-1, m\}$ relays must have decoded incorrectly. The diversity order therefore is $d = \min_j d_j = NN_d + NN_dK + N\min_j\{(N-N_d)|\bar{\Psi}_j|\} = NN_d + N^2K + N\min_j\{(N_d-N)|\Psi_j|\}$. Hence, at high SNR, considering only the dominant $K+1$ terms mentioned above in (37) and neglecting the remaining terms, the total PEP can be further approximated as,

$$P_e \approx \sum_{i=1}^{|\mathcal{C}|}\sum_{j=0}^{K} \chi_j \xi_{i,j} \left(\frac{\eta_0}{P}\right)^{NN_d+NN_dK+jN(N-N_d)}, \tag{38}$$

where the terms $\chi_j$ and $\xi_{i,j}$ are defined as,

$$\chi_j = \prod_{k=1}^{j}\left[\frac{(4N)^{N^2}\zeta(N^2)}{\left(\beta_0\left(\delta_{sr}^{(k)}\right)^2\right)^{N^2}}\left(\sum_{l=1}^{|\mathcal{C}|}\frac{1}{\left(\prod_{n=1}^{N}\lambda_{ln}^2\right)^N}\right)\right],$$

$$\xi_{i,j} = \frac{(4N)^{NN_d}\zeta\left(NN_d + NN_d(K-j)\right)}{\left(\prod_{n=1}^{N}\lambda_{in}^2\right)^{N_d}\left(\delta_{sd}^2\beta_0\right)^{NN_d}} \times \left(\prod_{w=j+1}^{K}\left(\frac{(4N)^{NN_d}}{\left(\prod_{n=1}^{N}\lambda_{in}^2\right)^{N_d}\left(\left(\delta_{rd}^{(w)}\right)^2\beta_w\right)^{NN_d}}\right)\right).$$

It can be seen that the achieved diversity order is $d = NN_d + NN_dK + \min_{0\leq j\leq K}\{jN(N-N_d)\}$. Also, interestingly, this does not depend on $m$, the maximum number of previous relay transmissions. Hence, one can use the $C(1)$ protocol which requires fewer channel estimates compared to $C(K-1)$, while achieving an identical diversity gain.

### B. Diversity Order Analysis and Optimal Source Relay Power Allocation

We now derive the diversity order and develop the framework for optimal source relay power allocation for the $C(m)$ protocol based cooperative communication system. We begin by considering the scenario with $N_d > N$, for which $\min_{0\leq j\leq T}\{jN(N-N_d)\}$ occurs when $j = K$, implying that all the relays decode correctly. Considering the dominant term corresponding to $j = K$ and neglecting all the other terms in (38), the PEP can be further approximated at high SNR as,

$$\begin{aligned} P_e &\approx \sum_{i=1}^{|\mathcal{C}|}\left(\frac{\eta_0}{P}\right)^{NN_d+KN^2} \\ &\quad\times \frac{(4N)^{NN_d}\zeta(NN_d)}{\left(\sigma_i^2\right)^{N_d}\left(\delta_{sd}^2\beta_0\right)^{NN_d}}\prod_{k=1}^{K}\left(\frac{(4N)^{N^2}\zeta(N^2)}{\left(\beta_0\left(\delta_{sr}^{(k)}\right)^2\right)^{N^2}}\kappa\right) \\ &\leq |\mathcal{C}|\left(\frac{\eta_0}{P}\right)^{NN_d+KN^2} \\ &\quad\times \frac{(4N)^{NN_d+N^2K}\zeta(NN_d)\left(\zeta(N^2)\right)^K\kappa^K}{\left(\sigma_{\min}^2\right)^{N_d}\left(\delta_{sd}^2\right)^{NN_d}(\beta_0)^{NN_d+N^2K}\prod_{k=1}^{K}\left(\left(\delta_{sr}^{(k)}\right)^2\right)^{N^2}}. \end{aligned} \tag{39}$$

Hence, the achieved diversity order is $d = NN_d + KN^2$. Further, the optimization problem for optimal power allocation is,

$$\begin{aligned} &\min_{\beta_0,\beta_1,\ldots,\beta_K} \frac{1}{\beta_0^{NN_d+N^2K}} \\ &\text{s.t.} \quad \beta_0 + \sum_{k=1}^{K}\beta_k = 1 \\ &\qquad \beta_0 \geq 0, \beta_k \geq 0, 1 \leq k \leq K, \end{aligned} \tag{40}$$

$$\begin{aligned} P_D(i,j) &\approx G\left(\prod_{n=1}^{N}\left[\left(\frac{P_0\lambda_{in}^2\delta_{sd}^2}{4\eta_0 N\sin^2\theta}\right)^{N_d}\prod_{k'\in\Psi_j}\left(\frac{P^{(k')}\lambda_{in}^2\left(\delta_{rd}^{(k')}\right)^2}{4\eta_0 N\sin^2\theta}\right)^{N_d}\right]\right) \\ &= \underbrace{\frac{(4N)^{NN_d+NN_d|\Psi_j|}\zeta\left(NN_d+NN_d|\Psi_j|\right)}{\left(\prod_{n=1}^{N}\lambda_{in}^2\right)^{N_d+N_d|\Psi_j|}\left(\delta_{sd}^2\beta_0\right)^{NN_d}}\left(\prod_{k'\in\Psi_j}\left(\frac{1}{\left(\delta_{rd}^{(k')}\right)^2\beta_{k'}}\right)^{NN_d}\right)}_{Y_{i,j}}\left(\frac{\eta_0}{P}\right)^{NN_d+NN_d|\Psi_j|} \end{aligned} \tag{36}$$

from which it can be seen that the optimal power factor $\beta_0 \approx 1$, i.e., allocating all the power to the source is optimal. For $N_d < N$, $\min_{0\le j\le T}\{jN(N-N_d)\}$ occurs for $j=0$, corresponding to all the relays decoding incorrectly. Therefore, the term corresponding to $j=0$ dominates the PEP expression, which is approximated at high SNR as,

$$\begin{aligned}P_e &\approx \sum_{i=1}^{|\mathcal{C}|}\left(\frac{\eta_0}{P}\right)^{NN_d+NN_dK}\frac{(4N)^{NN_d}\zeta(NN_d+NN_dK)}{\left(\sigma_i^2\right)^{N_d}\left(\delta_{sd}^2\beta_0\right)^{NN_d}}\\ &\quad\times\left(\prod_{w=1}^{K}\left(\frac{(4N)^{NN_d}}{\left(\sigma_i^2\right)^{N_d}\left(\left(\delta_{rd}^{(w)}\right)^2\beta_w\right)^{NN_d}}\right)\right)\\ &\le |\mathcal{C}|\left(\frac{\eta_0}{P}\right)^{NN_d+NN_dK}\frac{(4N)^{NN_d+NN_dK}\zeta(NN_d+NN_dK)}{\left(\sigma_{\min}^2\right)^{N_d+N_dK}\left(\delta_{sd}^2\beta_0\right)^{NN_d}}\\ &\quad\times\prod_{w=1}^{K}\left(\left(\delta_{rd}^{(w)}\right)^2\beta_w\right)^{-NN_d}.\end{aligned}$$

The corresponding problem for optimal power allocation can be formulated similar to (40) by replacing the optimization objective by $\frac{1}{\beta_0^{NN_d}\prod_{w=1}^{K}\beta_w^{NN_d}}$. The solution to this optimization problem is seen to be given as $\beta_0=\beta_w=\frac{1}{K+1}$, $1\le w\le K$, i.e., equal power allocation to the source and relay nodes. The achieved diversity order is $NN_d+NN_dK$. Finally, when the source and destination have an equal number of antennas, i.e., $N_d=N$, all the $K+1$ terms will contribute to the PEP. The PEP for this scenario is given as (41), shown at the bottom of the page. Also, the optimization problem for optimal source relay power allocation can now be formulated as,

$$\begin{aligned}&\min_{\beta_0,\beta_1,\ldots,\beta_K}\sum_{j=0}^{K}\prod_{w=j+1}^{K}\frac{D_j}{(\beta_0)^{N^2(j+1)}(\beta_w)^{N^2}}\\ &\text{s.t.}\quad \beta_0+\sum_{w=1}^{K}\beta_w=1\\ &\qquad\quad \beta_0\ge 0,\ \beta_w\ge 0,\ 1\le w\le K,\end{aligned}\tag{42}$$

which is a GP similar to (30) for the $C(m)$ protocol. The diversity order for this scenario is $NN_d+NK\min\{N,N_d\}$.

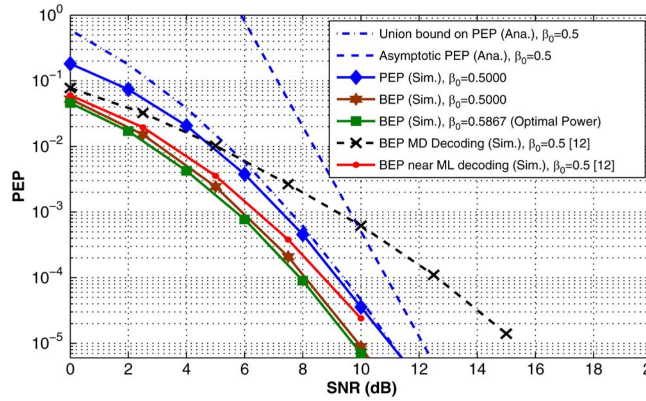


Fig. 4. Exact PEP, asymptotic upper bound, simulated PEP and BEP for a Alamouti-coded selective DF-MIMO cooperative communication with $K=1$, $N=N_d=2$ and $\delta_{s,d}^2=2$, $\delta_{s,r}^2=2$, $\delta_{r,d}^2=2$.

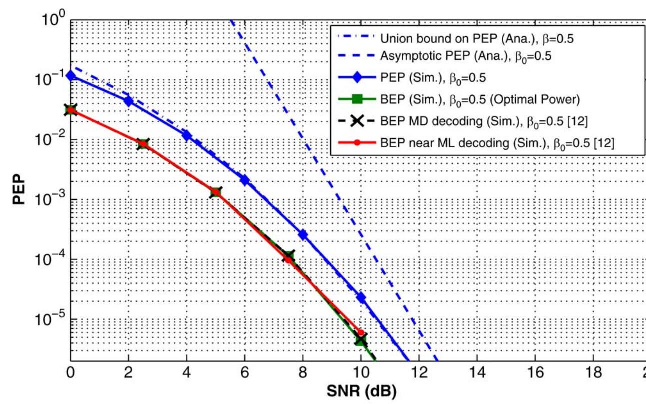


Fig. 5. Exact PEP, asymptotic upper bound, simulated PEP and BEP for a Alamouti-coded selective DF-MIMO cooperative communication with $K=1$, $N=N_d=2$ and $\delta_{s,d}^2=2$, $\delta_{s,r}^2=2000$, $\delta_{r,d}^2=2$.

## VI. Simulation Results

We consider a MIMO cooperative relaying system with $N=2$ antennas at the source and relay and $N_d=2$ antennas at the destination. The Alamouti STBC is employed with QPSK modulated symbols. Therefore, we have $|\mathcal{C}|=4^2=16$. Figs. 4–6 demonstrate the end-to-end PEP, symbol error probability (SEP) and bit error probability (BEP) for various source-relay, relay-destination and source-destination channel

$$\begin{aligned}P_e &\le |\mathcal{C}|\left(\frac{\eta_0}{P}\right)^{N^2+N^2K}\frac{(4N)^{N^2+N^2K}}{\left(\sigma_{\min}^2\right)^{N}\left(\delta_{sd}^2\right)^{N^2}}\sum_{j=0}^{K}\left(\prod_{w=j+1}^{K}\frac{1}{\left(\left(\delta_{rd}^{(w)}\right)^2\beta_w\right)^{N^2}}\prod_{k=1}^{j}\frac{1}{\left(\left(\delta_{sr}^{(k)}\right)^2\right)^{N^2}}\right)\frac{\zeta\left(N^2+N^2(K-j)\right)\left(\kappa\zeta(N^2)\right)^j}{\left(\sigma_{\min}^2\right)^{N(K-j)}(\beta_0)^{N^2+jN^2}}\\ &=\sum_{j=0}^{K}\prod_{w=j+1}^{K}\underbrace{|\mathcal{C}|\left(\frac{\eta_0}{P}\right)^{N^2(K+1)}\frac{(4N)^{N^2(K+1)}}{\left(\sigma_{\min}^2\right)^{N}\left(\delta_{sd}^2\right)^{N^2}}\frac{\zeta\left(N^2+N^2(K-j)\right)\left(\kappa\zeta(N^2)\right)^j}{\left(\sigma_{\min}^2\right)^{N(K-j)}\left(\left(\delta_{rd}^{(w)}\right)^2\right)^{N^2}}\prod_{k=1}^{j}\frac{1}{\left(\left(\delta_{sr}^{(k)}\right)^2\right)^{N^2}}}_{D_j}\frac{1}{(\beta_0)^{N^2(j+1)}(\beta_w)^{N^2}}\end{aligned}\tag{41}$$

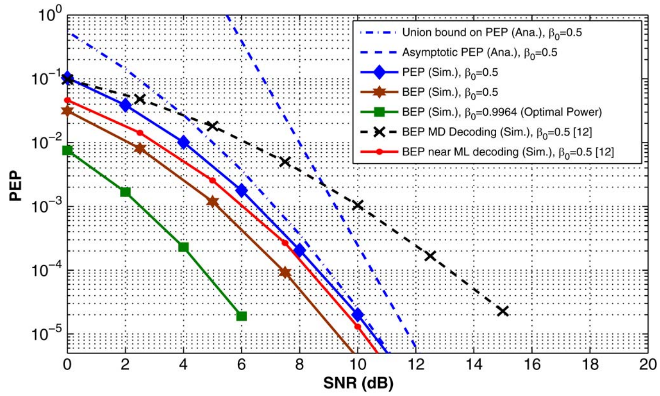


Fig. 6. Exact PEP, asymptotic upper bound, simulated PEP and BEP for a Alamouti-coded selective DF-MIMO cooperative communication with $K = 1$, $N = N_d = 2$ and $\delta_{s,d}^2 = 2, \delta_{s,r}^2 = 2$, $\delta_{r,d}^2 = 2000$.

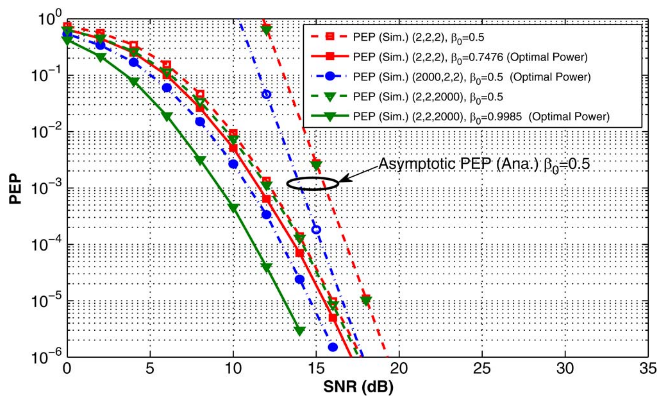


Fig. 7. Exact PEP, asymptotic upper bound, simulated PEP for a $2\times2$ Golden-coded selective DF-MIMO cooperative communication with $K = 1$ and $N = N_d = 2$.

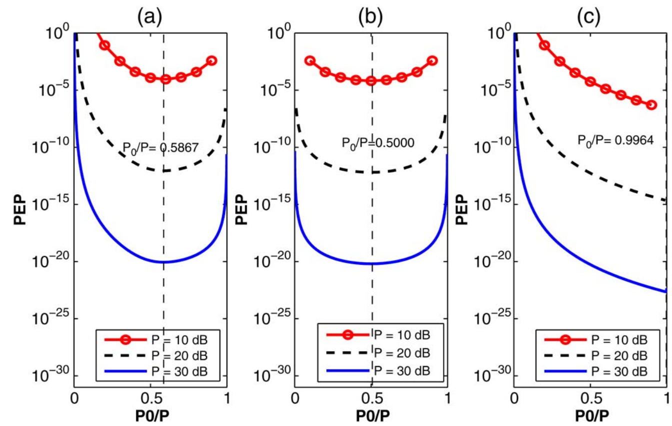


Fig. 8. PEP of Alamouti-coded selective DF-MIMO cooperative scheme for $K = 1, N = N_d = 2$ vs $P_0/P$ with $\delta_{sd}^2 = 2$ (a) $\delta_{sr}^2 = 2, \delta_{rd}^2 = 2$ (b) $\delta_{sr}^2 = 2000, \delta_{rd}^2 = 2$ (c) $\delta_{sr}^2 = 2, \delta_{rd}^2 = 2000$.

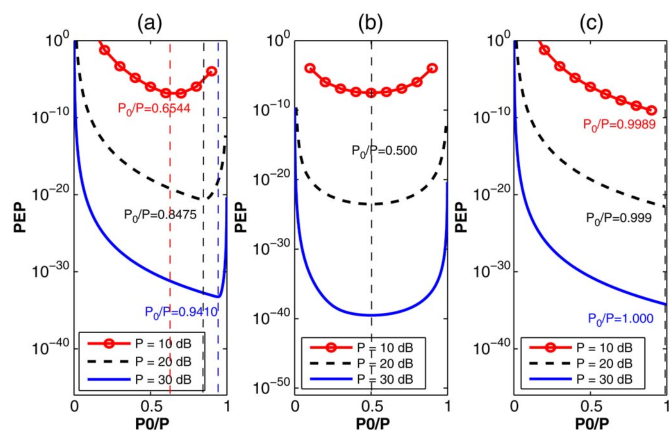


Fig. 9. PEP of Alamouti-coded selective DF-MIMO cooperative scheme for $K = 1, N = 2, N_d = 4$ vs $P_0/P$ with $\delta_{sd}^2 = 2$ (a) $\delta_{sr}^2 = 2, \delta_{rd}^2 = 2$ (b) $\delta_{sr}^2 = 2000, \delta_{rd}^2 = 2$ (c) $\delta_{sr}^2 = 2, \delta_{rd}^2 = 2000$.

conditions of a selective DF protocol based single relay system. The corresponding PEP bound and asymptotic PEP approximation derived in (9) and (11) respectively are also shown therein, which can be seen to be in close agreement with the simulation results. The plots also validate the result in (11) that the selective DF MIMO relaying system achieves a diversity order of $NN_d + N\min\{N, N_d\}$. Plots are also given for the BEP with the optimal power factor $\beta_0$ in each scenario, which can be seen to achieve a lower BEP compared to equal power allocation, i.e., $\beta_0 = 0.5$. Thus, the optimal power factors $\beta_0$ and $\beta_1$ yield a significant performance enhancement in terms of the BEP of decoding at the destination node in cooperative MIMO STBC systems. For comparison, we also plot the performance of the suboptimal fixed DF relaying based near ML decoder and the minimum distance (MD) decoder in [12], which can be seen to result in a higher BEP. This performance gain is more pronounced in scenarios such as the one considered in Fig. 6 where the source relay link is of poor quality. This arises due to the fact that the codeword matrix decoded at the relay is transmitted to the destination irrespective of the SNR conditions at the relay, which results in a high BEP at the destination when the source-relay link is of poor quality. Further, in order to demonstrate that the analytical framework developed in this paper is valid for general STBCs and not restricted to OSTBCs such as the Alamouti code, we also simulate the PEP performance of the system for the $2\times2$ full rate Golden code [19] as shown in Fig. 7. It has also been demonstrated that the end-to-end performance of the system with the Golden code can be further improved using the optimal power allocation derived in (14). In Figs. 8–10 we plot the PEP bound for the QPSK based cooperative STBC system considered above with respect to the power factor $\beta_0 = P_0/P$ for $P \in \{10, 20, 30\}$ dB, $N_d = 2, 4$ under several channel conditions. One can observe that the optimal power allocation obtained by solving the KKT equation in (14), is able to achieve the minimum PEP over a wide SNR range. Similar to the result in [8], the optimal power factors $\beta_0, \beta_1$ depend only on the average source-relay and relay-destination channel gains. Also, $\beta_0$ can be seen to satisfy $\beta_0 \geq 0.5$, leading to the conclusion that a dominant fraction of power has to be allocated to the source to optimize performance. For a stronger source-relay link, i.e., $\delta_{sr}^2 \gg \delta_{rd}^2$, the optimal value of $\beta_0 = 0.5$ as seen in Figs. 8(b)–10(b), implying that equal source-relay power allocation is optimal since decoding at the relay is accurate

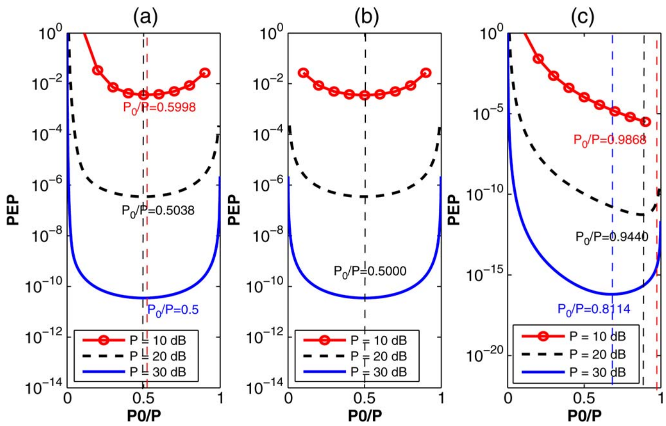


Fig. 10. PEP of Alamouti-coded selective DF-MIMO cooperative scheme for $K = 1, N = 2, N_d = 1$ vs $P_0/P$ with $\delta_{sd}^2 = 2$ (a) $\delta_{sr}^2 = 2, \delta_{rd}^2 = 2$ (b) $\delta_{sr}^2 = 2000, \delta_{rd}^2 = 2$ (c) $\delta_{sr}^2 = 2, \delta_{rd}^2 = 2000$.

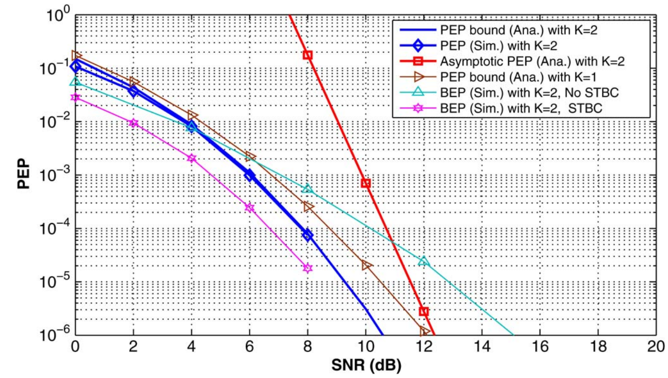


Fig. 12. Exact PEP, asymptotic upper bound, simulated PEP and BEP for a $C(0)$ based Alamouti-coded selective DF-MIMO cooperative communication with $K = N = N_d = 2$ and $\delta_{s,d}^2 = 2, \left(\delta_{s,r}^{(k)}\right)^2 = 2000, \left(\delta_{r,d}^{(k)}\right)^2 = 2, \forall\ 1 \le k \le 2$.

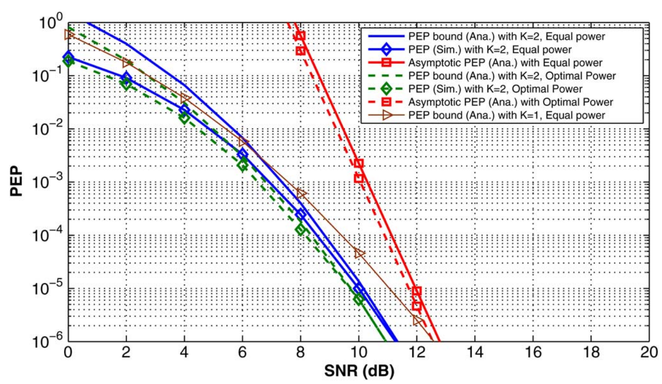


Fig. 11. Exact PEP, asymptotic upper bound and simulated PEP for a $C(0)$ based Alamouti-coded selective DF-MIMO cooperative communication with $K = N = N_d = 2$ and $\delta_{s,d}^2 = 2, \left(\delta_{s,r}^{(k)}\right)^2 = 2, \left(\delta_{r,d}^{(k)}\right)^2 = 2, \forall\ 1 \le k \le 2$.

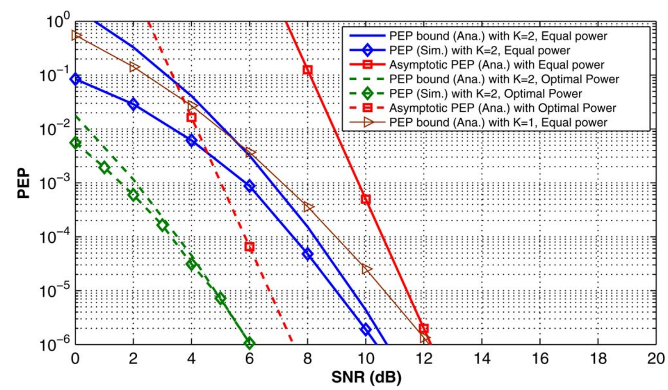


Fig. 13. Exact PEP, asymptotic upper bound and simulated PEP for a $C(0)$ based Alamouti-coded selective DF-MIMO cooperative communication with $K = N = N_d = 2$ and $\delta_{s,d}^2 = 2, \left(\delta_{s,r}^{(k)}\right)^2 = 2, \left(\delta_{r,d}^{(k)}\right)^2 = 2000, \forall\ 1 \le k \le 2$.

with very high probability. On the other hand, as seen from Figs. 8(c)–10(c), the optimal power factor $\beta_0 \approx 1$ when $\delta_{sr}^2 \ll \delta_{rd}^2$, implying that a substantial fraction of the power is allocated to the source. Further, very interestingly, even when $\delta_{sr}^2 \ll \delta_{rd}^2$, if $N_d < N$, the power factor approaches 0.5, which implies that the diversity gain due to the large number of antennas at the relay offsets the poor source-relay channel condition, making equal source-relay power allocation optimal.

Figs. 11–13 show the performance of the proposed $C(0)$ protocol based multiple relay cooperative system, with $K = 2$ relays, described in Section IV. The results demonstrate that the diversity order is indeed equal to $NN_d + N_d K \min\{N, N_d\}$ as derived in (24). Further the performance of the system is significantly improved with the optimal power factors $\beta_0, \beta_k, 1 \le k \le 2$ obtained by solving the optimization problem in (30). This is formulated as a GP and solved using the CVX solver for the various channel conditions described in Table I. The performance gain with respect to that of a $C(0)$ based uncoded multi-relay system is also shown therein as a benchmark. Figs. 14–16 show the performance of the cooperative multi-relay system for the $C(m)$ protocol with $m = 1$. The analytical PEP from (38) and the asymptotic bound from (41) are in close agreement with the simulated performance. Further, the performance significantly improves with the optimal power factors $\beta_0, \beta_k, 1 \le k \le 2$ compared to the suboptimal factors $\beta_0 = \beta_1 = \beta_2 = \frac{1}{3}$ corresponding to equal power allocation. Also, the performance of the single antenna $C(1)$ based system presented in [8] is given in the figure for purposes of comparison, which can be seen to achieve a diversity that is significantly lower than the one corresponding to the MIMO-STBC cooperative system. To demonstrate the effect of $m$ on the performance of the proposed $C(m)$ system, we consider a scenario with $K = 3$ relays. It can be clearly observed from Fig. 17 that increasing the value of $m$ results only in a coding gain as demonstrated analytically in Section V-A. Further, it can be observed from Table I that when source-relay links are relatively strong in comparison to the relay-destination links, equal power allocation at each node is optimal for both $C(0)$ and $C(1)$ schemes. On the other hand, when the source-relay links have a relatively poor strength in comparison to the relay-destination links or when the source-relay and relay-destination links have approximately similar strengths, more power is allocated to the source in comparison to the relays in both $C(0)$ and $C(1)$ schemes. The remaining power is equally distributed among both the relays in the $C(0)$ scheme, whereas in the $C(1)$ scheme, the power allocated to the

TABLE I

OPTIMAL POWER ALLOCATION FOR $C(0)$ AND $C(m)$, $m = 1$ WITH $K = N_d = N = 2, \left(\delta_{sd}^{(k)}\right)^2 = 2,\ \forall k$

| Channel Condition for SR and RD links | $\beta_0$, $C(0)$ | $\beta_1$, $C(0)$ | $\beta_2$, $C(0)$ | $\beta_0$, $C(1)$ | $\beta_1$, $C(1)$ | $\beta_2$, $C(1)$ |
|---|---|---|---|---|---|---|
| $\left(\delta_{sr}^{(k)}\right)^2 = \left(\delta_{rd}^{(k)}\right)^2 = 2,\ \forall\ k$ | 0.4250 | 0.2875 | 0.2875 | 0.4055 | 0.2754 | 0.3191 |
| $\left(\delta_{sr}^{(k)}\right)^2 = 20,\ \left(\delta_{rd}^{(k)}\right)^2 = 2,\ \forall\ k$ | 0.3334 | 0.3333 | 0.3333 | 0.3334 | 0.3333 | 0.3333 |
| $\left(\delta_{sr}^{(k)}\right)^2 = 2,\ \left(\delta_{rd}^{(k)}\right)^2 = 20,\ \forall\ k$ | 0.7951 | 0.1024 | 0.1024 | 0.8108 | 0.0761 | 0.1221 |
| $\left(\delta_{sr}^{(k)}\right)^2 = 2000,\ \left(\delta_{rd}^{(k)}\right)^2 = 2,\ \forall\ k$ | 0.3333 | 0.3333 | 0.3333 | 0.3333 | 0.3333 | 0.3333 |
| $\left(\delta_{sr}^{(k)}\right)^2 = 2,\ \left(\delta_{rd}^{(k)}\right)^2 = 2000,\ \forall\ k$ | 0.9934 | 0.0033 | 0.0033 | 0.9953 | 0.0013 | 0.0036 |

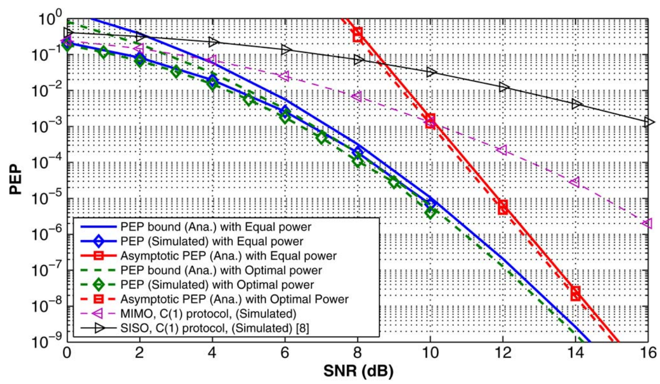


Fig. 14. Exact PEP, asymptotic upper bound and simulated PEP for a $C(m)$ based Alamouti-coded selective DF-MIMO cooperative communication with $m = 1, K = N = N_d = 2$ and $\delta_{s,d}^2 = 2$, $\left(\delta_{r,r}^{(1,2)}\right)^2 = 2$, $\left(\delta_{s,r}^{(k)}\right)^2 = 2$, $\left(\delta_{r,d}^{(k)}\right)^2 = 2,\ \forall 1 \le k \le 2$.

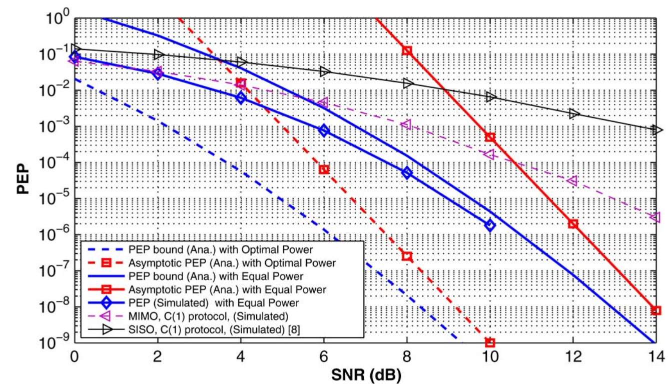


Fig. 16. Exact PEP, asymptotic upper bound and simulated PEP for a $C(m)$ based Alamouti-coded selective DF-MIMO cooperative communication with $m = 1, K = N = N_d = 2$ and $\delta_{s,d}^2 = 2$, $\left(\delta_{r,r}^{(1,2)}\right)^2 = 2$, $\left(\delta_{s,r}^{(k)}\right)^2 = 2$, $\left(\delta_{r,d}^{(k)}\right)^2 = 2000,\ \forall 1 \le k \le 2$.

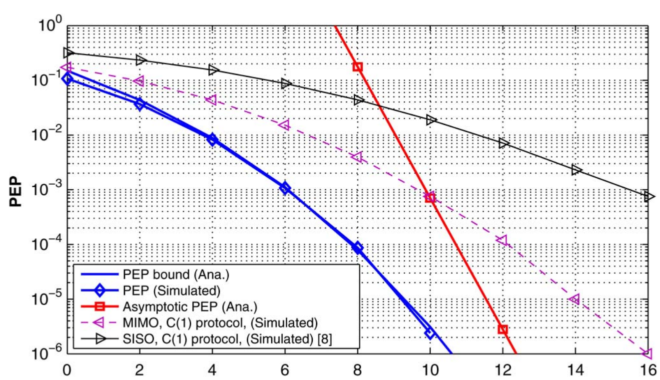


Fig. 15. Exact PEP, asymptotic upper bound and simulated PEP for a $C(m)$ based Alamouti-coded selective DF-MIMO cooperative communication with $m = 1, K = N = N_d = 2$ and $\delta_{s,d}^2 = 2$, $\left(\delta_{r,r}^{(1,2)}\right)^2 = 2$, $\left(\delta_{s,r}^{(k)}\right)^2 = 2000$, $\left(\delta_{r,d}^{(k)}\right)^2 = 2,\ \forall 1 \le k \le 2$.

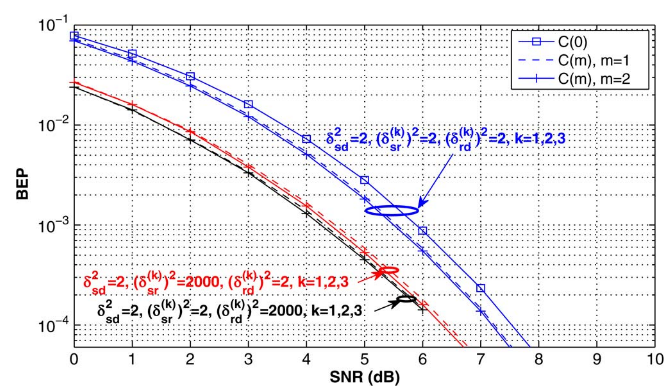


Fig. 17. Simulated BEP for $C(m)$, $m = 1, 2$ and $C(0)$ based Alamouti-coded selective DF-MIMO cooperative communication with equal power allocation, $K = 3, N = N_d = 2$ and $\left(\delta_{r,r}^{(1,2)}\right)^2 = \left(\delta_{r,r}^{(2,3)}\right)^2 = 2$.

second relay is higher than the power allocated to the first relay. This can be expected since the second relay is more reliable than the first relay in the $C(1)$ scheme, as it combines the signals from both the source and the first relay prior to decoding, while the first relay decodes employing only the signal received from the source.

## VII. CONCLUSION

This work presents a comprehensive analysis of a selective DF relaying based MIMO STBC cooperative wireless system. The end-to-end decoding performance, along with the diversity order and optimal source-relay power allocation has been presented initially for a single relay based cooperative scenario. Subsequently, the end-to-end performance results, along with the diversity order and optimal power allocation have been derived for the multiple relay scenario considering the two phase $C(0)$ cooperation protocol and also the general multiphase $C(m)$ protocol. Simulation results demonstrate a close agreement with the derived analytical results and also the performance benefits of optimal power allocation along with the improvement over existing schemes.

## APPENDIX A
## SIMPLIFICATION OF $P_{S\to R}(\mathbf{X}_0 \to \mathbf{X}_j)$

After employing the identity for $Q(\cdot)$ function, the expression for $P_{S\to R}(\mathbf{X}_0 \to \mathbf{X}_j)$ can be simplified as follows to yield the final expression in (3).

$$
\begin{aligned}
&P_{S\to R}(\mathbf{X}_0 \to \mathbf{X}_j)\\
&= \frac{1}{\pi}\int_{\theta=0}^{\pi/2}\prod_{n=1}^{N}\prod_{\tilde{n}=1}^{N}\int_{0}^{\infty}\frac{1}{\delta_{sr}^2}\exp\left(-\frac{\tilde{g}_{\tilde{n},n}}{\delta_{sr}^2}\right)\\
&\quad\times\exp\left(-\frac{P_0\lambda_{jn}^2\tilde{g}_{\tilde{n},n}}{4N\eta_0\sin^2\theta}\right)d\tilde{g}_{\tilde{n},n}d\theta\\
&= \frac{1}{\pi}\int_{\theta=0}^{\pi/2}\prod_{n=1}^{N}\prod_{\tilde{n}=1}^{N}\frac{1}{\delta_{sr}^2}\\
&\quad\times\int_{0}^{\infty}\exp\left(-\left(\frac{1}{\delta_{sr}^2}+\frac{P_0\lambda_{jn}^2}{4N\eta_0\sin^2\theta}\right)\tilde{g}_{\tilde{n},n}\right)d\tilde{g}_{\tilde{n},n}d\theta\\
&= \frac{1}{\pi}\int_{\theta=0}^{\pi/2}\prod_{n=1}^{N}\prod_{\tilde{n}=1}^{N}\frac{\frac{1}{\delta_{sr}^2}}{\left(\frac{1}{\delta_{sr}^2}+\frac{P_0\lambda_{jn}^2}{4N\eta_0\sin^2\theta}\right)}d\theta\\
&= \frac{1}{\pi}\int_{\theta=0}^{\pi/2}\frac{1}{\prod_{n=1}^{N}\prod_{\tilde{n}=1}^{N}\left(1+\frac{P_0\lambda_{jn}^2\delta_{sr}^2}{4N\eta_0\sin^2\theta}\right)}d\theta
\end{aligned}
$$

## APPENDIX B
## DIVERSITY ORDER ANALYSIS FOR SINGLE RELAY BASED MIMO-STBC SELECTIVE DF COOPERATIVE COMMUNICATION

In order to demonstrate the achieved diversity order of the system, the expression given in (11) can be written as,

$$
P_e \le C_1\left(\frac{\eta_0}{P}\right)^{NN_d+N^2} + C_2\left(\frac{\eta_0}{P}\right)^{2NN_d},
$$

where the terms $C_1$ and $C_2$ are appropriately defined constants. Now, consider the case when $N_d > N$, i.e., the number of antennas at the destination is greater than the number of antennas at the source and relay. For this case, under high SNR conditions, it can be readily observed that the first term dominates the above PEP expression and thus the net end-to-end PEP of the system with $N_d > N$ can be written as,

$$
P_e \le C_1\left(\frac{\eta_0}{P}\right)^{NN_d+N^2}.
$$

Now, using the above given equation one can readily derive the achieved diversity order as, $-\lim_{\frac{P}{\eta_0}\to\infty}\frac{\log(P_e)}{\log\left(\frac{P}{\eta_0}\right)} = NN_d + N^2$. On the other hand, for the case when $N_d < N$, the second term will dominate at high SNR and thus the achieved diversity order can be similarly derived as $2NN_d$. Finally, the general expression for the diversity order of the system can be written as, $\min\{NN_d + N^2, 2NN_d\} = NN_d + N\min\{N, N_d\}$. Note that for the case when $N = N_d$, both the terms in (11) will contribute and the achieved diversity order can be easily seen as $NN_d + N\min\{N, N_d\} = 2N^2$.

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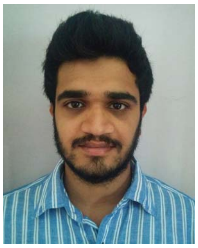

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