

Soft Copy Assignment–1 of ME-4237: Decision Modelling (LPP)

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Answer only the question allotted to you as shown in last page and cross check the answer using MATLAB software. Answer should be typed in MS Word and the word version be uploaded at moodle.nitrkl.ac.in by 7.00 AM on 26.08.24 (Monday). The file name should be in prescribed format eg. sca1q9air121me0444 means Soft Copy Assignment-1 Question No. 9a(i) submitted by the student whose roll no is 121ME0444.

1. A finished product must weigh exactly 150 g. The two raw materials used in manufacturing product are 'A' with a cost of Rs. 2 per unit and 'B' with a cost of Rs. 8 per unit. At least 14 units of 'B' and not more than 20 units of 'A' must be used. Each unit of 'A' and 'B' weighs 5 g and 10 g respectively. How much of each type of raw material should be used for each unit of the final product in order to minimize the cost. Use i) graphical method ii) simplex method. [Ans:  $z = 116$ ,  $a = 2$ ,  $b = 14$ ]
2. M/s Kirloskar Electric company produces two types of electric motors on two separate assembly lines. The respective daily capacities of the two lines are 600 and 700 motors. Type 1 motor uses 10 armatures and type 2 motor uses 8 armatures. The supplier of armatures can provide 8000 pieces a day. The profits per motor for types 1 and 2 are Rs. 60 and Rs. 40 respectively. Determine optimal production mix.
  - i) Solve it graphically and find out the range of ratio of unit profits that will keep the solution unchanged.
  - ii) Solve it by simplex method.[Ans:  $z = 46000$ ,  $a=600$ ;  $b =250$ ,]
3. A toy industry produces two types of toys. Type 1 toy requires twice as much labour time as Type 2. If all the available labour time is dedicated to Type 2 alone it can produce a total of 400 Type 2 hats a day. The maximum sales in the local market for the two types are 150 and 200 toys per day. The profit is Rs. 8 per Type 1 and Rs. 5 per Type 2 toy.
  - i) Use graphical solution to determine the number of toys of each type that would maximize profit. [Ans:  $z = 1800$ ,  $a = 100$ ,  $b = 200$ ]
  - ii) Solve the above problem using simplex method.
  - iii) Determine the worth of increasing the production capacity by one Type 2 toy and the range for which this result will be applicable. [Ans: Rs. 2, 200-500]
  - iv) If the daily demand limit on Type 1 toy is decreased to 120, use the unit worth of the resource to determine the corresponding effect on the optimal profit. [Ans: no effect; Rs. 0, 100- $\infty$ ]
  - v) What is the worth per unit increase in the market share of Type 2 toy? By how much can the market share be increased while yielding the computed worth per unit? [ Rs. 1, 100-400]
4. A company can advertise its products on radio or television. The advertisement budget is limited to Rs. 10,000 per month. Each minute of advertisement costs Rs. 15 and that of TV costs Rs. 300. It likes to use radio advertisement at least twice as much as TV. It's not practical to use more than 400 minutes of radio advertisement a month. Past experience shows that TV advertisement is estimated to be 25 times effective than that of radio.

- a) Determine the optimal allocation of the budget to radio and TV advertisements i) graphically ii) by simplex method [Ans:  $a = 60.61$ ,  $b = 30.3$ ,  $z = 818.11$ ]
- b) Find out worth per unit of increasing the monthly limit on radio advertisement [Ans:  $818.18/60.61$  when  $a$  is changed from 0 to 60.61 and 0 if  $a > 60.61$ ]
- c) If the monthly budget is increased to Rs. 15000, what would be the new optimum measure of advertisement effectiveness and find out corresponding worth per unit this resource? [Ans: 1227.27, 0.0818]

5. M/s Lee produces shirts and trousers. The process includes cutting, sewing and packaging. It employs 25, 25 and 5 workers in cutting, sewing and packaging departments. The factory works one 8-hour shift, 5 days a week. The following table gives the time requirements and profits per unit for the two garments.

Garment	Minutes/ unit			Profit Rs/ unit
	Cutting	Sewing	Packaging	
Shirt	20	70	12	8
Trouser	60	60	4	12

- a) Determine the optimal weekly production schedule i) graphically ii) by simplex.
- b) Find out worth per hour of cutting, sewing and packaging.  
[Ans:  $z = 200$ ,  $a = 0$ ,  $b = 16.67$ ]

6. A manufacturer can produce two different products A and B during a given time period. Each of these products requires four different manufacturing processes: Grinding, Turning, Assembly and Testing. Manufacturing requirements in hours per unit of product are given below for A and B

Products/Operations	Grinding	Turning	Assembly	Testing
A	2	4	7	6
B	3	2	4	5

The available capacities of these operations in hours for a given time period are: Turning- 80, Assembly- 300, Testing- 200 and Grinding- 40. The contribution to profit is Rs.8.00 for each unit of A and Rs.10.00 for each unit of B. Formulate a linear programming model and find its optimal solution using i) graphical method ii) simplex method. [Ans:  $z=160$ ,  $x=20$ ,  $y=0$ ]

7. The standard weight of a special purpose brick is 6 kg and it contains two ingredients p and q. p costs Rs.6.00 per kg and q costs Rs.9.00 per kg. Strength considerations dictate that the brick contains not more than 5 kg of p and a minimum of 3 kg of q since the demand for product is likely to be related to the price of the brick. Formulate the problem as a L.P. model and find its optimal solution using i) graphical method ii) simplex method. [Ans:  $z=45$ ,  $x=3$ ,  $y=3$ ]
8. Egg contains 7 units of vitamin A and 8 units of vitamin B per gram and costs Rs.10 per gram. Milk contains 9 units of vitamin A and 11 units of vitamin B per gram and costs Rs.12 per gram. The daily minimum requirement of vitamin A and vitamin B are 125 units and 118 units respectively. Formulate a L.P. model of the problem and find its optimal solution using i) graphical method ii) simplex method. [Ans:  $z=166.67$ ,  $x=0$ ,  $y=13.89$ ]

9. Solve the following LPPs i) graphically and ii) by simplex method

a. Maximize  $z = 2x_1 + 3x_2$

Subject to:  $x_1 + 3x_2 \leq 6$

$3x_1 + 2x_2 \leq 6$

and  $x_1, x_2 \geq 0$ .

[Ans:  $z = 48/7 = 6.85$ ,  $x_1 = 0.857$ ,  $x_2 = 1.714$ ]

b. Maximize  $z = 6x_1 + 8x_2$

Subject to:  $5x_1 + 2x_2 \leq 20$

$x_1 + 2x_2 \leq 10$

and  $x_1, x_2 \geq 0$ .

[Ans:  $z = 45$ ,  $x_1 = 2.5$ ,  $x_2 = 3.75$ ]

c. Maximize  $z = 20x_1 + 10x_2$

Subject to:  $x_1 + x_2 \leq 3$

$3x_1 + x_2 \leq 7$

and  $x_1, x_2 \geq 0$ .

[Ans:  $z = 50$ ,  $x_1 = 2$ ,  $x_2 = 1$ ]

d. Maximize  $z = 3x_1 + 2x_2$

Subject to:  $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

and  $x_1, x_2 \geq 0$ .

[Ans:  $z = 11$ ,  $x_1 = 3$ ,  $x_2 = 1$ ]

e. Maximize  $z = 3x_1 + 2x_2$

Subject to:  $4x_1 + 3x_2 \leq 12$

$4x_1 + x_2 \leq 8$

$4x_1 - x_2 \leq 8$

and  $x_1, x_2 \geq 0$ .

[Ans:  $z = 17/2$ ,  $x_1 = 3/2$ ,  $x_2 = 2$ ]

f. Maximize  $z = 3x_1 + 5x_2$

Subject to:  $x_1 \leq 4$

$x_2 \leq 6$

$3x_1 + 2x_2 \leq 18$

and  $x_1, x_2 \geq 0$ .

[Ans:  $z = 36$ ,  $x_1 = 2$ ,  $x_2 = 6$ ]

g. Maximize  $z = 2x_1 + 4x_2$

Subject to:  $2x_1 + 3x_2 \leq 60$

$x_1 + 3x_2 \leq 36$

$x_2 \leq 10$

and  $x_1, x_2 \geq 0$ .

[Ans:  $z = 64$ ,  $x_1 = 24$ ,  $x_2 = 4$ ]

h. Maximize  $z = 2x_1 + x_2$

Subject to:  $x_2 \leq 10$

$2x_1 + 5x_2 \leq 60$

$x_1 + x_2 \leq 18$

$3x_1 + x_2 \leq 44$

and  $x_1, x_2 \geq 0$ .

[Ans:  $z = 31$ ,  $x_1 = 13$ ,  $x_2 = 5$ ]

i. Maximize  $z = 7x_1 + 9x_2$

Subject to:  $2x_1 + x_2 \leq 100$

$x_1 + 2x_2 \leq 120$

and  $x_1, x_2 \geq 0$ .

[Ans:  $z = 1820/3, x_1 = 80/3, x_2 = 140/3$ ]

j. Maximize  $z = 5x_1 + 7x_2$

Subject to:  $x_1 + x_2 \leq 4$

$$3x_1 + 8x_2 \leq 24$$

$$10x_1 + 7x_2 \leq 35$$

and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 1.6, x_2 = 2.4, z = 24.8$ ]

k. Maximize  $z = x_1 - 3x_2$

Subject to:  $x_1 + x_2 \leq 300$

$$x_1 - 2x_2 \leq 200$$

$$2x_1 + x_2 \leq 100$$

$$x_2 \leq 200$$

and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 50, x_2 = 0, z = 50$ ]

l. Maximize  $z = 5x_1 + 8x_2$

Subject to:  $3x_1 + 2x_2 \leq 36$

$$x_1 + 2x_2 \leq 20$$

$$3x_1 + 4x_2 \leq 42$$

and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 2, x_2 = 9, z = 82$ ]

m. Maximize  $z = 3x_1 + 9x_2$

Subject to:  $x_1 + 4x_2 \leq 8$

$$x_1 + 2x_2 \leq 4$$

and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 0, x_2 = 2, z = 18$ ]

n. Maximize  $z = 2x_1 + x_2$

Subject to:  $4x_1 + 3x_2 \leq 12$

$$4x_1 + x_2 \leq 8$$

$$4x_1 - x_2 \leq 8$$

and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 1.5, x_2 = 2, z = 5$ ]

o. Maximize  $z = x_1 + 3x_2$

Subject to:  $x_1 + x_2 \leq 300$

$$x_1 - 2x_2 \leq 200$$

$$x_1 + x_2 \leq 100$$

$$x_2 \geq 200$$

and  $x_1, x_2 \geq 0$ .

[Ans:  $A = 100 \neq 0$ , so infeasible solution space]

p. Maximize  $z = 10x_1 + 6x_2$

Subject to:  $x_1 + x_2 \leq 2$

$$2x_1 + x_2 \leq 4$$

$$3x_1 + 8x_2 \leq 12$$

and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 2, x_2 = 0, z = 20$ ]

q. Maximize  $z = 8x_1 + 5x_2$

Subject to:  $x_1 + x_2 \leq 250$

$$x_1 \leq 150$$

$$x_2 \leq 250$$

and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 150, x_2 = 100, z = 1700$ ]

r. Maximize  $z = 6x_1 + 4x_2$   
 Subject to:  $-2x_1 + x_2 \leq 2$   
 $x_1 - x_2 \leq 2$   
 $3x_1 + 2x_2 \leq 9$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 13/5, x_2 = 3/5, z = 18$ ]

s. Maximize  $z = 3x_1 + 5x_2$   
 Subject to:  $x_1 \leq 4, x_2 \leq 6$   
 $3x_1 + 2x_2 \leq 18$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 2, x_2 = 6, z = 36$ ]

t. Maximize  $z = x_1 + 2x_2$   
 Subject to:  $x_1 + 2x_2 \leq 12$   
 $-x_1 + 2x_2 \leq 8$   
 $x_1 - 2x_2 \leq 3$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 2, x_2 = 5, z = 12$ ]

u. Maximise  $z = 8x_1 + 16x_2$

subject to  $x_1 + x_2 \leq 200$   
 $x_2 \leq 125$   
 $3x_1 + 6x_2 \leq 900$   
 $x_1, x_2 \geq 0$

[Ans: multiple solutions]

v. Maximize  $z = 20x_1 + 10x_2$   
 Subject to:  $x_1 + 2x_2 \leq 40$   
 $3x_1 + x_2 \leq 30$   
 $4x_1 + 3x_2 \leq 60$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 6, x_2 = 12, z = 240$ ]

10. Solve the following LPPs i) graphically and ii) by simplex method

a. Maximize  $z = 20x_1 + 10x_2$   
 Subject to:  $x_1 + 2x_2 \leq 40$   
 $3x_1 + x_2 \geq 30$   
 $4x_1 + 3x_2 \geq 60$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 40, x_2 = 0, z = 800$ ]

b. Maximize  $z = 3x_1 + 4x_2$   
 Subject to:  $5x_1 + 4x_2 \leq 200$   
 $3x_1 + 5x_2 \leq 150$   
 $5x_1 + 4x_2 \geq 100$   
 $2x_1 + x_2 \geq 20$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 30.8, x_2 = 11.5, z = 138.4$ ]

c. Maximize  $z = 2x_1 + 3x_2$   
 Subject to  $x_1 + 2x_2 \leq 4$   
 $x_1 + x_2 = 3$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 2, x_2 = 1, z = 7$ ]

d. Maximize  $z = 2x_1 + 5x_2$   
 Subject to  $3x_1 + 2x_2 \geq 6$   
 $2x_1 + x_2 \leq 2$

and  $x_1, x_2 \geq 0$

[Infeasible solution]

- e. Maximize  $z = -x_1 + 4x_2$   
Subject to  $-3x_1 + x_2 \leq 6$   
 $x_1 + 2x_2 \leq 4$   
 $x_2 \geq -3$

and  $x_2 \geq 0$ , no lower bound constraint for  $x_1$

[Ans:  $x_1 = -1.143, x_2 = 2.571, z = 80/7$ ]

- f. Maximize  $z = x_1 + 2x_2$   
Subject to:  $x_1 + 2x_2 \leq 10$   
 $x_1 + x_2 \geq 1, x_2 \leq 4$   
and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 10, x_2 = 0, z = 10$ ]

- g. Maximize  $z = 3x_1 + 2x_2$   
Subject to:  $5x_1 + x_2 \geq 10$   
 $x_1 + x_2 \geq 6$   
 $x_1 + 4x_2 \geq 12$   
and  $x_1, x_2 \geq 0$ .

[unbounded]

- h. Maximize  $z = 6x_1 + 4x_2$   
Subject to:  $2x_1 + 3x_2 \geq 30$   
 $3x_1 + 2x_2 \leq 24$   
 $x_1 + x_2 \geq 3$   
and  $x_1, x_2 \geq 0$ .

[infinite alternate solutions each with  $z = 48$ ]

- i. Maximize  $z = 2x_1 + x_2$   
Subject to:  $x_1 + x_2 \geq 2$   
 $x_1 + x_2 \leq 4$   
and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 4, x_2 = 0, z = 8$ ]

- j. Maximize  $z = x_1 + x_2$   
Subject to:  $x_1 + 2x_2 \leq 6$   
 $2x_1 + x_2 \geq 16$   
and  $x_1, x_2 \geq 0$ .

[Ans: Infeasible]

- k. Maximize  $z = 3x_1 + 2x_2$   
Subject to:  $x_1 - x_2 \leq 1$   
 $x_1 + x_2 \geq 3$   
and  $x_1, x_2 \geq 0$ .

[unbounded]

- l. Maximize  $z = x_1 + 2x_2$   
Subject to:  $x_1 + x_2 \leq 3$   
 $x_1 + 2x_2 \leq 5$   
 $3x_1 + x_2 \leq 6$   
and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 0, x_2 = 5/2, z = 5$ ]

- m. Maximize  $z = 10x_1 + 5x_2$   
Subject to:  $4x_1 + 2x_2 \leq 40$   
 $3x_1 + x_2 \leq 30$   
 $2x_1 + 6x_2 \leq 40$   
and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 10, x_2 = 0, z = 100$ ]

n. Maximize  $z = 3x_1 + 2x_2$   
Subject to:  $-2x_1 + x_2 \leq 1$   
 $x_1 \leq 2$   
 $x_1 + x_2 \leq 3$   
and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 2, x_2 = 1, z = 8$ ]

11. Solve the following LPPs by i) graphical method, ii) simplex method iii) solving its dual by simplex method, and iv) solving its dual by graphical method (if possible).

a. Minimize  $z = 12x_1 + 20x_2$   
Subject to:  $6x_1 + 8x_2 \geq 100$   
 $7x_1 + 12x_2 \geq 120$   
and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 15, x_2 = 5/4, z = 205$ ]

b. Minimize  $z = 3x_1 + 2x_2$   
Subject to:  $5x_1 + x_2 \geq 10$   
 $x_1 + x_2 \geq 6$   
 $x_1 + 4x_2 \geq 12$   
and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 1, x_2 = 5, z = 13$ ]

c. Minimize  $z = 4x_1 + x_2$   
Subject to  $3x_1 + x_2 = 3$   
 $4x_1 + 3x_2 \geq 6$   
 $x_1 + 2x_2 \leq 4$   
and  $x_1, x_2 \geq 0$

[Ans:  $z = 17/5, x_1 = 0.4, x_2 = 1.8$ ]

d. Minimize  $z = 3x_1 + 9x_2$   
Subject to  $x_1 + x_2 \geq 800$   
 $21x_1 - 30x_2 \leq 0$   
 $3x_1 - x_2 \geq 0$   
and  $x_1, x_2 \geq 0$

[Ans:  $x_1 = 470.59, x_2 = 329.41, z = 4376.47$ ]

e. Minimize  $z = 20x_1 + 10x_2$   
Subject to:  $x_1 + 2x_2 \leq 40$   
 $3x_1 + x_2 \geq 30$   
 $4x_1 + 3x_2 \geq 60$   
and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 6, x_2 = 12, z = 240$ ]

f. Minimize  $z = 4x_1 + x_2$   
Subject to:  $3x_1 + x_2 = 3$   
 $4x_1 + 3x_2 \geq 6$   
 $x_1 + 2x_2 \leq 4$   
and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 2/5, x_2 = 9/5, z = 17/5$ ]

g. Minimize  $z = 5x_1 + 3x_2$   
Subject to:  $x_1 + 2x_2 \leq 6$   
 $5x_1 + 2x_2 \geq 10$   
 $x_1 + x_2 = 5$   
and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 4, x_2 = 1, z = 23$ ]

h. Minimize  $z = 5x_1 + 6x_2$   
 Subject to:  $x_1 + x_2 \geq 2$   
 $4x_1 + x_2 \geq 4$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 2, x_2 = 0, z = 10$ ]

i. Minimize  $z = 3x_1 + x_2$   
 Subject to:  $x_1 + x_2 \geq 1$   
 $x_1 + 3x_2 \geq 2$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 0, x_2 = 1, z = 1$ ]

j. Minimize  $z = x_1 + x_2$   
 Subject to:  $x_1 + 2x_2 \geq 7$   
 $4x_1 + x_2 \geq 6$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 5/7, x_2 = 22/7, z = 27/7$ ]

k. Minimize  $z = 2x_1 + 3x_2$   
 Subject to:  $3x_1 + x_2 \geq 4$   
 $3x_1 + x_2 \leq 4$   
 $x_1 + 3x_2 \leq 6$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 4/3, x_2 = 0, z = 8/3$ ]

l. Minimize  $z = 3x_1 + 2x_2$   
 Subject to:  $x_1 + x_2 \geq 1$   
 $x_1 + x_2 \leq 7$   
 $x_1 + 2x_2 \geq 10$   
 $x_2 \leq 3$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 4, x_2 = 3, z = 18$ ]

m. Maximize  $z = 3x_1 - x_2$   
 Subject to:  $x_1 + x_2 \geq 1$   
 $2x_1 + 3x_2 \geq 2$   
 and  $x_1, x_2 \geq 0$ .

[unbounded]

n. Maximize  $z = x_1 + 2x_2$   
 Subject to:  $2x_1 + 5x_2 \geq 6$   
 $x_1 + x_2 \geq 2$   
 and  $x_1, x_2 \geq 0$ .

[unbounded]

o. Minimize  $z = 4x_1 + 3x_2$   
 Subject to:  $2x_1 + x_2 \geq 10$   
 $x_1 + x_2 \geq 6$   
 $-3x_1 + 2x_2 \leq 6$   
 and  $x_1, x_2 \geq 0$ .

[Ans:  $x_1 = 4, x_2 = 2, z = 22$ ]

p. Maximize  $z = 2x_1 + 3x_2 + 3x_3$   
 Subject to  $2x_1 - x_2 - x_3 \leq 3$   
 $x_1 - x_2 + x_3 \leq 2$   
 and  $x_1, x_2, x_3 \geq 0$ .

[unbounded]



12. Solve the following LPPs by i) simplex method ii) solving its dual by graphical if possible else by simplex method.

- a. Maximize  $z = 4x_1 + 3x_2 + 6x_3$   
 Subject to  $3x_1 + x_2 + 3x_3 \leq 30$   
 $2x_1 + 2x_2 + 3x_3 \leq 40$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=0, x_2=10, x_3=20/3, z=70$ ]

- b. Maximize  $z = 5x_1 + 2x_2 + 3x_3 - x_4 + x_5$   
 Subject to  $x_1 + 2x_2 + 2x_3 + x_4 = 8$   
 $3x_1 + 4x_2 + x_3 + x_5 = 7$   
 and  $x_1, x_2, x_3, x_4, x_5 \geq 0$ .

[Ans:  $x_1=1.2, x_3=3.4, x_4=x_5=x_2=0, z = 81/5$ ]

- c. Maximize  $z = 5x_1 + 3x_2 - x_3 + x_4$   
 Subject to  $x_1 + 2x_2 \leq 8$   
 $3x_1 + x_2 \leq 7$   
 and  $x_1, x_2, x_3, x_4 \geq 0$ .

[unbounded]

- d. Maximize  $z = x_1 + x_2 + 3x_3$   
 Subject to  $3x_1 + 2x_2 + x_3 \leq 3$   
 $2x_1 + x_2 + 2x_3 \leq 2$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=x_2=0, x_3=1, z = 3$ ]

- e. Maximize  $z = 2x_1 + 3x_2 + 10x_3$   
 Subject to  $x_1 + 2x_3 = 0$   
 $x_2 + x_3 = 1$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=x_3=0, x_2=1, z = 3$ ]

- f. Maximize  $z = 2x_1 + x_2 + 3x_3$   
 Subject to  $x_1 + x_2 + 2x_3 \leq 5$   
 $2x_1 + 3x_2 + 4x_3 = 12$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=3, x_2=2, x_3=0, z = 8$ ]

- g. Maximize  $z = 6x_1 - 2x_2 - 3x_3$   
 Subject to  $2x_1 - x_2 + 2x_3 \leq 2$   
 $x_1 + 4x_3 \leq 4$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=4, x_2=6, x_3=0, z = 12$ ]

- h. Maximize  $z = 3x_1 + 2x_2 + 5x_3$   
 Subject to  $x_1 + 2x_2 + x_3 \leq 430$   
 $3x_1 + 2x_3 \leq 260$   
 $x_1 + 4x_2 \leq 420$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=0, x_2=105, x_3=130, z = 860$ ]

- i. Maximize  $z = -x_1 + 3x_2 - 2x_3$   
 Subject to  $3x_1 - x_2 + 2x_3 \leq 7$   
 $-x_1 + 2x_2 \leq 6$   
 $-4x_1 + 3x_2 + 8x_3 \leq 10$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=4, x_2=5, x_3=0, z = 11$ ]

- j. Maximize  $z = x_1 + 2x_2 + x_3$   
 Subject to  $-2x_1 - x_2 + x_3 \leq 2$   
 $-2x_1 + x_2 - 5x_3 \leq 6$   
 $4x_1 + x_2 + x_3 \leq 6$   
 and  $x_1, x_2, x_3 \geq 0$ .  
 [Ans:  $x_1=0, x_2=6, x_3=0, z=12$ ]
- k. Maximize  $z = 5x_1 - 2x_2 + 3x_3$   
 Subject to  $2x_1 + 2x_2 - x_3 \geq 2$   
 $3x_1 - 4x_2 \leq 3$   
 $x_2 - 3x_3 \leq 5$   
 and  $x_1, x_2, x_3 \geq 0$ .  
 [unbounded]
- l. Maximize  $z = x_1 + 2x_2 + x_3$   
 Subject to  $2x_1 + x_2 - x_3 \leq 2$   
 $2x_1 - x_2 + 5x_3 \leq 6$   
 $4x_1 + x_2 + x_3 \leq 6$   
 and  $x_1, x_2, x_3 \geq 0$ .  
 [Ans:  $x_1=0, x_2=4, x_3=2, z=10$ ]
- m. Maximize  $z = 5x_1 - 4x_2 + 3x_3$   
 Subject to  $2x_1 + x_2 - 6x_3 = 20$   
 $6x_1 + 5x_2 + 10x_3 \leq 76$   
 $8x_1 - 3x_2 + 6x_3 \leq 50$   
 and  $x_1, x_2, x_3 \geq 0$ .  
 [Ans:  $x_1=55/7, x_2=30/7, x_3=0, z=155/7$ ]
- n. Maximize  $z = 3x_1 + 2x_2 + 5x_3$   
 Subject to  $x_1 + 2x_2 + x_3 \leq 430$   
 $3x_1 + 2x_3 \leq 460$   
 $x_1 + 4x_2 \leq 420$   
 and  $x_1, x_2, x_3 \geq 0$ .  
 [Ans:  $x_1=0, x_2=100, x_3=230, z=1350$ ]
- o. Maximize  $z = 15x_1 + 6x_2 + 9x_3 + 2x_4$   
 Subject to  $2x_1 + x_2 + 5x_3 + 6x_4 \leq 20$   
 $3x_1 + x_2 + 3x_3 + 25x_4 \leq 24$   
 $7x_1 + x_4 \leq 70$   
 and  $x_1, x_2, x_3, x_4 \geq 0$ .  
 [Ans:  $x_1=4, x_2=12, x_3=0, x_4=0, z=132$ ]
- p. Maximize  $z = x_1 + 2x_2 + 3x_3 - x_4$   
 Subject to  $x_1 + 2x_2 + 3x_3 = 15$   
 $2x_1 + x_2 + 5x_3 = 20$   
 $x_1 + 2x_2 + x_3 + x_4 = 10$   
 and  $x_1, x_2, x_3, x_4 \geq 0$ .  
 [Ans1:  $x_1=x_2=x_3=5/2, x_4=0, z=15$ ]

13. Solve the following LPPs by i) simplex method, ii) solving its dual by simplex method, iii) solving its dual by graphical method (if possible) and iv) solving its dual by dual simplex method.

- a. Maximize  $z = 10x_1 + 15x_2 + 12x_3$   
 Subject to  $5x_1 + 3x_2 + x_3 \leq 9$   
 $-5x_1 + 6x_2 + 15x_3 \leq 15$   
 $2x_1 + x_2 + x_3 \geq 5$

and  $x_1, x_2, x_3 \geq 0$ .

[Infeasible solution]

- b. Maximize  $z = 2x_1 + x_2 + x_3$   
 Subject to  $4x_1 + 6x_2 + 3x_3 \leq 8$   
 $3x_1 - 6x_2 - 4x_3 \leq 1$   
 $2x_1 + 3x_2 - 5x_3 \geq 4$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=9/7, x_2=10/21, x_3=0, z = 64/21$ ]

- c. Maximize  $z = 5x_1 - 2x_2 + 3x_3$   
 Subject to  $2x_1 + 2x_2 - x_3 \geq 2$   
 $3x_1 - 4x_2 \leq 3$   
 $x_2 + 3x_3 \leq 5$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=23/3, x_2=5, x_3=0, z = 85/3$ ]

- d. Maximize  $z = 4x_1 + 4x_2 + 5x_3 + 2x_4 + 5x_5$   
 Subject to  $3x_1 + 2x_2 - x_3 - 2x_4 + 4x_5 \leq 1$   
 $2x_1 + x_2 + 3x_3 + x_4 + 2x_5 \leq 1$   
 and  $x_1, x_2, x_3, x_4, x_5 \geq 0$ .

[Ans:  $x_1=x_3=x_5=0, x_2=3/4, x_4=1/4, z = 7/2$ ]

14. Solve the following LPPs by i) simplex method, ii) graphical method (if possible), iii) solving its dual by simplex method, and iv) solving its dual by graphical method (if possible).

- a. Minimize  $z = 2x_1 + 3x_2 + x_3$   
 Subject to  $x_1 + 4x_2 + 2x_3 \geq 8$   
 $3x_1 + 2x_2 \geq 6$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=0.8, x_2=1.8, x_3=0, z = 7$ ]

- b. Minimize  $z = 5x_1 + 4x_2 + 4x_3$   
 Subject to  $x_1 + x_2 + x_3 = 100$   
 $x_1 \leq 20$   
 $x_2 \geq 30$   
 $x_3 \leq 40$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=0, x_2=100, x_3=0, z = 400$ ]

- c. Minimize  $z = 4x_1 + 3x_2 + x_3$   
 Subject to  $x_1 + 2x_2 + 4x_3 \geq 12$   
 $3x_1 + 2x_2 + x_3 \geq 8$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=x_2=0, x_3=8, z = 8$ ]

- d. Minimize  $z = 4x_1 + 3x_2 + 9x_3$   
 Subject to  $2x_1 + 4x_2 + 6x_3 \geq 15$   
 $6x_1 + x_2 + 6x_3 \geq 12$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=3/2, x_2=3, x_3=0, z = 15$ ]

- e. Minimize  $z = 2x_1 + x_3$   
 Subject to  $x_1 + x_2 - x_3 \geq 5$   
 $x_1 - 2x_2 + 4x_3 \geq 8$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=0, x_2=14, x_3=9, z = 9$ ]

- f. Minimize  $z = 10x_1 + 6x_2 + 2x_3$

Subject to  $-x_1 + x_2 + x_3 \geq 1$   
 $3x_1 + x_2 - x_3 \geq 2$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=1/4, x_2=5/4, x_3=0, z=10$ ]

g. Minimize  $z = 2x_1 + 2x_2 + 4x_3$   
 Subject to  $2x_1 + 3x_2 + 5x_3 \geq 2$   
 $3x_1 + x_2 + 7x_3 \leq 3$   
 $x_1 + 4x_2 + 6x_3 \leq 5$   
 and  $x_1, x_2, x_3 \geq 0$ .

[Ans:  $x_1=0, x_2=2/3, x_3=0, z=4/3$ ]

15. Maximize  $z = 2x_1 + 3x_2 + 4x_3$

Subject to  $2x_1 + x_2 + x_3 \leq 30$   
 $x_1 + 2x_2 + x_3 \leq 20$   
 $x_1 + x_2 + 2x_3 \leq 30$

and  $x_1, x_2, x_3 \geq 0$  [Ans:  $x_1=0, x_2=10/3, x_3=40/3, z=190/3$ ]

Starting from the solution of above LPP, what will be the new solution if

- $2x_1$  in the objective function is changed to  $x_1$ . [A:  $x(0, 10/3, 40/3), z=190/3$ ]
- $3x_2$  in the objective function is changed to  $x_2$ . [A:  $x(0, 0, 15), z=60$ ]
- $4x_3$  in the objective function is changed to  $x_3$ . [A:  $x(40/3, 10/3, 0), z=110/3$ ]
- $2x_1$  in the objective function is changed to  $5x_1$  [A:  $x(10, 0, 10), z=90$ ]
- $3x_2$  in the objective function is changed to  $5x_2$  [A:  $x(0, 10/3, 40/3), z=70$ ]
- $4x_3$  in the objective function is changed to  $5x_3$  [A:  $x(0, 10/3, 40/3), z=230/3$ ]
- RHS in 1<sup>st</sup> constraint is changed from 30 to 20 [A:  $x(0, 10/3, 40/3), z=190/3$ ]
- RHS in 2<sup>nd</sup> constraint is changed from 20 to 10. [A:  $x(0, 0, 10), z=40$ ]
- RHS in 3<sup>rd</sup> constraint is changed from 30 to 20 [A:  $x(0, 20/3, 20/3), z=140/3$ ]
- RHS in 1<sup>st</sup> constraint is changed from 30 to 40 [A:  $x(0, 10/3, 40/3), z=190/3$ ]
- RHS in 2<sup>nd</sup> constraint is changed from 20 to 30. [A:  $x(0, 10, 10), z=70$ ]
- RHS in 3<sup>rd</sup> constraint is changed from 30 to 40. [A:  $x(0, 0, 20), z=80$ ]
- 1<sup>st</sup> constraint coefficient  $x_2$  is changed to  $2x_2$ . [A:  $x(0, 10/3, 40/3), z=190/3$ ]
- 2<sup>nd</sup> constraint coefficient  $x_1$  is changed to  $2x_1$ . [A:  $x(0, 10/3, 40/3), z=190/3$ ]
- 3<sup>rd</sup> constraint coefficient  $x_1$  is changed to  $2x_1$ . [A:  $x(0, 10/3, 40/3), z=190/3$ ]
- 3<sup>rd</sup> constraint coefficient  $x_3$  is changed to  $3x_3$ . [A:  $x(0, 6, 8), z=50$ ]
- A new constraint  $x_1 + x_2 + x_3 \leq 10$  is added. [A:  $x(0, 0, 10), z=40$ ]
- A new constraint  $3x_1 + 2x_2 + x_3 \leq 20$  is added. [A:  $x(0, 6, 8), z=50$ ]

16. An industry buys forgings A and B and sells them after machining and painting. They are bought at Rs. 3 and 4 per unit for A and B respectively and sold at Rs. 8 and 10 respectively. The maximum number of items processed per hour in the machining and painting set ups are as tabulated below:

Process	A	B
Machining	30	60
Painting	40	80

The cost of machining and painting are Rs. 30 and 40 per hour respectively. It is required to take a decision on the number of items of A and B to be produced so as to maximize total profit. Formulate it as a linear programming problem & solve it using graphical and simplex methods.

17. Solve the following LPP using Big M method.

Maximize  $z = 4a - b$

Subject to

$$2a + b \geq 4$$

$$a + 3b \leq 3$$

$$b \leq 2$$

and  $a, b \geq 0$  [Ans:  $a=3, b=0, z=12$ ]

Sl	Roll	Q	Sl	Roll	Q	Sl	Roll	Q
1	120ME0284	1	46	121ME0481	9p(ii)	91	121ME0727	12d
2	120ME0294	2	47	121ME0482	9q(ii)	92	121ME0728	12e
3	120ME0346	3	48	121ME0483	9r(ii)	93	121ME0798	12f
4	121ME0125	4	49	121ME0484	9s(ii)	94	121ME0799	12g
5	121ME0440	5	50	121ME0485	9t(ii)	95	121ME0801	12h
6	121ME0441	6	51	121ME0486	9u(ii)	96	121ME0846	12i
7	121ME0442	7	52	121ME0487	10a(i)	97	121ME0855	12j
8	121ME0443	8	53	121ME0488	10b(i)	98	121ME0856	12k
9	121ME0444	9a(i)	54	121ME0489	10c(i)	99	121ME0857	12l
10	121ME0445	9b(i)	55	121ME0490	10d(i)	100	121ME0858	12m
11	121ME0446	9c(i)	56	121ME0491	10e(i)	101	121ME0859	12n
12	121ME0447	9d(i)	57	121ME0492	10f(i)	102	121ME0860	12o
13	121ME0448	9e(i)	58	121ME0493	10a(ii)	103	121ME0861	12p
14	121ME0449	9f(i)	59	121ME0494	10b(ii)	104	121ME0921	13a
15	121ME0450	9g(i)	60	121ME0495	10c(ii)	105	121ME0923	13b
16	121ME0451	9h(i)	61	121ME0496	10d(ii)	106	121ME0924	13c
17	121ME0452	9i(i)	62	121ME0497	10e(ii)	107	121ME0925	13d
18	121ME0453	9j(i)	63	121ME0498	10f(ii)	108	121ME0926	14a
19	121ME0454	9k(i)	64	121ME0499	10g(i)	109	121ME0978	14b
20	121ME0455	9l(i)	65	121ME0500	10h(i)	110	121ME0980	14c
21	121ME0456	9m(i)	66	121ME0501	10i(i)	111	121ME0981	14d
22	121ME0457	9n(i)	67	121ME0502	10j(i)	112	121ME0982	14e
23	121ME0458	9o(i)	68	121ME0503	10k(i)	113	121ME0983	14f
24	121ME0459	9p(i)	69	121ME0504	10l(i)	114	121ME0984	14g
25	121ME0460	9q(i)	70	121ME0505	10m(i)	115	121ME0985	15a
26	121ME0461	9r(i)	71	121ME0506	10n(i)	116	121ME0986	15b
27	121ME0462	9s(i)	72	121ME0507	11a	117	121ME0987	15c
28	121ME0463	9t(i)	73	121ME0508	11b	118	121ME0988	15d
29	121ME0464	9u(i)	74	121ME0555	11c	119	121ME0989	15e
30	121ME0465	9v(i)	75	121ME0556	11d	120	121ME0990	15f
31	121ME0466	9a(ii)	76	121ME0557	11e	121	121ME0991	15g
32	121ME0467	9b(ii)	77	121ME0558	11f	122	121ME0993	15h
33	121ME0468	9c(ii)	78	121ME0559	11g	123	121ME0994	15i
34	121ME0469	9d(ii)	79	121ME0611	11h	124	121ME0995	15j
35	121ME0470	9e(ii)	80	121ME0711	11i	125	121ME0996	15k
36	121ME0471	9f(ii)	81	121ME0716	11j	126	121ME0997	15l
37	121ME0472	9g(ii)	82	121ME0717	11k	127	121ME0998	15m
38	121ME0473	9h(ii)	83	121ME0719	11l	128	121ME1026	15n
39	121ME0474	9i(ii)	84	121ME0720	11m	129	121ME1027	15o
40	121ME0475	9j(ii)	85	121ME0721	11n	130	121ME1029	15p
41	121ME0476	9k(ii)	86	121ME0722	11o	131	121ME1030	15q
42	121ME0477	9l(ii)	87	121ME0723	11p	132	121ME1031	15r
43	121ME0478	9m(ii)	88	121ME0724	12a	133	121ME1032	16
44	121ME0479	9n(ii)	89	121ME0725	12b	134	121ME1055	17
45	121ME0480	9o(ii)	90	121ME0726	12c	135	121ME1056	10g(ii)

[illegible]

