Unit 5

Turing Machine :

→ invented in 1936 by Alan Turing
→ accepts recursive enumerable language generated by type 0 grammer.

TM is defined as a collection of 7 tuples

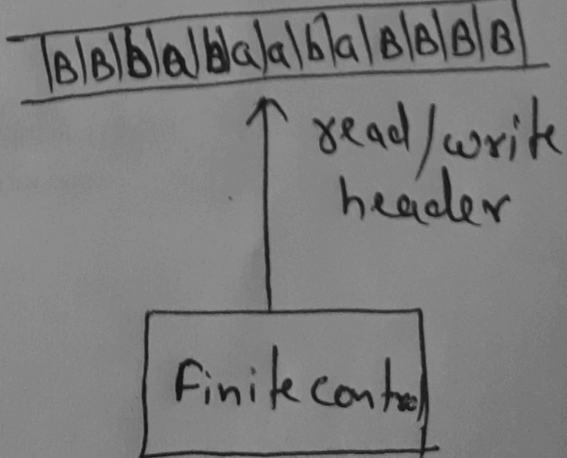
TM : $\{Q, \Sigma, \Gamma, q_0, F, B, \delta\}$

Q → finite set of states
Σ → Finite set of input symbols
Γ → Tape symbol
q_0 → initial state
F → set of final state
B → Blank symbol
δ → transition function
$\delta : Q \times \Gamma \to (Q, \Gamma, R/L)$

{ Note! $\Sigma \subseteq \Gamma$ }

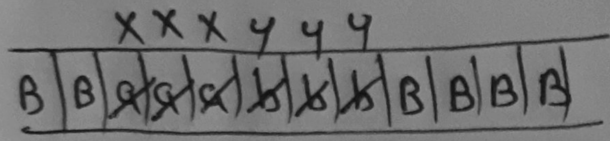
A Turing M/c has

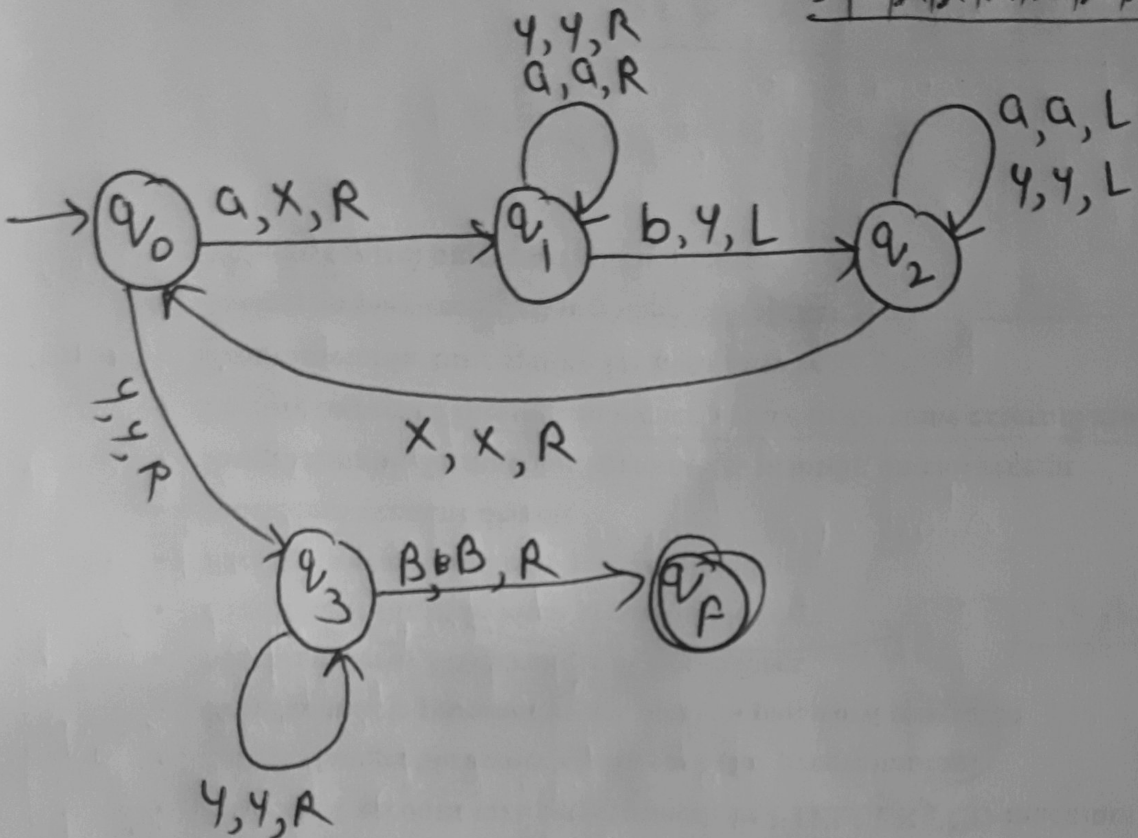
i) Both way infinite read/write tape
ii) Read write header
iii) Finite control



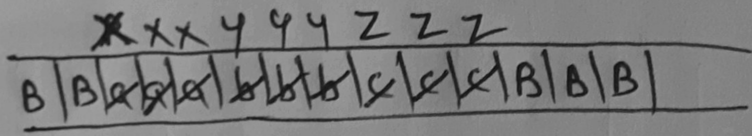
§ Turing Machine for

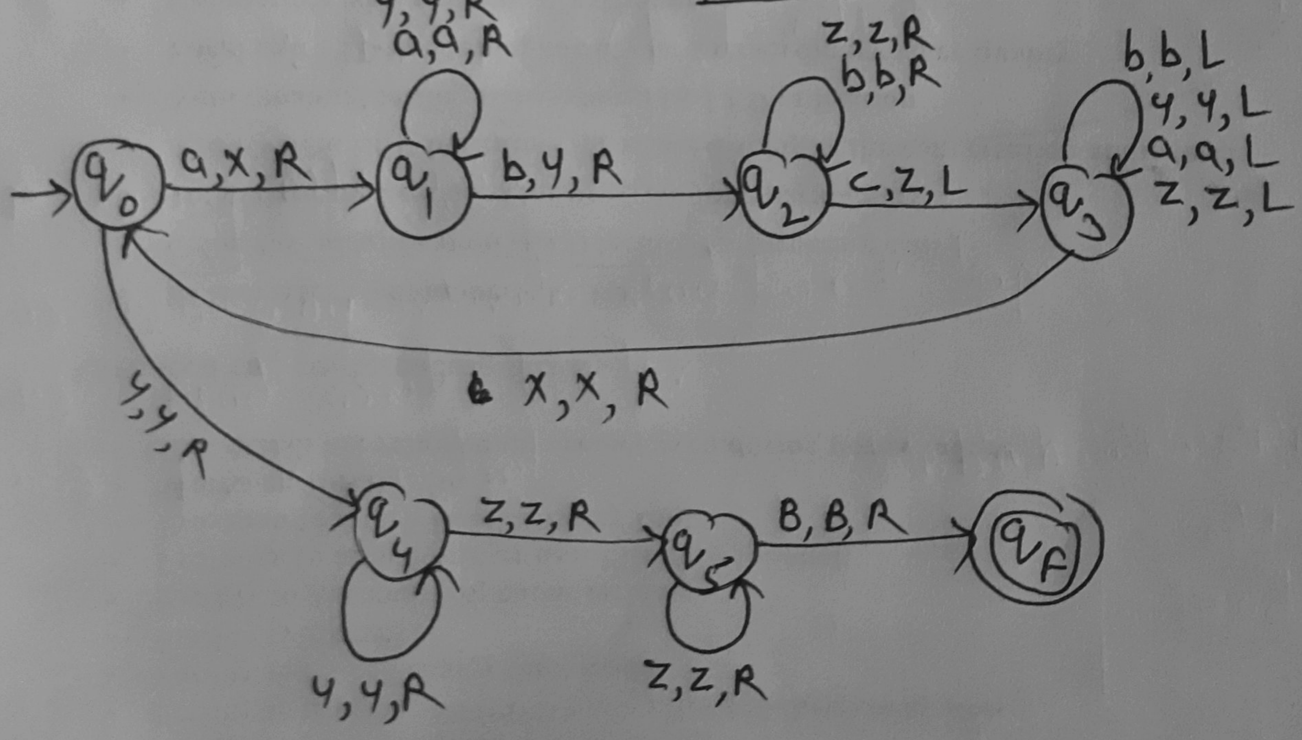
i) $a^n b^n \mid n \geq 1$



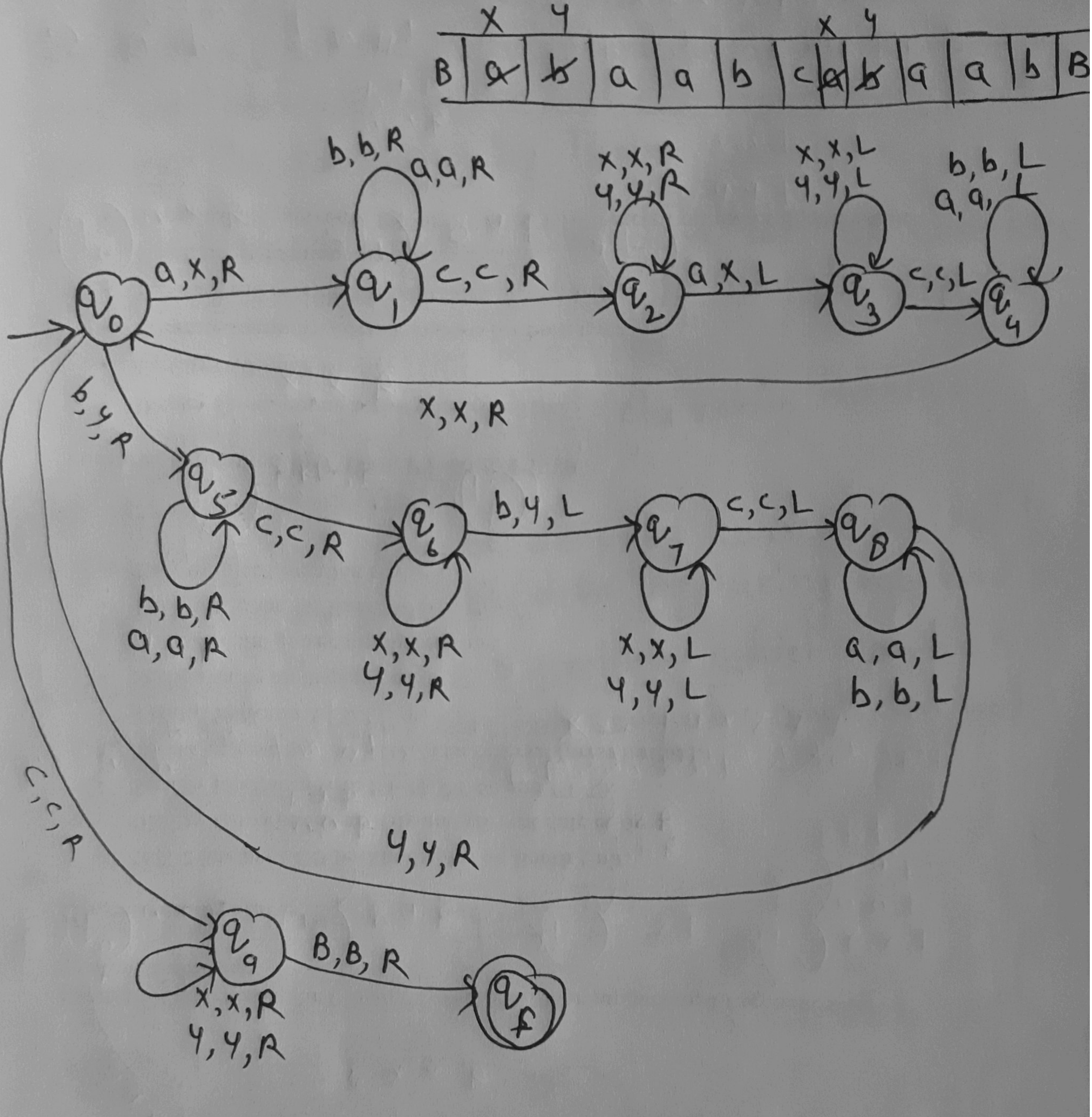


ii) $a^n b^n c^n \mid n \geq 1$





iii) ~~...~~ $wcw \mid w \in (a,b)^*$



§ church's Thesis: It is believed that there are no functions that can be defined by human, whose calculation can be described by any well defined mathamatical algorithm, that can not be computed by Turing Machine.

"No computational procedure will be consider-ed an algorithm unless it can be represen-ted as a turing machine"

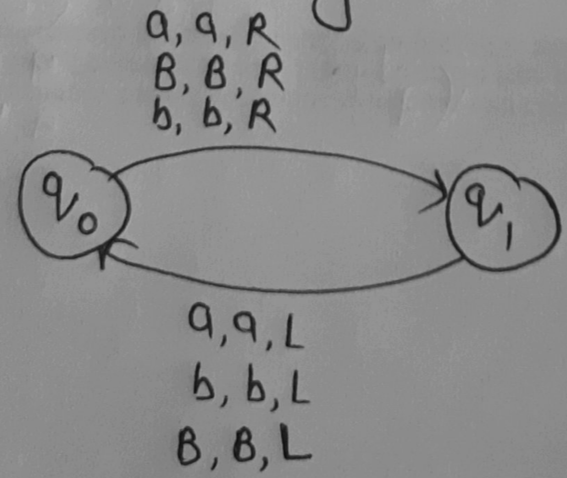
This statement is called church's thesis.

The conclusions are

→ any thing that can be done by current digital computer can also be done by turing machine.

→ currently there is no problem which can be solve by current digital computer and can not be solve by TM.

§ Halting / Non Halting in TM



L = a b

L = a a b b

Some time turing machine get into infinite loop and do not accept or reject. the string. It is called as non-halting problem.

Note: All the languages accepted by TM, are considered as REL (Recursively enumerable language). The languages which do not have halting problem on TM are called as RL (Recursive language)

Note! $\{RL \subseteq REL\}$

§ Difference between REL & RL

REL	RL
i) A Languag L is REL if there is TM for L.	i) A Language L is RL iff there is halting TM for L.
ii) In REL, three states are possible a) Halt & accept b) Halt & reject c) never halt & gets into loop	ii) In RL, two states are possible a) Halt & accept b) Halt & reject
iii) closed under all except set difference & complement	iii) closed under all except Homomorphism & substitution

§ Modification in TM :

i) TM with stay option
ii) TM with semi infinite tape
iii) off line TM
iv) Multidimensional TM
v) Non deterministic TM
vi) Universal TM
vii) Multi tape TM
viii) Multi head TM
ix) TM with three states
x) Jumping TM
xi) Non erasing TM
xii) Always writing TM

In all above 12 modification, the power will be unchanged.

xiii) TM without writing capacity = FA
xiv) TM with I/p size tape = LBA
xv) TM tape as stack = PDA
xvi) TM with finite tape = FA

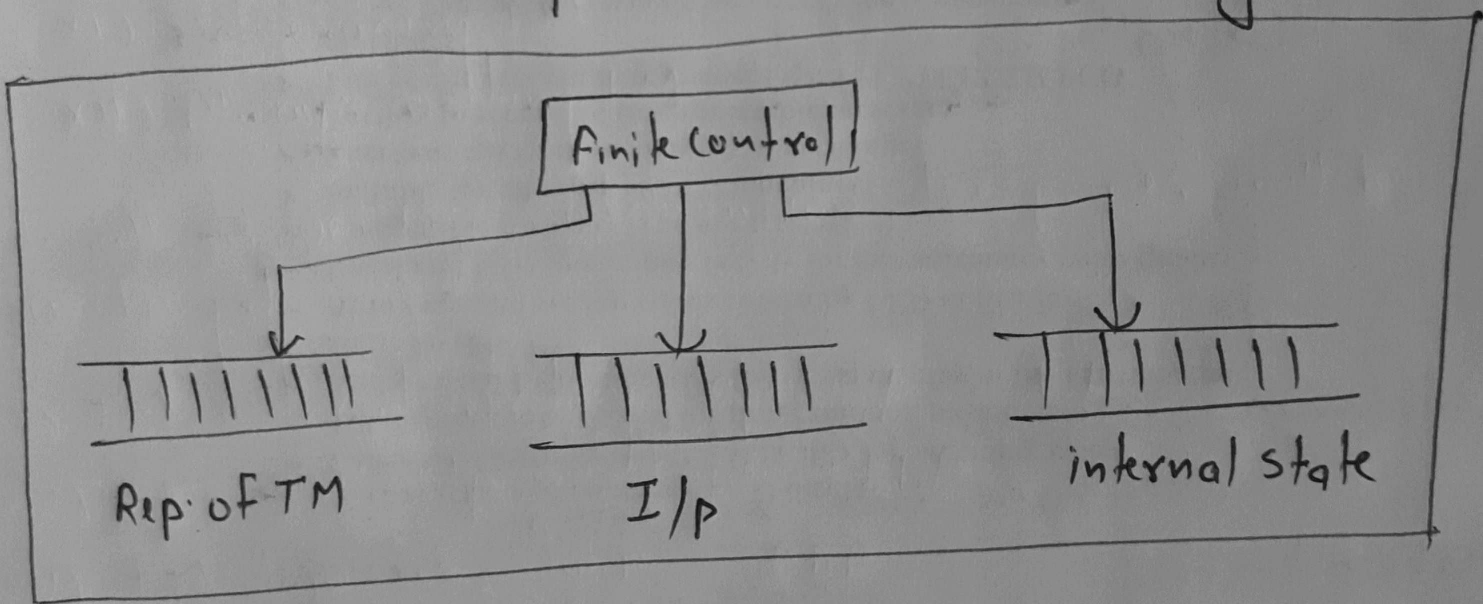
§ Universal TM :

→ acc. to church's thesis power of TM & digital comp. is equal

→ ~~but~~ digital comp. performs all operations in ~~sig~~ single unit but seperate TM required for specific operation

→ Universal TM is an arrangement which can act as digital computer i.e. can perform all operations in a single unit.

→ This arrangement comprises a finite control and three tapes - as shown in fig:



i) Tape.1 stores the present state

ii) Tape.2 stores the present input

iii) Tape.3 stores the representation of TM.

For ex) $Q = \{q_1, q_2, q_3 \text{---} q_n\}$

$\Gamma = \{a_1, a_2, a_3, \text{--} a_k\}$

Let $q_1 \to 1$ $\quad$ $a_1 \to 1$ $\quad$ $L \to 1$

$q_2 \to 11$ $\quad$ $a_2 \to 11$ $\quad$ $R \to 11$

$q_3 \to 111$ $\quad$ $a_3 \to 111$

$\delta : Q \times \Gamma \to Q \times \Gamma \times L/R$

Let $q_1 \times a_1 \to q_2 \times a_2 \times R$

it can be encoded as

$\boxed{0101011011011 0}$ → it will be store in tape. 3 (i.e. rep of TM)

Note: A UTM is an automata given as input the description of TM 'M', a string 'ω', can simulate the computation of M on ω.

§ Linear bound automata (LBA)

↳ Turing Machine with input size tape

↳ I/p alphabate contains 2 special symbols which serves as end markers.

↳ LBA is 8 tuple M/c

Left end marker

$LBA = \{Q, \Gamma, \Sigma, q_0, F, \delta, M_R, M_L\}$

Right end Marker

$\delta : Q \times \Gamma \to Q \times \Gamma \times (R/L)$

Post correspondence problem

↳ introduced by Emid post in 1946

↳ given two seq^n of strings

$$\left.\begin{aligned} A &= w_1, w_2, w_3 \ldots w_n \\ B &= x_1, x_2, x_3 \ldots x_n \end{aligned}\right\} \in \Sigma$$

Then instance of PCP has sol^n : if there is any seq^n of integer $i_1, i_2, i_3 \ldots i_m$ such that $w_{i_1}, w_{i_2}, w_{i_3} \ldots w_{i_m} = x_{i_1}, x_{i_2}, x_{i_3} \ldots x_{i_m}$

Ex)

	List A	List B
$i=1$	1	111
$i=2$	10111	10
$i=3$	10	0

⇒ 2 1 1 3 ← sol^n of PCP

A	10111	1	1	10
B	10	111	111	0