

## Problem A. Rikka with Nash Equilibrium

Input file: *standard input*  
 Output file: *standard output*  
 Time limit: 5 seconds  
 Memory limit: 512 mebibytes

Nash Equilibrium is an important concept in game theory.

Rikka and Yuta are playing a simple matrix game. At the beginning of the game, Rikka shows an  $n \times m$  integer matrix  $A$ . And then Yuta needs to choose an integer in  $[1, n]$ , Rikka needs to choose an integer in  $[1, m]$ . Let  $i$  be Yuta's number and  $j$  be Rikka's number, the final score of the game is  $A_{i,j}$ .

In the remaining part of this statement, we use  $(i, j)$  to denote the strategy of Yuta and Rikka.

For example, when  $n = m = 3$  and matrix  $A$  is

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 4 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

If the strategy is  $(1, 2)$ , the score will be 2; if the strategy is  $(2, 2)$ , the score will be 4.

A pure strategy Nash equilibrium of this game is a strategy  $(x, y)$  which satisfies neither Rikka nor Yuta can make the score higher by changing his(her) strategy unilaterally. Formally,  $(x, y)$  is a Nash equilibrium if and only if:

$$\begin{cases} A_{x,y} \geq A_{i,y} & \forall i \in [1, n] \\ A_{x,y} \geq A_{x,j} & \forall j \in [1, m] \end{cases}$$

In the previous example, there are two pure strategy Nash equilibriums:  $(3, 1)$  and  $(2, 2)$ .

To make the game more interesting, Rikka wants to construct a matrix  $A$  for this game which satisfies the following conditions:

1. Each integer in  $[1, nm]$  occurs exactly once in  $A$ .
2. The game has **at most one** pure strategy Nash equilibriums.

Now, Rikka wants you to count the number of matrixes with size  $n \times m$  which satisfy the conditions.

### Input

The first line contains a single integer  $t$  ( $1 \leq t \leq 20$ ), the number of the testcases.

The first line of each testcase contains three numbers  $n, m$  and  $K$  ( $1 \leq n, m \leq 80, 1 \leq K \leq 10^9$ ).

The input guarantees that there are at most 3 testcases with  $\max(n, m) > 50$ .

### Output

For each testcase, output a single line with a single number: the answer modulo  $K$ .

### Example

standard input	standard output
2	64
3 3 100	1170
5 5 2333	

## Problem B. Rikka with Seam

Input file: *standard input*  
 Output file: *standard output*  
 Time limit: 8 seconds  
 Memory limit: 512 mebibytes

Seam carving is a novel algorithm for resizing images while maintaining as much information as possible from the source image.

Now, Rikka is going to use seam carving method to deal with an  $n \times m$  black and white picture. We can abstract this picture into an  $n \times m$  01 matrix  $A$ .

A  $K$ -seam of this picture is an integer sequence  $a$  which satisfies the following conditions:

1.  $|a| = n$ ,  $a_i \in [1, m]$ .
2.  $|a_i - a_{i+1}| \leq K$ ,  $\forall i \in [1, n)$ .

After choosing a  $K$ -seam  $a$ , Rikka reduces the size of the picture by deleting pixels  $(i, a_i)$ , and then she gets a matrix  $A'$  of size  $n \times (m - 1)$ .

For example, if the chosen seam is  $[1, 2, 3]$  and the picture is

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

the result matrix will be

$$\begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Rikka finds that deleting different seams may get the same result. In the previous example, seam  $[1, 2, 3]$ ,  $[1, 2, 1]$ ,  $[1, 2, 2]$ ,  $[1, 1, 1]$  are equivalent.

Now Rikka wants to calculate the number of **different** matrixes she can get by deleting exactly one  $K$ -seam.

### Input

The first line contains a single integer  $t$  ( $1 \leq t \leq 10^3$ ), the numebr of testcases.

For each testcase, the first line contains three numbers  $n, m, K$  ( $2 \leq n, m \leq 2 \times 10^3$ ,  $1 \leq K \leq m$ ).

Then  $n$  lines follow, each line contains a 01 string of length  $m$  which describes one row of the matrix.

The input guarantees that there are at most 5 testcases with  $\max(n, m) > 300$ .

### Output

For each testcase, output a single line with a single number, the answer modulo 998244353.

## Example

standard input	standard output
3	2
2 2 1	70
00	199
10	
5 5 1	
00100	
10101	
00100	
01000	
11101	
5 5 2	
00100	
10101	
00100	
01000	
11101	

## Problem C. Rikka with APSP

Input file: *standard input*  
 Output file: *standard output*  
 Time limit: 10 seconds  
 Memory limit: 512 mebibytes

APSP (All Pair Shortest Path) is a basic problem in graph theory.

Since solving APSP for arbitrary graphs is not a simple task, Rikka wants to start with some particular graphs.

At first, Rikka has  $n$  vertices without any edges. And then for each pair of vertices  $(i, j) (i < j)$ , Rikka links an undirected edge between them with length  $f(i, j)$ . The value of  $f(i, j)$  is equal to the minimum **positive integer**  $k$  which satisfies  $ijk$  is a square number. (It is clear that  $f(i, j)$  always exists because  $ij(ij)$  must be a square number)

For example, if  $n = 4$ , the adjacency matrix of the graph will be:

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 6 & 2 \\ 3 & 6 & 1 & 3 \\ 1 & 2 & 3 & 1 \end{bmatrix}$$

Let  $d(i, j)$  be the length of the shortest path between vertex  $i, j$ . And now Rikka wants you to calculate:

$$\sum_{i=1}^n \sum_{j=i+1}^n d(i, j)$$

### Input

The first line contains one single integer  $t (1 \leq t \leq 50)$ , the number of the testcases.

For each testcase, the first line contains exactly one integer  $n (1 \leq n \leq 10^{10})$ .

The input guarantees that there are at most 3 testcases with  $n > 10^8$ .

### Output

For each testcase, output a single integer, the answer modulo 998244353.

### Example

standard input	standard output
3	16
4	243
10	190371
100	

## Problem D. Rikka with Stone-Paper-Scissors

Input file: *standard input*  
 Output file: *standard output*  
 Time limit: 1 second  
 Memory limit: 512 mebibytes

Did you watch the movie "Animal World"? There is an interesting game in this movie.

The rule is like traditional Stone-Paper-Scissors. At the beginning of the game, each of the two players receives several cards, and there are three types of cards: scissors, stone, paper. And then in each round, two players need to play out a card simultaneously. The chosen cards will be discarded and can not be used in the remaining part of the game.

The result of each round follows the basic rule: Scissors beat Paper, Paper beats Stone, Stone beats Scissors. And the winner will get 1 point, the loser will lose 1 point, and the points will not change in the case of a draw.

Now, Rikka is playing this game with Yuta. At first, Yuta gets  $a$  Scissors cards,  $b$  Stone cards and  $c$  Paper cards; Rikka gets  $a'$  Scissors cards,  $b'$  Stone cards,  $c'$  Paper cards. The parameters satisfy  $a + b + c = a' + b' + c'$ . And then they will play the game exactly  $a + b + c$  rounds (i.e., they will play out all the cards).

Yuta's strategy is "random". Each round, he will choose a card among all remaining cards with equal probability and play it out.

Now Rikka has got the composition of Yuta's cards (i.e., she has got the parameters  $a, b, c$ ) and Yuta's strategy (random). She wants to calculate the maximum expected final points she can get, i.e., the expected final points she can get if she plays optimally.

**Hint: Rikka can make decisions using the results of previous rounds and the types of cards Yuta has played.**

### Input

The first line contains a single number  $t (1 \leq t \leq 10^4)$ .

For each testcase, the first line contains three numbers  $a, b, c$  and the second line contains three numbers  $a', b', c' (0 \leq a, b, c, a', b', c' \leq 10^9, a + b + c = a' + b' + c' > 0)$ .

### Output

For each testcase, if the result is an integer, print it in a line directly.

Otherwise, if the result equals to  $\frac{a}{b} (|\gcd(a, b)| = 1, b > 0, a \text{ and } b \text{ are integers})$ , output " $a/b$ " (without the quote) in a single line.

### Example

standard input	standard output
4	2
2 0 0	0
0 2 0	-1
1 1 1	3552/19
1 1 1	
1 0 0	
0 0 1	
123 456 789	
100 200 1068	

## Problem E. Rikka with Rain

Input file: *standard input*  
 Output file: *standard output*  
 Time limit: 10 seconds  
 Memory limit: 512 mebibytes

There will always be sudden heavy rain in summer.

When the heavy rain begins, there are  $m$  students on the campus. We can consider each student as a circle with radius  $r$  and center  $(a_i, b_i)$ .

There is only one building in the school. This building can be considered as a simple polygon with  $n$  vertices  $(x_i, y_i)$ . To get out of the rain, the students need to run into the building as quickly as possible.

As a member of the students, Rikka wants to calculate the minimum possible running distance for each student.

The following are some supplements to this task:

- You may assume that students will not interact with each other and the circles may overlap.
- A student is inside the building if and only if the circle is completely inside the simple polygon.
- Students can run in any direction, and the running distance is the Euclidean distance between the initial position and the target position of the center.

### Input

The first line contains a single integer  $t(1 \leq t \leq 10)$ , the numebr of the testcases.

For each testcase, the first line contains three integers  $n, m, R(3 \leq n, m \leq 200, 1 \leq R \leq 10^6)$ .

Then  $n$  lines follows, each line contains two integers  $(x_i, y_i)(|x_i|, |y_i| \leq 10^6)$ , which describe the simple polygon in counter-clockwise.

Then  $m$  lines follows, each line contains two integers  $(a_i, b_i)(|a_i|, |b_i| \leq 10^6)$ , which describe the initial position of the students.

The input guarantees that it is possible for each student to run into the building.

### Output

For each query, let  $w$  be the minimum running distance of the student. To avoid precision problem, you need to round  $w$  to the nearest integer and print the result.

The input guarantees that the first decimal digit of the answer will not be 4 or 5 and the answer will not change if we add or subtract  $R$  by 0.1.

### Example

standard input	standard output
1	2
4 4 2	2
0 0	11
4 0	5
4 4	
0 4	
1 0	
2 0	
10 10	
-1 -2	

## Problem F. Rikka with Spanning Tree

Input file: *standard input*  
 Output file: *standard output*  
 Time limit: 2 seconds  
 Memory limit: 512 mebibytes

Rikka is interested in researching traditional problems on particular graphs. Today, she chooses the task “counting the number of spanning trees in an undirected graph”.

With an array  $a$  of length  $n$ , Rikka constructs an undirected graph with  $\sum_{i=1}^n a_i$  vertices in the following way:

1. Construct an auxiliary array  $B$ :  $B_0 = 0, B_i = B_{i-1} + a_i, \forall i \in [1, n]$ .
2. Assign a color to each vertex. The color of vertex  $i$  is an integer  $j$  which satisfies  $i \in (B_{j-1}, B_j]$ .
3. For each pair  $(i, j) (i < j)$ , if vertex  $i$  and  $j$  have the same color, link an undirected graph between  $i$  and  $j$ .

In other words, Rikka constructs a graph which contains  $n$  cliques, and the  $i$ th clique's size is  $a_i$ .

Rikka finds that if  $n > 1$ , the graph cannot be connected, and there must not be any spanning trees in it. To avoid this, Rikka adds extra  $m$  edges  $(u_i, v_i)$  to this graph.

Now, Rikka wants to count the number of different spanning trees in this graph.

**Two spanning trees are different if and only if there is one edge which occurs in exactly one of the two trees.**

### Input

The first line contains a single integer  $t (1 \leq t \leq 50)$ , the number of the testcases.

For each testcase, the first line contains two integers  $n, m (1 \leq n \leq 200, 0 \leq m \leq 200)$ . The second line contains  $n$  integers  $a_i (1 \leq a_i \leq 10^6)$ .

Then  $m$  lines follows, each line contains two integers  $u_i, v_i (1 \leq u_i, v_i \leq \sum_{k=1}^n a_k)$ , which describes an extra edge.

The input guarantees that:

1. There are at most 5 testcases with  $\max(n, m) > 50$ .
2. The graph does not have multiplicate edges and self circle. i.e.,  $u_i \neq v_i$ ,  $u_i, v_i$  have different colors and unordered pairs  $(u_i, v_i)$  are different from each other.
3. The final graph is connected.

### Output

For each testcase, output a single line with a single integer, the answer modulo 998244353.

### Example

standard input	standard output
3	125
1 0	15625
5	296
2 1	
5 5	
1 6	
4 4	
1 2 3 4	
1 2	
3 4	
6 7	
10 1	

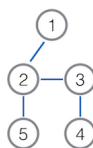
## Problem G. Rikka with Treasure

Input file: *standard input*  
 Output file: *standard output*  
 Time limit: 12 seconds  
 Memory limit: 512 mebibytes

By coincidentally, Rikka found a treasure map. She wants to employ several expedition teams to help her find the treasure.

There are  $n$  caves in the treasure maps and  $n-1$  undirected roads between the caves. For each cave pair  $(i, j) (i \neq j)$ , it is guaranteed that there is exactly one simple path between them, and the distance  $d(i, j)$  between these two caves is defined by the number of caves in this path.

For example, in the following treasure map,  $d(1, 3) = 3$ ,  $d(2, 5) = 2$  and  $d(1, 4) = 4$ .



Some of the caves have depots near them. Each expedition team can explore all the caves on the path  $(s_i, t_i)$  ( $s_i$  may be equal to  $t_i$ ) which satisfies there are depots near  $s_i, t_i$ , and  $s_i, t_i$  are stipulated by Rikka. After the exploration, Rikka needs to pay  $d(s_i, t_i) \times C$  dollars to this team.

Each cave has treasure in it and the treasure in the  $i$ th cave worths  $a_i$  dollars. Rikka can get  $i$ th treasure if and only if there are at least one expedition teams which have explored  $i$ th cave.

For example, in the previous treasure map, if  $a_i = i, C = 1$ , all caves have depots near them, Rikka employs two expedition teams and the first team explores path  $(1, 5)$ , the second team explores  $(2, 3)$ , then Rikka need pay 3 dollars to the first team, 2 dollars to the second team, and Rikka can get treasure 1, 2, 3, 5. So the Rikka's income will be  $1 + 2 + 3 + 5 - 3 - 2 = 6$  dollars.

Now, for each  $K$  from 1 to  $n$ , Rikka wants to calculate the maximum possible income she can get if she employs at most  $K$  expedition teams.

### Input

The first line contains a single integer  $t (1 \leq t \leq 10^3)$ , the number of the testcases.

For each testcase, the first line contains three integers  $n, C (1 \leq n \leq 3000, 1 \leq C \leq 10^7)$ .

The second line contains  $n$  01 integers  $p_i$ .  $p_i = 1$  if and only if there is a depot near the  $i$ th cave.

The third line contains  $n$  integers  $a_i (1 \leq a_i \leq 10^7)$ , which describes the value of the treasure.

Then  $n-1$  lines follow, each line contains two integers  $u_i, v_i$ , which describes one road in the map.

The input guarantees that there are at least one depots and there are at most 5 testcases with  $n > 200$ .

### Output

For each testcase, output a single line with a  $n$  numbers — the answer.



## Example

standard input	standard output
5	7 7 7 7 7
5 1	10 10 10 10 10
1 0 1 0 1	10 10 10 10 10
1 2 3 4 5	4 4 4 4 4
1 2	2 2 2 2 2
2 3	
2 5	
3 4	
5 1	
1 0 1 1 1	
1 2 3 4 5	
1 2	
2 3	
2 5	
3 4	
5 1	
1 1 1 1 1	
1 2 3 4 5	
1 2	
2 3	
2 5	
3 4	
5 2	
1 0 1 0 1	
1 2 3 4 5	
1 2	
2 3	
2 5	
3 4	
5 3	
1 0 1 0 1	
1 2 3 4 5	
1 2	
2 3	
2 5	
3 4	

## Problem H. Rikka with Line Graph

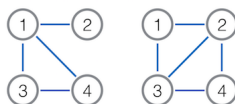
Input file: *standard input*  
 Output file: *standard output*  
 Time limit: 4 seconds  
 Memory limit: 512 mebibytes

Line Graph  $L(G)$  can be considered as an operator on an undirected graph  $G$  just like Complementary Graph and Dual Graph.

Rikka generalizes Line Graph to edge-weighted undirected graphs. For a graph  $G = \langle V, E \rangle$ ,  $L(G)$  is still an edge-weighted undirected graph which is constructed in the following way:

1.  $L(G)$  has  $|E|$  vertices and the  $i$ th vertex corresponds to the  $i$ th edge in  $G$ .
2. There is an edge between  $i, j$  in  $L(G)$  if and only if edge  $i$  and  $j$  have at least one common vertices in  $G$ . And the edge weight is equal to the sum of the weights of edge  $i$  and  $j$  in  $G$ .

For example, in the following picture, the right graph is the line graph of the left one. Vertex 1, 2, 3, 4 in  $L(G)$  correspond to edge (1, 2), (1, 4), (1, 3), (3, 4) in  $G$ . And if all edges in the left graph have weight 1, the edges in the right graph will have weight 2.



Now, Rikka has an edge-weighted tree  $T$  with  $n$  vertices. And she constructs a graph  $G = L(L(T))$ . It is clear that  $G$  is connected.

Let  $d(i, j)$  be the length of the shortest path between vertex  $i, j$  in  $G$  (the length of each edge is equal to the weight),  $m$  be the number of vertices in  $G$ , Rikka wants you to calculate  $\sum_{i=1}^m \sum_{j=i+1}^m d(i, j)$ .

### Input

The first line contains a single number  $t$  ( $1 \leq t \leq 100$ ), the number of the testcases.

For each testcase, the first line contains one single integer  $n$  ( $1 \leq n \leq 10^5$ ).

Then  $n - 1$  lines follow, each line contains three integers  $u_i, v_i, w_i$  ( $1 \leq u_i, v_i \leq 10^5, 1 \leq w_i \leq 10^9$ ), describe an edge with weight  $w_i$  between  $u_i$  and  $v_i$ .

The input guarantees that  $G$  has at least one vertices and there are at most 5 testcases with  $n > 10^3$ .

### Output

For each testcase, output a single line with a single number, the answer modulo 998244353.

## Example

standard input	standard output
3	24
4	166
1 2 1	420
1 3 2	
1 4 3	
5	
1 2 1	
2 3 10	
2 5 7	
3 4 2	
10	
1 2 1	
1 3 1	
2 4 1	
2 5 1	
2 6 1	
3 7 1	
7 8 1	
5 9 1	
6 10 1	

## Problem I. Rikka with Bubble Sort

Input file: *standard input*  
 Output file: *standard output*  
 Time limit: 30 seconds  
 Memory limit: 512 mebibytes

Bubble sort is a simple but beautiful algorithm. And this problem is a simple data structure task which is relative to bubble sort.

Rikka has an integer array  $A$  of length  $n$ , and then she makes  $m$  operations on it.

There are two types of operations:

- 1 L R, Rikka runs the following C program on  $A$ .  

```
for (int i=L; i<R; i++) if (A[i]>A[i+1]) swap(A[i], A[i+1]);
```
- 2 L R, Rikka wants to know the interval sum of  $[L, R]$ , i.e.,  $\sum_{i=L}^R A_i$ .

### Input

The first line contains two integers  $n, m$  ( $1 \leq n \leq 10^6, 1 \leq m \leq 3 \times 10^4$ ), and the second line contains  $n$  integers  $A_i$  ( $1 \leq A_i \leq 10^9$ ).

And then  $m$  lines follow, each line contains three integers  $t, L, R$  ( $t \in \{1, 2\}, 1 \leq L \leq R \leq n$ ).

### Output

For each query, output a single line with a single integer, the answer.

### Example

standard input	standard output
5 6	10
1 5 4 5 1	6
1 3 5	9
2 2 4	
1 2 4	
2 1 3	
1 1 5	
2 3 4	

### Example explanation

There are 3 operations in the first type, and the result arrays are  $[1, 5, 4, 1, 5], [1, 4, 1, 5, 5], [1, 1, 4, 5, 5]$ .

## Problem J. Rikka with Time Complexity

Input file: *standard input*  
 Output file: *standard output*  
 Time limit: 1 second  
 Memory limit: 512 mebibytes

Calculating and comparing time complexity for algorithms are the most important necessary skills for CS students. This semester, Rikka applies for the assistant of course "Algorithm Analysis". Now Rikka needs to set problems for the final examination, and she is going to set some tasks about time complexity.

Let  $f_a(n) = \log \dots \log n$  (there are exactly  $a$  log in this function, and log uses base 2). And then, for an integer array  $A$ , Rikka defines  $g_A(n)$  in the following way ( $B$  is the suffix of  $A$  with length  $|A| - 1$ ):

$$g_A(n) = \begin{cases} f_{A_1}(n) & |A| = 1 \\ f_{A_1}(n)^{g_B(n)} & |A| > 1 \end{cases}$$

For example,  $g_{[1,2]}(n) = \log n^{\log \log n}$  and  $g_{[3,1,1]}(n) = \log \log \log n^{\log n^{\log n}}$ .

Now, given integer arrays  $A$  and  $B$ , Rikka wants you to compare  $g_A(n)$  with  $g_B(n)$ . i.e., let  $k$  be  $\lim_{n \rightarrow +\infty} \frac{g_A(n)}{g_B(n)}$ . If  $k = 0$ , output  $-1$ ; if  $k = +\infty$ , output  $1$ ; otherwise output  $0$ .

### Input

The first line contains a single number  $t$  ( $1 \leq t \leq 10^5$ ), the number of testcases.

For each testcase, the first line contains two integers  $a, b$  ( $1 \leq a, b \leq 3$ ), the length of  $A$  and  $B$ .

The second line contains  $a$  integers  $A_i$  and the third line contains  $b$  integers  $B_i$  ( $1 \leq A_i, B_i \leq 10^9$ ), which describe  $A$  and  $B$ .

### Output

For each testcase, output a single line with a single integer, the answer.

### Example

standard input	standard output
3	1
1 1	-1
1	-1
2	
2 2	
1 2	
2 1	
1 3	
1	
1000000000 3 3	

## Problem K. Rikka with Badminton

Input file: *standard input*  
Output file: *standard output*  
Time limit: 1 second  
Memory limit: 512 mebibytes

In the last semester, Rikka joined the badminton club.

There are  $n$  students in the badminton club, some of them have rackets, and some of them have balls. Formally, there are  $a$  students have neither rackets nor balls,  $b$  students have only rackets,  $c$  students have only balls, and  $d$  students have both rackets and balls. ( $a + b + c + d = n$ )

This week, the club is going to organize students to play badminton. Each student can choose to take part in or not freely. So there are  $2^n$  possible registration status.

To play badminton, there must be at least two students who have rackets and at least one students who have balls. So if there aren't enough balls or rackets, the activity will fail.

Now, Rikka wants to calculate the number of the status among all  $2^n$  possible registration status which will make the activity fail.

### Input

The first line contains a single number  $t$  ( $1 \leq t \leq 10^3$ ), the number of testcases.

For each testcase, the first line contains four integers  $a, b, c, d$  ( $0 \leq a, b, c, d \leq 10^7, a + b + c + d \geq 1$ ).

### Output

For each testcase, output a single line with a single integer, the answer modulo 998244353.

### Example

standard input	standard output
3	12
1 1 1 1	84
2 2 2 2	2904
3 4 5 6	