**Department of Mathematics**
**Indian Institute of Technology Guwahati**
**MA 473 Computational Finance - Lab - VII**

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a). Solve the following two-point Boundary-Value problem by using finite element method:

$$\begin{cases} -\dfrac{d}{dx}\left((x+1)\dfrac{du}{dx}\right) + (2+x^2)u(x) = x^2 - 4, \quad x \in (0,1) \\ u(0) = u(1) = 0, \end{cases}$$

by using piecewise linear polynomials and using trapezoidal rule and Simpsons rule for the numerical quadrature.

b). Solve the following two-point Boundary-Value problem by using finite element method:

$$\begin{cases} -\dfrac{d}{dx}\left((x^2-2)\dfrac{du}{dx}\right) + (1+2x)u(x) = x^2 + 4x - 5, \quad x \in (0,1) \\ u(0) = 2, \quad u'(1) = 0, \end{cases}$$

by using piecewise linear polynomials and using trapezoidal rule and Simpsons rule for the numerical quadrature.

c). Consider the following Black-Scholes PDE for European call:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2 V}{\partial S^2} + (r-\delta)S\dfrac{\partial V}{\partial S} - rV = 0, \quad (0,\infty)\times(0,T],\ T>0 \\ V(S,t) = 0, \quad \text{for } S = 0, \\ V(S,t) = S - Ke^{-r(T-t)}, \quad \text{for } S \to \infty \\ \text{with suitable initial condition } V(S,0). \end{cases}$$

1. Solve the transformed PDE $yt = y_{xx}$ with suitable initial and boundary conditions by using finite elements mentioned in problem (a)and the Crank-Nicolson scheme.
2. Plot $V(S,t)$ for $T = 1$, $K = 10$, $r = 0.06$, $\sigma = 0.3$, and the payoff.

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