

1. Consider the following Black-Scholes PDE for European call:

$$\left\{ \begin{array}{l} \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - \delta)S \frac{\partial V}{\partial S} - rV = 0, \quad (0, \infty) \times (0, T], \quad T > 0 \\ V(S, t) = 0, \quad \text{for } S = 0, \\ V(S, t) = S - Ke^{-r(T-t)}, \quad \text{for } S \rightarrow \infty \\ \text{with suitable initial/terminal condition } V(S, 0) \text{ or } V(S, T). \end{array} \right.$$

Solve the above Black-Scholes PDE by the following schemes:

- (i) Forward-Euler for time & central difference for space (FTCS) scheme.
- (ii) Backward-Euler for time & central difference for space (BTCS) scheme.
- (iii) Crank-Nicolson finite difference scheme

The values of the parameters are $T = 1$, $K = 10$, $r = 0.06$, $\sigma = 0.3$ and $\delta = 0$.

2. Consider the following Black-Scholes PDE for European put:

$$\left\{ \begin{array}{l} \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - \delta)S \frac{\partial V}{\partial S} - rV = 0, \quad (0, \infty) \times (0, T], \quad T > 0 \\ V(S, t) = Ke^{-r(T-t)} - S, \quad \text{for } S = 0, \\ V(S, t) = 0, \quad \text{for } S \rightarrow \infty \\ \text{with suitable initial/terminal condition } V(S, 0) \text{ or } V(S, T). \end{array} \right.$$

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