

1. Solve the following parabolic PDE by using FTCS, BTCS and Crank-Nicolson finite difference schemes:

$$\begin{cases} \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, & (x, t) \in (0, 1) \times (0, T), \quad T > 0 \\ u(0, t) = u(1, t) = 0, & t \in (0, T], \\ u(x, 0) = f(x), & x \in (0, 1), \end{cases}$$

for the following values of f :

- (a) $f(x) = \sin(\pi x)$
- (b) $f(x) = x^3$
- (c) $f(x) = x(1 - x)$.

Plot the solution for various values of δx and δt .

2. Determine the numerical solution of the following one-dimensional parabolic IBVP:

$$\begin{cases} u_t - u_{xx} = 0, & (x, t) \in (0, 1/2) \times (0, T), \quad T > 0 \\ u(0, t) = 0, \quad u_x(1/2, t) = -\frac{1}{2}u, & \forall t \in (0, T] \\ u(x, 0) = x(1 - x), & x \in (0, 1/2), \end{cases}$$

by using the FTCS, BTCS and Crank-Nicolson schemes. Plot the numerical solution for various δx and δt values.