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MA 473: Computational Finance Lab – VIII 26/03/2019

1. The PDE modelling the *Asian option* for the *European arithmetic average strike call* (for $S > 0$, $A > 0$, $0 \leq t \leq T$) is given by

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} + S\frac{\partial V}{\partial A} - rV = 0.$$

By using the transformation $V(S, A, t) = \widetilde{V}(S, R, t) = S \cdot H(R, t)$, with $R = A/S$, transform the above PDE into the following form:

$$\begin{cases} \dfrac{\partial H}{\partial t} + \dfrac{1}{2}\sigma^2 R^2 \dfrac{\partial^2 H}{\partial R^2} + (1 - rR)\dfrac{\partial H}{\partial R} = 0, \\ H = 0, \quad \text{for } R \to \infty, \\ \dfrac{\partial H}{\partial t} + \dfrac{\partial H}{\partial R} = 0, \quad \text{for } R = 0, \\ H(R_T, T) = \left(1 - \dfrac{R_T}{T}\right)^+ \end{cases}$$

(a) Solve the above transformed PDE by the following schemes:

(i) Forward-Euler for time & central difference for space (FTCS) scheme.

(ii) Backward-Euler for time & central difference for space (BTCS) scheme.

(iii) Crank-Nicolson finite difference scheme with a suitable second-order approximation for the left boundary condition.

(b) Plot the solution surfaces for $K = 100$, $T = 0.2$, $r = 0.05$, $\sigma = 0.25$ at different time levels.