MATHEMATICS

ONE MARKS QUESTIONS (1-20)

1. The dimension of the vector space $V = \{A = (a_{ij})_{n \times n}; a_{ij} \in □, a_{ij} = -a_{ji}\}$ over field □ is
   a. n^2
   b. $n^2 - 1$
   c. $n^2 - n$
   d. $\frac{n^2}{2}$

2. The minimal polynomial associated with the matrix $\begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ is
   a. $x^3 - x^2 - 2x - 3$
   b. $x^3 - x^2 + 2x - 3$
   c. $x^3 - x^2 - 3x - 3$
   d. $x^3 - x^2 + 3x - 3$

3. For the function $f(z) = \sin\left(\frac{1}{\cos(1/z)}\right)$, the point $z = 0$ is
   a. a removable singularity
   b. a pole
   c. an essential singularity
   d. a non-isolated singularity

4. Let $f(z) = \sum_{n=0}^{15} z^n$ *for* $z \in □$. If $C : |z - i| = 2$ then $\oint_C \frac{f(z)\,dz}{(z-i)^{15}} =$
   a. $2\pi i(1 + 15i)$
   b. $2\pi i(1 - 15i)$
   c. $4\pi i(1 + 15i)$
   d. $2\pi i$

5. For what values of α and β, the quadrature formula $\int_{-1}^{1} f(x)\,dx \approx \alpha f(-1) + f(\beta)$ is exact for all polynomials of degree ≤ 1?
   a. $\alpha = 1, \beta = 1$
   b. $\alpha = -1, \beta = 1$
   c. $\alpha = 1, \beta = -1$
   d. $\alpha = -1, \beta = -1$

6. Let $f : [0,4] \to □$ be a three times continuously differentiable function. Then the value of $f[1,2,3,4]$ is
   a. $\frac{f''(\xi)}{3}$ for some $\xi \in (0,4)$
   b. $\frac{f''(\xi)}{6}$ for some $\xi \in (0,4)$
   c. $\frac{f'''(\xi)}{3}$ for some $\xi \in (0,4)$
   d. $\frac{f'''(\xi)}{6}$ for some $\xi \in (0,4)$

7. Which one of the following is TRUE?
   a. Every linear programming problem has a feasible solution.
   b. If a linear programming problem has an optimal solution then it is unique.
   c. The union of two convex sets is necessarily convex.
   d. Extreme points of the disk $x^2 + y^2 \leq 1$ are the point on the circle $x^2 + y^2 = 1$.

8. The dual of the linear programming problem:
   Minimize $c^T x$ subject to $Ax \geq b$ and $x \geq 0$ is
   a. Maximize $b^T w$ subject to $A^T w \geq c$ and $w \geq 0$
   b. Maximize $b^T w$ subject to $A^T w \leq c$ and $w \geq 0$
   c. Maximize $b^T w$ subject to $A^T w \leq c$ and w is unrestricted
   d. Maximize $b^T w$ subject to $A^T w \geq c$ and w is unrestricted

9. The resolvent kernel for the integral equation $u(x) = F(x) + \int_{\log 2}^{x} e^{(t-x)} u(t)\,dt$ is
   a. $\cos(x - t)$
   b. 1
   c. e^{t-x}

d. $e^{2(t-s)}$

10. Consider the metrics $d_2(f,g)=\left(\int_a^b |f(t)-g(t)|^2\,dt\right)^{1/2}$ and $d_\infty(f,g)=\sup_{t\in[a,b]}|f(t)-g(t)|$ on the space $X=C[a,b]$ of all real valued continuous functions on $[a,b]$. Then which of the following is TRUE?

a. Both (X,d_2) and (X,d_∞) are complete.

b. (X,d_2) is complete but (X,d_∞) is NOT complete.

c. (X,d_∞) is complete but (X,d_2) is NOT complete.

d. Both (X,d_2) and (X,d_∞) are NOT complete.

11. A function $f:\square\to\square$ need NOT be Lebesgue measurable if

a. f is monotone

b. $\{x\in\square : f(x)\ge\alpha\}$ is measurable for each $\alpha\in\square$

c. $\{x\in\square : f(x)=\alpha\}$ is measurable for each $\alpha\in\square$

d. For each open set G is $\square$, $f^{-1}(G)$ is measurable

12. Let $\{e_n\}_{n=1}^{\infty}$ be an orthonormal sequence in a Hilbert space H and let $x(\neq 0)\in H$. Then

a. $\lim_{n\to\infty}\langle x,e_n\rangle$ does not exist

b. $\lim_{n\to\infty}\langle x,e_n\rangle=\|x\|$

c. $\lim_{n\to\infty}\langle x,e_n\rangle=1$

d. $\lim_{n\to\infty}\langle x,e_n\rangle=0$

13. The subspace $\square\times[0,1]$ of $\square^2$ (with the usual topology) is

a. dense is $\square^2$

b. connected

c. separable

d. compact

14. $\square_2[x]/\langle x^3+x^2+1\rangle$ is

a. a field having 8 elements

b. a field having 9 elements

c. an infinite field

d. NOT a field

15. The number of element of a principal ideal domain can be

a. 15

b. 25

c. 35

d. 36

16. Let, F, G and H be pair wise independent events such that $P(F)=P(G)=P(H)=\frac{1}{3}$ and $(F\cap G\cap H)=\frac{1}{4}$ Then the probability that at least one event among F, G and H occurs is

a. $\frac{11}{12}$

b. $\frac{7}{12}$

c. $\frac{5}{12}$

d. $\frac{3}{4}$

17. Let X be a random variable such that $E(X^2)=E(X)=1$. Then $E(X^{100})=$

a. 0

b. 1

c. 2^{100}

d. $2^{100}+1$

18. For which of the following distributions, the weak law of large numbers does NOT hold?

a. Normal

b. Gamma

c. Beta

d. Cauchy

19. If $D\equiv\frac{d}{dx}$ then the value of $\frac{1}{(xD+1)}(x^{-1})$ is

a. $\log x$

b. $\frac{\log x}{x}$

c. $\frac{\log x}{x^2}$

d. $\frac{\log x}{x^3}$

20. The equation

$\left(\alpha xy^3 + y\cos x\right)dx + \left(x^2y^2 + \beta\right)dy = 0$

is exact for

a. $\alpha = \frac{3}{2}, \beta 1$

b. $\alpha = 1, \beta = \frac{3}{2}$

c. $\alpha = \frac{2}{3}, \beta = 1$

d. $\alpha = 1, \beta = \frac{2}{3}$

TWO MARKS QUESTIONS (21-60)

21. If $A = \begin{pmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1-i\sqrt{3}}{2} \end{pmatrix}$, then the trace of A^{102} is

a. 0
b. 1
c. 2
d. 3

22. Which of the following matrices is NOT diagonalizable?

a. $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

b. $\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$

c. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

d. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

23. Let V be the column space of the matrix $A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -1 \end{pmatrix}$. Then the orthogonal projection of $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ on V is

a. $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

b. $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

c. $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

d. $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

24. Let $\sum_{n=-\infty}^{\infty} a_n (z-1)^n$ be the Laurent series expansion of $f(z) = \sin\left(\frac{z}{z+1}\right)$. Then $a_{-2} =$

a. 1
b. 0
c. cos(1)
d. $\frac{-1}{2}\sin(1)$

25. Let $u(x,y)$ be the real part of an entire function $f(z) = u(x,y) + iv(x,y)$ for $z = x+iy \in \mathbb{C}$. If C is the positively oriented boundary of a rectangular region R in $\mathbb{R}^2$, then $\oint_C \left[\frac{\partial u}{\partial y}dx - \frac{\partial u}{\partial x}dy\right] =$

a. 1
b. 0
c. 2π
d. π

26. Let $\phi : [0,1] \to \mathbb{R}$ be three times continuously differentiable. Suppose that the iterates defined by $x_{n+1} = \phi(x_n), n \ge 0$ converge to the fixed point ξ of ϕ. If the order of convergence is three then

a. $\phi'(\xi) = 0, \phi''(\xi) = 0$

b. $\phi'(\xi) \neq 0, \phi''(\xi) = 0$

c. $\phi'(\xi) = 0, \phi''(\xi) \neq 0$

d. $\phi'(\xi) \neq 0, \phi''(\xi) \neq 0$

27. Let $f:[0,2]\to\mathbb{R}$ be a twice continuously differentiable function. If $\int_0^2 f(x)dx \approx 2f(1)$, then the error in the approximation is

a. $\frac{f'(\xi)}{12}$ for some $\xi\in(0,2)$

b. $\frac{f'(\xi)}{2}$ for some $\xi\in(0,2)$

c. $\frac{f''(\xi)}{3}$ for some $\xi\in(0,2)$

d. $\frac{f''(\xi)}{6}$ for some $\xi\in(0,2)$

28. For a fixed $t\in\mathbb{R}$, consider the linear programming problem:

Maximize $z=3x+4y$

Subject to $x+y\le 100$

$x+3y\le t$

and $x\ge 0, y\ge 0$

The maximum value of z is 400 for t =

a. 50
b. 100
c. 200
d. 300

29. The minimum value of

$z=2x_1-x_2+x_3-5x_4+22x_5$ subject to

$x_1-2x_4+x_5=6$

$x_2+x_4-4x_5=3$

$x_3+3x_4+2x_5=10$

$x_j\ge 0, j=1,2,\dots,5$

is

a. 28
b. 19
c. 10
d. 9

30. Using the Hungarian method, the optimal value of the assignment problem whose cost matrix is given by

5	23	14	8
10	25	1	23
35	16	15	12
16	23	11	7

is

a. 29
b. 52
c. 26
d. 44

31. Which of the following sequence $\{f_n\}_{n=1}^{\infty}$ of functions does NOT converge uniformly on [0, 1]?

a. $f_n(x)=\frac{e^{-x}}{n}$

b. $f_n(x)=(1-x)^n$

c. $f_n(x)=\frac{x^2+nx}{n}$

d. $f_n(x)=\frac{\sin(nx+n)}{n}$

32. Let $E=\{(x,y)\in\mathbb{R}^2 : 0<x<y\}$. Then $\iint_E ye^{-(x+y)}dxdy=$

a. $\frac{1}{4}$

b. $\frac{3}{2}$

c. $\frac{4}{3}$

d. $\frac{3}{4}$

33. Let $f_n(x)=\frac{1}{n}\sum_{k=0}^{n}\sqrt{k(n-k)}\binom{n}{k}x^k(1-x)^{n-k}$ for $x\in[0,1], n=1,2,\dots$. If $\lim_{n\to\infty} f_n(x)=f(x)$ for $x\in[0,1]$, then the maximum value of $f(x)$ on [0,1] is

a. 1

b. $\frac{1}{2}$

c. $\frac{1}{3}$

d. $\frac{1}{4}$

34. Let $f:(c_{00},\|\cdot\|_1)\to\mathbb{R}$ be a non zero continuous linear functional. The number of Hahn-Banach extensions of f to $(\ell^1,\|\cdot\|_1)$ is

a. One
b. Two
c. Three

d. infinite

35. If $I:\left(l^1,\|\cdot\|_2\right)\to\left(l^1,\|\cdot\|_1\right)$ is the identity map, then
   a. Both I and I^{-1} are continuous
   b. I is continuous but I^{-1} is NOT continuous
   c. I^{-1} is continuous but I is NOT continuous
   d. Neither I and I^{-1} is continuous

36. Consider the topology $\tau=\{G\subseteq □ : □ \setminus G$ is compact in $(□,\tau_u)\}\cup\{\phi, □\}$ on □, where τ_u is the usual topology on □ and ϕ is the empty set. Then $(□,\tau)$ is
   a. a connected Hausdorff space
   b. connected but NOT Hausdorff
   c. hausdorff but NOT connected
   d. neither connected nor Hausdorff

37. Let
   $\tau_1=\{G\subseteq □ : G$ is finite or □ \ G is finite$\}$
   and
   $\tau_2=\{G\subseteq □ : G$ is contable or □ \ G is contable$\}$
   Then
   a. neither τ_1 nor τ_2 is a topology on □
   b. τ_1 is a topology on □ but τ_2 is NOT a topology on □
   c. τ_2 is a topology on □ but τ_1 is NOT a topology on □
   d. both τ_1 and τ_2 are topologies on □

38. Which one of the following ideals of the ring □$[i]$ of Gaussian integers is NOT maximal?
   a. $\langle 1+i\rangle$
   b. $\langle 1-i\rangle$
   c. $\langle 2+i\rangle$
   d. $\langle 3+i\rangle$

39. If Z(G) denotes the centre of a group G, then the order of the quotient group G/Z(G) cannot be
   a. 4
   b. 6
   c. 15
   d. 25

40. Let Aut(G) denote the group of automorphism of a group G. Which one of the following is NOT a cyclic group?
   a. $Aut(□_4)$
   b. $Aut(□_6)$
   c. $Aut(□_8)$
   d. $Aut(□_{10})$

41. Let X be a non-negative integer valued random variable with $E\left(X^2\right)=3$ and $E(X)=1$. Then $\sum_{i=1}^{\infty} iP(X\geq i)=$
   a. 1
   b. 2
   c. 3
   d. 4

42. Let X be a random variable with probability density function $f\in\{f_0,f_1\}$, where

$$f_0(x)=\begin{cases}2x, & if\ 0<x<1\\ 0, & otherwise\end{cases}\ \text{and}$$

$$f_1(x)=\begin{cases}3x^2, & if\ 0<x<1\\ 0, & otherwise\end{cases}$$

   For testing the null hypothesis $H_0: f=f_0$ against the alternative hypothesis $H_1: f=f_1$ at level of significance α = 0.19, the power of the most powerful test is
   a. 0.729
   b. 0.271
   c. 0.615
   d. 0.385

43. Let X and Y be independent and identically distributed U(0, 1) random variables. Then $P\left(Y<\left(X-\frac{1}{2}\right)^2\right)=$
   a. $\frac{1}{12}$
   b. $\frac{1}{4}$
   c. $\frac{1}{3}$
   d. $\frac{2}{3}$

44. Let X and Y be Banach spaces and let $T: X \to Y$ be a linear map. Consider the statements:

P: If $x_n \to x$ in X then $Tx_n \to Tx$ in Y.

Q: If $x_n \to x$ in X and $Tx_n \to y$ in Y then $Tx = y$.

Then

a. P implies Q and Q implies P
b. P implies Q but Q does not imply P
c. Q implies P but P does not imply Q
d. Neither P implies Q nor Q implies P

45. If $y(x) = x$ is a solution of the differential equation $y'' - \left(\frac{2}{x^2} + \frac{1}{x}\right)(xy' - y) = 0, 0 < x < \infty$, then its general solution is

a. $\left(\alpha + \beta e^{-2x}\right)x$
b. $\left(\alpha + \beta e^{2x}\right)x$
c. $\alpha x + \beta e^{x}$
d. $\left(\alpha e^{x} + \beta\right)x$

46. Let $P_n(x)$ be the Legendra polynomial of degree n such that $P_n(1) = 1, n = 1, 2, \ldots$ If $\int_{-1}^{1}\left(\sum_{j=1}^{n}\sqrt{j(2j+1)}P_j(x)\right)^2 dx = 20$, then n =

a. 2
b. 3
c. 4
d. 5

47. The integral surface satisfying the equation $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x^2 + y^2$ and passing through the curve $x = 1 - t, y = 1 + t, z = 1 + t^2$ is

a. $z = xy + \frac{1}{2}\left(x^2 - y^2\right)^2$
b. $z = xy + \frac{1}{4}\left(x^2 - y^2\right)^2$
c. $z = xy + \frac{1}{8}\left(x^2 - y^2\right)^2$
d. $z = xy + \frac{1}{16}\left(x^2 - y^2\right)^2$

48. For the diffusion problem $u_{xx} = u_t\,(0 < x < \pi, t > 0)$, $u(0,t) = 0$, $u(\pi, t) = 0$ and $u(x, 0) = 3\sin 2x$, the solution is given by

a. $3e^{-t}\sin 2x$
b. $3e^{-4t}\sin 2x$
c. $3e^{-9t}\sin 2x$
d. $3e^{-2t}\sin 2x$

49. A simple pendulum, consisting of a bob of mass m connected with a string of length a, is oscillating in a vertical plane. If the string is making an angle θ with the vertical, then the expression for the Lagrangian is given as

a. $ma^2\left(\theta^2 - \frac{2g}{a}\sin^2\left(\frac{\theta}{2}\right)\right)$
b. $2mga\sin^2\left(\frac{\theta}{2}\right)$
c. $ma^2\left(\frac{\theta^2}{2} - \frac{2g}{a}\sin^2\left(\frac{\theta}{2}\right)\right)$
d. $\frac{ma}{2}\left(\theta^2 - \frac{2g}{a}\cos\theta\right)$

50. The extremal of the functional $\int_0^1\left(y + x^2 + \frac{y'^2}{4}\right)dx, y(0) = 0, y(1) = 0$ is

a. $4\left(x^2 - x\right)$
b. $3\left(x^2 - x\right)$
c. $2\left(x^2 - x\right)$
d. $x^2 - x$

Common Data for Questions (51 & 52)

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$T(x_1, x_2, x_3) = (x_1 + 3x_2 + 2x_3, 3x_1 + 4x_2 + x_3, 2x_1 + x_2 - x_3)$

51. The dimension of the range space of T^2 is

a. 0
b. 1
c. 2
d. 3

52. The dimension of the null space of T^3 is

a. 0
b. 1
c. 2
d. 3

Common Data for Questions (53 & 54)

Let $y_1(x)=1+x$ and $y_2(x)=e^x$ be two solutions of $y''(x)+P(x)y'(x)+Q(x)y(x)=0$.

53. $P(x)=$

a. $1+x$
b. $-1-x$
c. $\frac{1+x}{x}$
d. $\frac{-1-x}{x}$

54. The set of initial conditions for which the above differential equation has NO solution is

a. $y(0)=2, y'(0)=1$
b. $y(1)=0, y'(1)=1$
c. $y(1)=1, y'(1)=0$
d. $y(2)=1, y'(2)=2$

Common Data for Questions (55 & 56)

Let X and Y be random variables having the joing probability density function

$$f(x,y)=\begin{cases}\frac{1}{\sqrt{2\pi y}}e^{\frac{-1}{2y}(x-y)^2}, & if -\infty<x<\infty, 0<y<1\\ 0, & otherwise\end{cases}$$

55. The variance of the random variable X is

a. $\frac{1}{12}$
b. $\frac{1}{4}$
c. $\frac{7}{12}$
d. $\frac{5}{12}$

56. The covariance between the random variables X and Y is

a. $\frac{1}{3}$
b. $\frac{1}{4}$
c. $\frac{1}{6}$
d. $\frac{1}{12}$

Statement for Linked Answer Question (57 and 58)

Consider the function $f(z)=\frac{e^{iz}}{z(z^2+1)}$.

57. The residue of f at the isolated singular point in the upper half plane $\{z=x+iy\in\mathbb{C}: y>0\}$ is

a. $\frac{-1}{2e}$
b. $\frac{-1}{e}$
c. $\frac{e}{2}$
d. 2

58. The Cauchy Principal Value of the integral $\int_{-\infty}^{\infty}\frac{\sin x dx}{x(x^2+1)}$ is

a. $-2\pi(1+2e^{-1})$
b. $\pi(1+e^{-1})$
c. $2\pi(1+e)$
d. $-\pi(1+e^{-1})$

Statement for Linked Answer Question (59 and 60)

Let $f(x,y)=kxy-x^3y-xy^3$ for $(x,y)\in\mathbb{R}^2$, where k is a real constant. The directional derivative of f at the point (1, 2) in the direction of the unit vector $u=\left(\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$ is $\frac{15}{\sqrt{2}}$.

59. The value of k is

a. 2
b. 4
c. 1
d. -2

60. The value of f at a local minimum in the rectangular region $R=\left\{(x,y)\in\mathbb{R}^2: |x|<\frac{3}{2}, |y|<\frac{3}{2}\right\}$ is

a. - 2
b. - 3
c. $\frac{-7}{8}$
d. 0