Q. 1 – Q. 25 carry one mark each.

Q.1 Consider a system of linear equations:

$$x-2y+3z=-1,$$
$$x-3y+4z=1, \text{ and}$$
$$-2x+4y-6z=k.$$

The value of k for which the system has infinitely many solutions is ______.

Q.2 A function $f(x)=1-x^2+x^3$ is defined in the closed interval $[-1,1]$. The value of x, in the open interval $(-1,1)$ for which the mean value theorem is satisfied, is

(A) $-1/2$ (B) $-1/3$ (C) $1/3$ (D) $1/2$

Q.3 Suppose A and B are two independent events with probabilities $P(A) \neq 0$ and $P(B) \neq 0$. Let $\overline{A}$ and $\overline{B}$ be their complements. Which one of the following statements is FALSE?

(A) $P(A \cap B) = P(A)P(B)$ (B) $P(A|B) = P(A)$
(C) $P(A \cup B) = P(A) + P(B)$ (D) $P(\overline{A} \cap \overline{B}) = P(\overline{A})P(\overline{B})$

Q.4 Let $z=x+iy$ be a complex variable. Consider that contour integration is performed along the unit circle in anticlockwise direction. Which one of the following statements is **NOT TRUE**?

(A) The residue of $\frac{z}{z^2-1}$ at $z=1$ is 1/2

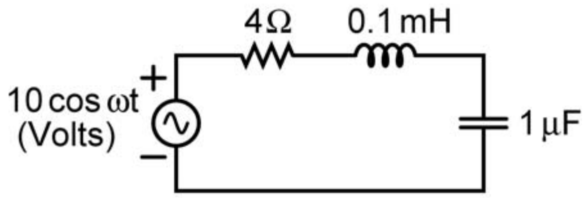
(B) $\oint_C z^2 dz = 0$

(C) $\frac{1}{2\pi i}\oint_C \frac{1}{z} dz = 1$

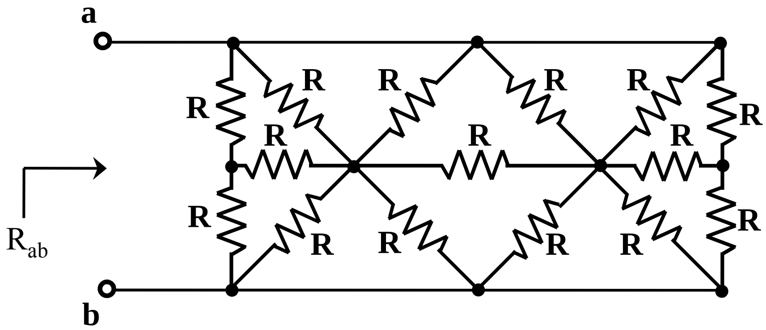
(D) $\overline{z}$ (complex conjugate of z) is an analytical function

Q.5 The value of p such that the vector $\begin{bmatrix}1\\2\\3\end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix}4 & 1 & 2\\ p & 2 & 1\\ 14 & -4 & 10\end{bmatrix}$ is _______.

Q.6 In the circuit shown, at resonance, the amplitude of the sinusoidal voltage (in Volts) across the capacitor is ________.

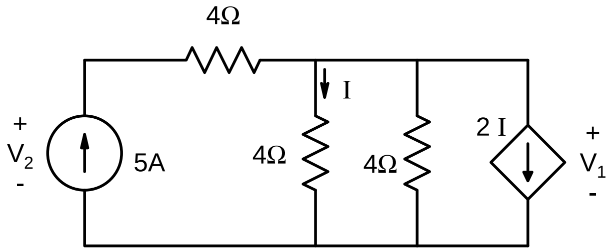


Q.7 In the network shown in the figure, all resistors are identical with R = 300 Ω. The resistance R_{ab} (in Ω) of the network is ______.



R = 300 Ω

Q.8 In the given circuit, the values of V_1 and V_2 respectively are



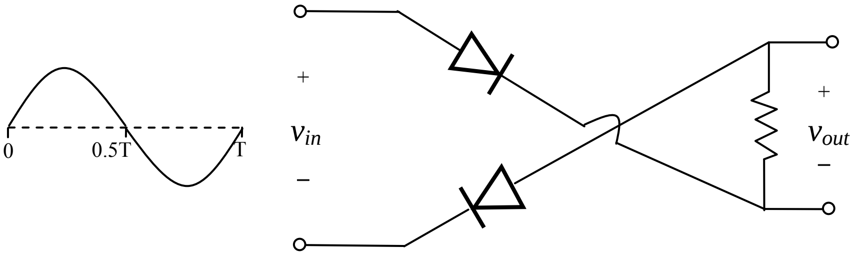
(A) 5 V, 25 V (B) 10 V, 30 V (C) 15 V, 35 V (D) 0 V, 20 V

Q.9 A region of negative differential resistance is observed in the current voltage characteristics of a silicon PN junction if

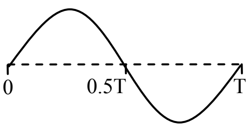
(A) both the P-region and the N-region are heavily doped
(B) the N-region is heavily doped compared to the P-region
(C) the P-region is heavily doped compared to the N-region
(D) an intrinsic silicon region is inserted between the P-region and the N-region

Q.10 A silicon sample is uniformly doped with donor type impurities with a concentration of 10^{16} /cm^3. The electron and hole mobilities in the sample are 1200 cm^2/V-s and 400 cm^2/V-s respectively. Assume complete ionization of impurities. The charge of an electron is 1.6×10^{-19} C. The resistivity of the sample (in Ω-cm) is ____________.

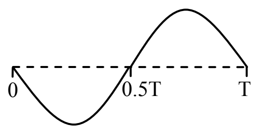
Q.11 For the circuit with ideal diodes shown in the figure, the shape of the output (v_{out}) for the given sine wave input (v_{in}) will be



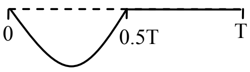
(A)



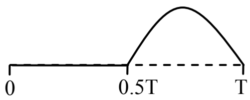
(B)



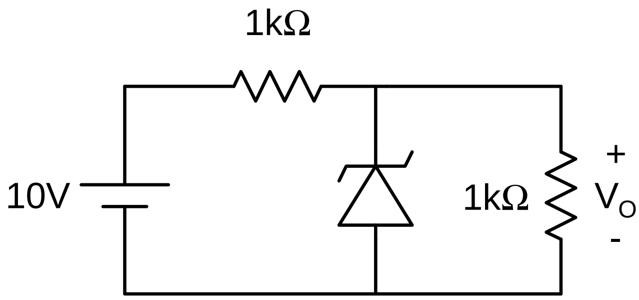
(C)



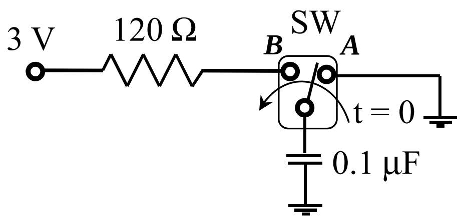
(D)



Q.12 In the circuit shown below, the Zener diode is ideal and the Zener voltage is 6 V. The output voltage V_o (in volts) is_______.



Q.13 In the circuit shown, the switch SW is thrown from position *A* to position *B* at time t = 0. The energy (in μJ) taken from the 3 V source to charge the 0.1 μF capacitor from 0 V to 3 V is



(A) 0.3 (B) 0.45 (C) 0.9 (D) 3

Q.14 In an 8085 microprocessor, the shift registers which store the result of an addition and the overflow bit are, respectively

(A) B and F
(B) A and F
(C) H and F
(D) A and C

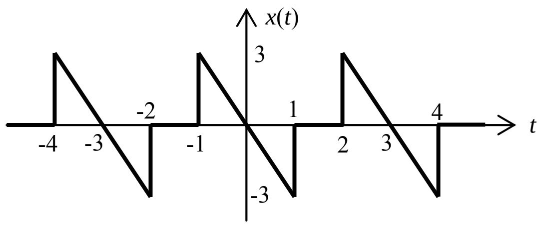
Q.15 A 16 Kb (=16,384 bit) memory array is designed as a square with an aspect ratio of one (number of rows is equal to the number of columns). The minimum number of address lines needed for the row decoder is _______.

Q.16 Consider a four bit D to A converter. The analog value corresponding to digital signals of values 0000 and 0001 are 0 V and 0.0625 V respectively. The analog value (in Volts) corresponding to the digital signal 1111 is ________.

Q.17 The result of the convolution $x(-t) * \delta(-t - t_0)$ is

(A) $x(t + t_0)$ (B) $x(t - t_0)$ (C) $x(-t + t_0)$ (D) $x(-t - t_0)$

Q.18 The waveform of a periodic signal $x(t)$ is shown in the figure.



A signal $g(t)$ is defined by $g(t) = x\left(\frac{t-1}{2}\right)$. The average power of $g(t)$ is _______.

Q.19 Negative feedback in a closed-loop control system **DOES NOT**

(A) reduce the overall gain
(B) reduce bandwidth
(C) improve disturbance rejection
(D) reduce sensitivity to parameter variation

Q.20 A unity negative feedback system has the open-loop transfer function $G(s) = \frac{K}{s(s+1)(s+3)}$. The value of the gain K (>0) at which the root locus crosses the imaginary axis is ________.

Q.21 The polar plot of the transfer function $G(s) = \frac{10(s+1)}{s+10}$ for $0 \leq \omega < \infty$ will be in the

(A) first quadrant
(B) second quadrant
(C) third quadrant
(D) fourth quadrant

Q.22 A sinusoidal signal of 2 kHz frequency is applied to a delta modulator. The sampling rate and step-size Δ of the delta modulator are $20,000$ samples per second and 0.1 V, respectively. To prevent slope overload, the maximum amplitude of the sinusoidal signal (in Volts) is

(A) $\frac{1}{2\pi}$ (B) $\frac{1}{\pi}$

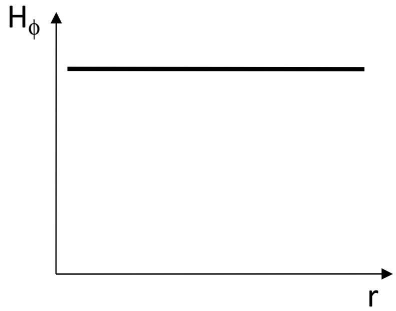
(C) $\frac{2}{\pi}$ (D) π

Q.23 Consider the signal $s(t) = m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)$ where $\hat{m}(t)$ denotes the Hilbert transform of $m(t)$ and the bandwidth of $m(t)$ is very small compared to f_c. The signal $s(t)$ is a

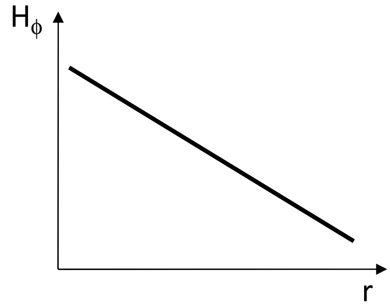
(A) high-pass signal
(B) low-pass signal
(C) band-pass signal
(D) double sideband suppressed carrier signal

Q.24 Consider a straight, infinitely long, current carrying conductor lying on the *z*-axis. Which one of the following plots (in linear scale) qualitatively represents the dependence of H_ϕ on r, where H_ϕ is the magnitude of the azimuthal component of magnetic field outside the conductor and r is the radial distance from the conductor?

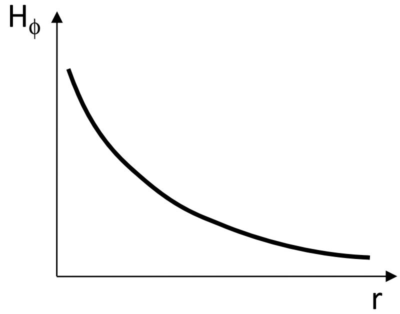
(A)



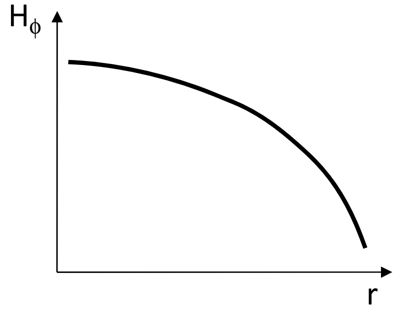
(B)



(C)



(D)



Q.25 The electric field component of a plane wave traveling in a lossless dielectric medium is given by $\vec{E}(z,t) = \hat{a}_y 2\cos\left(10^8 t - \frac{z}{\sqrt{2}}\right)$ V/m. The wavelength (in m) for the wave is ___________.

Q. 26 – Q. 55 carry two marks each.

Q.26 The solution of the differential equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$ with $y(0) = y'(0) = 1$ is

(A) $(2-t)e^t$ (B) $(1+2t)e^{-t}$

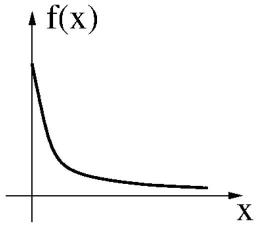
(C) $(2+t)e^{-t}$ (D) $(1-2t)e^t$

Q.27 A vector $\vec{P}$ is given by $\vec{P} = x^3y\,\vec{a}_x - x^2y^2\,\vec{a}_y - x^2yz\,\vec{a}_z$. Which one of the following statements is **TRUE**?

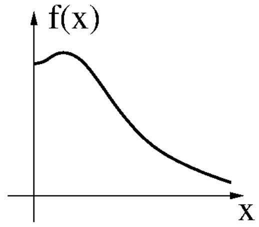
(A) $\vec{P}$ is solenoidal, but not irrotational
(B) $\vec{P}$ is irrotational, but not solenoidal
(C) $\vec{P}$ is neither solenoidal nor irrotational
(D) $\vec{P}$ is both solenoidal and irrotational

Q.28 Which one of the following graphs describes the function $f(x) = e^{-x}(x^2 + x + 1)$?

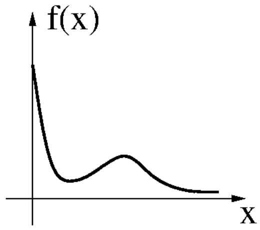
(A)



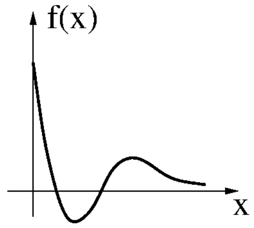
(B)



(C)



(D)

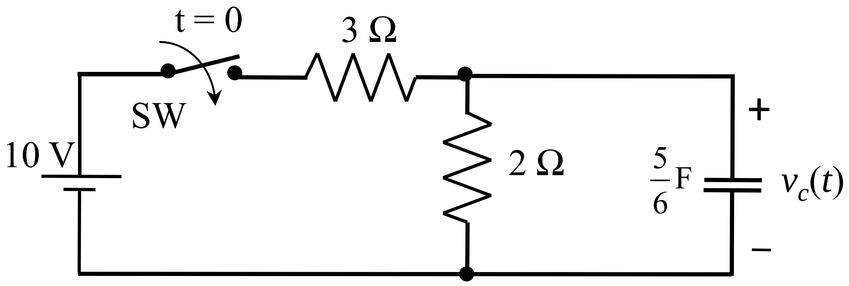


Q.29 The maximum area (in square units) of a rectangle whose vertices lie on the ellipse $x^2 + 4y^2 = 1$ is _______.

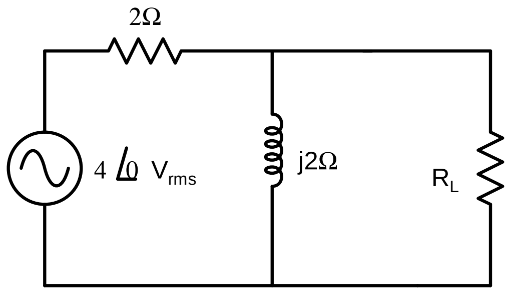
Q.30 The damping ratio of a series RLC circuit can be expressed as

(A) $\frac{R^2C}{2L}$ (B) $\frac{2L}{R^2C}$ (C) $\frac{R}{2}\sqrt{\frac{C}{L}}$ (D) $\frac{2}{R}\sqrt{\frac{L}{C}}$

Q.31 In the circuit shown, switch SW is closed at $t = 0$. Assuming zero initial conditions, the value of $v_c(t)$ (in Volts) at $t = 1$ sec is ________.



Q.32 In the given circuit, the maximum power (in Watts) that can be transferred to the load R_L is _______.



Q.33 The built-in potential of an abrupt p-n junction is 0.75 V. If its junction capacitance (C_J) at a reverse bias (V_R) of 1.25 V is 5 pF, the value of C_J (in pF) when $V_R = 7.25$ V is ________.

Q.34 A MOSFET in saturation has a drain current of 1 mA for $V_{DS} = 0.5$ V. If the channel length modulation coefficient is 0.05 V^{-1}, the output resistance (in kΩ) of the MOSFET is _________.

Q.35 For a silicon diode with long P and N regions, the accepter and donor impurity concentrations are 1×10^{17} cm^{-3} and 1×10^{15} cm^{-3}, respectively. The lifetimes of electrons in P region and holes in N region are both 100 µs. The electron and hole diffusion coefficients are 49 cm^2/s and 36 cm^2/s, respectively. Assume kT/q = 26 mV, the intrinsic carrier concentration is 1×10^{10} cm^{-3}, and $q = 1.6 \times 10^{-19}$ C. When a forward voltage of 208 mV is applied across the diode, the hole current density (in nA/cm^2) injected from P region to N region is _____________.

Q.36 The Boolean expression $F(X,Y,Z) = \bar{X}\,Y\,\bar{Z} + X\,\bar{Y}\,\bar{Z} + X\,Y\,\bar{Z} + X\,Y\,Z$ converted into the canonical product of sum (POS) form is

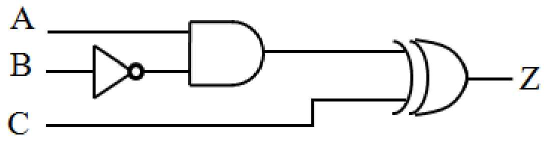
(A) $(X+Y+Z)(X+Y+\bar{Z})(X+\bar{Y}+\bar{Z})(\bar{X}+Y+\bar{Z})$

(B) $(X+\bar{Y}+Z)(\bar{X}+Y+\bar{Z})(\bar{X}+\bar{Y}+Z)(\bar{X}+\bar{Y}+\bar{Z})$

(C) $(X+Y+Z)(\bar{X}+Y+\bar{Z})(X+\bar{Y}+Z)(\bar{X}+\bar{Y}+\bar{Z})$

(D) $(X+\bar{Y}+\bar{Z})(\bar{X}+Y+Z)(\bar{X}+\bar{Y}+Z)(X+Y+Z)$

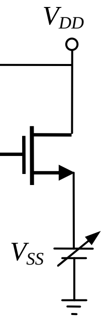
Q.37 All the logic gates shown in the figure have a propagation delay of 20 ns. Let A = C = 0 and B = 1 until time t = 0. At t = 0, all the inputs flip (i.e., A = C = 1 and B = 0) and remain in that state. For t > 0, output Z = 1 for a duration (in ns) of ______________.



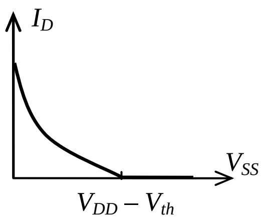
Q.38 A 3-input majority gate is defined by the logic function $M(a,b,c) = ab + bc + ca$. Which one of the following gates is represented by the function $M(\overline{M(a,b,c)}, M(a,b,\bar{c}), c)$?

(A) 3-input NAND gate
(B) 3-input XOR gate
(C) 3-input NOR gate
(D) 3-input XNOR gate

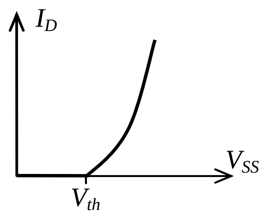
Q.39 For the NMOSFET in the circuit shown, the threshold voltage is V_{th} , where $V_{th} > 0$. The source voltage V_{SS} is varied from 0 to V_{DD}. Neglecting the channel length modulation, the drain current I_D as a function of V_{SS} is represented by



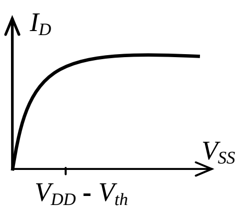
(A)



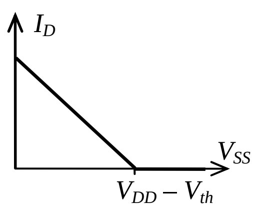
(B)



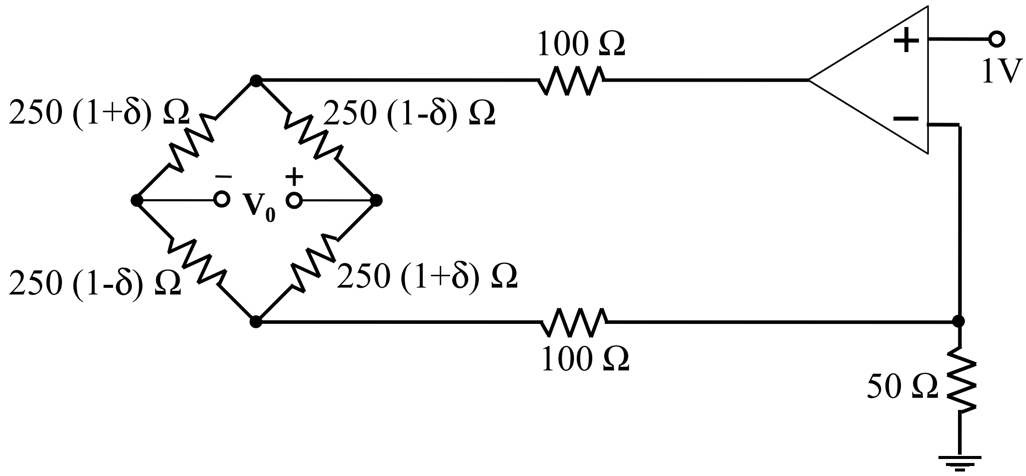
(C)



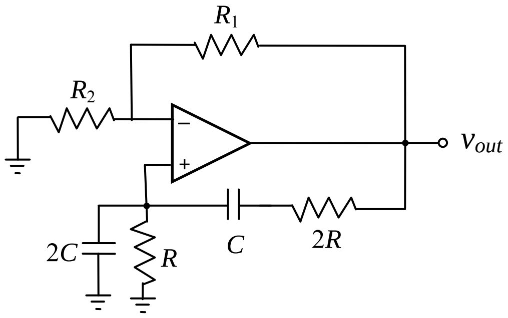
(D)



Q.40 In the circuit shown, assume that the opamp is ideal. The bridge output voltage V_0 (in mV) for $\delta = 0.05$ is _______.



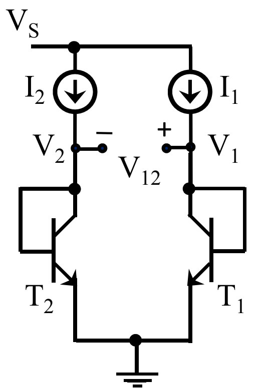
Q.41 The circuit shown in the figure has an ideal opamp. The oscillation frequency and the condition to sustain the oscillations, respectively, are



(A) $\frac{1}{CR}$ and $R_1 = R_2$ (B) $\frac{1}{CR}$ and $R_1 = 4R_2$

(C) $\frac{1}{2CR}$ and $R_1 = R_2$ (D) $\frac{1}{2CR}$ and $R_1 = 4R_2$

Q.42 In the circuit shown, $I_1 = 80$ mA and $I_2 = 4$ mA. Transistors T_1 and T_2 are identical. Assume that the thermal voltage V_T is 26 mV at 27 °C. At 50 °C, the value of the voltage $V_{12} = V_1 - V_2$ (in mV) is _______.



Q.43 Two sequences $[a, b, c]$ and $[A, B, C]$ are related as,

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{-1} & W_3^{-2} \\ 1 & W_3^{-2} & W_3^{-4} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad \text{where } W_3 = e^{j\frac{2\pi}{3}}.$$

If another sequence $[p, q, r]$ is derived as,

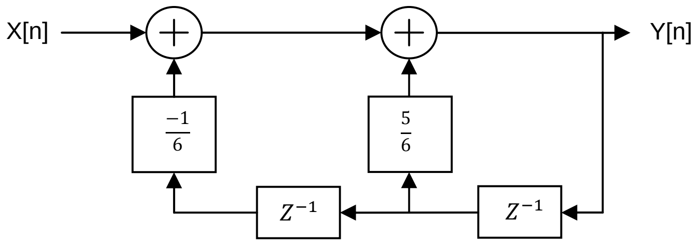
$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{1} & W_3^{2} \\ 1 & W_3^{2} & W_3^{4} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & W_3^{2} & 0 \\ 0 & 0 & W_3^{4} \end{bmatrix} \begin{bmatrix} A/3 \\ B/3 \\ C/3 \end{bmatrix},$$

then the relationship between the sequences $[p, q, r]$ and $[a, b, c]$ is

(A) $[p, q, r] = [b, a, c]$ (B) $[p, q, r] = [b, c, a]$

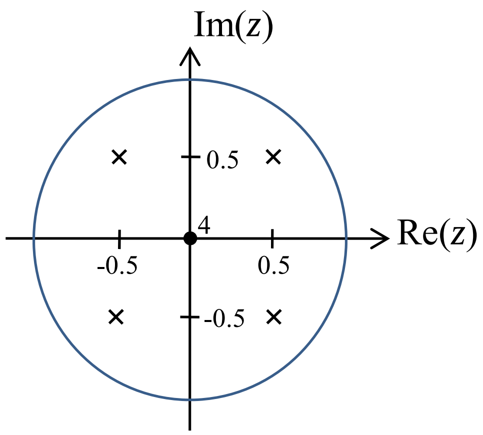
(C) $[p, q, r] = [c, a, b]$ (D) $[p, q, r] = [c, b, a]$

Q.44 For the discrete-time system shown in the figure, the poles of the system transfer function are located at



(A) 2, 3 (B) $\frac{1}{2}$, 3 (C) $\frac{1}{2}$, $\frac{1}{3}$ (D) 2, $\frac{1}{3}$

Q.45 The pole-zero diagram of a causal and stable discrete-time system is shown in the figure. The zero at the origin has multiplicity 4. The impulse response of the system is $h[n]$. If $h[0] = 1$, we can conclude



(A) $h[n]$ is real for all n
(B) $h[n]$ is purely imaginary for all n
(C) $h[n]$ is real for only even n
(D) $h[n]$ is purely imaginary for only odd n

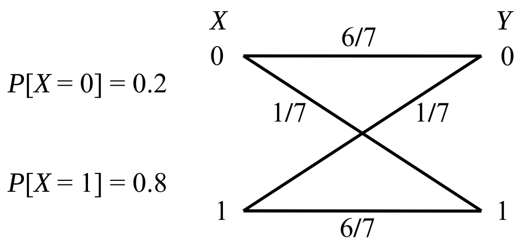
Q.46 The open-loop transfer function of a plant in a unity feedback configuration is given as $G(s)=\dfrac{K(s+4)}{(s+8)(s^2-9)}$. The value of the gain $K(>0)$ for which $-1+j2$ lies on the root locus is _______.

Q.47 A lead compensator network includes a parallel combination of R and C in the feed-forward path. If the transfer function of the compensator is $G_c(s)=\dfrac{s+2}{s+4}$, the value of RC is ________.

Q.48 A plant transfer function is given as $G(s)=\left(K_P+\dfrac{K_I}{s}\right)\dfrac{1}{s(s+2)}$. When the plant operates in a unity feedback configuration, the condition for the stability of the closed loop system is

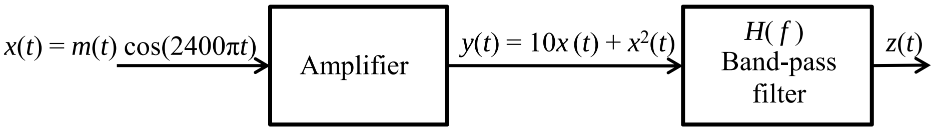
(A) $K_P>\dfrac{K_I}{2}>0$ (B) $2K_I>K_P>0$ (C) $2K_I<K_P$ (D) $2K_I>K_P$

Q.49 The input X to the Binary Symmetric Channel (BSC) shown in the figure is '1' with probability 0.8. The cross-over probability is 1/7. If the received bit Y = 0, the conditional probability that '1' was transmitted is _______.

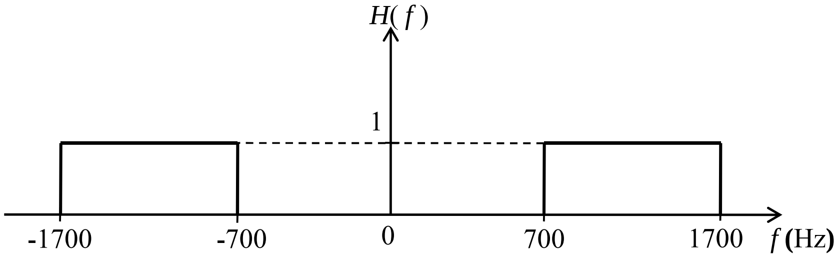


Q.50 The transmitted signal in a GSM system is of 200 kHz bandwidth and 8 users share a common bandwidth using TDMA. If at a given time 12 users are talking in a cell, the total bandwidth of the signal received by the base station of the cell will be at least (in kHz) ________.

Q.51 In the system shown in Figure (a), $m(t)$ is a low-pass signal with bandwidth W Hz. The frequency response of the band-pass filter $H(f)$ is shown in Figure (b). If it is desired that the output signal $z(t) = 10x(t)$, the maximum value of W (in Hz) should be strictly less than ________.



(a)



(b)

Q.52 A source emits bit 0 with probability $\frac{1}{3}$ and bit 1 with probability $\frac{2}{3}$. The emitted bits are communicated to the receiver. The receiver decides for either 0 or 1 based on the received value R. It is given that the conditional density functions of R are as

$$f_{R|0}(r) = \begin{cases} \frac{1}{4}, & -3 \le x \le 1, \\ 0, & \text{otherwise}, \end{cases} \quad \text{and} \quad f_{R|1}(r) = \begin{cases} \frac{1}{6}, & -1 \le x \le 5, \\ 0, & \text{otherwise}. \end{cases}$$

The minimum decision error probability is

(A) 0 (B) 1/12 (C) 1/9 (D) 1/6

Q.53 The longitudinal component of the magnetic field inside an air-filled rectangular waveguide made of a perfect electric conductor is given by the following expression

$$H_z(x, y, z, t) = 0.1\ \cos(25\pi x)\ \cos(30.3\,\pi y)\ \cos(12\pi \times 10^9 t - \beta z)\ \ (A/m)$$

The cross-sectional dimensions of the waveguide are given as $a = 0.08\ m$ and $b = 0.033\ m$. The mode of propagation inside the waveguide is

(A) TM_{12} (B) TM_{21}

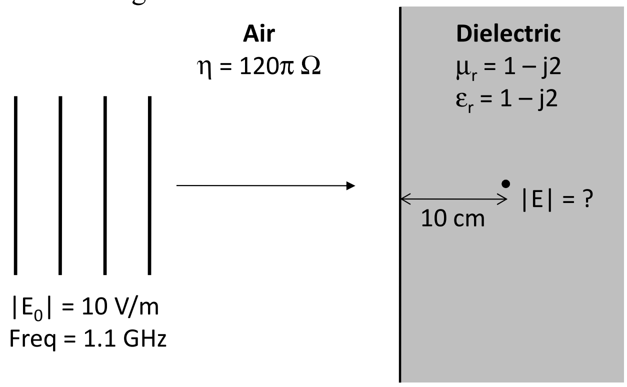
(C) TE_{21} (D) TE_{12}

Q.54 The electric field intensity of a plane wave traveling in free space is given by the following expression

$$\mathbf{E}(x, t) = \mathbf{a}_y\, 24\,\pi\ \cos(\omega t - k_0 x)\ \ (\text{V/m})$$

In this field, consider a square area 10 cm x 10 cm on a plane $x + y = 1$. The total time-averaged power (in mW) passing through the square area is _______.

Q.55 Consider a uniform plane wave with amplitude (E_0) of 10 V/m and 1.1 GHz frequency travelling in air, and incident normally on a dielectric medium with complex relative permittivity (ε_r) and permeability (μ_r) as shown in the figure.



The magnitude of the transmitted electric field component (in V/m) after it has travelled a distance of 10 cm inside the dielectric region is _________.

END OF THE QUESTION PAPER