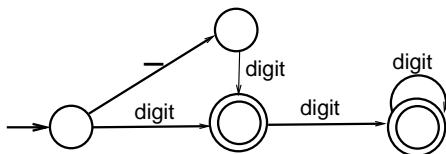
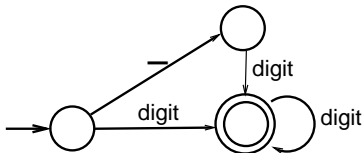


MINIMIZING THE NUMBER OF STATES

- The DFA constructed by the procedure mentioned earlier does not result in a minimum DFA.
- The DFA that we had seen earlier:

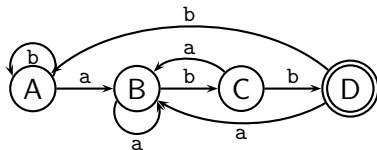
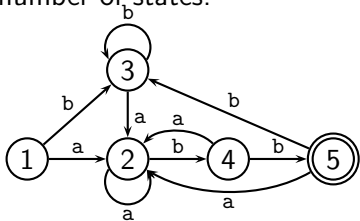


is not minimum. There is another DFA for the same regular expression with lesser number of states.



MINIMIZING THE NUMBER OF STATES

- The DFA constructed for $(b|\epsilon)(a|b)^*abb$.
- There is another DFA for the same regular expression with lesser number of states.

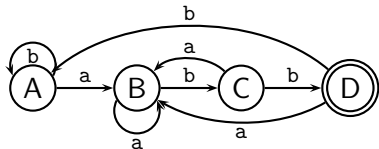
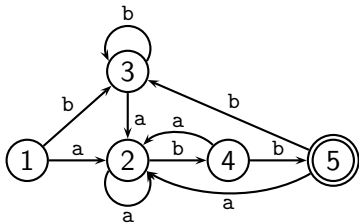


MINIMIZING THE NUMBER OF STATES

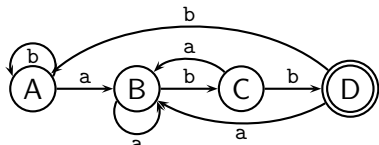
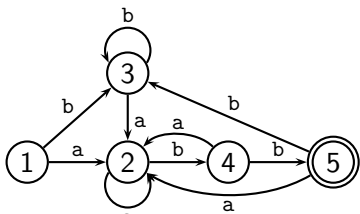
- For a typical language, the number of states of the DFA is in order of hundreds.
- Therefore we should try to minimize the number of states.

MINIMIZING THE NUMBER OF STATES

- The second DFA has been obtained by merging states 1 and 3 of the first DFA.
- Under what conditions can this merging take place?



MINIMIZING THE NUMBER OF STATES

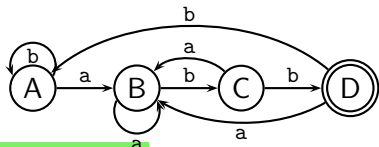
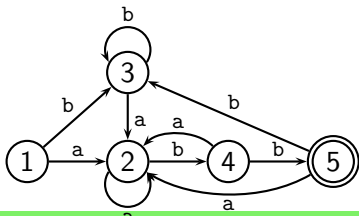


- The string **bb** takes both states 1 and 3 to a **non-final state**.
- The string **aba** takes both states 1 and 3 to a **non-final state**.
- The string ϵ takes both states 1 and 3 to a **non-final state**.
- The string **bbabb** takes both states 1 and 3 to a **final state**.

Observation:

*Any string that takes state 1 to a final state also takes 3 to a final state.
Conversely, any string that takes state 1 to a non-final state also takes 3 to a non-final state.*

MINIMIZING THE NUMBER OF STATES



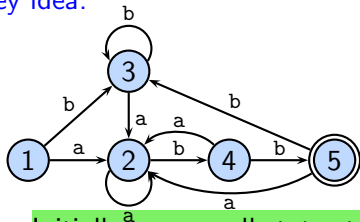
- States 1 and 3 are said to be *indistinguishable*.
- Minimization strategy:
 - Find indistinguishable states.
 - Merge them.
- Question: How does one find indistinguishable states?

MINIMIZING THE NUMBER OF STATES

Key idea:

MINIMIZING THE NUMBER OF STATES

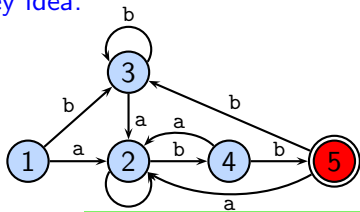
Key idea:



- Initially assume all states to be indistinguishable. Put them in a single set.

MINIMIZING THE NUMBER OF STATES

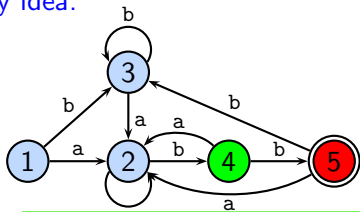
Key idea:



- The string ϵ distinguishes between final states and non-final states.
Create two partitions.

MINIMIZING THE NUMBER OF STATES

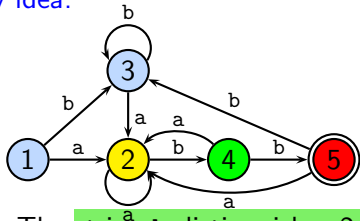
Key idea:



- b takes 4 to a red partition and retains other blue states in blue partition. Put 4 in a separate partition.

MINIMIZING THE NUMBER OF STATES

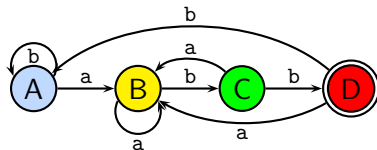
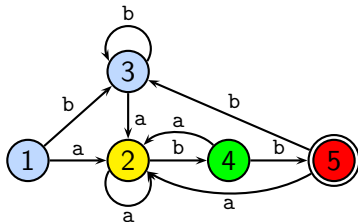
Key idea:



- The string **b** distinguishes 2 from other states in the blue partition.

MINIMIZING THE NUMBER OF STATES

Key idea:



- No other partition possible. Merge all states in the same partition.

Summary of the Method

1. Construct an initial partition $\pi = \{S - F, F_1, \dots, F_n\}$, where $F = F_1 \cup F_2 \cup \dots \cup F_n$, and each F_i is the set of final states for some token i .
2. for each set G in π do
 partition G into subsets such that two states s and t of G are in the same subset if and only if for all input symbols a , states s and t have transitions onto states in the same set of π ;
 replace G in π_{new} by the set of all subsets formed
3. If $\pi_{new} = \pi$, let $\pi_{final} := \pi$ and continue with step 4. Otherwise repeat step 2 with $\pi := \pi_{new}$.
4. Merge states in the same set of the partition.
5. Remove any dead states.