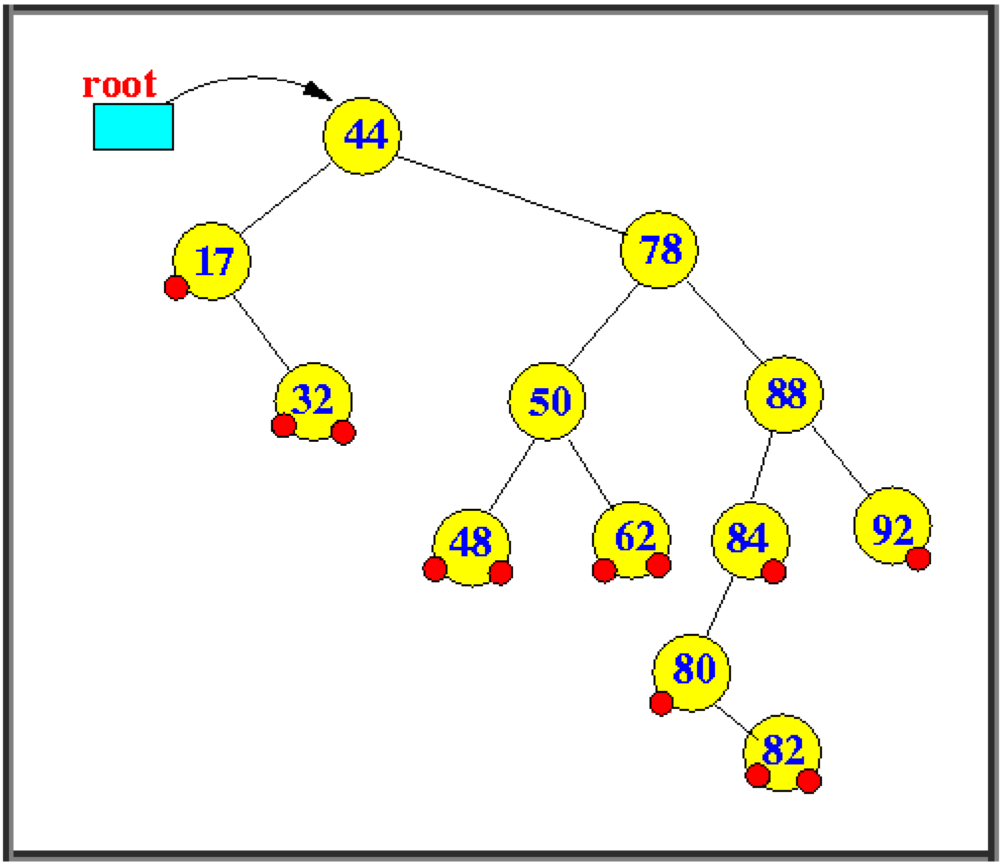
Delete operations on AVL trees

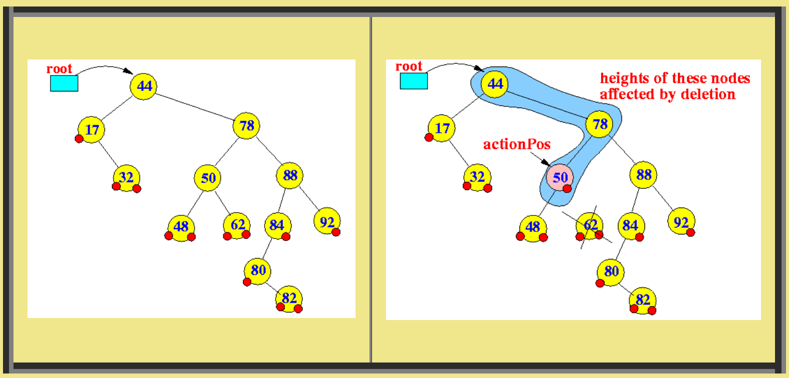
mathcs.emory.edu/~cheung/Courses/323/Syllabus/Trees/AVL-delete.html

- **Review: deleting an entry from a *binary search tree***
  - **Example *binary search tree*:**



  - Deleting a ***leaf* node** (no children nodes): **easy, just delete away....**
    Example:

Delete 62:

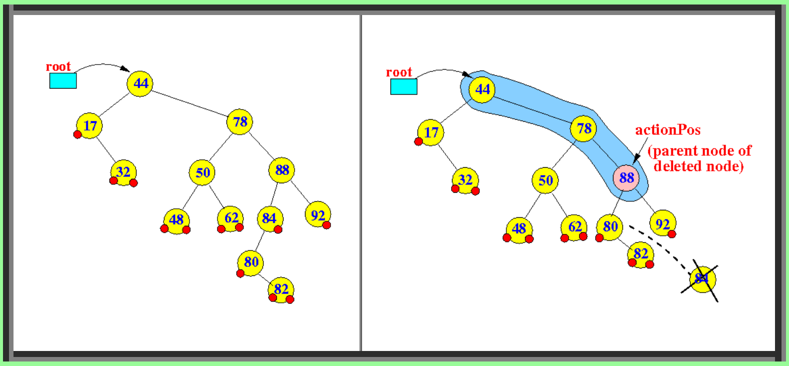


Note: action position

- The **action position** is a **reference** to the **parent node** from which a **node** has been ***physically* removed**
- The **action position** indicate the **first node** whose **height** has been **affected (possibly changed)** by the **deletion**
  (This will be **important in the re-balancing phase to adjust the tree back to an AVL tree)**

- Deleting a **node** with ***1 child node:*** **easy, connect its parent and child....**
  Example:

Delete 84:



- Deleting a **node** with ***2 children nodes:***

- **Replace** the **(to-delete) node** with its **in-order predecessor** or **in-order successor**
- Then **delete** the **in-order predecessor** or **in-order successor**

where:

- A node's **in-order successor** of a **node with 2 children** is the **left-most child** of its **right subtree**
- A node's **in-order predecessor** of a **node with 2 children** is the **right-most child** of its **left subtree**

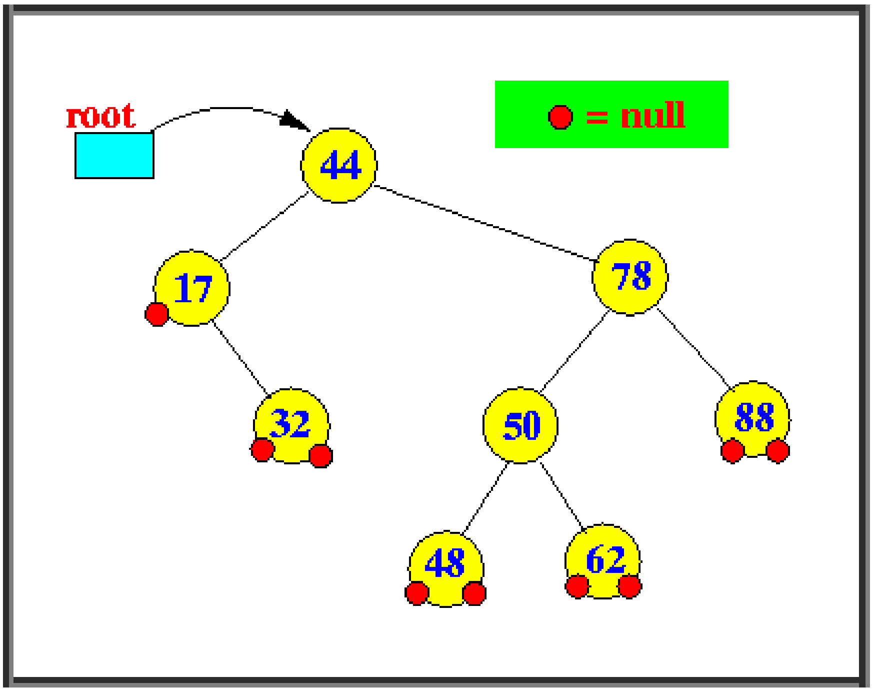
Example: deleting using the **to-delete node** with its **in-order successor**

Delete 78:

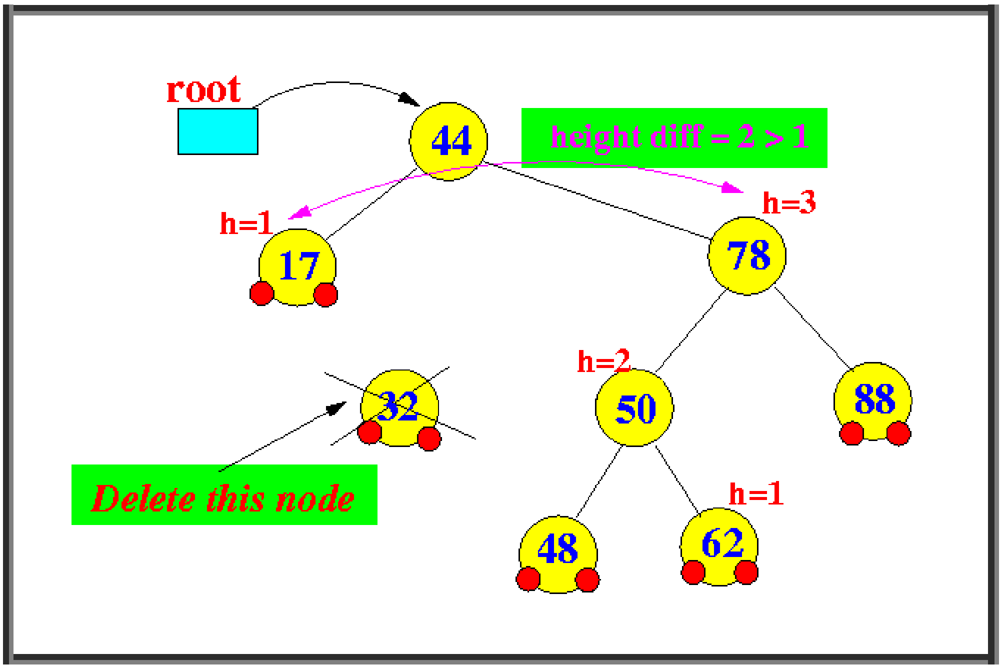
Note:

To find out more on how to **find** the **successor node** (or the **predecessor node**), see: click here

- *Deletion* in an AVL tree can *also* cause imbalance
  - Sample AVL tree:



  - **Deleting** an **entry (node)** can **also** cause an **AVL tree** to become **height unbalanced**:

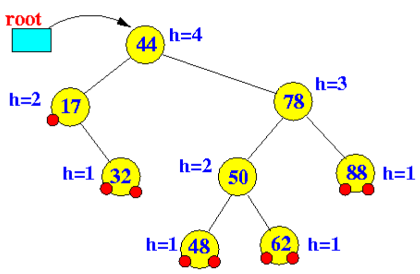
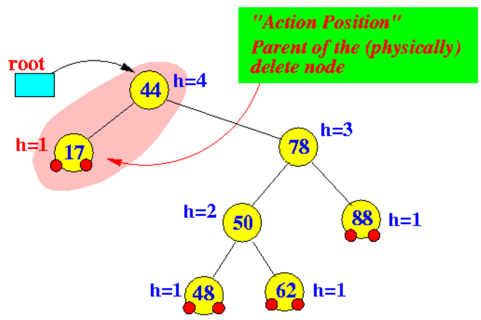


(The **resulting tree** is **no longer** an **AVL tree** !!)

- **Re-balancing the AVL tree after a *deletion* ---- an introductory example**
  - **Recall** that:

The **height** changes at ***only*** nodes between the **root** and the **parent node** of the ***physically*** **deleted node**

- **Example:**

Before **deleting node 32**	***After*** **deleting node 32**
	

Notes:

- The **actionPos (action position)** in a **delete operation** is the **parent node** of the **"deleted" node**
(By **"deleted"**, I mean the node that is ***actually*** **unlinked** - **not** the node that was ***substituted*** by a **successor node** if there was a substitution === see the **deleting a *full internal* node** case above - **click here**)

- Just like **insert**, the **height** of the nodes between the **"Action Position"** and the **root node *may*** change.
The **change** in this case is: **decrease by 1**

- **Re-balancing** the **AVL tree** after a **delete operation**:

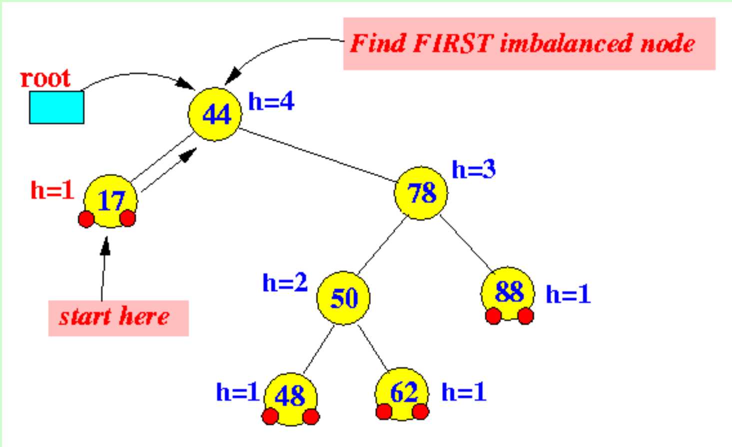
Just like **insert operation**, we can use the **tri-node restructure operations** to **re-balance** an **out-of-balanced** AVL tree.

Important difference:

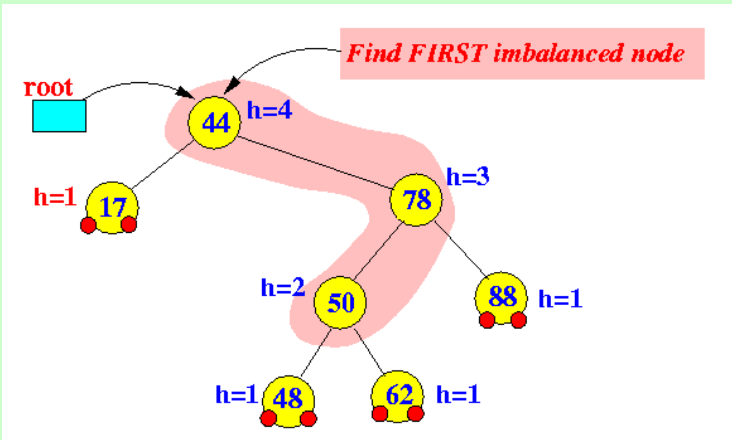
How to **apply** the **tri-node restructure operations** is a bit ***tricky*** in the **delete operation**.

- **Example: re-balancing** an AVL tree after a **deletion**:

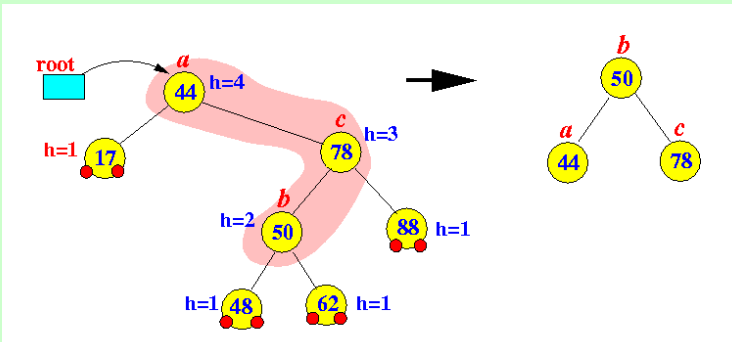
- **Starting** at the **action position** (= **parent node** of the ***physically* deleted node**), find the ***first*** **imbalanced node**
(This step is **exactly** the same as in **insert**)



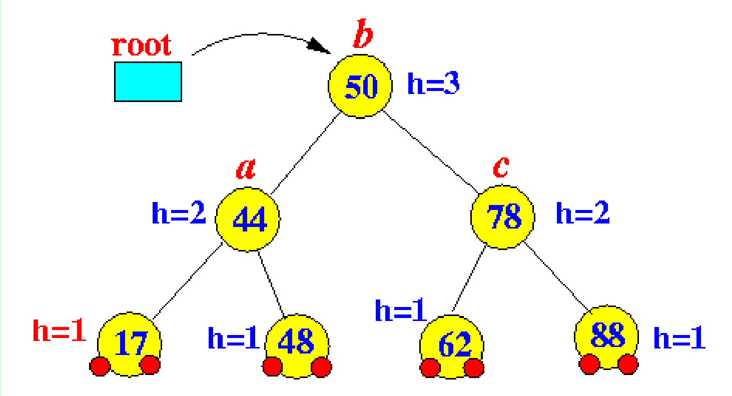
- Perform a **tri-node restructure operation** using ***these* 3 nodes (shaded)**:



- For **clarity sake**, I have depicted the **movement** of the **3 nodes** in this figure first:

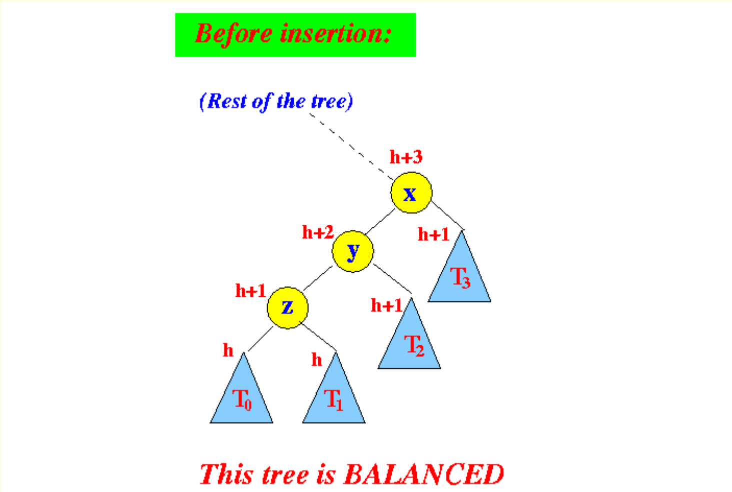


- The tree ***after*** the **tri-node restructure operation** is:

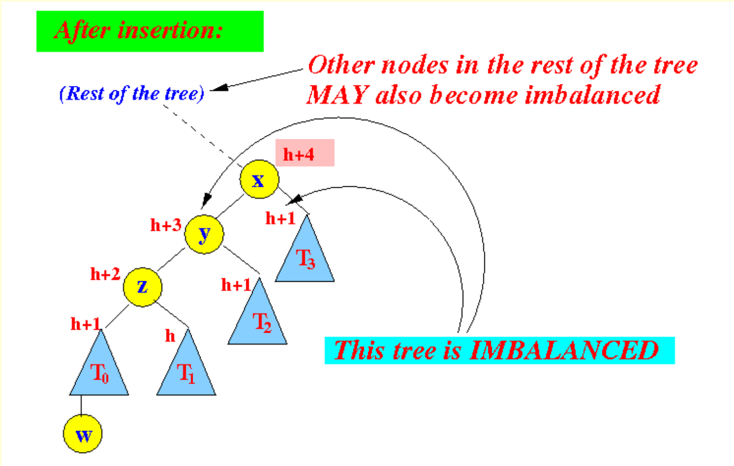


You can see it is an **AVL tree**....

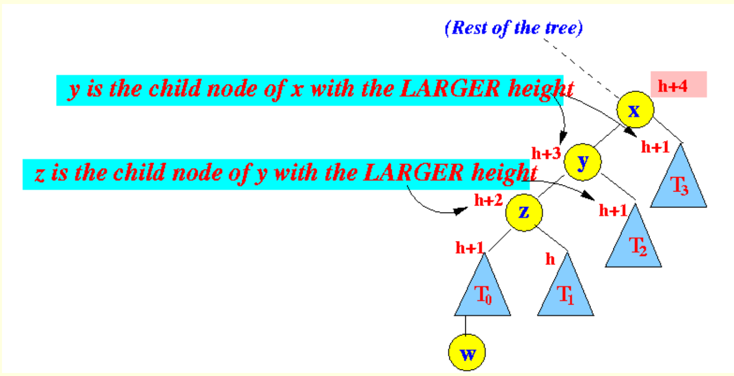
- ***Pre-condition*** **for applying a tri-node restructure operation**
  - Let us look at the **state** of the **imbalanced AVL tree prior** to applying the **tri-node restructure operation** in the ***insert*** **operation**:
    - **AVL tree** ***before*** **inserting** a new node:



    - **AVL tree** ***after*** **inserting** a new node:



- **Notice** that:



- **Pre-conditions** for **apply** a **tri-node restructure operation** to **re-balance** an AVL tree:

  - **Node x** = the ***first*** **imbalanced node** from the **action position** (= **parent** of the ***physically*** **inserted/deleted node**) to the **root**.
  - **Node y** = the **child node** of **node x** that has the ***higher*** **height**
  - **Node z** = the **child node** of **node y** that has the ***higher*** **height**

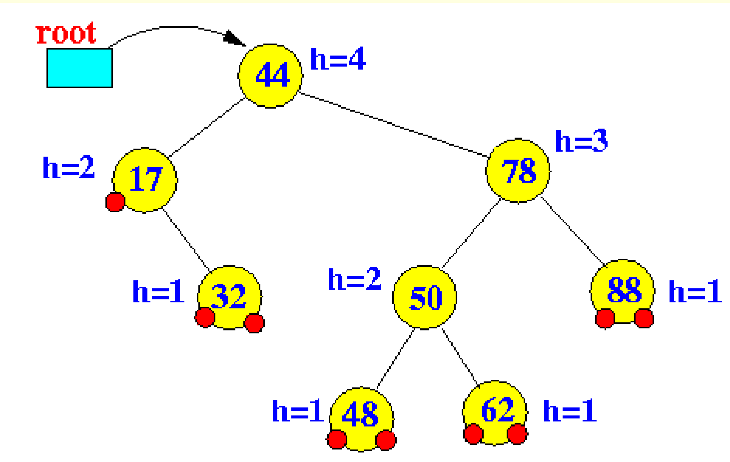
- ***Preliminary*** **(incomplete version of the *re-balancing* algorithm for delete**

  A ***preliminary*** **version** (***incomplete*** **!!**) of the ***re-balance*** **algorithm** for the ***delete*** **operation:**

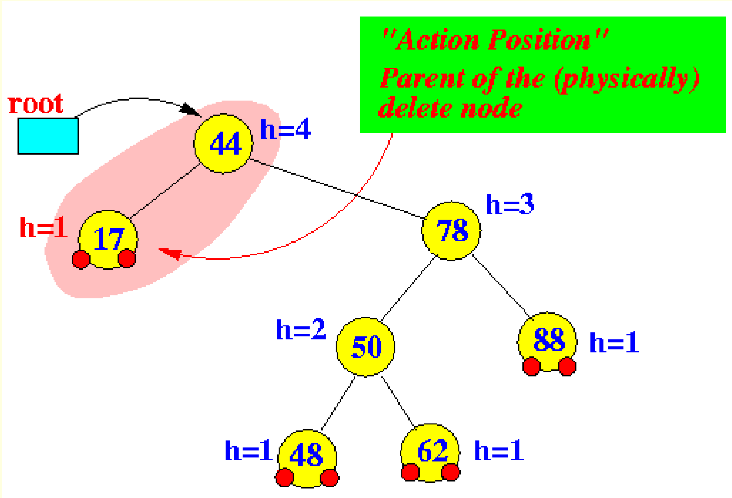
```
    Starting at the action position node
    (= parent node of the physically delete node):
    {
        find the first node p that is imbalanced
    }

    /* ----------------------------------------------
       Identify grandchild node of the unbalanced node
       for the restructure operation
------------------------------------------------ */
    x = p;
    y = the taller Child of x;
    z = the taller Child of y;

    tri-node-restructure( x, y, z );    // Apply the tri-node reconf. op.
```

Example:

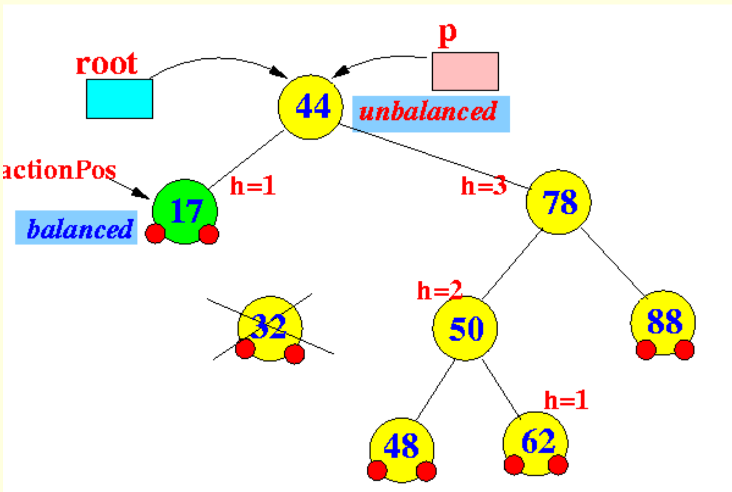
- **Delete** node **32**:



- Start the search at the **action position node (17)** (= **parent node** of the ***physically*** **deleted node (32)**):



- **traverse** up the tree towards the **root node** to find the ***first* node p** that is **unbalanced**

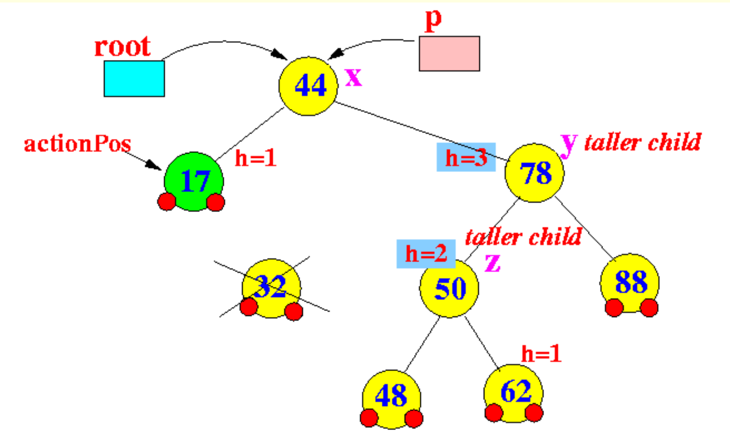


Found: **node 44**

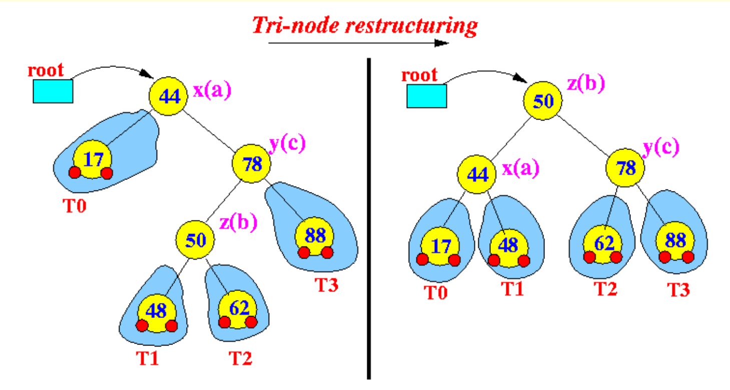
- **Determine x**, **y** and **z** for the **tri-node restructure operation**:

```
x = p;                    ====> x = 44
y = taller child of x;    ====> y = 78
z = taller child of y;    ====> z = 50
```

Graphically:



- **Now** apply ***tri-node-restructure(x,y,z)***:



- ***Further* re-balancing required for the *delete* operation**
  - **Facts:**

- We just saw that:

  > The **imbalance** at the ***first*** **imbalance node** due to a **deletion operation** ***can be restored*** using a **tri-node restructure operation**

- **However**:

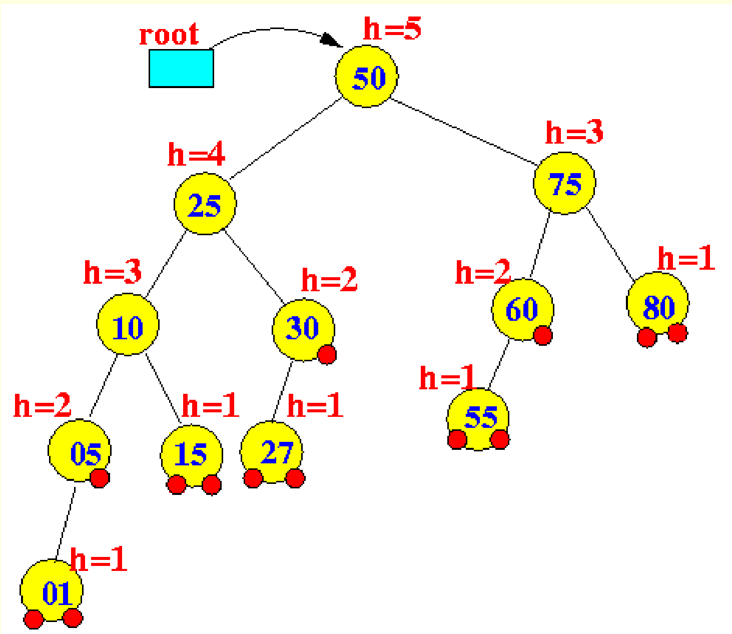
  > The ***resulting*** **subtree does** ***not*** have the ***same*** **height** as the ***original*** **subtree** !!!

- **Consequently**:

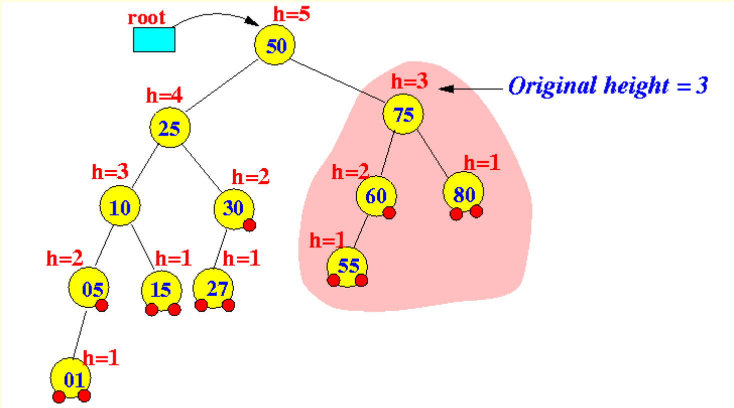
  > **Node** that are ***further*** **up the tree** are ***not*** **re-balanced** by the **re-balancing of the** ***first*** **imbalanced node** !!!

- **Example:**

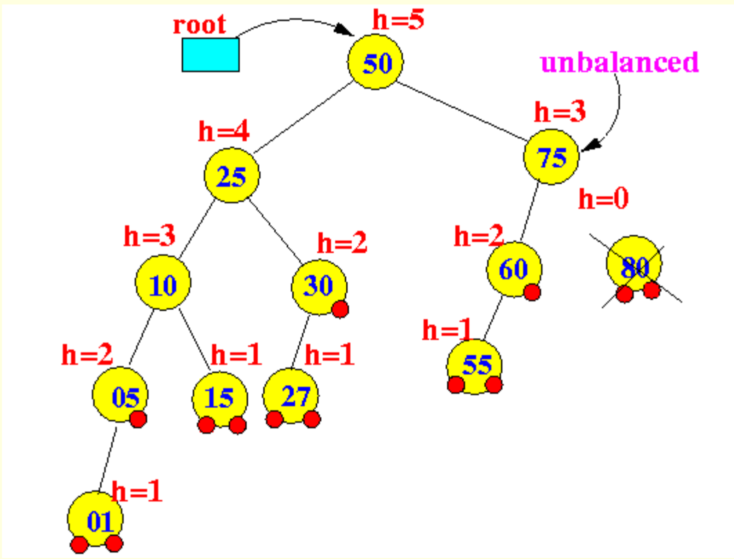
  - Consider the following **AVL tree**:



Notice that the **original height** of this (**shaded**) subtree is **3**:

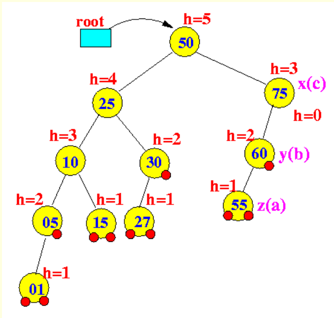


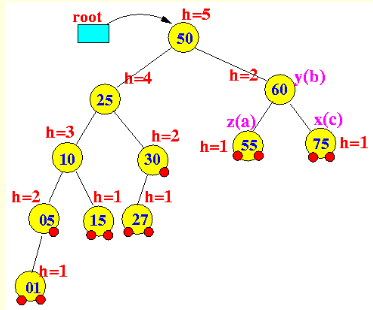
- **Delete** the node **80**:



- **Perform a Tri-node restructuring:**

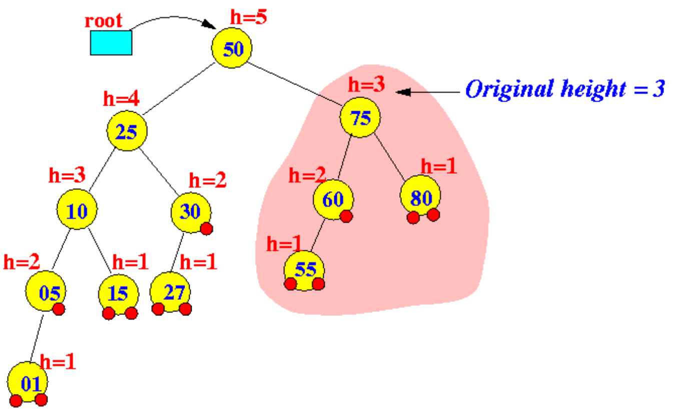
Tree *before* restructuring	Tree *after* restructuring



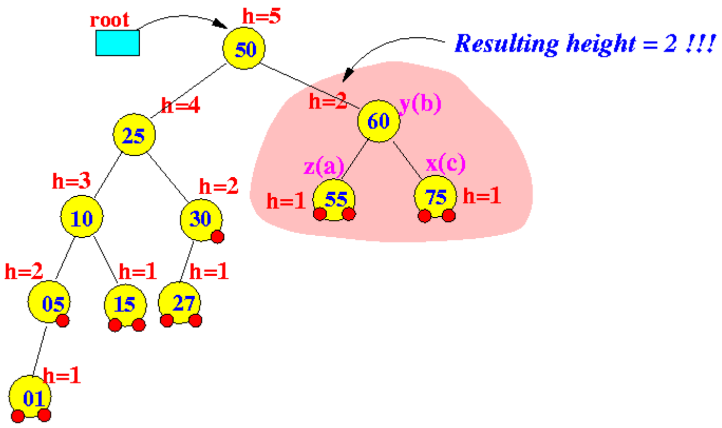


- **Notice** that the ***resulting*** **subtree** is ***shorter*** than the ***original*** **subtree !!!**

Original **subtree**

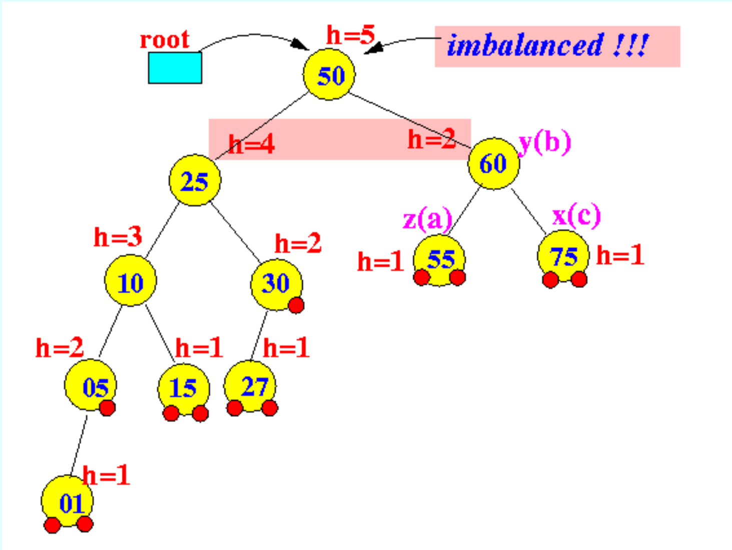


Resulting **subtree**



Result:

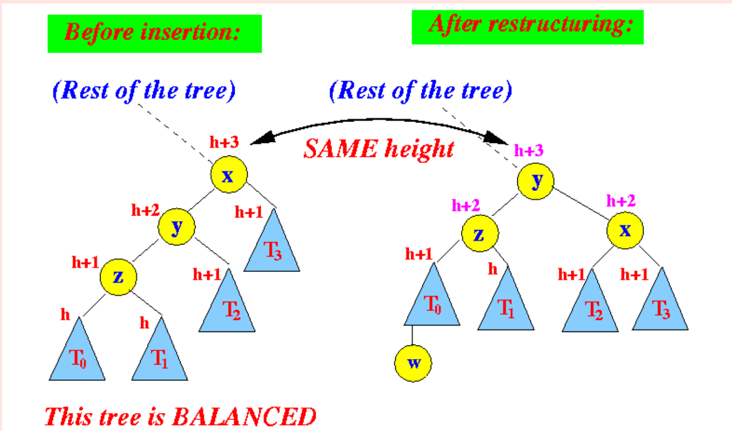
Nodes ***further up*** **in the tree** ***can*** become **imbalanced** !!!
Example:



- **Comment:**

  - ***This*** **problem** (see above) does ***not*** **occur** in the **insert operation** because the **height** of the ***resulting*** **subtree** was **equal** to the **original tree**

  - We has **this situation** in an **insert operation**:



  - So we **don't have to** perform the **tri-node restructure operation** on **node** ***further*** **up the tree** in an **insert operation** !

- **The (complete)** ***re-balance*** **procedure for a** ***delete*** **operation**

- **Re-balancing algorithm for the deletion:**

```
    p = action position;     // Starting point

    while ( p != root )      // Travel all the way up the
tree !!!
    {
 if ( p is unbalanced )
 {
          /* -----------------------------------------------
      This the is first imbalanced node (i.e., x)
      ----------------------------------------------- */
          x = p;

    /* ---------------------------------------------------
      Identify nodes y and z for the restructure operation
      --------------------------------------------------- */
    y = the taller Child of x;
    z = the taller Child of y;

          p = tri-node-restructure(x, y, z);
       // NOTE: we MODIFY tri-node-restructure() to
       //       return the root node of the NEW subtree !!!
       }

       p = p.parent;              // traverse twards the
root

    }
```

- **Java implementation:**

```
   public void rebalance(BSTEntry p)  // The starting point is passed as
parameter !!!
   {
      BSTEntry x, y, z, q;

      while ( p != null )
      {
         if ( diffHeight(p.left, p.right) > 1 )
         {

            x = p;
            y = tallerChild( x );
            z = tallerChild( y );

            System.out.println("tri_node_restructure: " + x + y + z);

            p = tri_node_restructure( x, y, z );
         }

         p = p.parent;
      }
   }
```

- The (***slightly***) **modified tri-node restructure** method:

```
   public BSTEntry tri_node_restructure( BSTEntry x, BSTEntry y, BSTEntry
z)
   {

       .... Same old code .....  (See: click here)


       return b;   // We return the root of the new subtree
   }
```

- **The *remove* method for the AVL tree**

    remove() in **Java**: I have **high lighted** the **re-balance()** calls

```
   /* ======================================================
      This is the SAME remove method as BST tree, but
      with rebalance() calls inserted after a deletion
      to rebalance the BST....
      ====================================================== */

   public void remove(String k)
   {
       BSTEntry p, q;     // Help variables
       BSTEntry parent;   // parent node
       BSTEntry succ;     // successor node

       /* --------------------------------------------
          Find the node with key == "key" in the BST
          -------------------------------------------- */
       p = findEntry(k);

       if ( ! k.equals( p.key ) )
          return;                       // Not found ==> nothing to
delete....


       /* ====================================================
          Hibbard's Algorithm
          ==================================================== */

       if ( p.left == null && p.right == null ) // Case 0: p has no
children
       {
          parent = p.parent;

          /* --------------------------------
             Delete p from p's parent
             -------------------------------- */
          if ( parent.left == p )
             parent.left = null;
          else
             parent.right = null;

          /* --------------------------------------------
             Recompute the height of all parent nodes...
             -------------------------------------------- */
          recompHeight( parent );

          /* --------------------------------------------
             Re-balance AVL tree starting at ActionPos
             -------------------------------------------- */
          rebalance ( parent );     // Rebalance AVL tree after delete at
parent
          return;
       }

       if ( p.left == null )                 // Case 1a: p has 1 (right)
child
       {
          parent = p.parent;

          /* ----------------------------------------------
             Link p's right child as p's parent child
             ---------------------------------------------- */
          if ( parent.left == p )
             parent.left = p.right;
          else
             parent.right = p.right;

          /* --------------------------------------------
             Recompute the height of all parent nodes...
             -------------------------------------------- */
          recompHeight( parent );

          /* --------------------------------------------
             Re-balance AVL tree starting at ActionPos
             -------------------------------------------- */
          rebalance ( parent );     // Rebalance AVL tree after delete at
parent
          return;
       }

       if ( p.right== null )                 // Case 1b: p has 1 (left)
```

```
child
       {
          parent = p.parent;

          /* -------------------------------------------
             Link p's left child as p's parent child
             ------------------------------------------- */
          if ( parent.left == p )
             parent.left = p.left;
          else
             parent.right = p.left;

          /* ------------------------------------------
             Recompute the height of all parent nodes...
             ------------------------------------------ */
          recompHeight( parent );

          /* ------------------------------------------
             Re-balance AVL tree starting at ActionPos
             ------------------------------------------ */
          rebalance ( parent );     // Rebalance AVL tree after delete at
parent
          return;
       }

      /* ================================================================
          Tough case: node has 2 children - find successor of p

          succ(p) is as as follows:  1 step right, all the way left

          Note: succ(p) has NOT left child !
          ================================================================
*/
       succ = p.right;                  // p has 2 children....

       while ( succ.left != null )
           succ = succ.left;

       p.key = succ.key;                // Replace p with successor
       p.value = succ.value;


       /* --------------------------------
          Delete succ from succ's parent
          -------------------------------- */
       parent = succ.parent;            // Prepare for deletion

       parent.left = succ.right;        // Link right tree to parent's left

       /* --------------------------------------------
          Recompute the height of all parent nodes...
          -------------------------------------------- */
       recompHeight( parent );

       /* --------------------------------------------
          Re-balance AVL tree starting at ActionPos
          -------------------------------------------- */
       rebalance ( parent );     // Rebalance AVL tree after delete at
parent

       return;

   }
```

- **Example program**
  - **Example Program:** (Demo the insert operation in AVL tree)
    - The **AVL Tree** class file: click here
    - The **BSTEntry.java** class file: click here
    - **Remove Test program 1** (No propagation of the re-structuring operation): click here
    - **Remove Test program 2:** (**with propagation** of the re-structuring operation): click here

Example

- **Output of *Remove* Test Prog2:**
  (I made some **annotations** to the output...)