National Institute of Technology, Tiruchirappalli - 15
Department of Computer Science and Engineering

Cycle Test 1

CSPE32 – Combinatorics and Graph Theory

Course/Department	: B.Tech./CSE	Batch	: 2021-2025
Semester/Section	: III B	Session	: July/2022
Date and Time	: 21-09-2022 & 04.00 PM – 05.00 PM	Marks	: 15

Answer ALL Questions with proper steps and justification.
Draw diagrams wherever necessary.

1. An ice cream shop has various ice cream flavors like strawberry, chocolate, mango, kiwi, tender coconut, and vanilla. Sprinkles, caramel, whipped cream, marshmallow and oreos are the toppings available. A customer can place order by selecting an ice cream flavor and various toppings as add-ons. In how many ways can an order be placed? (1)

2. Determine the coefficient of $m^4y^3z^{-3}$ in $(3m+4x+3y-4z^{-1}+5)^{12}$. (1)

3. Given positive integers m, n with $m \geq n$. Show that the number of ways to distribute m identical objects into n distinct containers with no containers left empty is (1)

$$C(m-1, m-n) = C(m-1, n-1)$$

4. Find the sequence generated by the following exponential generating function. (1)

$$f(x) = \frac{7}{(1-3x^2)} + e^{2x} - 3x^4 + 5$$

5. 24 children are to be seated around 3 round tables. The first table has a seating capacity of 9, and that of second and third are 8 and 7 respectively. How many different seating arrangements are possible? (1)

6. A librarian has to place 36 books in 6 shelves so that each shelf has at least 2 books. Consider that the books on each shelf are placed one after the other from left to right. In how many these 36 books be placed? (2)

7. In how many ways can the letters in "OCCURRENCE" be arranged so that (2)

   a) there is no pair of consecutive identical letters

   b) there are exactly two pairs of consecutive identical letters.

8. 40 identical robots are present in a factory which has 6 assembly lines. In how many ways can these robots be assigned such that each assembly line should have at least 4 but no more than 8 robots. Write the generating function for the given scenario and solve the problem using it. (2)

9. Draw the Ferrer's graph for any distinct partition of 9. Using a Ferrer's graph, show that the number of partitions of n is equal to the number of partitions of $2n$ into n summands. (2)

10. State and prove Derangement formula. (2)