

Consider the following six subsets of **Z**.

$$A = \{2m + 1 | m \in \mathbb{Z}\}\$$
 $B = \{2n + 3 | n \in \mathbb{Z}\}\$

$$C = \{2p - 3 | p \in \mathbb{Z}\}$$

$$E = \{3s + 2 | s \in \mathbf{Z}\}$$

$$B = \{2n + 3 | n \in \mathbf{Z}\}$$

$$D = \{3r + 1 | r \in \mathbf{Z}\}$$

$$F = \{3t - 2 | t \in \mathbf{Z}\}$$

Which of the following statements are true and which are false?

a)
$$A = B$$

$$\mathbf{b}) \ A = C$$

c)
$$B = C$$

d)
$$D = E$$

e)
$$D = F$$

f)
$$E = F$$

- a) True
- d) False
- b) True
- e) True
- c) True
- f) False

2. Let $A = \{1, \{1\}, \{2\}\}$. Which of the following statements are true?

a)
$$1 \in A$$

c)
$$\{1\} \subseteq A$$

e)
$$\{2\} \in A$$

g)
$$\{\{2\}\}\subseteq A$$

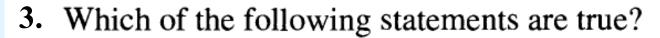
b)
$$\{1\} \in A$$

d)
$$\{\{1\}\}\subseteq A$$

$$\mathbf{f}$$
) $\{2\} \subseteq A$

h)
$$\{\{2\}\}\subset A$$

- a) True
- e) True
- b) True
- f) False
- c) True
- g) True
- d) True
- h) True



a) $\emptyset \in \emptyset$

b) $\emptyset \subset \emptyset$

c) $\emptyset \subseteq \emptyset$

d) $\emptyset \in \{\emptyset\}$

e) $\emptyset \subset \{\emptyset\}$

f) $\emptyset \subseteq \{\emptyset\}$

- a) False
- b) False
- c) True
- d) True
- e) True
- f) True

4. 抄寫 Set of Numbers

Z: integers

N: nonnegative integers/natural numbers

Z⁺: positive integers

Q: rational numbers

Q⁺: positive rational numbers

Q*: nonzero rational numbers

R: real numbers

R⁺: positive real numbers

R*: nonzero real numbers

C: complex numbers

C*: nonzero complex numbers

 $\mathbf{Z}_n : \{ x \in \mathbf{N} \mid x \le n \}$

5. For $\mathcal{U} = \{1, 2, 3, \dots, 9, 10\}$ let $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 4, 8\}$, $C = \{1, 2, 3, 5, 7\}$, and $D = \{2, 4, 6, 8\}$. Determine each of the following:

a)
$$\overline{C \cap D}$$

b)
$$(A \cup B) - C$$

a)
$$\mathcal{U}$$
-{2} or {1,3,4,5,6,7,8,9,10}

6. If A = [0, 3], B = [2, 7), with $\mathcal{U} = \mathbf{R}$, determine each of the following:

a) $A \cap B$

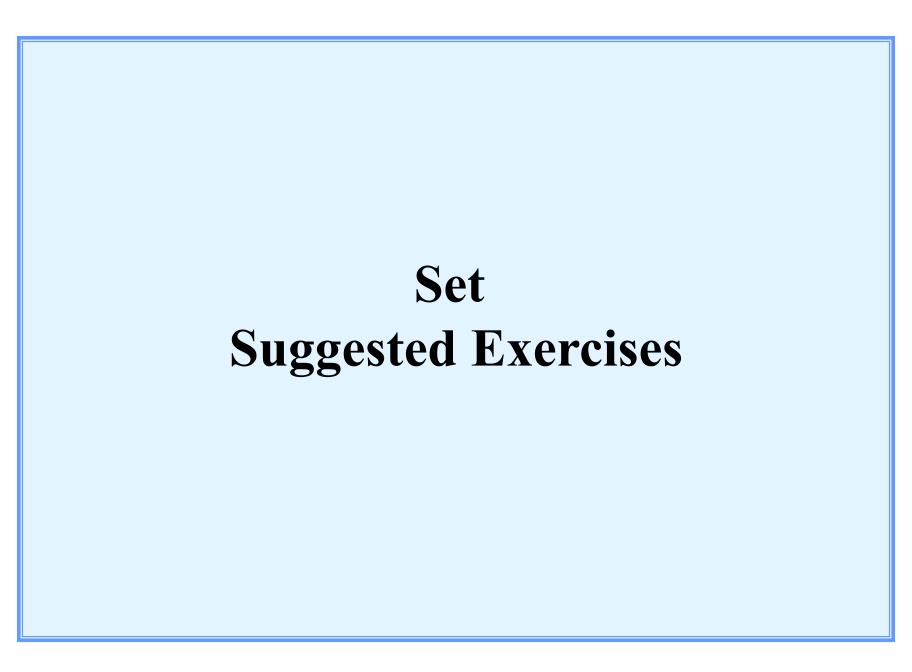
b) $A \cup B$

c) \overline{A}

d) B - A

a) [2.3] or
$$\{x | x \in \mathcal{R}, 2 \le x \le 3\}$$

- b) [0,7) or $\{x | x \in \mathbb{R}, 0 \le x < 7\}$
- c) $(-\infty, 0) \cup (3, +\infty)$ or $\{x | x \in \mathbb{R}, (x < 0) \lor (x > 3)\}$
- d) (3,7) or $\{x | x \in \mathcal{R}, 3 < x < 7\}$



- 1. Let $A = \{1, 2, 3, 4, 5, 7, 8, 10, 11, 14, 17, 18\}.$
 - a) How many subsets of A contain six elements?
 - **b)** How many six-element subsets of A contain four even integers and two odd integers?
 - c) How many subsets of A contain only odd integers?

a)
$$\binom{12}{6}$$
 = 924

b)
$$\binom{6}{4}\binom{6}{2} = 225$$

c)
$$2^6 - 1 = 63$$

2. Let $A, B, C \subseteq \mathcal{U}$. Prove the following.

If $A \subset B$ and $B \subset C$, then $A \subset C$.

 $A \subset B$ $\therefore \forall x, x \in A \Longrightarrow x \in B$. But $\exists y, y \in B \land y \notin A$, where $x, y \in \mathcal{U}$ $: B \subset C \quad : \forall 3, , 3 \in B \implies 3 \in C.$ But $\exists w, w \in C \land w \notin B$, where $3, w \in \mathcal{U}$. By the Law of the Syllogism, we conclude $\forall x, x \in A \Longrightarrow x \in C$, and thus $A \subseteq C$. Let $w \in C \land w \notin A$, where $w \in \mathcal{U}$. *If* $w \in A$, then $w \in b$, contradiction! Thus, $w \notin A$. $: w \in C \land w \notin A.$ We may conclude $A \subset C$.

3. For any universe \mathcal{U} and any sets $A, B \subseteq \mathcal{U}$

Prove that $A \cup B = B \implies A \cap B = A$

We need to show both ANB SA and ASANB. For $\chi \in \mathcal{U}$, let $\chi \in A \cap B$, then $\chi \in A$, so $A \cap B \subseteq A$. Furthermore, if $x \in A$, then $x \in AUB$, and "AUB = B, This implies $\forall x \in A$, $x \in A \cap B$. Hence we have $A \subseteq A \cap B$. therefore $\chi \in B$, also $\chi \in A \cap B$. Since we have both ANB SA and ASANB, it leads to

4. For any universe \mathcal{U} and any sets $A, B \subseteq \mathcal{U}$

Prove that

$$\overline{B} \subseteq \overline{A} \implies A \subseteq B$$

For
$$x \in \mathcal{U}$$
, let $x \in A$, then $x \notin \overline{A}$.

Now, if $x \in B$, then $x \in \overline{A}$ because $B \subseteq \overline{A}$.

But it is impossible due to that $x \notin \overline{A}$, so $x \notin \overline{B}$.

Hence, $x \in B$. Therefore, if $x \in A$, then $x \in B$,

we may conclude $A \subseteq B$.

5. Determine which of the following statements are true and which are false.

a)
$$\mathbf{Z}^+ \subseteq \mathbf{Q}^+$$

c)
$$Q^+ \subseteq R$$

$$e) Q^+ \cap R^+ = Q^+$$

g)
$$\mathbf{R}^+ \cap \mathbf{C} = \mathbf{R}^+$$

i)
$$Q^* \cap Z = Z$$

b)
$$\mathbf{Z}^+ \subseteq \mathbf{Q}$$

d)
$$\mathbf{R}^+ \subseteq \mathbf{Q}$$

f)
$$Z^+ \cup R^+ = R^+$$

h)
$$C \cup R = R$$

- a) True f) True
- b) True g) True
- c) True h) False
- d) False i) False
- e) True

6. Prove

For sets, A, B, $C \subseteq \mathcal{U}$, $A \triangle C = B \triangle C \Rightarrow A = B$.

7. Using the laws of set theory, simplify each of the following:

$$(A - B) \cup (A \cap B)$$

$$(A - B) \cup (A \cap B)$$
 Reasons
 $= (A \cap \overline{B}) \cup (A \cap B)$ By definition
 $= A \cap (\overline{B} \cup B)$ Distributive Laws
 $= A \cap U$ Inverse Laws
 $= A$ Identity Laws