Set
In-class Exercises

1. Consider the following six subsets of $\mathbf{Z}$.

$A = \{2m + 1 \mid m \in \mathbf{Z}\}$ $\quad$ $B = \{2n + 3 \mid n \in \mathbf{Z}\}$

$C = \{2p - 3 \mid p \in \mathbf{Z}\}$ $\quad$ $D = \{3r + 1 \mid r \in \mathbf{Z}\}$

$E = \{3s + 2 \mid s \in \mathbf{Z}\}$ $\quad$ $F = \{3t - 2 \mid t \in \mathbf{Z}\}$

Which of the following statements are true and which are false?

a) $A = B$ $\quad$ **b)** $A = C$ $\quad$ **c)** $B = C$

d) $D = E$ $\quad$ **e)** $D = F$ $\quad$ **f)** $E = F$

a) True
b) True
c) True

d) False
e) True
f) False

2. Let $A = \{1, \{1\}, \{2\}\}$. Which of the following statements are true?

a) $1 \in A$

b) $\{1\} \in A$

c) $\{1\} \subseteq A$

d) $\{\{1\}\} \subseteq A$

e) $\{2\} \in A$

f) $\{2\} \subseteq A$

g) $\{\{2\}\} \subseteq A$

h) $\{\{2\}\} \subset A$

a) True
b) True
c) True
d) True

e) True
f) False
g) True
h) True

3. Which of the following statements are true?

a) $\emptyset \in \emptyset$ **b)** $\emptyset \subset \emptyset$ **c)** $\emptyset \subseteq \emptyset$

d) $\emptyset \in \{\emptyset\}$ **e)** $\emptyset \subset \{\emptyset\}$ **f)** $\emptyset \subseteq \{\emptyset\}$

a) False
b) False
c) True
d) True
e) True
f) True

4. 抄寫 Set of Numbers

$\mathbf{Z}$: integers

$\mathbf{N}$: nonnegative integers/natural numbers

$\mathbf{Z}^+$: positive integers

$\mathbf{Q}$: rational numbers

$\mathbf{Q}^+$: positive rational numbers

$\mathbf{Q}^*$: nonzero rational numbers

$\mathbf{R}$: real numbers

$\mathbf{R}^+$: positive real numbers

$\mathbf{R}^*$: nonzero real numbers

$\mathbf{C}$: complex numbers

$\mathbf{C}^*$: nonzero complex numbers

$\mathbf{Z}_n$: $\{x \in \mathbf{N} \mid x < n\}$

5. For $\mathcal{U} = \{1, 2, 3, \ldots, 9, 10\}$ let $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 4, 8\}$, $C = \{1, 2, 3, 5, 7\}$, and $D = \{2, 4, 6, 8\}$. Determine each of the following:

a) $\overline{C \cap D}$ **b)** $(A \cup B) - C$

a) $\mathcal{U} - \{2\}$ or $\{1,3,4,5,6,7,8,9,10\}$ b) $\{4,8\}$

6. If $A = [0, 3]$, $B = [2, 7)$, with $\mathcal{U} = \mathbf{R}$, determine each of the following:

a) $A \cap B$ **b)** $A \cup B$

c) $\overline{A}$ **d)** $B - A$

a) $[2,3]$ or $\{x | x \epsilon \mathcal{R},\ 2 \leq x \leq 3\}$
b) $[0,7)$ or $\{x | x \epsilon \mathcal{R},\ 0 \leq x < 7\}$
c) $(-\infty, 0) \cup (3, +\infty)$ or $\{x | x \epsilon \mathcal{R}, (x < 0) \vee (x > 3)\}$
d) $(3,7)$ or $\{x | x \epsilon \mathcal{R}, 3 < x < 7\}$

Set
Suggested Exercises

1. Let $A = \{1, 2, 3, 4, 5, 7, 8, 10, 11, 14, 17, 18\}$.

a) How many subsets of A contain six elements?

b) How many six-element subsets of A contain four even integers and two odd integers?

c) How many subsets of A contain only odd integers?

a) $\binom{12}{6} = 924$

b) $\binom{6}{4}\binom{6}{2} = 225$

c) $2^6 - 1 = 63$

2. Let A, B, $C \subseteq \mathcal{U}$. Prove the following.

If $A \subset B$ and $B \subset C$, then $A \subset C$.

$\because A \subset B \quad \therefore \forall x, x \in A \Rightarrow x \in B.$

$But\ \exists y, y \in B \wedge y \notin A, where\ x, y \in \mathcal{U}$

$\because B \subset C \quad \therefore \forall z,, z \in B \Rightarrow z \in C.$

$But\ \exists w, w \in C \wedge w \notin B,$

$where\ z, w \in \mathcal{U}.$

By the Law of the Syllogism, we conclude

$\forall x, x \in A \Rightarrow x \in C$, *and thus* $A \subseteq C$.

Let $w \in C \wedge w \notin A$, *where* $w \in \mathcal{U}$.

If $w \in A$, *then* $w \in b$, *contradiction!*

Thus, $w \notin A$.

$\therefore w \in C \wedge w \notin A.$

We may conclude $A \subset C$.

3. For any universe $\mathcal{U}$ and any sets $A, B \subseteq \mathcal{U}$

Prove that $A \cup B = B \implies A \cap B = A$

We need to show both $A \cap B \subseteq A$ and $A \subseteq A \cap B$.

For $x \in \mathcal{U}$, let $x \in A \cap B$, then $x \in A$, so $A \cap B \subseteq A$.

Furthermore, if $x \in A$, then $x \in A \cup B$, and $\because A \cup B = B$, therefore $x \in B$, also $x \in A \cap B$.

This implies $\forall x \in A$, $x \in A \cap B$. Hence we have $A \subseteq A \cap B$.

Since we have both $A \cap B \subseteq A$ and $A \subseteq A \cap B$, it leads to $A \cap B = A$.

4. For any universe $\mathcal{U}$ and any sets A, $B \subseteq \mathcal{U}$

Prove that $\overline{B} \subseteq \overline{A} \Rightarrow A \subseteq B$

For $x \in \mathcal{U}$, let $x \in A$, then $x \notin \overline{A}$.

Now, if $x \in \overline{B}$, then $x \in \overline{A}$ because $\overline{B} \subseteq \overline{A}$.

But it is impossible due to that $x \notin \overline{A}$, so $x \notin \overline{B}$.

Hence, $x \in B$. Therefore, if $x \in A$, then $x \in B$,

we may conclude $A \subseteq B$.

5. Determine which of the following statements are true and which are false.

a) $\mathbf{Z}^+ \subseteq \mathbf{Q}^+$
b) $\mathbf{Z}^+ \subseteq \mathbf{Q}$
c) $\mathbf{Q}^+ \subseteq \mathbf{R}$
d) $\mathbf{R}^+ \subseteq \mathbf{Q}$
e) $\mathbf{Q}^+ \cap \mathbf{R}^+ = \mathbf{Q}^+$
f) $\mathbf{Z}^+ \cup \mathbf{R}^+ = \mathbf{R}^+$
g) $\mathbf{R}^+ \cap \mathbf{C} = \mathbf{R}^+$
h) $\mathbf{C} \cup \mathbf{R} = \mathbf{R}$
i) $\mathbf{Q}^* \cap \mathbf{Z} = \mathbf{Z}$

a) True
b) True
c) True
d) False
e) True
f) True
g) True
h) False
i) False

6. Prove

For sets, A, B, $C \subseteq \mathcal{U}$, $A \,\Delta\, C = B \,\Delta\, C \Rightarrow A = B$.

For $x \in \mathcal{U}$, let $x \in A$. consider two cases:

<u>Case I</u>: $x \in C$

$\Rightarrow x \notin A \,\Delta\, C$ $\quad \because x \in A \cap C$

$\Rightarrow x \notin B \,\Delta\, C$ $\quad \because A \Delta C = B \Delta C$

$\Rightarrow x \in B$ $\quad \because (x \notin B) \to (x \in B \Delta C)$

<u>Case II</u>: $x \notin C$ $\quad$ by definition.

$\Rightarrow x \in A \Delta C$ $\quad \because A \Delta C = B \Delta C$

$\Rightarrow x \in B \Delta C$ $\quad \because (x \notin B) \to (x \notin B \Delta C)$

$\Rightarrow x \in B$

In either case $A \subseteq B$. Similarly, we may have $B \subseteq A$.

Hence, $A = B$.

7. Using the laws of set theory, simplify each of the following:

$$(A - B) \cup (A \cap B)$$

	Reasons
$(A - B) \cup (A \cap B)$	
$= (A \cap \bar{B}) \cup (A \cap B)$	By definition
$= A \cap (\bar{B} \cup B)$	Distributive Laws
$= A \cap \mathcal{U}$	Inverse Laws
$= A$	Identity Laws