4-3 In-Class Exercise

1. Determine whether the polynomial $1 + x + x^2$ is a linear combination of

$$\mathbf{p}_1 = 2 + x + x^2,\ \mathbf{p}_2 = 1 - x^2,\ \mathbf{p}_3 = 1 + 2x.$$

4-3 Suggested Exercises

1. Determine whether the following polynomials span P_2.

$$\mathbf{p}_1 = 1 + x, \qquad \mathbf{p}_2 = 1 - x,$$
$$\mathbf{p}_3 = 1 + x + x^2, \quad \mathbf{p}_4 = 2 - x^2$$

2. Express the vector $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

as a linear combination of

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; D = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$$

3. Suppose that $\mathbf{v}_1 = (2, 1, 0, 3)$, $\mathbf{v}_2 = (3, -1, 5, 2)$, and $\mathbf{v}_3 = (-1, 0, 2, 1)$. Which of the following vectors are in $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

a. $(2, 3, -7, 3)$

b. $(1, 1, 1, 1)$

4. Determine whether the matrices span M_{22}.

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

5. Let W be the solution space to the system $A\mathbf{x} = \mathbf{0}$. Determine whether the set $\{\mathbf{u}, \mathbf{v}\}$ spans W.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\mathbf{u} = (1,\ 0,\ -1,\ 0),\ \mathbf{v} = (0,\ 1,\ 0,\ -1)$$

6. In each part, let $T_A : R^3 \to R^2$ be multiplication by A, and let $\mathbf{u}_1 = (0,1,1)$ and $\mathbf{u}_2 = (2,-1,1)$ and $\mathbf{u}_3 = (1,1,-2)$. Determine whether the set $\{T_A(\mathbf{u}_1), T_A(\mathbf{u}_2), T_A(\mathbf{u}_3)\}$ spans R^2.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$