1-8 In-Class Exercise

1. Find the standard matrix for the transformation and use it to compute $T(\mathbf{x})$.

$$T(x_1, x_2, x_3) = (x_1, x_2 - x_3, x_2);\ \mathbf{x} = (1, 0, 5)$$

2. Find the standard matrix A for the linear transformation $T : R^2 \to R^2$ for which

$$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -2 \end{bmatrix},\ T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

1-8 Suggested Exercise

1. Find the domain and codomain of the transformation defined by the equations.

$$\begin{aligned} w_1 &= 5x_1 - 7x_2 \\ w_2 &= 6x_1 + x_2 \\ w_3 &= 2x_1 + 3x_2 \end{aligned}$$

2. Find the domain and codomain of the transformation T defined by the formula.

a) $T(x_1, x_2, x_3) = (x_1, x_2 - x_3, x_2)$

b) $$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ x_1 - x_3 \\ 0 \end{bmatrix}$$

3. Find the standard matrix for the transformation defined by the equation or formula.

a) $$\begin{aligned} w_1 &= 2x_1 - 3x_2 + x_3 \\ w_2 &= 3x_1 + 5x_2 - x_3 \end{aligned}$$

b) $T(x_1, x_2, x_3) = (0, 0, 0, 0, 0)$

4. Find the standard matrix for the transformation $T : R^4 \to R^2$ defined by.

$$\begin{aligned} w_1 &= 2x_1 + 3x_2 - 5x_3 - x_4 \\ w_2 &= x_1 - 5x_2 + 2x_3 - 3x_4 \end{aligned}$$

and then compute $T(1, -1, 2, 4)$ by directly substituting in the equations and then by matrix multiplication.

5. Find $T_A(\mathrm{x})$, and express your answer in matrix form.

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix};\ \mathbf{x} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$$

6. Use Theorem 1.8.2 to show that T is a matrix transformation.

$$T(x, y, z) = (x + y, y + z, x)$$

7. Use Theorem 1.8.2 to show that T is <u>not</u> a matrix transformation.

$$T(x, y) = (x, y + 1)$$

8. The images of the standard basis vectors for R^3 are given for a linear transformation $T : R^3 \to R^3$. Find the standard matrix for the transformation, and find $T(\mathbf{x})$.

$$T(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \quad T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad T(\mathbf{e}_3) = \begin{bmatrix} 4 \\ -3 \\ -1 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$