1-1 In-Class Exercise

1. Apply Elementary Row Operation on its augmented matrix to solve the following SLE

$$2x + z = 1$$
$$x - 2y + 4z = 7$$
$$4x + y - z = 0$$

$x = 1$
$y = -5$
$z = -1$

1-1 Suggested Exercise

1. Under what conditions on a and b will the linear system have no solutions, one solution, infinitely many solutions?

$$\begin{aligned} 2x - 3y &= a \\ 4x - 6y &= b \end{aligned}$$

We can eliminate x from the second equation by adding -2 times the first equation to the second. This yields the system

$$\begin{aligned} 2x - 3y &= a \\ 0 &= b - 2a \end{aligned}$$

If $b-2a=0$ (i.e., $b=2a$) then the second equation imposes no restriction on x and y; consequently, the system has infinitely many solutions.

If $b-2a\neq 0$ (i.e., $b\neq 2a$) then the second equation becomes contradictory thus the system has no solutions.

There are no values of a and b for which the system has one solution.

2. Solve the following SLE and use parametric equations to describe the solution set.

$$\begin{aligned} x_1 + 3x_2 - x_3 &= -4 \\ 3x_1 + 9x_2 - 3x_3 &= -12 \\ -x_1 - 3x_2 + x_3 &= 4 \end{aligned}$$

$$x_1 + 3x_2 - x_3 = -4$$

$$x_1 = -4 - 3r + s, \quad x_2 = r, \quad x_3 = s$$

where the parameters r and s are arbitrary real numbers.

3. Find all values of k for which the given augmented matrix corresponds to a consistent linear system.

$$\begin{bmatrix} k & 1 & -2 \\ 4 & -1 & 2 \end{bmatrix}$$

Add the first row to the second to obtain $\begin{bmatrix} k & 1 & -2 \\ 4+k & 0 & 0 \end{bmatrix}$ which corresponds to the system

$$\begin{aligned} kx + y &= -2 \\ (4+k)x &= 0 \end{aligned}$$

If $k = -4$ then the second equation becomes $0 = 0$, which does not impose any restriction on x and y therefore we can omit it and proceed to determine the solution set using the first equation. There are infinitely many solutions in this set.

If $k \neq -4$ then the second equation yields $x = 0$ and the first equation becomes $y = -2$.

Consequently, for all values of k the given augmented matrix corresponds to a consistent linear system.

4. Find all values of k for which the given augmented matrix corresponds to a consistent linear system.

$$\begin{bmatrix} 1 & k & -4 \\ 4 & 8 & 2 \end{bmatrix}$$

Add -4 times the first row to the second to obtain $\begin{bmatrix} 1 & k & -4 \\ 0 & 8-4k & 18 \end{bmatrix}$ which corresponds to the system

$$x + ky = -4$$

$$(8-4k)y = 18$$

If $k = 2$ then the second equation becomes $0 = 18$, which is contradictory thus the system becomes inconsistent.

If $k \neq 2$ then we can solve the second equation for y and proceed to substitute this value into the first equation and solve for x.

Consequently, for all values of $k \neq 2$ the given augmented matrix corresponds to a consistent linear system.