4-9 In-Class Exercise

1. In each part, use the information in the table to:

i. find the dimensions of the row space of A, column space of A, null space of A, and null space of A^T;

ii. determine whether the linear system $A\mathbf{x} = \mathbf{b}$ is consistent;

iii. find the number of parameters in the general solution of each system in (ii) that is consistent.

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Size of A	3×3	3×3	3×3	5×9	5×9	4×4	6×2
Rank(A)	3	2	1	2	2	0	2
Rank$[A \mid \mathbf{b}]$	3	3	1	2	3	0	2

		(a)	(b)	(c)	(d)	(e)	(f)	(g)
	Size of A:	3×3	3×3	3×3	5×9	5×9	4×4	6×2
	rank(A)	3	2	1	2	2	0	2
	rank($A \mid \mathbf{b}$)	3	3	1	2	3	0	2
(i)	dimension of the row space of A							
	dimension of the column space of A							
	dimension of the null space of A							
	dimension of the null space of A^T							
(ii)	is the system $A\mathbf{x} = \mathbf{b}$ consistent?							
(iii)	number of parameters in the general solution of $A\mathbf{x} = \mathbf{b}$							

2. Find the dimensions and bases for the four fundamental spaces of the matrix.

$$A = \begin{bmatrix} 0 & -1 & -4 \\ -1 & 0 & -4 \\ -2 & 3 & 4 \end{bmatrix}$$

4-9 Suggested Exercises

1. Find bases for the four fundamental spaces of the matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 & 1 \\ 2 & 8 & 0 & 1 & 2 \\ 0 & 4 & -6 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Discuss how the rank of A varies with t.

$$A = \begin{bmatrix} 1 & 1 & t \\ 1 & t & 1 \\ t & 1 & 1 \end{bmatrix}$$

3.

a. If A is a 3×5 matrix, then the rank of A is at most __________. Why?

b. If A is a 3×5 matrix, then the nullity of A is at most __________. Why?

c. If A is a 3×5 matrix, then the rank of A^T is at most __________. Why?

d. If A is a 3×5 matrix, then the nullity of A^T is at most __________. Why?

4. Let A be a 5×7 matrix with rank 4.

a. What is the dimension of the solution space of $A\mathbf{x} = \mathbf{0}$?

b. Is $A\mathbf{x} = \mathbf{b}$ consistent for all vectors $\mathbf{b}$ in R^5? Explain.