4-6 In-Class Exercise

1. In each part, find a basis for the given subspace of R^4, and state its dimension.

a. All vectors of the form $(a, b, c, 0)$.

b. All vectors of the form (a, b, c, d), where $d = a + b$ and $c = a - b$.

c. All vectors of the form (a, b, c, d), where $a = b = c = d$.

4-6 Suggested Exercises

1. Find a basis for the solution space of the homogeneous linear system, and find the dimension of that space.

(a)
$$\begin{aligned} x_1 - 4x_2 + 3x_3 - x_4 &= 0 \\ 2x_1 - 8x_2 + 6x_3 - 2x_4 &= 0 \end{aligned}$$

(b)
$$\begin{aligned} x + y + z &= 0 \\ 3x + 2y - 2z &= 0 \\ 4x + 3y - z &= 0 \\ 6x + 5y + z &= 0 \end{aligned}$$

2. Find the dimension of each of the following vector spaces.

a. The vector space of all diagonal $n \times n$ matrices.

b. The vector space of all symmetric $n \times n$ matrices.

c. The vector space of all upper triangular $n \times n$ matrices.

3. Show that the set W of all polynomials in P_2 such that $p(1) = 0$ is a subspace of P_2.

4. Find a standard basis vector for R^3 that can be added to the set $\{\mathbf{v}_1, \mathbf{v}_2\}$ to produce a basis for R^3.

$$\mathbf{v}_1 = (-1, 2, 3),\ \mathbf{v}_2 = (1, -2, -2)$$

5. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for a vector space V. Show that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is also a basis, where $\mathbf{u}_1 = \mathbf{v}_1$, $\mathbf{u}_2 = \mathbf{v}_1 + \mathbf{v}_2$, and $\mathbf{u}_3 = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$.

6. The vectors $\mathbf{v}_1 = (1, -2, 3)$ and $\mathbf{v}_2 = (0, 5, -3)$ are linearly independent. Enlarge $\{\mathbf{v}_1, \mathbf{v}_2\}$ to a basis for R^3.