1-7 In-Class Exercise

1. Find all values of x for which A is invertible.

$$A = \begin{bmatrix} x - \frac{1}{2} & 0 & 0 \\ x & x - \frac{1}{3} & 0 \\ x^2 & x^3 & x + \frac{1}{4} \end{bmatrix}$$

1-7 Suggested Exercise

1. Compute the product by inspection.

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}\begin{bmatrix} 5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

2. Use what you have learned in this section about multiplying by diagonal matrices to compute the product by inspection.

a. $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} u & v \\ w & x \\ y & z \end{bmatrix}$ **b.** $\begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

3. Show that if A is a symmetric $n \times n$ matrix and B is any $n \times m$ matrix, then the following products are symmetric:

$$B^T B, \quad BB^T, \quad B^T AB$$

4. Find a diagonal matrix A that satisfies the given condition.

$$A^{-2} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Let A be an $n \times n$ symmetric matrix.

a) Show that A^2 is symmetric.

b) Show that $2A^2 - 3A + I$ is symmetric.

6. Let $A = [a_{ij}]$ be an $n \times n$ matrix. Determine whether A is symmetric.

a. $a_{ij} = i^2 + j^2$ **b.** $a_{ij} = i^2 - j^2$

c. $a_{ij} = 2i + 2j$ **d.** $a_{ij} = 2i^2 + 2j^3$

7. Find an upper triangular matrix that satisfies

$$A^3 = \begin{bmatrix} 1 & 30 \\ 0 & -8 \end{bmatrix}$$