2-3 In-Class Exercise

1. use the adjoint method to find its inverse

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

2. solve by Cramer's rule

$$\begin{aligned} 4x + 5y \phantom{{}+2z} &= 2 \\ 11x + y + 2z &= 3 \\ x + 5y + 2z &= 1 \end{aligned}$$

1. $\det(A) = (2)(1)(2) = 4 \neq 0$ therefore A is invertible

The cofactors of A are:

$$C_{11} = \begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix} = 2 \qquad C_{12} = -\begin{vmatrix} 0 & -3 \\ 0 & 2 \end{vmatrix} = 0 \qquad C_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{21} = -\begin{vmatrix} -3 & 5 \\ 0 & 2 \end{vmatrix} = 6 \qquad C_{22} = \begin{vmatrix} 2 & 5 \\ 0 & 2 \end{vmatrix} = 4 \qquad C_{23} = -\begin{vmatrix} 2 & -3 \\ 0 & 0 \end{vmatrix} = 0$$

$$C_{31} = \begin{vmatrix} -3 & 5 \\ 1 & -3 \end{vmatrix} = 4 \qquad C_{32} = -\begin{vmatrix} 2 & 5 \\ 0 & -3 \end{vmatrix} = 6 \qquad C_{33} = \begin{vmatrix} 2 & -3 \\ 0 & 1 \end{vmatrix} = 2$$

The matrix of cofactors is $\begin{bmatrix} 2 & 0 & 0 \\ 6 & 4 & 0 \\ 4 & 6 & 2 \end{bmatrix}$ and the adjoint matrix is $\operatorname{adj}(A) = \begin{bmatrix} 2 & 6 & 4 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{bmatrix}$.

we have $A^{-1} = \frac{1}{\det(A)}\operatorname{adj}(A) = \frac{1}{4}\begin{bmatrix} 2 & 6 & 4 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} & 1 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$.

2.

$$\det(A)=\begin{vmatrix}4 & 5 & 0\\ 11 & 1 & 2\\ 1 & 5 & 2\end{vmatrix}=(8+10+0)-(0+40+110)=-132\,,$$

$$\det(A_1)=\begin{vmatrix}2 & 5 & 0\\ 3 & 1 & 2\\ 1 & 5 & 2\end{vmatrix}=(4+10+0)-(0+20+30)=-36\,,$$

$$\det(A_2)=\begin{vmatrix}4 & 2 & 0\\ 11 & 3 & 2\\ 1 & 1 & 2\end{vmatrix}=(24+4+0)-(0+8+44)=-24\,,$$

$$\det(A_3)=\begin{vmatrix}4 & 5 & 2\\ 11 & 1 & 3\\ 1 & 5 & 1\end{vmatrix}=(4+15+110)-(2+60+55)=12\,;$$

$$x=\frac{\det(A_1)}{\det(A)}=\frac{-36}{-132}=\frac{3}{11}\,,\quad y=\frac{\det(A_2)}{\det(A)}=\frac{-24}{-132}=\frac{2}{11}\,,\quad z=\frac{\det(A_3)}{\det(A)}=\frac{12}{-132}=-\frac{1}{11}\,.$$

2-3 Suggested Exercise

1. Find the values of k for which the matrix A is invertible.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ k & 1 & k \\ 0 & 2 & 1 \end{bmatrix}$$

$$\det(A) = (1+0+0) - (0+2k+2k) = 1-4k.$$

A is invertible if $k \neq \frac{1}{4}$.

2. Let

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Assuming that $\det(A) = -7$, find $\det(2A^{-1})$

$$\det\left(2A^{-1}\right) = 2^3 \det\left(A^{-1}\right) = \frac{8}{\det(A)} = \frac{8}{-7} = -\frac{8}{7}$$

3. Given that A is a 4×4 matrix and $\det(A) = -2$, find the determinant of $\det(2A^T)$.

$$\det\left(2A^T\right) = 2^4 \det\left(A^T\right) = 16\det(A) = -32$$

4. Given that A is a 3×3 matrix and $\det(A) = 7$, find the determinant of $\det((2A)^{-1})$.

$$\det\left((2A)^{-1}\right) = \frac{1}{\det(2A)} = \frac{1}{2^3 \det(A)} = \frac{1}{(8)(7)} = \frac{1}{56}$$