1-3 In-Class Exercise

Given $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix},$

$$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, \quad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

1. Calculate $\mathrm{tr}(C^T A^T + 2E^T)$

2. Calculate $(-AC)^T + 5D^T$

1.

$$\operatorname{tr}\left(\begin{bmatrix}1&3\\4&1\\2&5\end{bmatrix}\begin{bmatrix}3&-1&1\\0&2&1\end{bmatrix}+2\begin{bmatrix}6&-1&4\\1&1&1\\3&2&3\end{bmatrix}\right)$$

$$\operatorname{tr}\left(\begin{bmatrix}(1\cdot3)+(3\cdot0)&-(1\cdot1)+(3\cdot2)&(1\cdot1)+(3\cdot1)\\(4\cdot3)+(1\cdot0)&-(4\cdot1)+(1\cdot2)&(4\cdot1)+(1\cdot1)\\(2\cdot3)+(5\cdot0)&-(2\cdot1)+(5\cdot2)&(2\cdot1)+(5\cdot1)\end{bmatrix}+\begin{bmatrix}2\cdot6&2\cdot(-1)&2\cdot4\\2\cdot1&2\cdot1&2\cdot1\\2\cdot3&2\cdot2&2\cdot3\end{bmatrix}\right)$$

$$\operatorname{tr}\left(\begin{bmatrix}3&5&4\\12&-2&5\\6&8&7\end{bmatrix}+\begin{bmatrix}12&-2&8\\2&2&2\\6&4&6\end{bmatrix}\right)=\operatorname{tr}\left(\begin{bmatrix}15&3&12\\14&0&7\\12&12&13\end{bmatrix}\right)=15+0+13=28$$

2.

$$\left(-\begin{bmatrix}(3\cdot1)+(0\cdot3)&(3\cdot4)+(0\cdot1)&(3\cdot2)+(0\cdot5)\\-(1\cdot1)+(2\cdot3)&-(1\cdot4)+(2\cdot1)&-(1\cdot2)+(2\cdot5)\\(1\cdot1)+(1\cdot3)&(1\cdot4)+(1\cdot1)&(1\cdot2)+(1\cdot5)\end{bmatrix}\right)^T+5\begin{bmatrix}1&-1&3\\5&0&2\\2&1&4\end{bmatrix}$$

$$=\left(-\begin{bmatrix}3&12&6\\5&-2&8\\4&5&7\end{bmatrix}\right)^T+\begin{bmatrix}5\cdot1&5\cdot(-1)&5\cdot3\\5\cdot5&5\cdot0&5\cdot2\\5\cdot2&5\cdot1&5\cdot4\end{bmatrix}=\begin{bmatrix}-3&-5&-4\\-12&2&-5\\-6&-8&-7\end{bmatrix}+\begin{bmatrix}5&-5&15\\25&0&10\\10&5&20\end{bmatrix}$$

$$=\begin{bmatrix}-3+5&-5+(-5)&-4+15\\-12+25&2+0&-5+10\\-6+10&-8+5&-7+20\end{bmatrix}=\begin{bmatrix}2&-10&11\\13&2&5\\4&-3&13\end{bmatrix}$$

1-3 Suggested Exercise

Given $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$,

$$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, \quad E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

1. Calculate $(2E^T - 3D^T)^T$ $\begin{bmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -6 \end{bmatrix}$

2. Calculate $B^T(CC^T - A^TA)$ $\begin{bmatrix} 40 & 72 \\ 26 & 42 \end{bmatrix}$

3. $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ x-y \\ 0 \end{bmatrix}$ How many 3x3 matrices A can you find for which the equation is satisfied for all choices of x, y, and z?

Setting the left hand side $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y + a_{13}z \\ a_{21}x + a_{22}y + a_{23}z \\ a_{31}x + a_{32}y + a_{33}z \end{bmatrix}$ equal to $\begin{bmatrix} x+y \\ x-y \\ 0 \end{bmatrix}$ yields

$$\begin{aligned} a_{11}x + a_{12}y + a_{13}z &= x + y \\ a_{21}x + a_{22}y + a_{23}z &= x - y \\ a_{31}x + a_{32}y + a_{33}z &= 0 \end{aligned}$$

Assuming the entries of A are real numbers that do not depend on x, y, and z, this requires that the coefficients corresponding to the same variable on both sides of each equation must match. Therefore, the only matrix satisfying the given condition is $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

4. Find all values of k, if any, that satisfy the equation.

$$\begin{bmatrix} 2 & 2 & k \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix} = 0$$

$$\begin{bmatrix} 2 & 2 & k \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ k \end{bmatrix} = \begin{bmatrix} 2 & 2 & k \end{bmatrix} \begin{bmatrix} 6 \\ 3k+4 \\ k+6 \end{bmatrix} = k^2 + 12k + 20 = (k+10)(k+2)$$

The values of k that satisfy the equation are $k = -10$ and $k = -2$.

5. Suppose that type I items cost \$1 each, type II items cost \$2 each, and type III items cost \$3 each. Also, suppose that the accompanying table describes the number of items of each type purchased during the first four months of the year.

	Type I	Type II	Type III
Jan.	3	4	3
Feb.	5	6	0
Mar.	2	9	4
Apr.	1	1	7

What information is represented by the following product?

$$\begin{bmatrix} 3 & 4 & 3 \\ 5 & 6 & 0 \\ 2 & 9 & 4 \\ 1 & 1 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

The given matrix product represents $\begin{bmatrix} \text{the total cost of items purchased in January} \\ \text{the total cost of items purchased in February} \\ \text{the total cost of items purchased in March} \\ \text{the total cost of items purchased in April} \end{bmatrix}$.