3-1 True-False Exercises

1. The vectors (a, b) and $(a, b, 0)$ are equivalent. ✗
2. If k is a scalar and $\mathbf{v}$ is a vector, then $\mathbf{v}$ and $k\mathbf{v}$ are parallel if and only if $k \geq 0$. ✗
3. The vectors $\mathbf{v} + (\mathbf{u} + \mathbf{w})$ and $(\mathbf{w} + \mathbf{v}) + \mathbf{u}$ are the same. ○
4. If $\mathbf{u} + \mathbf{v} = \mathbf{u} + \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$. ○
5. If a and b are scalars such that $a\mathbf{u} + b\mathbf{v} = \mathbf{0}$, then $\mathbf{u}$ and $\mathbf{v}$ are parallel vectors. ✗

6. If $(a, b, c) + (x, y, z) = (x, y, z)$, then (a, b, c) must be the zero vector. ○

7. If k and m are scalars and $\mathbf{u}$ and $\mathbf{v}$ are vectors, then $(k + m)(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + m\mathbf{v}$ ×

8. If the vectors $\mathbf{v}$ and $\mathbf{w}$ are given, then the vector equation
$3(2\mathbf{v} - \mathbf{x}) = 5\mathbf{x} - 4\mathbf{w} + \mathbf{v}$
can be solved for $\mathbf{x}$. ○

9. The linear combinations $a_1\mathbf{v}_1 + a_2\mathbf{v}_2$ and $b_1\mathbf{v}_1 + b_2\mathbf{v}_2$ can only be equal if $a_1 = b_1$ and $a_2 = b_2$. ×

1. False. According to Definition 2, equivalent vectors must have the same number of components.
2. False. $\mathbf{v}$ and $k\mathbf{v}$ are parallel for any k.
3. True. This is a consequence of Theorem 3.1.1.
4. True. This is a consequence of Theorem 3.1.1.
5. False. At least one of the scalars must be nonzero for the vectors to be parallel.
6. True.
7. False. $(k+m)(\mathbf{u}+\mathbf{v})=(k+m)\mathbf{u}+(\mathrm{k}+m)\mathbf{v}$.
8. True. $\mathbf{x}=\frac{5}{8}\mathbf{v}+\frac{1}{2}\mathbf{w}$.
9. False. For instance, if $\mathbf{v}_2=2\mathbf{v}_1$ then $4\mathbf{v}_1+2\mathbf{v}_2=2\mathbf{v}_1+3\mathbf{v}_2$.

3-2 True-False Exercises

1. If each component of a vector in R^3 is doubled, the norm of that vector is doubled.

2. Every vector in R^n has a positive norm.

3. If $\mathbf{v}$ is a nonzero vector in R^n, there are exactly two unit vectors that are parallel to $\mathbf{v}$.

4. The expressions $(\mathbf{u} \cdot \mathbf{v}) + \mathbf{w}$ and $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$ are both meaningful and equal to each other.

5. If $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$.

6. If $\mathbf{u} \cdot \mathbf{v} = 0$, then either $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

1. True. By Theorem 3.2.1(b), $\|2\mathbf{v}\| = |2|\,\|\mathbf{v}\| = 2\|\mathbf{v}\|$.

2. False. Norm can be zero for the zero vector.

3. True. The two vectors are $\frac{1}{\|\mathbf{v}\|}\mathbf{v}$ and $-\frac{1}{\|\mathbf{v}\|}\mathbf{v}$.

4. False. The first expression does not make sense since the scalar $\mathbf{u}\cdot\mathbf{v}$ cannot be added to a vector.

5. False. For example, let $\mathbf{u} = (1,0)$, $\mathbf{v} = (0,1)$, and $\mathbf{w} = (0,2)$. We have $\mathbf{v} \neq \mathbf{w}$ even though $\mathbf{u}\cdot\mathbf{v} = \mathbf{u}\cdot\mathbf{w}$.

6. False. For example, for $\mathbf{u} = (1,1) \neq (0,0)$ and $\mathbf{v} = (1,-1) \neq (0,0)$ we have $\mathbf{u}\cdot\mathbf{v} = 0$.

3-3 True-False Exercises

1. The vectors (3, −1, 2) and (0, 0, 0) are orthogonal. ○
2. If **u** and **v** are orthogonal vectors, then for all nonzero scalars *k* and *m*, $k\mathbf{u}$ and $m\mathbf{v}$ are orthogonal vectors. ○
3. The orthogonal projection of **u** on **a** is perpendicular to the vector component of **u** orthogonal to **a**. ○
4. If **a** and **b** are orthogonal vectors, then for every nonzero vector **u**, we have $\text{proj}_{\mathbf{a}}(\text{proj}_{\mathbf{b}}(\mathbf{u})) = \mathbf{0}$ ○
5. If **a** and **u** are nonzero vectors, then $\text{proj}_{\mathbf{a}}(\text{proj}_{\mathbf{a}}(\mathbf{u})) = \text{proj}_{\mathbf{a}}(\mathbf{u})$ ○
6. If the relationship $\text{proj}_{\mathbf{a}}\mathbf{u} = \text{proj}_{\mathbf{a}}\mathbf{v}$ holds for some nonzero vector **a**, then $\mathbf{u} = \mathbf{v}$. ×

1. True. $(3,-1,2)\cdot(0,0,0)=0$.
2. True. By Theorem 3.2.2(c) and Theorem 3.2.3(e), $(k\mathbf{u})\cdot(m\mathbf{v})=(km)(\mathbf{u}\cdot\mathbf{v})=(km)(0)=0$.
3. True. This follows from Theorem 3.3.2.
4. True. $\operatorname{proj}_{\mathbf{a}}(\operatorname{proj}_{\mathbf{b}}(\mathbf{u}))=\frac{\left(\frac{\mathbf{u}\cdot\mathbf{b}}{\|\mathbf{b}\|^2}\mathbf{b}\right)\cdot\mathbf{a}}{\|\mathbf{a}\|^2}\mathbf{a}=\frac{\frac{\mathbf{u}\cdot\mathbf{b}}{\|\mathbf{b}\|^2}(\mathbf{b}\cdot\mathbf{a})}{\|\mathbf{a}\|^2}\mathbf{a}=\mathbf{0}\mathbf{a}=\mathbf{0}$

   ($\operatorname{proj}_{\mathbf{b}}(\mathbf{u})$ has the same direction as $\mathbf{b}$, so it is also orthogonal to $\mathbf{a}$).
5. True. $\operatorname{proj}_{\mathbf{a}}(\operatorname{proj}_{\mathbf{a}}(\mathbf{u}))=\frac{\frac{\mathbf{u}\cdot\mathbf{a}}{\|\mathbf{a}\|^2}\mathbf{a}\cdot\mathbf{a}}{\|\mathbf{a}\|^2}\mathbf{a}=\frac{\frac{\mathbf{u}\cdot\mathbf{a}}{\|\mathbf{a}\|^2}\|\mathbf{a}\|^2}{\|\mathbf{a}\|^2}\mathbf{a}=\frac{\mathbf{u}\cdot\mathbf{a}}{\|\mathbf{a}\|^2}\mathbf{a}=\operatorname{proj}_{\mathbf{a}}(\mathbf{u})$

   ($\operatorname{proj}_{\mathbf{a}}(\mathbf{u})=k\mathbf{a}$ for some scalar k and then $\operatorname{proj}_{\mathbf{a}}(k\mathbf{a})=k\mathbf{a}$).
6. False. For instance, let $\mathbf{u}$ be a nonzero vector orthogonal to $\mathbf{a}$. Then $\operatorname{proj}_{\mathbf{a}}(\mathbf{u})=\operatorname{proj}_{\mathbf{a}}(2\mathbf{u})=\mathbf{0}$ even though $\mathbf{u}\neq 2\mathbf{u}$.

3-4 True-False Exercises

1. The points lying on a line through the origin in R^2 or R^3 are all scalar multiples of any nonzero vector on the line. ○

2. All solution vectors of the linear system $A\mathbf{x} = \mathbf{b}$ are orthogonal to the row vectors of the matrix A if and only if $\mathbf{b} = \mathbf{0}$. ○

3. If $\mathbf{x}_1$ and $\mathbf{x}_2$ are two solutions of the nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$, then $\mathbf{x}_1 - \mathbf{x}_2$ is a solution of the corresponding homogeneous linear system. ○

1. True. '

2. True.
   If $\mathbf{b}=\mathbf{0}$ then by Theorem 3.4.3, all solution vectors of $A\mathbf{x}=\mathbf{b}$ are orthogonal to the row vectors of A.
   If all solution vectors of $A\mathbf{x}=\mathbf{b}$ are orthogonal to the row vectors of A, $\mathbf{r}_1,\ldots,\mathbf{r}_m$ then the ith component of the product $A\mathbf{x}$ is $\mathbf{r}_i \cdot \mathbf{x}=0$, so we must have $\mathbf{b}=\mathbf{0}$.

3. True. Subtracting $A\mathbf{x}_1=\mathbf{b}$ from $A\mathbf{x}_2=\mathbf{b}$ yields $A\mathbf{x}_1-A\mathbf{x}_2=\mathbf{b}-\mathbf{b}$, i.e., $A(\mathbf{x}_1-\mathbf{x}_2)=\mathbf{0}$.

3-5 True-False Exercises

1. The cross product of two nonzero vectors **u** and **v** is a nonzero vector if and only if **u** and **v** are not parallel. ○
2. A normal vector to a plane can be obtained by taking the cross product of two nonzero and noncollinear vectors lying in the plane. ○
3. The scalar triple product of **u**, **v**, and **w** determines a vector whose length is equal to the volume of the parallelepiped determined by **u**, **v**, and **w**. ✗
4. For all vectors **u**, **v**, and **w** in 3-space, the vectors $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ and $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ are the same. ✗
5. If **u**, **v**, and **w** are vectors in $R3$, where **u** is nonzero and $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$. ✗

1. True. This follows from Formula (6): for nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, $\|\mathbf{u}\times\mathbf{v}\|=\|\mathbf{u}\|\|\mathbf{v}\|\sin\theta$ is zero if and only if $\sin\theta=0$ (i.e., the vectors are parallel).
2. True. The cross product of two nonzero noncollinear vectors in a plane is a nonzero vector perpendicular to both vectors, and therefore to the entire plane.
3. False. The scalar triple product is a scalar, rather than a vector.
4. False. These two triple vector products are generally not the same, as evidenced by parts (d) and (e) of Theorem 3.5.1.
5. False. For instance, let $\mathbf{u}=\mathbf{v}=\mathbf{i}$ and $\mathbf{w}=2\mathbf{i}$. We have $\mathbf{u}\times\mathbf{v}=\mathbf{u}\times\mathbf{w}=0$ even though $\mathbf{v}\neq\mathbf{w}$.