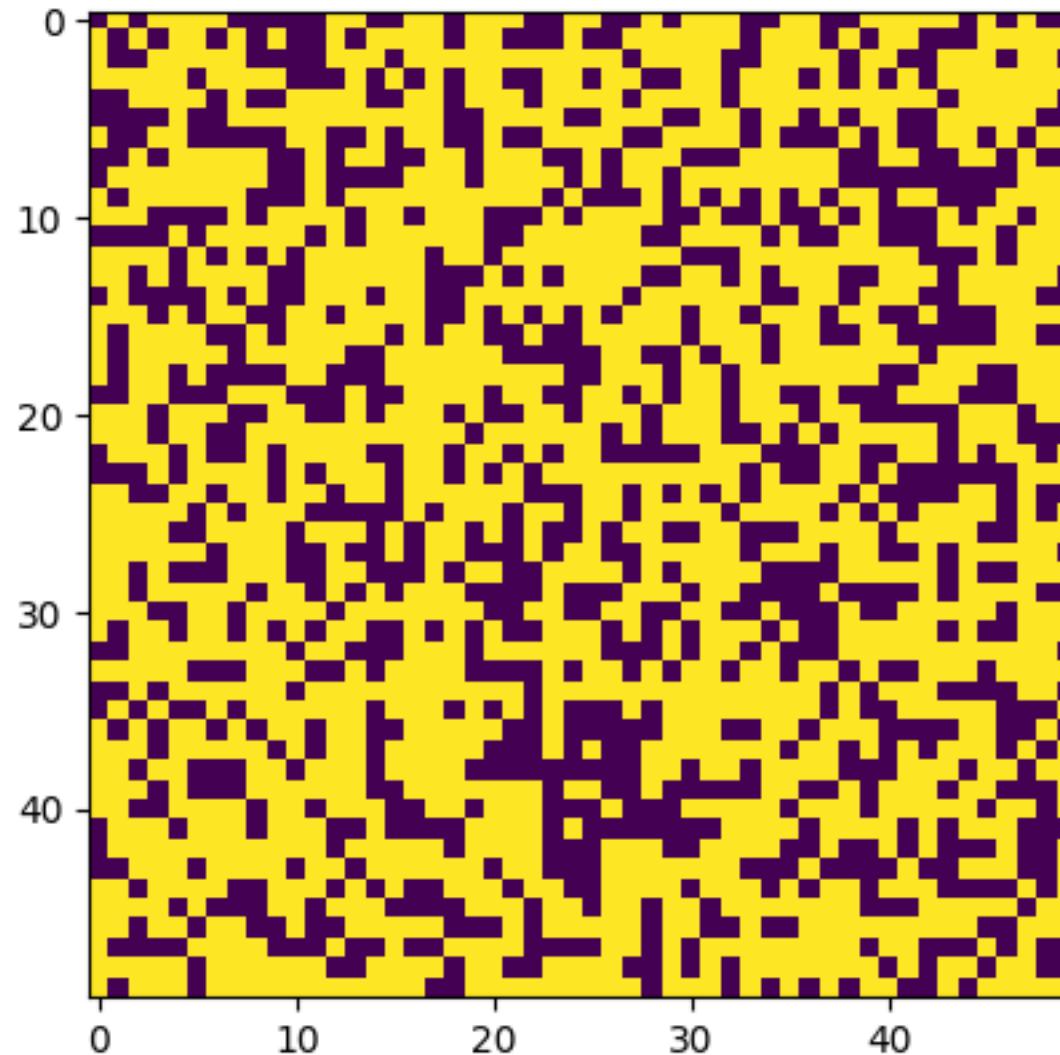


We are small enough - time to introduce yourselves (and apologies in advance when I forget your name)

Before we get to physics and computers...²

Any questions?



Ideally you would be working on a UNIX/LINUX command line. Some of you likely use Windows. This is fine, and I'm not going to make you dual-boot your computers, and as such, not going to teach you any Linux shell commands

Your assignments can be handed in using **Python** or **C++**. If you are only comfortable with a different programming language, come talk to me, since C++ is a pre-requisite for this course. Matlab is not an option.

We will use github classroom for assignments. We will briefly cover how to use github, which anyway is very useful to know these days

If you have a github account

6

Great!

Ungraded homework for today
(but do it today!) is to go to
github.com and create an
account. It's pretty
straightforward

If you want to work in C++, by all means, please do so. That is great. We will often discuss things in pseudo-code in this course, but examples will be in Python.

If you want to submit C++ code for assignments, each submission then has to have a Makefile that compiles on my laptop, and you are responsible for finding any needed visualization packages. It's your choice.

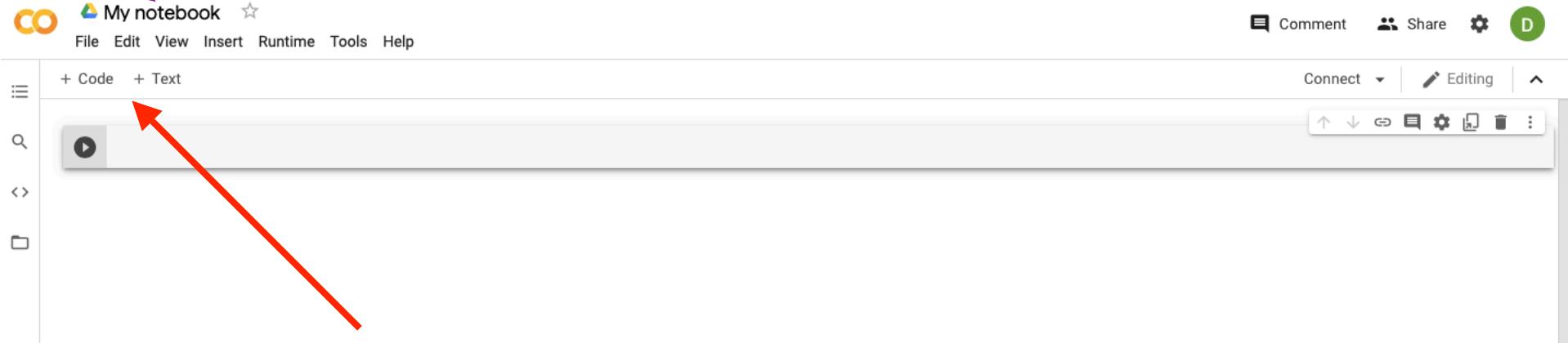
Let's all now visit <https://colab.research.google.com/notebooks/intro.ipynb>

You need a google account to use this. If you don't have one, please create one (and feel free to then use gmail and complain like I do about O365 and Outlook)

Please review the material there before the next course. It's straightforward (there's also a nice video). These are “Jupyter notebooks” that are hosted on Google Colab servers

Our colab notebook

Name of my notebook (can be changed), and you can see from the logo that it is currently stored on my google cloud drive



I can choose whether I am inserting text or code (so that you can nicely document code with formatting!)

Colab text

1

+ Code + Text

Connect ▾ | Editing

8

[

10

1

Edit here

When in text mode I have lots of usual text formatting options, including the ability to **bold text** and add [links](#) and images that I upload



Updates here

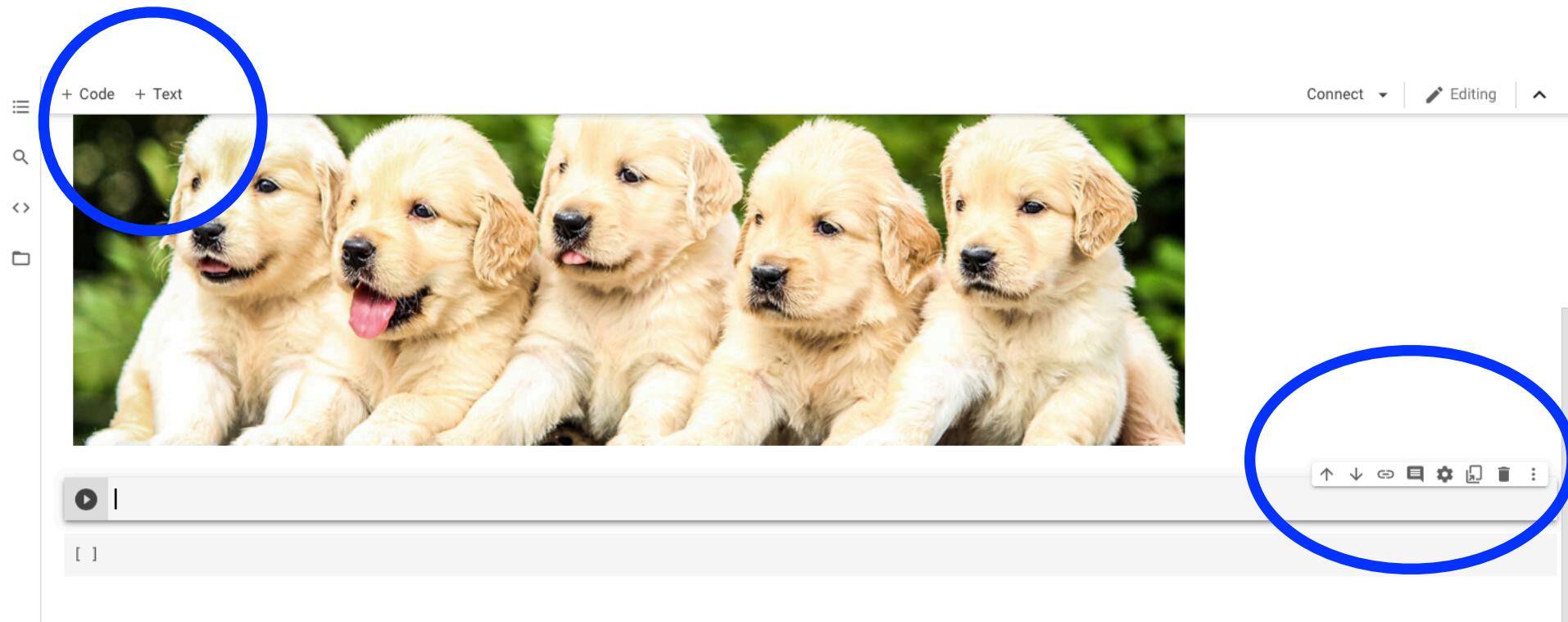
[]

When in text mode I have lots of usual text formatting options, including the ability to **bold text** and add [links](#) and images that I upload.



When we're
all done (we can
always edit it again
later)

Colab code

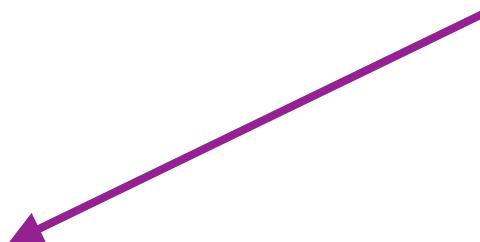


Next I clicked on “+ Code” to add some code. The **cell** gets placed below the last one. But note that I can delete these cells, you can have multiple cells of the same type in a row, or you can move them around. And you can type whatever code you want here

```
[6] This is not good code, it should not work
```

How to run code

Hit SHIFT-Enter or else click the “Play” button and then wait



```
[6] This is not good code, it should not work
```



```
This is not good code, it should not work
```



```
File "<ipython-input-6-a7b6366c138d>", line 1
```

```
    This is not good code, it should not work
```

^

```
SyntaxError: invalid syntax
```

Not surprising!!

SEARCH STACK OVERFLOW

If your code doesn't run, edit the same cell and try again

```
[7] print("This is now good code, it should work")
```

↳ This is now good code, it should work

```
[3] x=5  
     print(x)
```

↳ 5

```
[4] print(x)
```

↳ 5

```
[8] def func(val):  
     print(val*val)
```

```
[9] print(func(4))
```

↳ 16
None

When code runs it moves to the next cell. Note that `print(x)` works in the next cell. Why?! Only one smell at a time runs but anything you import and load, and any variables and functions that have been defined, remain in memory!

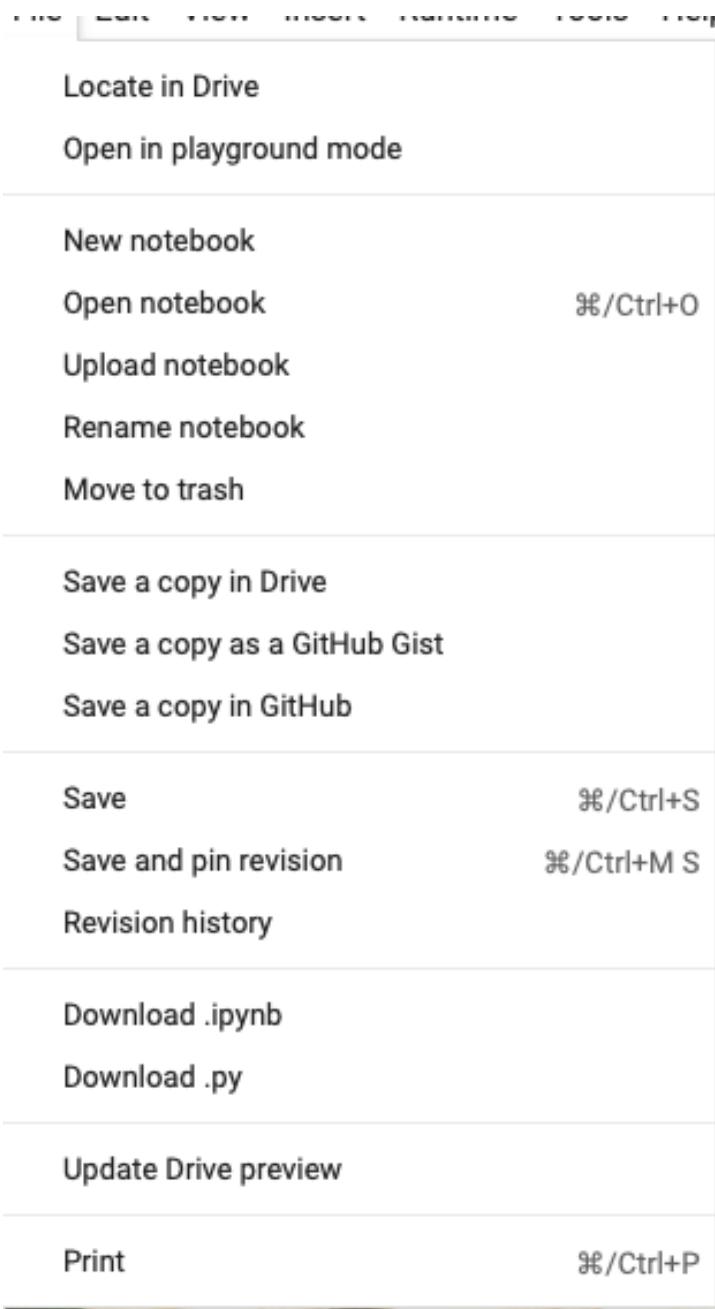
Makes it easy to “split up” your code and make small changes one piece at a time

And if you are submitting C++?

15

Play along with colab, and submit
C++ code with a makefile as
mentioned in github. Though I
strongly urge you to learn
Python, too

Don't forget to save your code



Under the file menu you have the usual list of options for opening new notebooks, downloading or printing them, saving them to Google Drive or saving to GitHub!

OK, let's get back to github

Join the classroom:

NIUCompPhys-classroom-1

To join the GitHub Classroom for this course, please select yourself from the list below to associate your GitHub account with your school's identifier (i.e., your name, ID, or email).

Can't find your name? [Skip to the next step →](#)

Identifiers	
Student 1	>
Student 8675309	>
Student AweSome Sauce	>

Once we log in to our github accounts we need to link it to a student identifier. Pick your name! I added you all before the start of class, but if you aren't here, skip the step and let me know

OK, let's get back to github

This should be your name!

Accept this assignment!

Your account is linked to Student Awesome Sauce on the roster. If this is wrong, please reach out to your instructor.

X

NIUCompPhys-classroom-1

Accept the assignment — Test Assignment

Once you accept this assignment, you will be granted access to the `test-assignment-jahreda` repository in the [NIUCompPhys](#) organization on GitHub.

There will be a repository for each of you created for each assignment

Accept this assignment

Accepting the assignment is not as bad as this

19



We accepted it!

← → C classroom.github.com/assignment-invitations/9f57acca0b6ad0177a114a885606799e/status ☆ ABP 1 🔍 💬 ⏱ ⚙️ ⚡ 🛠️

GitHub Classroom

GitHub Education



You're ready to go!

You accepted the assignment, **Test Assignment**.

Your assignment repository has been created:

 <https://github.com/NIUCompPhys/test-assignment-jahreda>

We've configured the repository associated with this assignment ([update](#)).

 Your assignment is due by **Oct 6, 2020, 12:00 CDT**

Note: You may receive an email invitation to join [NIUCompPhys](#) on your behalf. No further action is necessary.





Join the GitHub Student Developer Pack

Verified students receive free GitHub Pro plus thousands of dollars worth of the best real-world tools and training from GitHub Education partners — for free. [Learn more](#)

[Apply](#)

This has been created for us. It
is where we will store and
submit our work

OK, we can now view our assignment

← → C github.com/NIUCompPhys/test-assignment-davidwrighta ⭐ ABP 🔍 1 💬 ∞ 🚧 ⚙️ 🌐

 Search or jump to... / Pull requests Issues Marketplace Explore

Learn Git and GitHub without any code!

Using the Hello World guide, you'll start a branch, write comments, and open a pull request.

[Read the guide](#)

Cool! This is where we will submit things. Each of you will have one separately for each assignment

NIUCompPhys / test-assignment-davidwrighta Private Watch 0 Star 0 Fork 0

Code Issues Pull requests 1 Actions Projects Security Insights Settings

main 2 branches 0 tags Go to file Add file Code

davidwrighta Create Hello World b639472 1 hour ago 5 commits

.github GitHub Classroom Feedback 1 hour ago

Hello World Create Hello World 1 hour ago

Help people interested in this repository understand your project by adding a README. Add a README

About test-assignment-davidwrighta created by GitHub Classroom

Releases No releases published Create a new release

We can make a copy of it locally (clone it) if we want

 [NIUCompPhys / test-assignment-davidwrighta](#) Private

Cursor over test-assignment-davidwrighta (another example student) gives you the link you want for “git clone”

```
~/Desktop % git clone https://github.com/NIUCompPhys/test-assignment-davidwrighta
Cloning into 'test-assignment-davidwrighta'...
remote: Enumerating objects: 10, done.
remote: Counting objects: 100% (10/10), done.
remote: Compressing objects: 100% (6/6), done.
remote: Total 10 (delta 3), reused 0 (delta 0), pack-reused 0
Unpacking objects: 100% (10/10), done.
~/Desktop % cd test-assignment-davidwrighta
~/Desktop/test-assignment-davidwrighta % ls
Hello World
~/Desktop/test-assignment-davidwrighta %
```

Workflow on git

```
~/Desktop/test-assignment-davidwrighta % more Hello\ World  
Hello world and NIU
```

I've edited my file to include some important updates (you will update with code). Check that there are changes to update with git status

```
~/Desktop/test-assignment-davidwrighta % git status  
On branch main  
Your branch is up-to-date with 'origin/main'.  
Changes not staged for commit:  
(use "git add <file>..." to update what will be committed)  
(use "git checkout -- <file>..." to discard changes in working directory)  
  
modified:   Hello World  
  
no changes added to commit (use "git add" and/or "git commit -a")  
~/Desktop/test-assignment-davidwrighta %
```

Add/commit/Push

Want to add our files to change
Then commit them
Then push them

```
~/Desktop/test-assignment-davidwrighta % git add Hello\ World
```

```
~/Desktop/test-assignment-davidwrighta % git commit -m "Important update comment"
[main 043357d] Important update comment
 1 file changed, 1 insertion(+), 1 deletion(-)
~/Desktop/test-assignment-davidwrighta % git push
Counting objects: 3, done.
Delta compression using up to 4 threads.
Compressing objects: 100% (2/2), done.
Writing objects: 100% (3/3), 308 bytes | 0 bytes/s, done.
Total 3 (delta 0), reused 0 (delta 0)
To https://github.com/NIUCompPhys/test-assignment-davidwrighta
  b639472..043357d main -> main
~/Desktop/test-assignment-davidwrighta %
```

Back to the repo on the web

← → ⌂ github.com/NIUCompPhys/test-assignment-davidwrighta

It was just updated! And we see
our comment!
Let's click on it

Read the guide

NIUCompPhys / [test-assignment-davidwrighta](#) Private

[Code](#) [Issues](#) [Pull requests 1](#) [Actions](#) [Projects](#) [Security](#) [Insights](#) [Settings](#)

[main](#) ▾

[2 branches](#)

[0 tags](#)

[Go to file](#)

[Add file ▾](#)

[Code ▾](#)

 jahreda	Important update comment	043357d 2 minutes ago	6 commits
 .github	GitHub Classroom Feedback	1 hour ago	
 Hello World	Important update comment	2 minutes ago	

Help people interested in this repository understand your project by adding a README.

[Add a README](#)

Success!

 [NIUCompPhys / test-assignment-davidwrighta](#) Private

 Code

 Issues

 Pull requests 1

 Actions

 Projects

 Security

 main ▾

[test-assignment-davidwrighta / Hello World](#)



jahreda Important update comment

 2 contributors



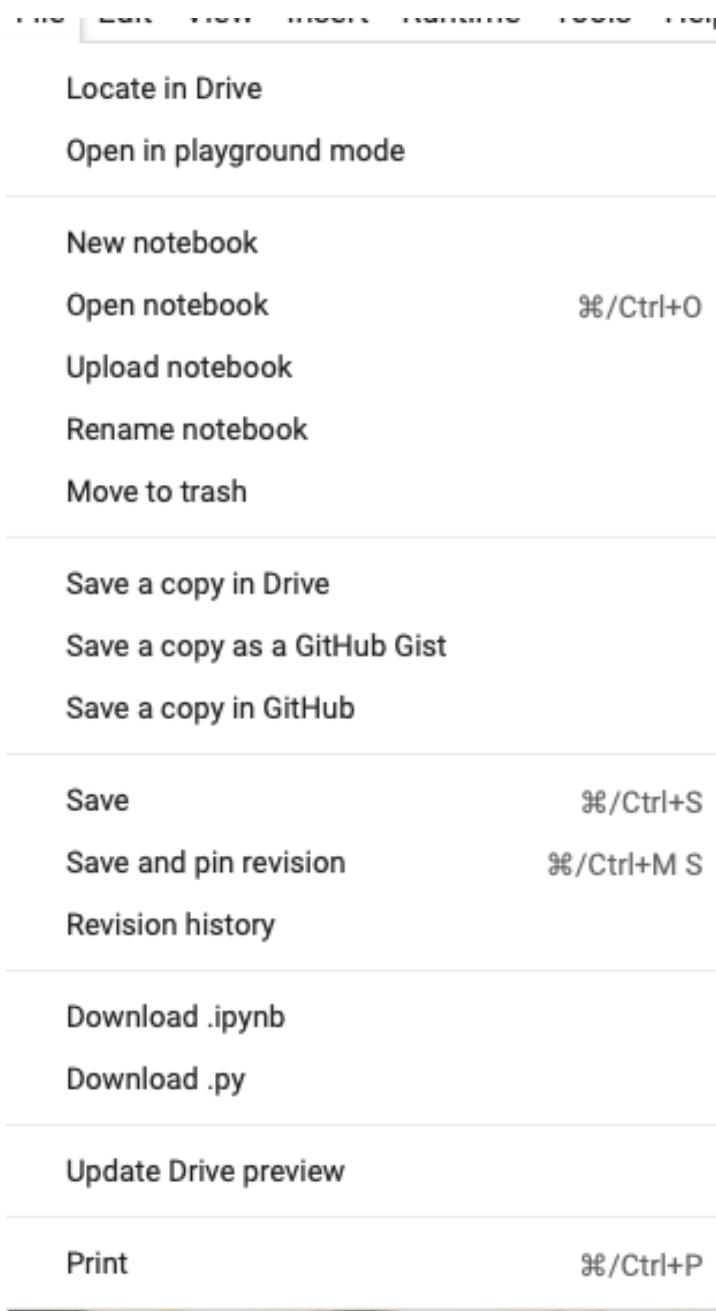
1 lines (1 sloc) | 20 Bytes

1 Hello world and NIU

Success!



Nice integration of two systems



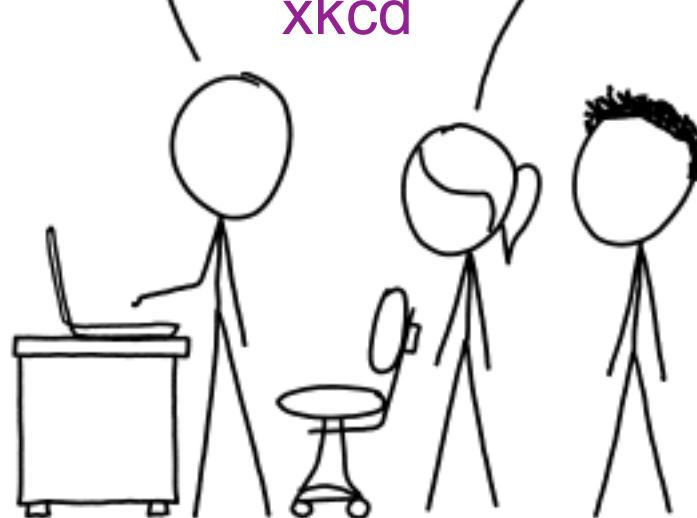
You can ask Colab to save a copy in GitHub for you (not required, of course, you can edit things locally). And then you can open from GitHub back into Colab. There's a nice discussion here that you can read at your own pace: <https://colab.research.google.com/github/googlecolab/colabtools/blob/master/notebooks/colab-github-demo.ipynb>

What is this git ? Why are we using it?

THIS IS GIT. IT TRACKS COLLABORATIVE WORK ON PROJECTS THROUGH A BEAUTIFUL DISTRIBUTED GRAPH THEORY TREE MODEL.

COOL. HOW DO WE USE IT?

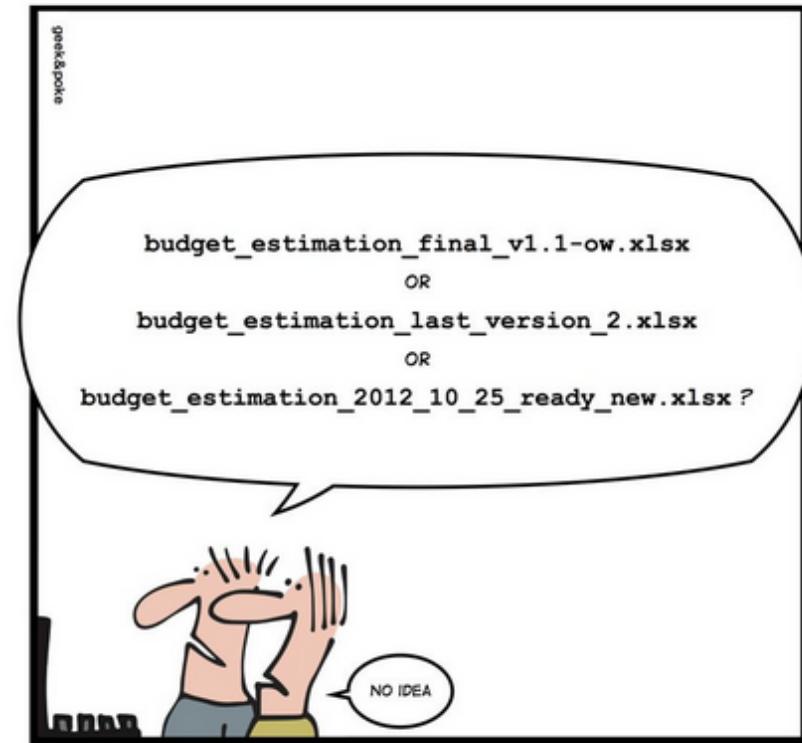
NO IDEA. JUST MEMORIZIZE THESE SHELL COMMANDS AND TYPE THEM TO SYNC UP. IF YOU GET ERRORS, SAVE YOUR WORK ELSEWHERE, DELETE THE PROJECT, AND DOWNLOAD A FRESH COPY.



<https://www.groovecommerce.com/ecommerce-blog/guide-to-version-control-for-magento-using-git-and-beanstalk/>

Version control!

SIMPLY EXPLAINED

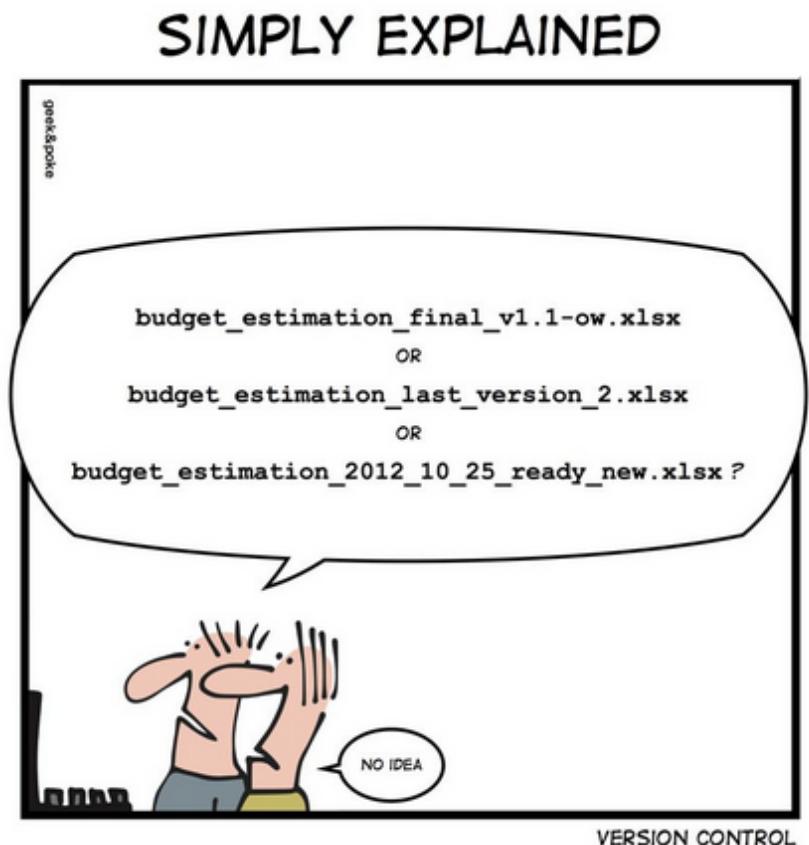


Version control

Think of it as a way to **back up your work** while having **different versions** to track what you did and when (and what others did and when)

I started out using CVS (concurrent versions system), then moved to SVN (subversion) and now git

We will use only the simplest features of git, which can be quite complex



Re: git

en.wikipedia.org/wiki/Git

Naming [edit]

Torvalds sarcastically quipped about the name *git* (which means "unpleasant person" in British English slang): "I'm an egotistical bastard, and I name all my projects after myself. First 'Linux', now 'git'."^{[24][25]} The man page describes Git as "the stupid content tracker".^[26] The read-me file of the source code elaborates further:^[27]

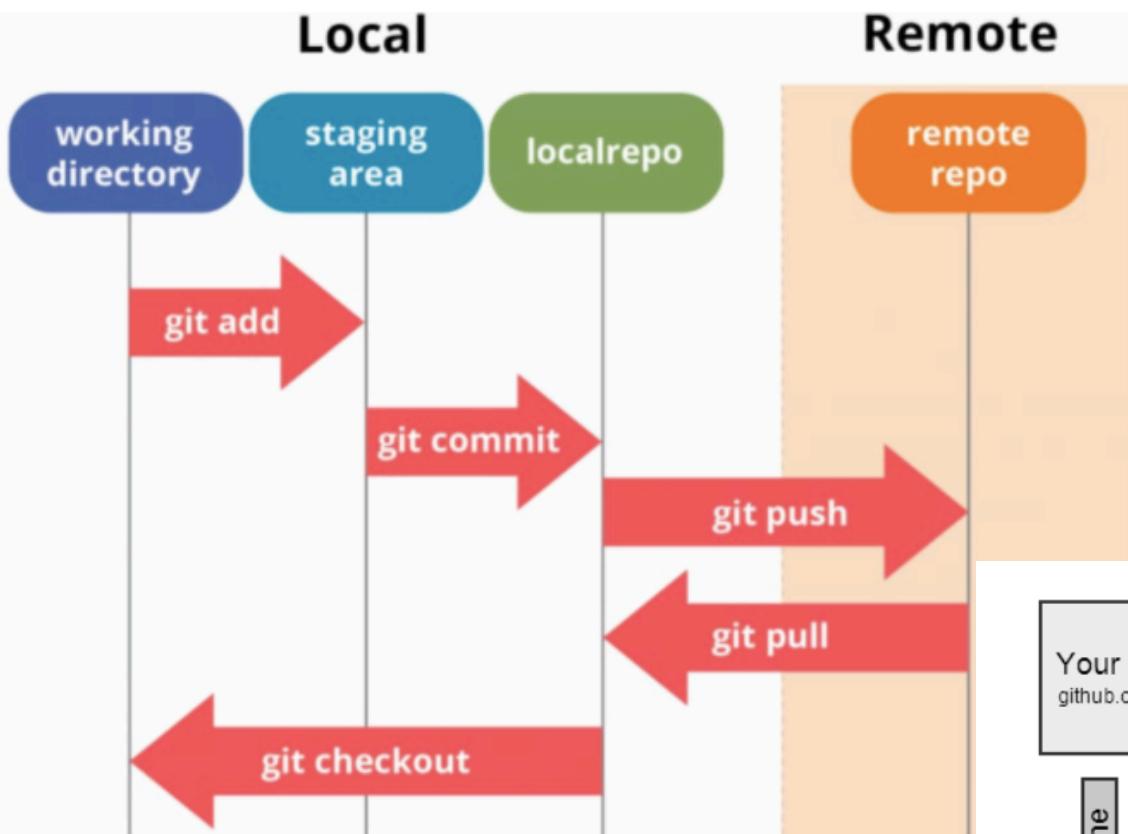
"git" can mean anything, depending on your mood.

- random three-letter combination that is pronounceable, and not actually used by any common UNIX command. The fact that it is a mispronunciation of "get" may or may not be relevant.
- stupid. contemptible and despicable. simple. Take your pick from the dictionary of slang.
- "global information tracker": you're in a good mood, and it actually works for you. Angels sing, and a light suddenly fills the room.
- "goddamn idiotic truckload of sh*t": when it breaks

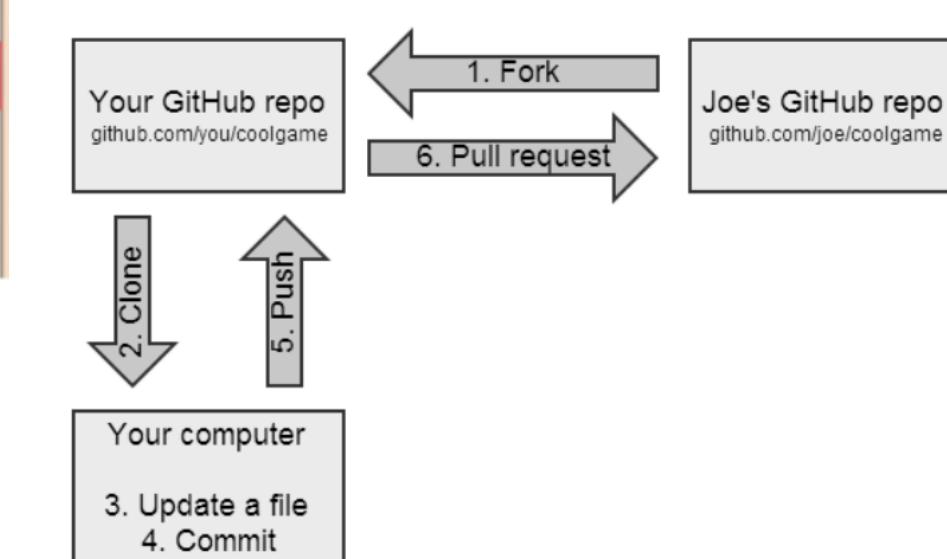
No comment

Workflow on git

<https://dev.to/mollynem/git-github--workflow-fundamentals-5496>



Not worrying about branches (different versions of the same code for different development and use) so things should be straightforward



Why the complication?

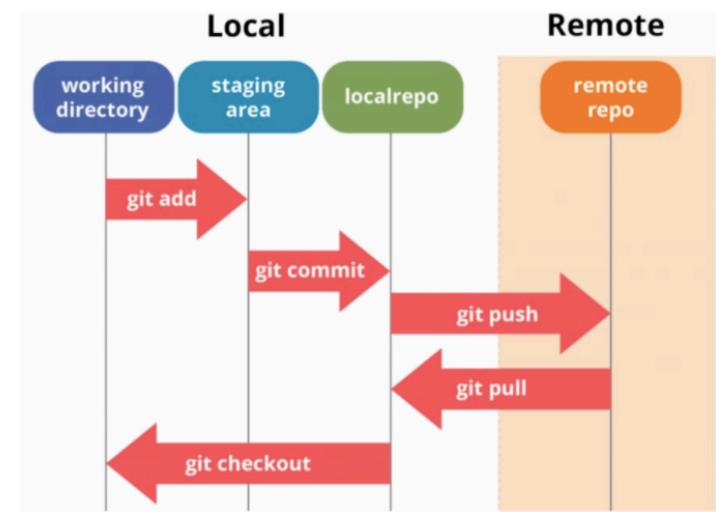
Your local areas is YOURS. You can do what you want with it, make your own branches, make mistakes, fix things and only when things are really working do you “push” to the central, remote repository of code

Can access other people’s code and suggest updates too!

Reminder: We will use only the simplest features of git, but it’s very useful to know

Useful to understand workflow:

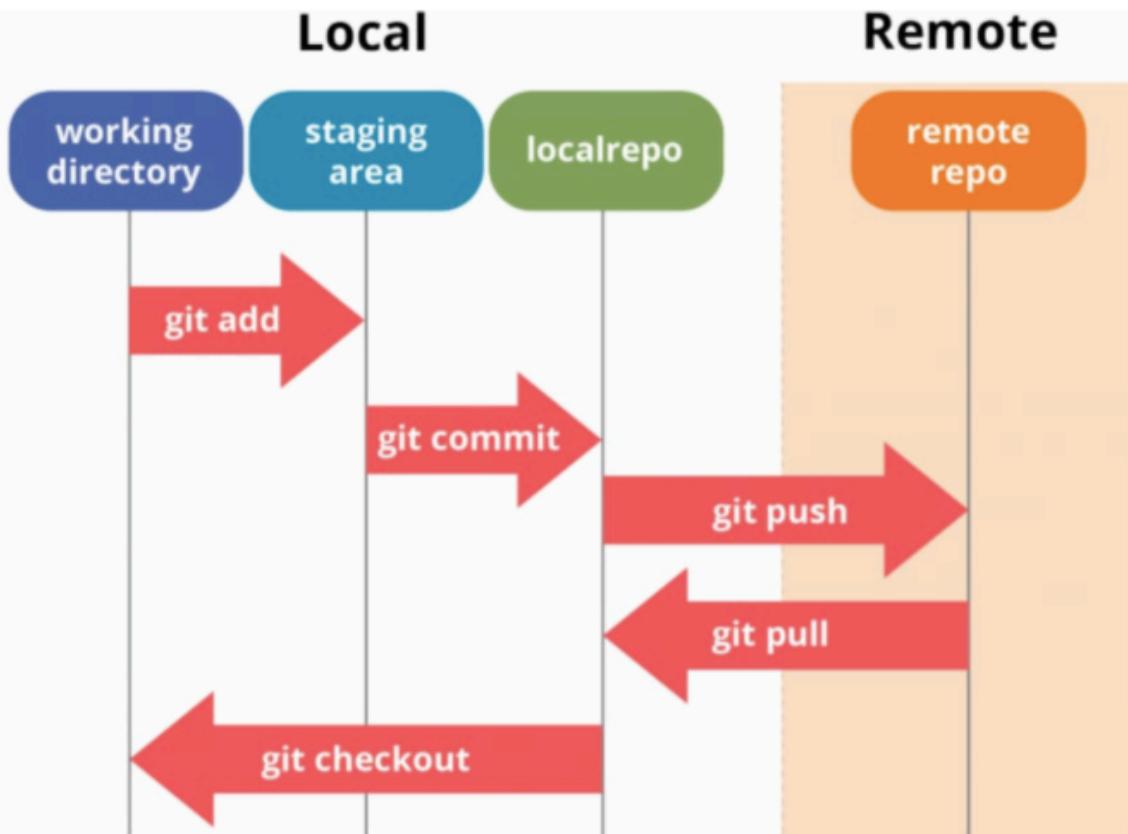
<https://guides.github.com/introduction/flow/>



Useful to understand how to use it:

<https://guides.github.com/activities/hello-world/>

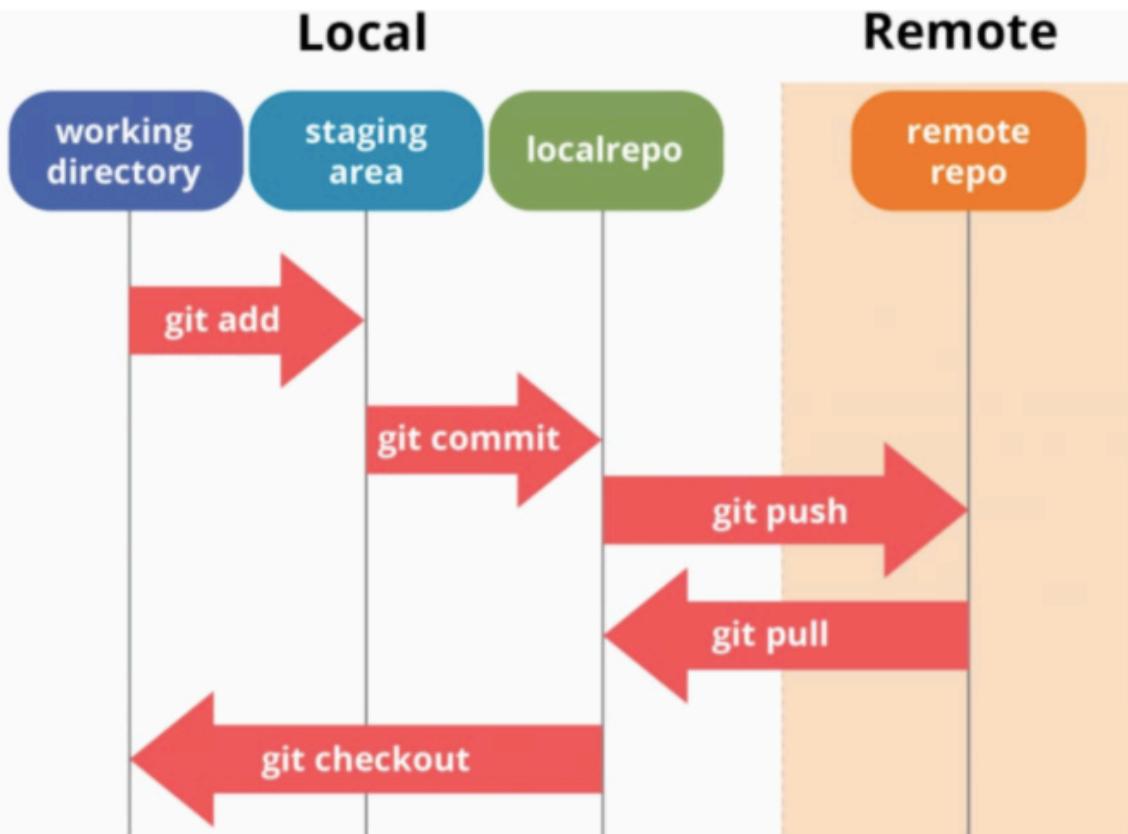
More on git and assignments



I will have access to anything you have pushed to your repo for any assignment. I will not look at it until the due date, but if you need help in advance, you can push updates for me to look at

Create your accounts and submit “test” assignment (hello world) right away so you don’t get stuck when we have real assignments to hand in

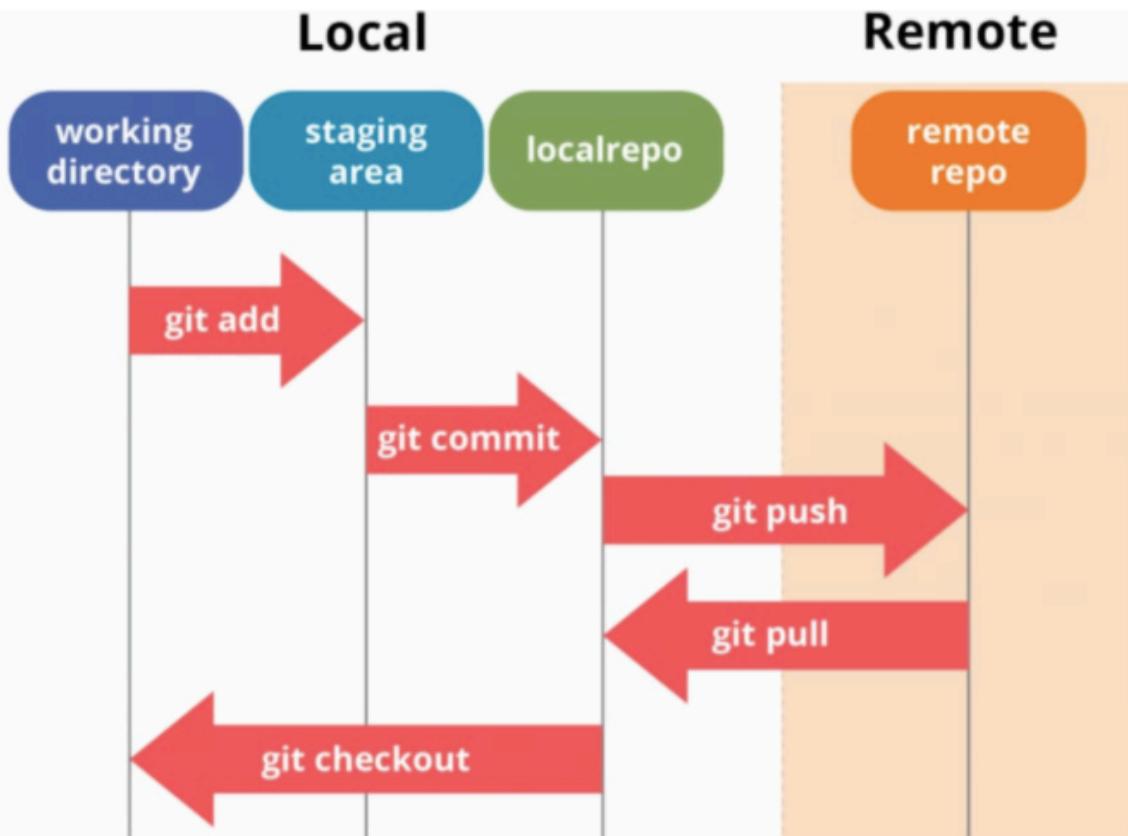
More on git and assignments



I will have access to anything you have pushed to your repo for any assignment. I will not look at it until the due date, but if you need help in advance, you can push updates for me to look at

You do NOT have to do python development on colab, though we will use it a bit and it is quite intuitive and easy. But you can do python development in a different environment and just upload the python file or notebook, if you prefer (though I will test in colab, so you should as well!)

More on git and assignments

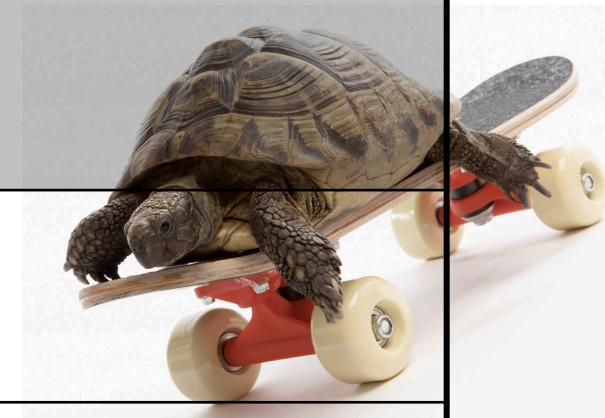
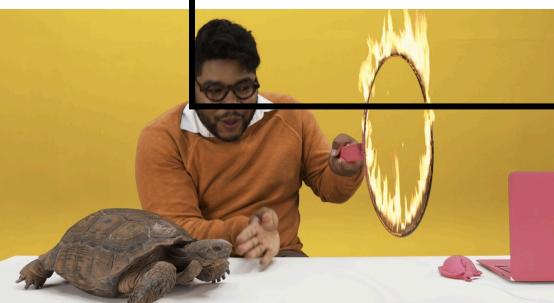


I will have access to anything you have pushed to your repo for any assignment. I will not look at it until the due date, but if you need help in advance, you can push updates for me to look at

You will have homework that isn't strictly coding. Just upload a PDF of your work! For some assignments I am also happy for you to submit things as Colab text

More on our choice of programming language

C++	Python
Compiled	Interpreted
Difficult to develop	Fast to develop (interpreted code helps with this!)
Often not so intuitive, difficult to debug	More intuitive, easier debugging
Fast execution	Slow to execute, but we'll see ways around this



- Write all the algorithms in Python
- Ensure that Python is using libraries pre-compiled in C++
- Best of both worlds! This is what numpy and scipy (which we will see) do
- If you don't know Python, please read the first few chapters of the textbook, which are easy to read and very instructive

- matlab is not free, see no reason to use it (though we'll use plotting code inspired by it!)
- Fortran (in various iterations) is very old, not used by many people anymore outside of obscure areas of academia
- There are many other new, interesting programming languages. But I can't help you with them, so you will be on your own if you decide to use them

Pseudocode

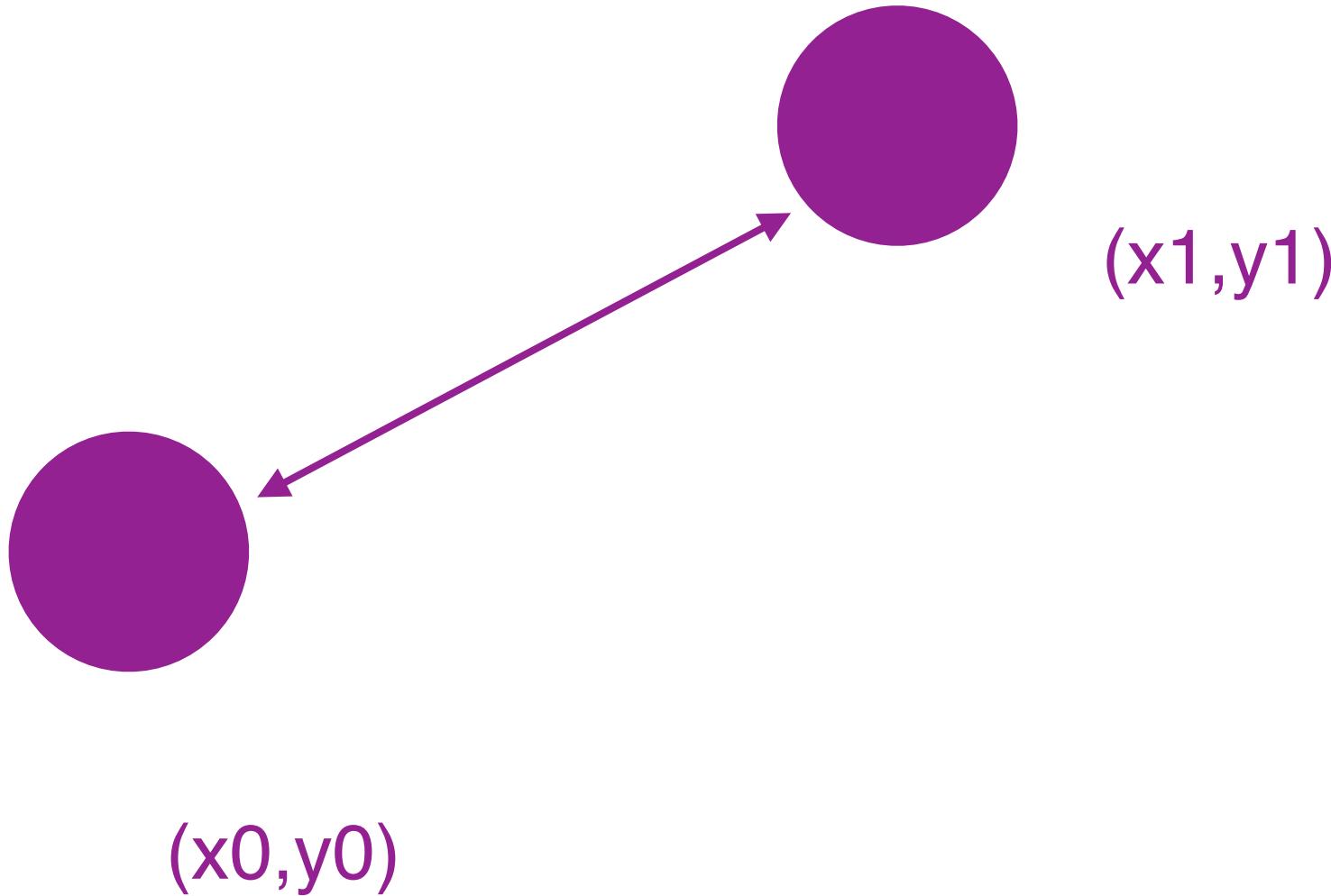
We will often discuss algorithms and tools and techniques using **pseudocode**, which is not for any code that is specific to any one programming language, but is instead designed to be easy to read and follow by humans (as opposed to be computer interpreters or computer compilers)

```
def studentGrade:  
    hw = average(HW grades)  
    quiz = average(quizzes)  
    exam = average(exam scores)  
    finalgrade = 25%*hw +  
        10%*quiz + 20%*exams +  
        45%*final  
    if (perfectAttendance)  
        finalGrade = finalGrade+Bonus  
    if (cheated) letterGrade = F  
    else if (finalGrade > 90%)  
        letterGrade = A  
    else if (finalGrade > 80%)  
        letterGrade = B  
    else if (finalGrade > 70%)  
        letterGrade = C  
    else if (finalGrade > 60%)  
        letterGrade = D  
    else  
        letterGrade = F
```

- Step 1: Write things down on paper
- Step 2: If you don't understand everything goto step 1
- Step 3: Write Pseudocode
- Step 4: Write code
- Step 5: Check code with unit tests
 - Check all criteria
 - If unit tests fail, goto step 4
- Step 6: Publish

What we'll try to do now

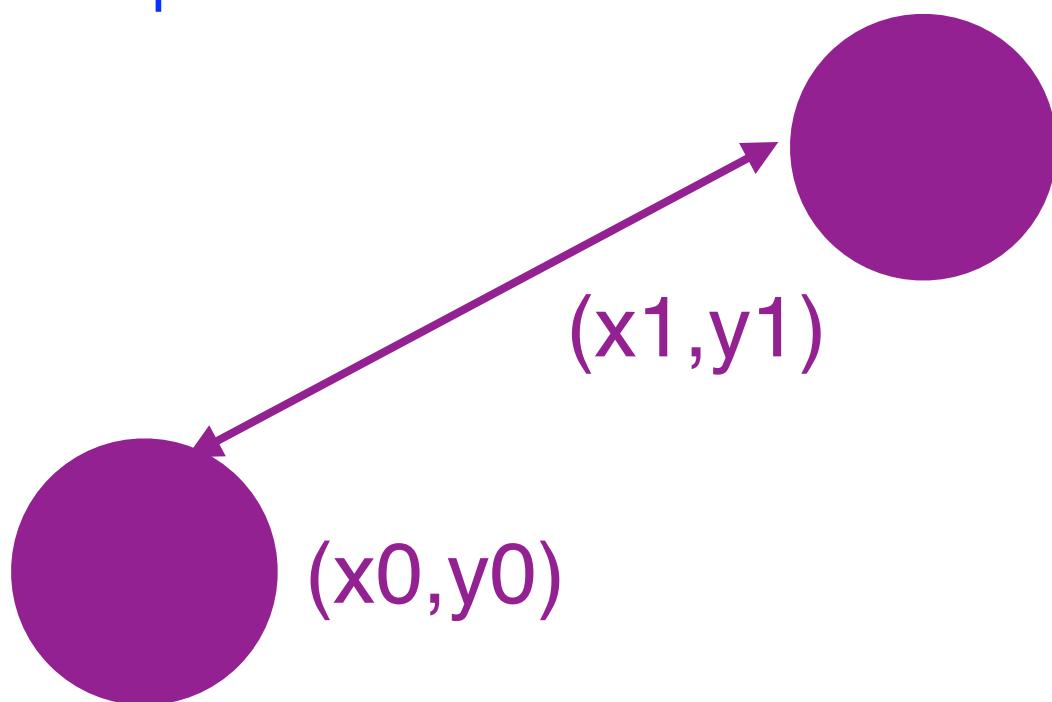
What is the slope between these two points?



Stages of development (this and below from UB course)

- Step 1: Write things down on paper
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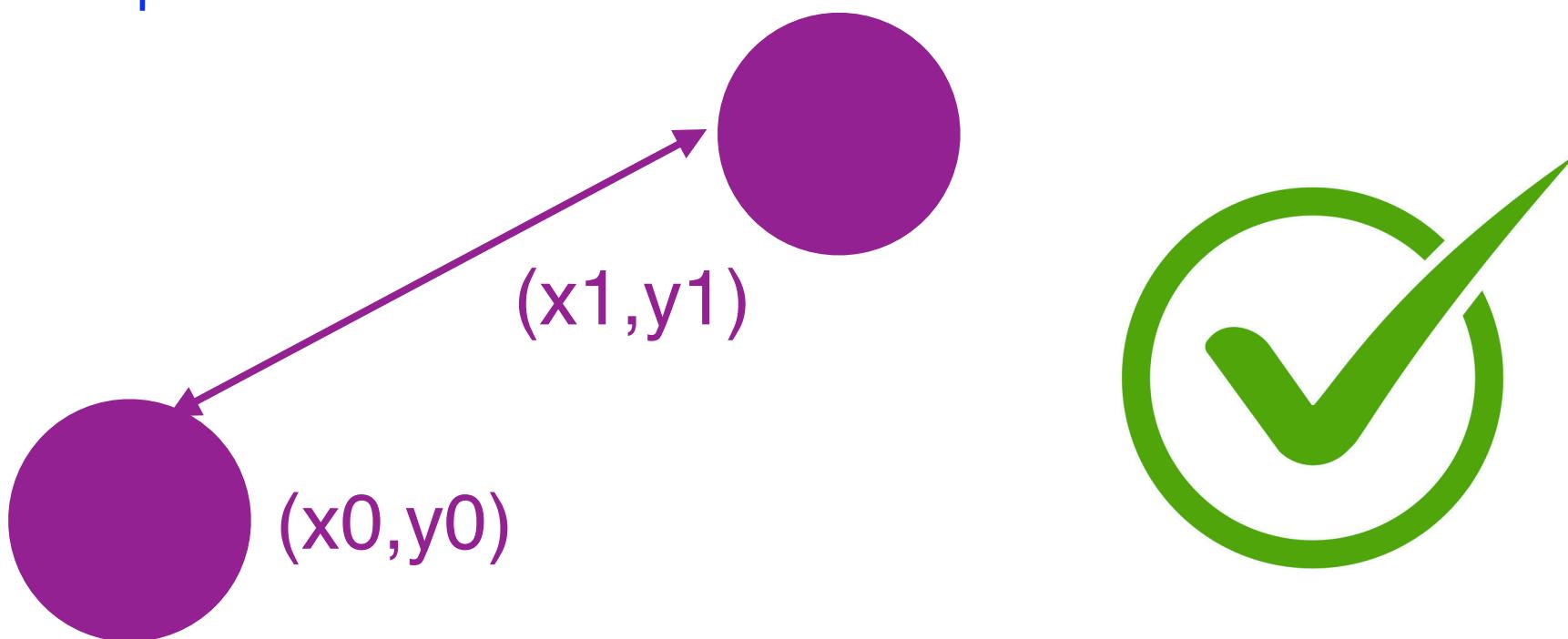
$$\text{Slope} = \frac{\Delta(y)}{\Delta(x)} = \frac{(y_1 - y_0)}{(x_1 - x_0)}$$



Stages of development (this and below from UB course)

- Step 1: Write things down on paper
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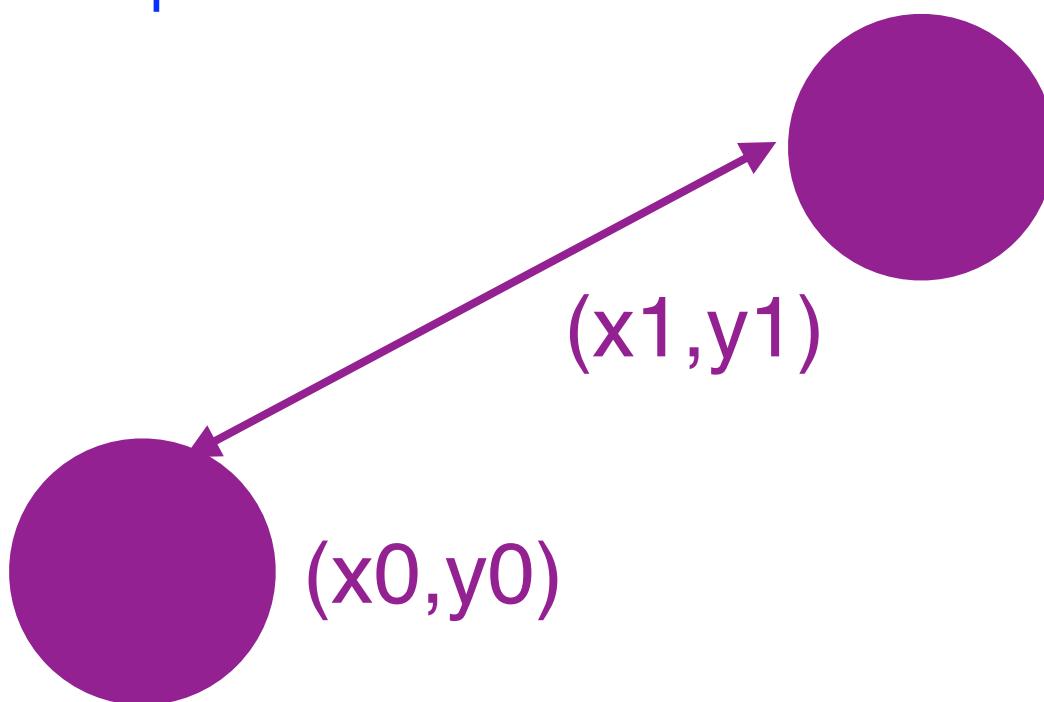
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Stages of development (this and below from UB course)

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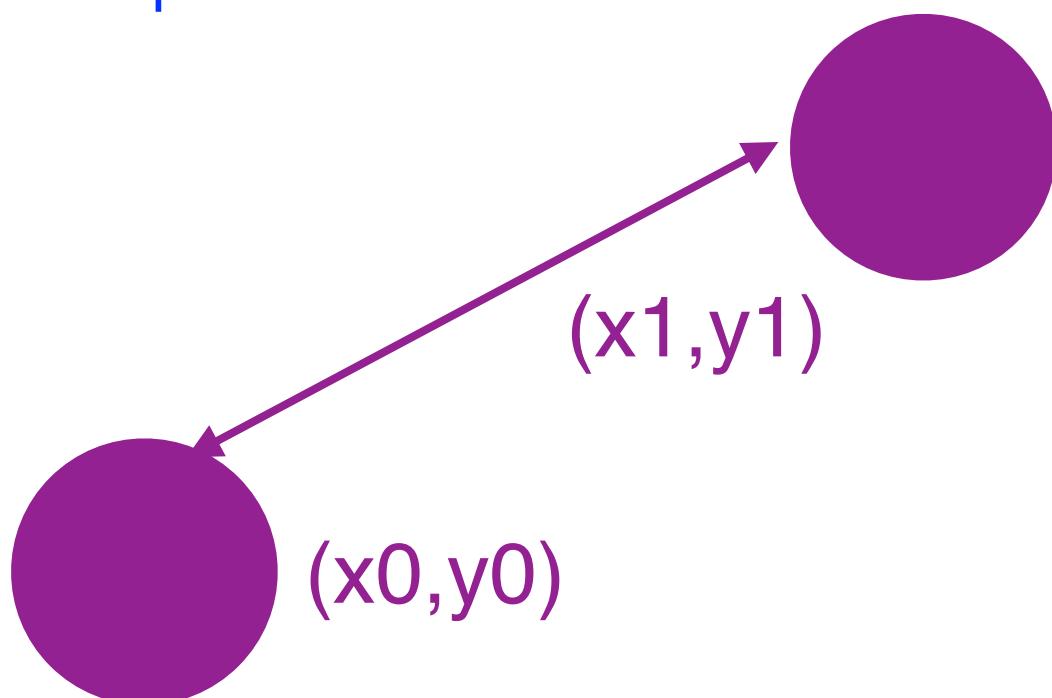
```
slope = (y1-y0)/(x1-x0)  
return slope
```



Stages of development (this and below from UB course)

- Step 1: Write things down on paper
- Step 2: If you don't understand everything goto step 1
- Step 3: Write Pseudocode
- Step 4: Write code
- Step 5: Check code with unit tests
 - Check all criteria
 - If unit tests fail, goto step 4
- Step 6: Publish

```
def getSlope(x0,y0,x1,y1):  
    slope = (y1-y0)/(x1-x0)  
    return slope
```



Stages of development (this and below from UB course)

- Step 1: Write things down on paper
- Step 2: If you don't understand everything goto step 1
- Step 3: Write Pseudocode
- Step 4: Write code
- **Step 5: Check code with unit tests**
 - Check all criteria
 - If unit tests fail, goto step 4
- Step 6: Publish



```
>>> def getSlope(x0,y0,x1,y1):  
...     slope = (y1-y0)/(x1-x0)  
...     return slope  
...  
>>> slope = getSlope(0.0,0.0,2.0,4.0)  
>>> print(slope)  
2.0
```

Stages of development (this and below from UB course)

- Step 1: Write things down on paper
- Step 2: If you don't understand everything goto step 1
- Step 3: Write Pseudocode
- Step 4: Write code
- Step 5: Check code with unit tests
 - Check all criteria
 - If unit tests fail, goto step 4
- Step 6: Publish



```
>>> slope = getSlope(2.0,3.0,2.0,4.0)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
    File "<stdin>", line 2, in getSlope
ZeroDivisionError: float division by zero
```

Stages of development (this and below from UB course)

- Step 1: Write things down on paper
- Step 2: If you don't understand everything goto step 1
- Step 3: Write Pseudocode
- Step 4: Write code
- Step 5: Check code with unit tests
 - Check all criteria
 - If unit tests fail, goto step 4
- Step 6: Publish

```
>>> from math import *
>>> epsilon = 1e-6
>>> def getSlope(x0,y0,x1,y1):
...     dy = y1-y0
...     dx = x1-x0
...     if abs(dx) > epsilon: ## only calculate if den != 0
...         return dy/dx
...     else:
...         print("Can't get the slope, dx is too close to zero")
...     return None
```

Stages of development (this and below from UB course)

- Step 1: Write things down on paper
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- Step 3: Write Pseudocode
- Step 4: Write code
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```
>>> from math import *
>>> epsilon = 1e-6
>>> def getSlope(x0,y0,x1,y1):
...     dy = y1-y0
...     dx = x1-x0
...     if abs(dx) > epsilon: ## only calculate if den != 0
...         return dy/dx
...     else:
...         print("Can't get the slope, dx is too close to zero")
...         return None
...
>>> getSlope(0.0,0.0,2.0,4.0)
2.0
>>> getSlope(2.0,3.0,2.0,4.0)
Can't get the slope, dx is too close to zero
```



Stages of development (this and below from UB course)

- Step 1: Write things down on paper
- Step 2: If you don't understand everything goto step 1
- Step 3: Write Pseudocode
- Step 4: Write code
- Step 5: Check code with unit tests
 - Check all criteria
 - If unit tests fail, goto step 4
- Step 6: Publish

Use github to submit assignment, go celebrate

- Computers do not understand infinite precision
 - See: upcoming chapters
- Computers follow instructions, no more and no less
 - They are not smarter than us!
- Stepping back and thinking a bit before writing code will ultimately save you a lot of time in the long run

C++ version

```
/Users/jahreda/Desktop/temp cat slope.C
#include <iostream>
#include <math.h>
using std::cout;
using std::endl;

// Better slope calculator : Error handling when
// denominator is zero
float getSlope( float x0, float y0, float x1, float y1, float epsilon = 1e-6 ) {
    cout << "Getting slope for x0 = " << x0 << " and x1 = " << x1 << " and y0 = " << y0 << " and y1 = " << y1
<< endl;
    float dy = y1 - y0;
    float dx = x1 - x0;
    if ( fabs(dx) > epsilon )
    {
        return dy / dx;
    }
    else {
        cout << "----> invalid input, setting to -99999.99" << endl;
        return -99999.99;
    }
}

int main(int argc, char *argv[])
{
    float slope = getSlope( 0.0,0.0,2.0,4.0 );
    cout << slope << endl;
    slope = getSlope(2.0,3.0,2.0,4.0);
    cout << slope << endl;
    return 0;
}
/Users/jahreda/Desktop/temp g++ -o slope slope.C
/Users/jahreda/Desktop/temp ./slope
Getting slope for x0 = 0 and x1 = 2 and y0 = 0 and y1 = 4
2
Getting slope for x0 = 2 and x1 = 2 and y0 = 3 and y1 = 4
----> invalid input, setting to -99999.99
-100000
```

Already see that Python is easier to read
and follow than C :)

Back to C++ vs Python

A lot of the examples that we'll use this semester will be shown in Python, but you don't have to submit any Python code at all if you don't want to. C/C++ is fine, too, though you need clear instructions for how I am to run the code, and in many cases you will need visualization tools. ROOT is the tool used in HEP, but there are plenty others. Up to you.

Why Python? Looks a lot like pseudocode! So you should be able to follow this even if you don't know Python

```
>>> from math import *
>>> epsilon = 1e-6
>>> def getSlope(x0,y0,x1,y1):
...     dy = y1-y0
...     dx = x1-x0
...     if abs(dx) > epsilon:
...         return dy/dx
...     else:
...         print("Can't get the slope, dx is too close to zero")
...         return None
```

What are these languages doing?

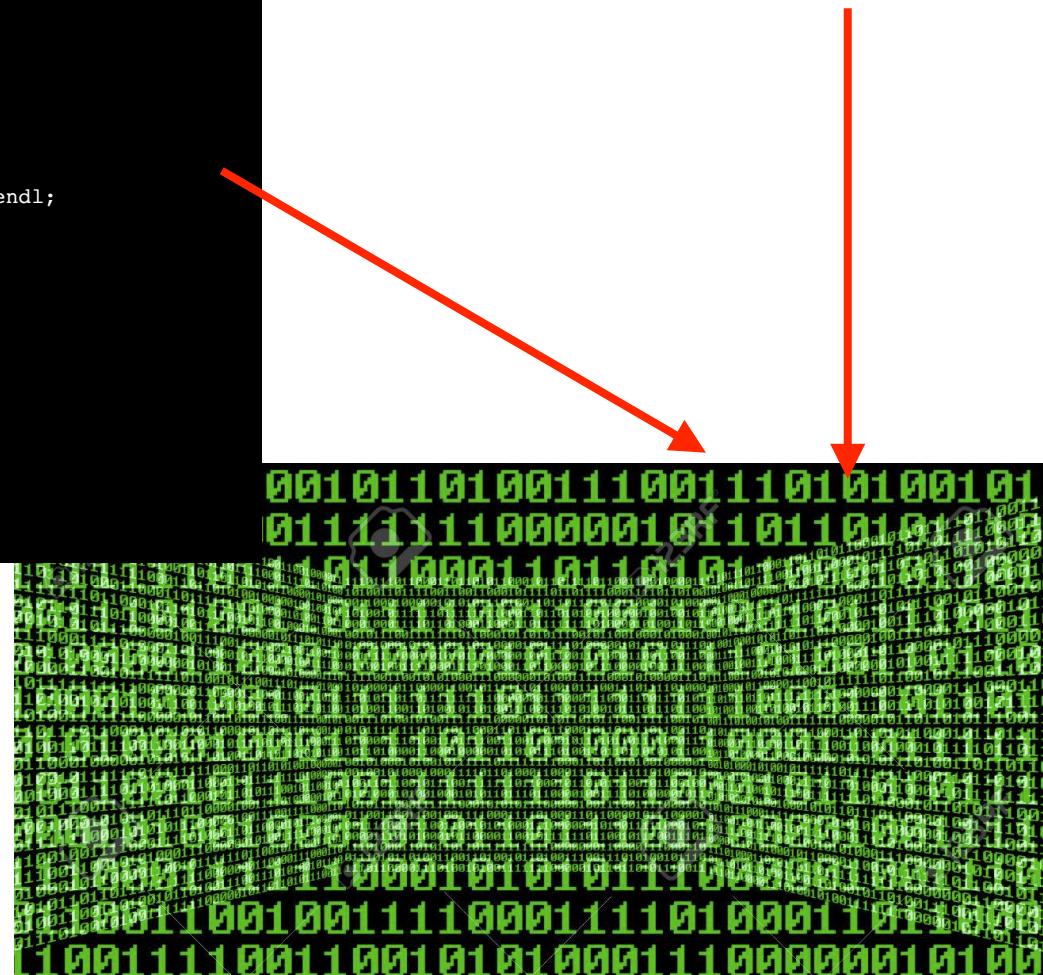
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// Better slope calculator : Error handling when
// denominator is zero
float getSlope( float x0, float y0, float x1, float y1, float epsilon = 1e-6 ) {
    cout << "Getting slope for x0 = " << x0 << " and x1 = " << x1 << " and y0 = "
    << y0 << " and y1 = " << y1 << endl;
    float dy = y1 - y0;
    float dx = x1 - x0;
    if ( fabs(dx) > epsilon )
    {
        return dy / dx;
    }
    else {
        cout << "----> invalid input, setting to -99999.99" << endl;
        return -99999.99;
    }
}

int main(int argc, char *argv[])
{
    float slope = getSlope( 0.0,0.0,2.0,4.0 );
    cout << slope << endl;
    slope = getSlope(2.0,3.0,2.0,4.0);
    cout << slope << endl;
    return 0;
}
/Users/jahreda/Desktop/temp g++ -o slope slope.C
```

```
>>> from math import *
>>> epsilon = 1e-6
>>> def getSlope(x0,y0,x1,y1):
...     dy = y1-y0
...     dx = x1-x0
...     if abs(dx) > epsilon:
...         return dy/dx
...     else:
...         print("Can't get the slope, dx is too close to zero")
...         return None
```

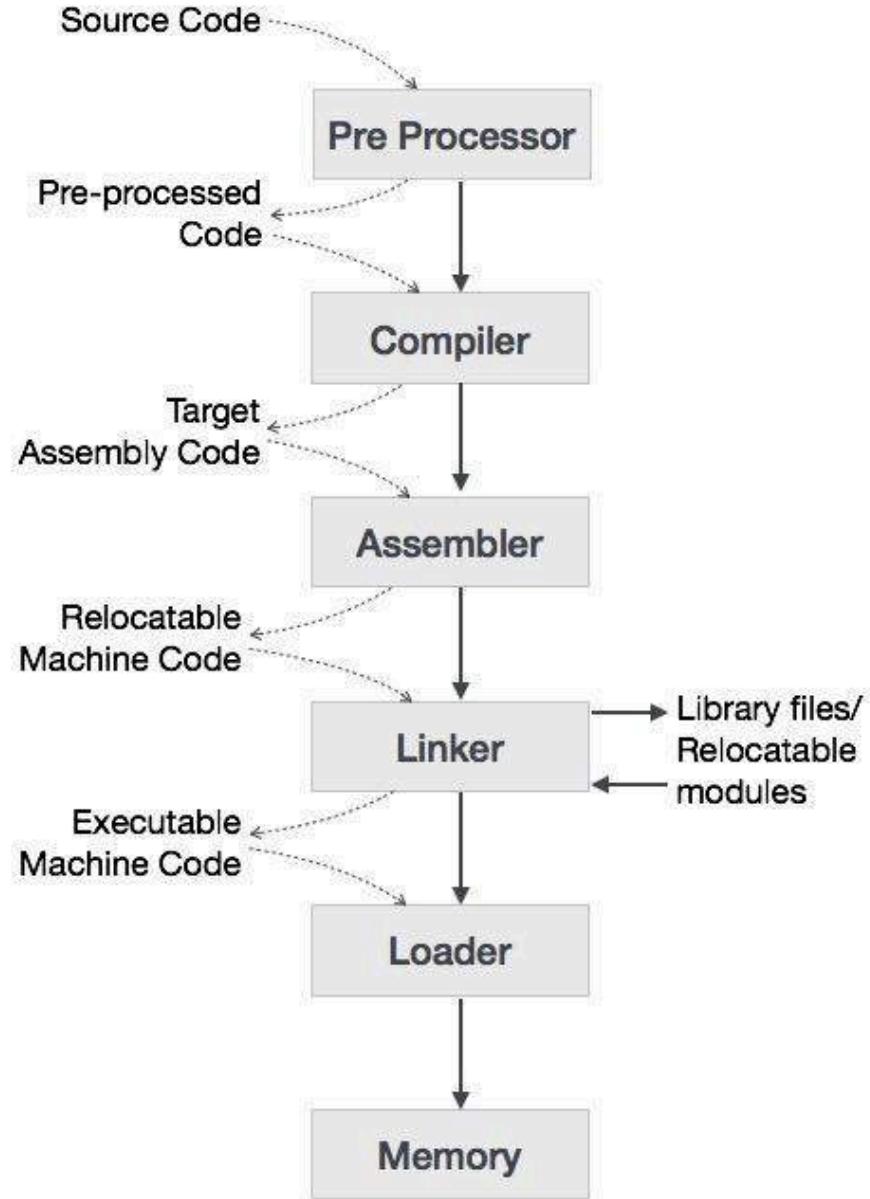
No matter what they are doing,
ultimately your program will be
executed on your phone/laptop/
computer, likely on a CPU but
maybe on some other
processing unit that
only knows 0's and 1's



What is compiled code (like C++)?

<https://sites.google.com/site/mscsystemsoftware/compiler-design>

- The compiler converts source code to assembly language (low-level language)
- This is then translated into machine code
- A linker links/combines the pieces of the program for execution
- The program is loaded into memory, and then executed



Assembly language?

https://commons.wikimedia.org/wiki/File:Motorola_6800_Assembly_Language.png

- No one wants to write assembly code. For very good reason. But it is a set of low-level basic instructions: Move Register 1 into Register 2, Add Register 1 contents to Register 2 contents, store these in Register 3
- Assembly language is specific to a hardware architecture, so the assembler needs to know what hardware you are using
- Assembly language is then closely connected to the exact hardware instructions run on your machine. But it's ... ugly :)

An assembly language listing for a Motorola 6800 8-bit microprocessor. This is a page from a "Monitor" program that communicates to a serial terminal connected to a MC6850 Asynchronous Communications Interface Adapter (ACIA). The routines here initialize the ACIA (INITA), read an ASCII character (INCH) and read a hexadecimal digit (INHEX).

```

MONITOR FOR 6802 1.4          9-14-80  TSC ASSEMBLER PAGE  2

C000          ORG    ROM+$0000 BEGIN MONITOR
C000 8E 00 70  START LDS   #STACK

*****
* FUNCTION: INITA - Initialize ACIA
* INPUT: none
* OUTPUT: none
* CALLS: none
* DESTROYS: acc A

0013        RESETA EQU    %00010011
0011        CTLREG EQU    %00010001

C003 86 13  INITA   LDA A #RESETA  RESET ACIA
C005 B7 80 04      STA A ACIA
C008 86 11      LDA A #CTLREG  SET 8 BITS AND 2 STOP
C00A B7 80 04      STA A ACIA

C00D 7E C0 F1  JMP    SIGNON  GO TO START OF MONITOR

*****
* FUNCTION: INCH - Input character
* INPUT: none
* OUTPUT: char in acc A
* DESTROYS: acc A
* CALLS: none
* DESCRIPTION: Gets 1 character from terminal

C010 B6 80 04  INCH    LDA A ACIA   GET STATUS
C013 47          ASR A   #1      SHIFT RDRTF FLAG INTO CARRY
C014 24 FA          BCC    INCH   RECEIVE NOT READY
C016 B6 80 05      LDA A ACIA+1 GET CHAR
C019 84 7F          AND A  #$7F  MASK PARITY
C01B 7E C0 79      JMP    OUTCH  ECHO & RTS

*****
* FUNCTION: INHEX - INPUT HEX DIGIT
* INPUT: none
* OUTPUT: Digit in acc A
* CALLS: INCH
* DESTROYS: acc A
* Returns to monitor if not HEX input

C01E 8D F0  INHEX   BSR    INCH   GET A CHAR
C020 81 30      CMP A  #'0  ZERO
C022 2B 11      BMI    HEXERR NOT HEX
C024 81 39      CMP A  #'9  NINE
C026 2F 0A      BLE    HEXRTS GOOD HEX
C028 81 41      CMP A  #'A
C02A 2B 09      BMI    HEXERR NOT HEX
C02C 81 46      CMP A  #'F
C02E 2E 05      BGT    HEXERR
C030 80 07      SUB A  #7  FIX A-F
C032 84 0F      HEXRTS AND A #$0F CONVERT ASCII TO DIGIT
C034 39          RTS

C035 7E C0 AF  HEXERR  JMP    CTRL   RETURN TO CONTROL LOOP

```

What about Python?

https://commons.wikimedia.org/wiki/File:Motorola_6800_Assembly_Language.png

- Even as an interpreted language, it still needs to execute those same basic instructions on a CPU, but it does so by executing (ie interpreting) your code line-by-line
- The python interpreter is a Virtual Machine that interprets your python scripts and executables (though it does some small conversion to python bytecode first)
 - Naively may be slower, though there are ways around this

```

MONITOR FOR 6802 1.4          9-14-80  TSC ASSEMBLER PAGE  2

C000          ORG    ROM+$0000 BEGIN MONITOR
C000 8E 00 70  START   LDS    #STACK

*****
* FUNCTION: INITA - Initialize ACIA
* INPUT: none
* OUTPUT: none
* CALLS: none
* DESTROYS: acc A

0013        RESETA EQU    %00010011
0011        CTLREG EQU    %00010001

C003 86 13  INITA   LDA A #RESETA  RESET ACIA
C005 B7 80 04          STA A ACIA
C008 86 11          LDA A #CTLREG  SET 8 BITS AND 2 STOP
C00A B7 80 04          STA A ACIA

C00D 7E C0 F1        JMP     SIGNON  GO TO START OF MONITOR

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* DESCRIPTION: Gets 1 character from terminal

C010 B6 80 04  INCH    LDA A ACIA    GET STATUS
C013 47          ASR A      SHIFT RDRT FLAG INTO CARRY
C014 24 FA          BCC INCH   RECEIVE NOT READY
C016 B6 80 05          LDA A ACIA+1  GET CHAR
C019 84 7F          AND A #$7F   MASK PARITY
C01B 7E C0 79          JMP     OUTCH   ECHO & RTS

*****
* FUNCTION: INHEX - INPUT HEX DIGIT
* INPUT: none
* OUTPUT: Digit in acc A
* CALLS: INCH
* DESTROYS: acc A
* Returns to monitor if not HEX input

C01E 8D F0  INHEX   BSR     INCH    GET A CHAR
C020 81 30          CMP A #'0  ZERO
C022 2B 11          BMI     HEXERR  NOT HEX
C024 81 39          CMP A #'9  NINE
C026 2F 0A          BLE     HEXRTS  GOOD HEX
C028 81 41          CMP A #'A
C02A 2B 09          BMI     HEXERR  NOT HEX
C02C 81 46          CMP A #'F
C02E 2E 05          BGT     HEXERR
C030 80 07          SUB A #7  FIX A-F
C032 84 0F          HEXRTS  AND A #$0F  CONVERT ASCII TO DIGIT
C034 39          RTS

C035 7E C0 AF  HEXERR  JMP     CTRL   RETURN TO CONTROL LOOP

```

An assembly language listing for a Motorola 6800 8-bit microprocessor. This is a page from a "Monitor" program that communicates to a serial terminal connected to a MC6850 Asynchronous Communications Interface Adapter (ACIA). The routines here initialize the ACIA (INITA), read an ASCII character (INCH) and read a hexadecimal digit (INHEX).

What is being executed, again?

Everything comes down to specific memory locations in your computer/ phone/Playstation, which are transistors (physics!) that store things as on/off, 1/0, yes/no, one/zero. We typically use 1/0 when we talk about **binary (base 2)**. Each of these numbers is stored in a specific **memory address**

Bits = Binary Digits

Address	Value
0	10000001
1	10101010
2	11111111
3	10000000

Sidebar on pointers (for the C++ crowd)

Great slide for the C++ folks from Sal Rappoccio.... please keep in mind!

C++: ptrs/refs in functions

- Pass by value:
 - COPIES value into a temporary variable called “x”
 - Pass by reference:
 - Pass REFERENCE to variable, temporarily called “x”
 - Pass by pointer
 - Pass POINTER to variable, pointer is called “x”
- | | | |
|---------------------------------|--------------------------------------|----------------------------------|
| <code>void func(int x);</code> | <code>void func(int &x);</code> | <code>void func(int *x);</code> |
|---------------------------------|--------------------------------------|----------------------------------|
- Use when you don't want to modify value, and cheap to copy
 - Use when you want to modify value, and expensive to copy, ptr=0 disallowed
 - Use when you want to modify value, and expensive to copy, ptr=0 allowed

Also usable with CONST

Binary

How do we store actual numbers in binary representation? A reminder that 127 in decimal really means:

$$1*10^2 + 2*10^1 + 7*10^0$$

How to use binary instead? In binary 1010 really means:

$$1*2^3 + 0*2^2 + 1*2^1 + 0*2^0 = 8+0+2+0 = 10 \text{ (in decimal)}$$

In other words, when using decimals, each added digit (to the left, ie of more significance) is worth an extra power of ten

Of course, 1010 in decimal means.... 1010 (in decimal). So we need to know which base we're using!

How to know what base we're using?

Without any indication otherwise, the assumption is we're using base 10 (of course, there is a good reason that this is how we count!)

A 0b prefix implies binary
(so 0b1010 = 10)

But writing out numbers in binary and reading them is cumbersome and a pain to us 10-digit machines



```
>>> bin(1234567890)  
'0b1001001100101100000001011010010'
```

Other useful bases

Machines use base 2, which don't easily match up to base 10 since neither is a simple exponent of the other ($0b1010 = 10$). So we often use two bases that are more compatible with binary and are closer to base 10, octal and hexadecimal

A 0 prefix implies octal (we otherwise don't write the leading zero!) and a 0x prefix implies hexadecimal

$$0b1000 = 8 = 010 \text{ (simpler!)}$$

$$0b10000 = 16 = 0x10 \text{ (simpler!)}$$

Digits in octal go from 0-7. Digits in hexadecimal go from 0-f !
($0xa = 10$, $0xb = 11$, $0xc = 12$, $0xd = 13$, $0xe = 14$, $0xf = 15$)

Comparison of bases

Decimal	Binary	Octal	Hex
1	0b1	01	0x1
7	0b111	07	0x7
8	0b1000	010	0x8
15	0b1111	017	0xf
22	0b10110	026	0x16
39	0b100111	047	0x27

Let's check all these together carefully!

There are 10 types
of people in the
world.

Those who understand
binary, and those who
don't.

On egg salad

The novel further describes an intra-Lilliputian quarrel over the practice of breaking eggs. Traditionally, Lilliputians broke boiled eggs on the larger end; a few generations ago, an Emperor of Lilliput, the Present Emperor's great-grandfather, had decreed that all eggs be broken on the smaller end after his son cut himself breaking the egg on the larger end. The differences between Big-Endians (those who broke their eggs at the larger end) and Little-Endians had given rise to "six rebellions ... wherein one Emperor lost his life, and another his crown". The Lilliputian religion says an egg should be broken on the convenient end, which is now interpreted by the Lilliputians as the smaller end. The Big-Endians gained favour in Blefuscus.

Gulliver and the Emperor of Lilliput, from a French edition of *Gulliver's Travels* (1850s)

The Big-Endian/Little-Endian controversy reflects, in a much simplified form, British quarrels over religion. Less than 200 years previously, England had been a Catholic (Big-Endian) country; but a series of reforms beginning in the 1530s under King Henry VIII (reigned 1509–1547), Edward VI (1547–1553), and Queen Elizabeth I (1558–1603) had converted most of the country to Protestantism (Little-Endianism), in the episcopalian form of the Church of England. At the same time, revolution and reform in Scotland (1560) had also converted that country to Presbyterian Protestantism, which led to fresh difficulties when England and Scotland were united under one ruler, James I (1603–1625).^{[13]:31}



Wikipedia: [https://en.wikipedia.org/wiki/Lilliput_and_Blefuscu_\(Gulliver's_travels\)](https://en.wikipedia.org/wiki/Lilliput_and_Blefuscu_(Gulliver's_travels))

What do deviled eggs have to do with us?



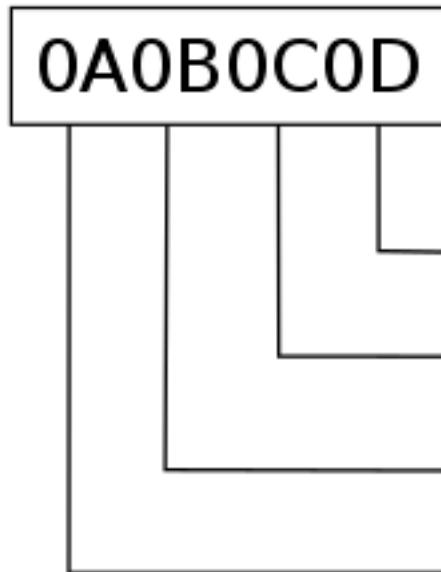
How do we decide how to store our bits in the transistors of the computer (or in any other binary system)?

Computer memory has no concept of “leftmost digits” or “rightmost digit” but rather “address a, address a+1, address a+2, etc”

Big endian: Most significant digit is first
Little endian: Least significant digit is first

Endianness, pictorially

32 bits of data

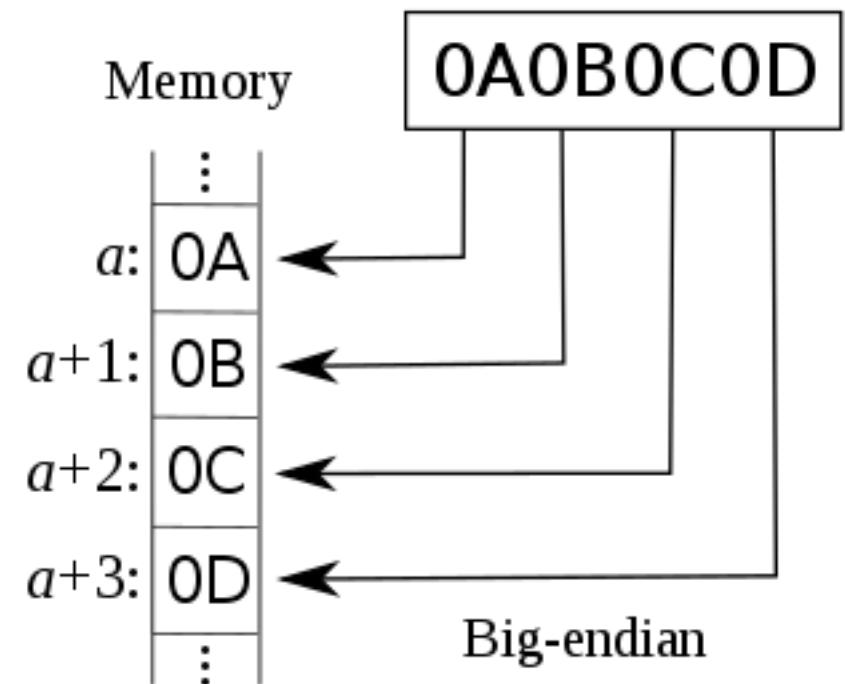


Little-endian

Memory

	:
a :	0D
$a+1$:	0C
$a+2$:	0B
$a+3$:	0A
	:

32 bits of data



Big-endian

<https://en.wikipedia.org/wiki/Endianness>

Just conventions, neither one is better or more right than the other, but important in many places to know which one your machinery is using!

And how do we deal with negative numbers?

Decimal	Binary
1	0b1
7	0b111
8	0b1000
15	0b1111
22	0b10110
39	0b100111
-39	0b-100111 = 0b1100111

Great, but how do we store a negative sign? Well, we can just use another bit (choose one, for example the most significant bit, 1 = “negative”, 0 = “positive”, though this is just one choice)

Let's try some elementary school math

Decimal: $6+2 = 8$, Binary: $0b0110 + 0b0010 = 0b1000$

Let's check that we see this together

Decimal: $6 - 2 = 4$, Binary: $0b0110 - 0b0010 = 0b0100$

Let's check that we see this together, this works

And something else you learn in elementary school:

Decimal: $6 + (-2) = 4$

Binary (adding in signs in **bold**): $0b\textbf{0}0110 + 0b\textbf{1}0010 = 0b\textbf{1}1000$

Let's check this together... but that clearly isn't right!!!!

We want: $0b\textbf{0}0110 + 0b\textbf{1}0010 = 0b\textbf{0}0100$



A better way to store negative numbers

Two's complement to store negative numbers: If we have N bits available to represent a number in binary, subtract the negative of the number we want the two's complement of from 2^N

Example: $12 = 0b01100$ (5 bits)

Two's complement of $-12 = 2^5 - 12 = 32 - 12 = 20 = 0b010100$

Let's check that we see this together

Naive storage of $-12: 0b11100$

Two's complement version: $0b10100$ (drop extra bit)

A (much) easier way to get the two's complement: flip the bits of the negative of the number and add 1:

$12 = 0b01100$ (start with negative of the number, ie $+12$)

$0b10011$ (bits flipped)

$0b10100$ (add one)

Why is two's complement useful?

Let's check what $12-5$ is... using 5 bits.
I hope you can do this in decimal :)

In two's complement, positive numbers remain as expected, so

$$12 = 0b01100$$

What is $12-5$? It is equal to $12+(-5)$

What is -5 in two's complement?

$$-(-5) = 5 = 0b00101$$

Flip the digits $\rightarrow 0b11010$

Add 1: $0b11011$

So $12-5 = 0b01100 + 0b11011 = 0b100111$, and we truncate to get $0b00111 = 7$! It works!!!! It's nice when simple arithmetic gives the expected results

What does two's complement work?

First, a reminder that to get the right number using two's complement, you must know the number of bits available (since inverting a 0 means you get a 1 in that place!)

Next, remember that the negative of a number is equivalent to 0 minus that number. Now let's go back to our example from the last slide and work out together what happens if we do that.

In order to correctly deal with negative numbers, we also need to deal with overflow and truncation. So instead of subtracting from zero, we can subtract from 1000000000..... (where we have N zeros, so the 1 is an overflow extra bit). And how do we write that number we subtract from?

It is $(1 + 1111111\dots)$ Subtracting from $1111111\dots$ is the equivalent of inverting the bits of the digits. And then we add back in that last one!

Two's complement, some examples

Assuming 8 bits! Let's check a few of these together

Decimal	Binary
1	0b00000001
7	0b00000111
33	0b00100001
-1	0b11111111
-7	0b11111001
-33	0b11011111
0	0b00000000

Other useful bases

One bit is the smallest unit on a computer, but it's very small and not typically useful. We typically refer to a byte as 8 bits (though this can vary on architectures)

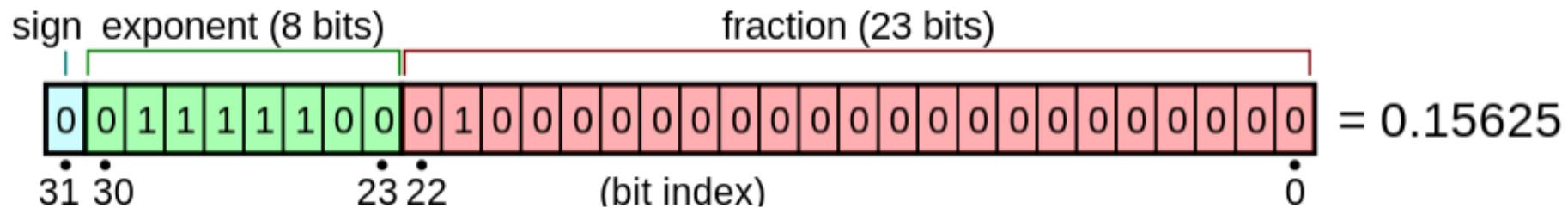
So on the previous slide, each collection of 8 bits was one byte. A byte can store $2^8 = 256$ different numbers. If unsigned, 0-255. If signed, -128 to +127!

We can cleanly represent a byte as 0x0-0xff, and we can put bytes together to make bigger quantities. If we use 2 bytes (16 bits, 0x0-0xffff) we have access to 2^{16} numbers, so 0-65535 or -32768 to +32767

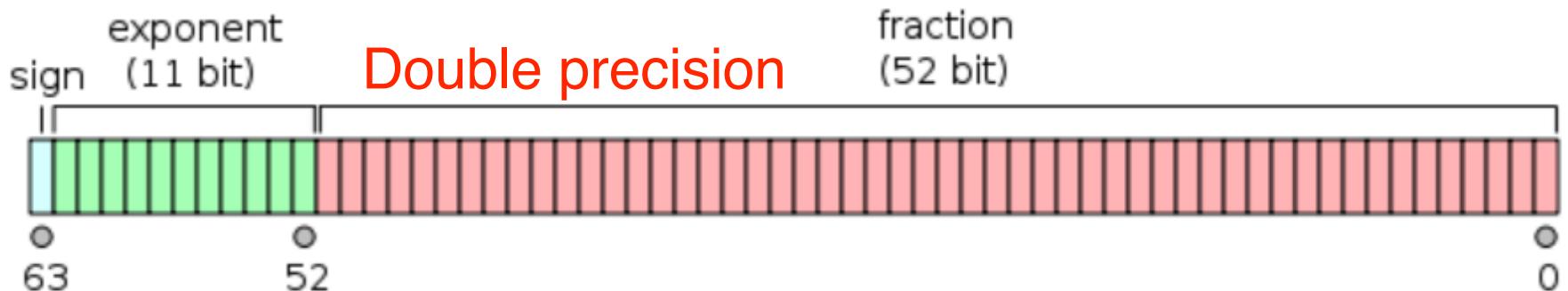
What about non-integer numbers?

How to store 3.14159 in discrete bits? Use scientific notation! Typically use 32 bits/4 bytes or 64 bits / 8 bytes. One of the bits is often used for the sign, so:

Single precision



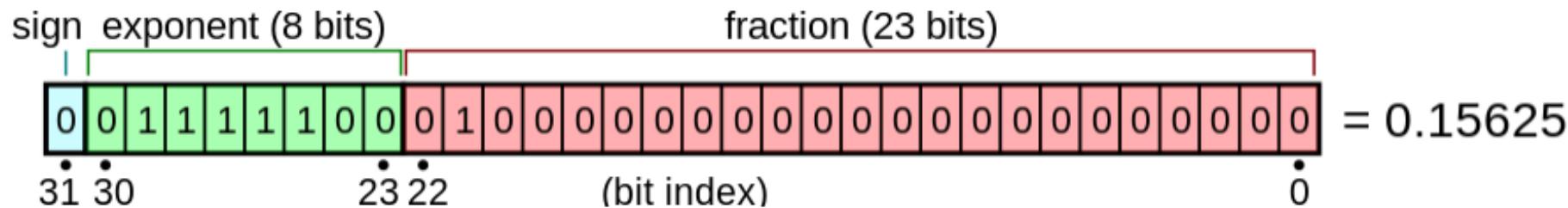
(Wikipedia)



How does that IEEE single precision work?

1.f = mantissa. Note that we can only compare floating points within the precision of the mantissa! Better to assign a tolerance level you are willing to live with: if($\text{abs}(x-\pi) < \text{epsilon}$):

Here's a simple example from Wikipedia



(Wikipedia)

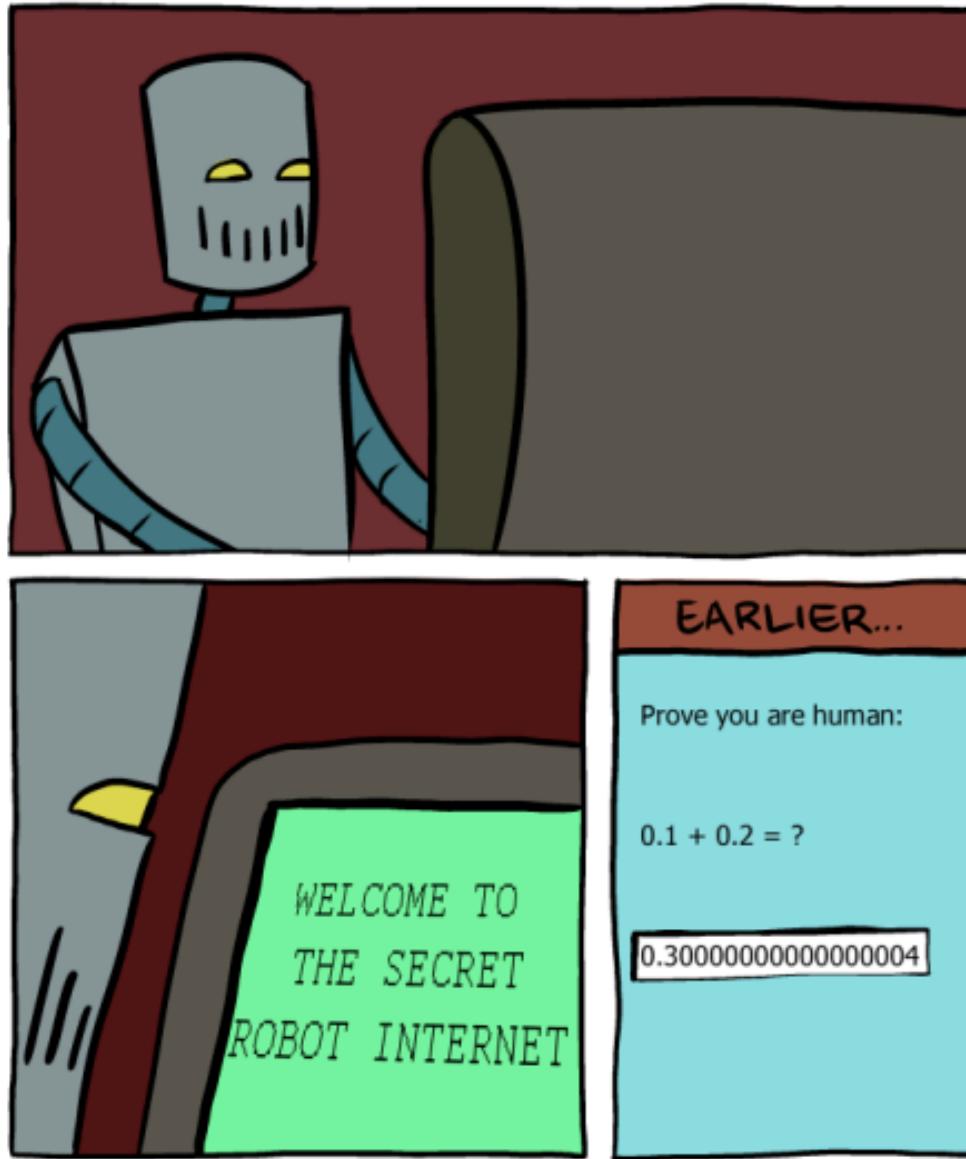
$$\text{value} = (-1)^{\text{sign}} \times 2^{(e-127)} \times \left(1 + \sum_{i=1}^{23} b_{23-i} 2^{-i} \right)$$

Sign is $(-1)^0 = 1$

Exponent is $0b01111100 = 124$

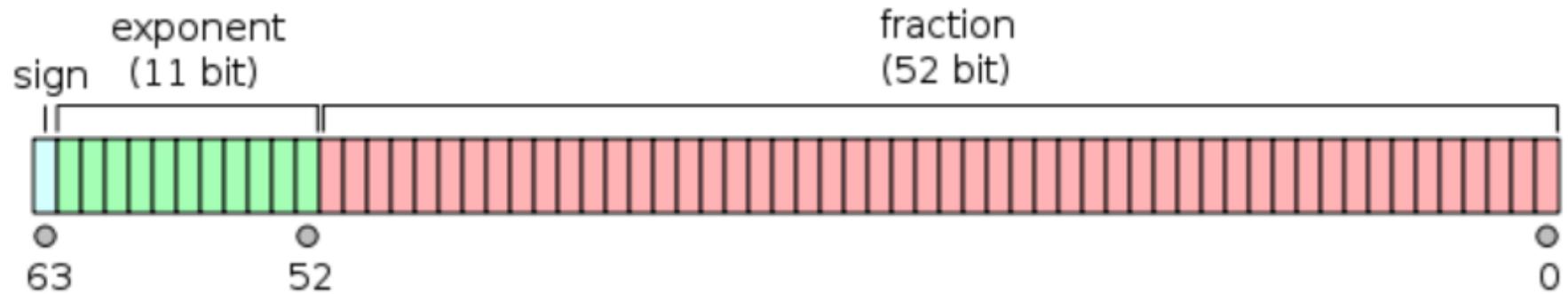
Fraction is $1+1*2^{-2} = 1+0.25=1.25$ Value is $1*2^{-3}*1.25 = 0.15625$

Comparisons of floating points



How does that IEEE double precision work?

Here's a simple example from Wikipedia



$$(-1)^{\text{sign}} \left(1 + \sum_{i=1}^{52} b_{52-i} 2^{-i} \right) \times 2^{e-1023}$$

(Wikipedia)

Note that in either format, integers can be exactly represented (if small enough). But otherwise, there is an implicit precision involved for floating point numbers! Keep in mind as you use them

How do we add numbers in these formats?

Can't just take the bits and add them as expected! Our IEEE formats are really (binary) scientific notation. How do we add numbers in scientific notation? We need to make sure we add the numbers with the same exponents

Take the number with the smaller exponent and shift the mantissa so that it has the same exponent as the number with the larger exponent. Then we can add things as expected

And subtraction is then using two's complement and doing the same thing. Multiplication and division? Add/subtract exponents and then work on the mantissas

C++ vs Python

C++ is a strongly typed language. You have to declare in advance the sort of object and number you are representing - Is it signed? Unsigned? Is it an integer? Or a floating point? How many bits do you want to use to represent it?

This is often a source of confusion (and occasional misery!), though many of the types can easily be converted from one to another. C++ prefers that you declare this in advance so that you know what the computer is doing and why.

Types in Python are not static and can change!

```
>>> x=4/3.  
>>> type(x)  
<type 'float'>  
>>> x  
1.3333333333333333  
>>> x=4/3  
>>> type(x)  
<type 'int'>  
>>> x  
1
```

For example, if you are using two bytes to store a number, the difference in the range of possible values depending on whether it is signed or not (-32768 to +32767 vs 0 to 65535) is important!

TRUNCATION!
Let's discuss

Python2 vs Python3

```
/Users/jahreda python3
Python 3.5.2 (v3.5.2:4def2a2901a5, Jun 26 2016, 10:47:25)
[GCC 4.2.1 (Apple Inc. build 5666) (dot 3)] on darwin
Type "help", "copyright", "credits" or "license" for more information.
>>> 2/7
0.2857142857142857
>>> 2//7
0
>>> ^D
/Users/jahreda python2
Python 2.7.14 |Anaconda, Inc.| (default, Dec  7 2017, 11:07:58)
[GCC 4.2.1 Compatible Clang 4.0.1 (tags/RELEASE_401/final)] on darwin
Type "help", "copyright", "credits" or "license" for more information.
>>> 2/7
0
>>> 2//7
0
```

C++ types

Type specifier	Equivalent type	Width in bits by data model				
		C++ standard	LP32	ILP32	LLP64	LP64
<code>short</code>						
<code>short int</code>						
<code>signed short</code>	<code>short int</code>	at least 16	16	16	16	16
<code>signed short int</code>						
<code>unsigned short</code>	<code>unsigned short int</code>					
<code>unsigned short int</code>						
<code>int</code>	<code>int</code>	at least 16	16	32	32	32
<code>signed</code>						
<code>signed int</code>	<code>int</code>					
<code>unsigned</code>						
<code>unsigned int</code>	<code>unsigned int</code>					
<code>long</code>						
<code>long int</code>	<code>long int</code>	at least 32	32	32	32	64
<code>signed long</code>						
<code>signed long int</code>	<code>long int</code>					
<code>unsigned long</code>						
<code>unsigned long int</code>	<code>unsigned long int</code>					
<code>long long</code>						
<code>long long int</code>	<code>long long int</code> (C++11)	at least 64	64	64	64	64
<code>signed long long</code>						
<code>signed long long int</code>	<code>long long int</code> (C++11)					
<code>unsigned long long</code>						
<code>unsigned long long int</code>	<code>unsigned long long int</code> (C++11)					

C++ types

Type	Size in bits	Format	Value range	
			Approximate	Exact
character	8	signed		-128 to 127
		unsigned		0 to 255
	16	unsigned		0 to 65535
	32	unsigned		0 to 1114111 (0x10ffff)
integer	16	signed	$\pm 3.27 \cdot 10^4$	-32768 to 32767
		unsigned	0 to $6.55 \cdot 10^4$	0 to 65535
	32	signed	$\pm 2.14 \cdot 10^9$	-2,147,483,648 to 2,147,483,647
		unsigned	0 to $4.29 \cdot 10^9$	0 to 4,294,967,295
	64	signed	$\pm 9.22 \cdot 10^{18}$	-9,223,372,036,854,775,808 to 9,223,372,036,854,775,807
		unsigned	0 to $1.84 \cdot 10^{19}$	0 to 18,446,744,073,709,551,615
floating point	32	IEEE-754 	<ul style="list-style-type: none"> min subnormal: $\pm 1.401,298,4 \cdot 10^{-45}$ min normal: $\pm 1.175,494,3 \cdot 10^{-38}$ max: $\pm 3.402,823,4 \cdot 10^{38}$ 	<ul style="list-style-type: none"> min subnormal: $\pm 0x1p-149$ min normal: $\pm 0x1p-126$ max: $\pm 0x1.fffffep+127$
	64	IEEE-754 	<ul style="list-style-type: none"> min subnormal: $\pm 4.940,656,458,412 \cdot 10^{-324}$ min normal: $\pm 2.225,073,858,507,201,4 \cdot 10^{-308}$ max: $\pm 1.797,693,134,862,315,7 \cdot 10^{308}$ 	<ul style="list-style-type: none"> min subnormal: $\pm 0x1p-1074$ min normal: $\pm 0x1p-1022$ max: $\pm 0x1.fffffffffffffp+1023$

Why do we care about all this stuff?

If you want to know how long it will take for my car to go from point A to point B, it probably doesn't matter that your floating point answer has finite precision.

But finite precision is a source of **error**, and in many cases, these sources of error can build up and accumulate and become quite critical or even dominant for many calculations

Can consider the absolute error on a number in absolute terms. If x_0 is the true value and x is the stored/estimated value in the computer, then the **absolute error is $|x_0 - x|$** . This is useful in many cases, but an error of 1 earth-year on the age of the Earth is probably not a big deal for the age of the Universe, even if it is absolutely a long time. So we can also consider the relative or fractional error, $|x_0 - x|/x_0$

Quadratic equations example

Exercise 4.2 in the book is an interesting one. Let's look at the solutions to quadratic equations

$$x = \frac{[-b \pm \sqrt{b^2 - 4ac}]}{2a}$$

Let $\sqrt{b^2 - 4ac} = s$

Then $x = \frac{[-b \pm s]}{2a}$

Multiply x by 1 = $\frac{[-b \pm s]}{2a} \cdot \frac{[-b \pm s]}{[-b \pm s]}$ to get

$$x = \frac{[b^2 \pm bs - b^2 - bs - s^2]}{[-2ab \pm 2as]}$$

$$x = \frac{[b^2 - s^2]}{[-2ab \pm 2as]}$$

But $s^2 = b^2 - 4ac$ so

$$x = \frac{[b^2 - b^2 + 4ac]}{[-2ab \pm 2as]}$$

$$x = \frac{[4ac]}{[2ab \pm 2as]} = \frac{[2c]}{[-b \pm s]}$$

Two “equivalent” ways to write the solutions to a quadratic equation.
But what do they give?

```
[8] from math import sqrt
def quad(a,b,c):
    print("")
    print(a,b,c)
    squareroot = sqrt(b*b-4*a*c)
    sol1 = (-b + squareroot)/(2*a)
    sol2 = (-b - squareroot)/(2*a)
    sol1_prime = 2*c/(-b-squareroot)
    sol2_prime = 2*c/(-b+squareroot)
    print("Standard approach:",sol1,sol2)
    print("And standard approach we get back:",a*sol1*sol1+b*sol1+c,a*sol2*sol2+b*sol2+c)
    print("Alternate approach:",sol1_prime,sol2_prime)
    print("And alternate approach we get back:",a*sol1_prime*sol1_prime+b*sol1_prime+c,a*sol2_prime*sol2_prime+b*sol2_prime+c)

quad(2,10,5)
quad(0.001,1000,0.001)
```

Quadratic equations example

```
quad(2,10,5)
quad(0.001,1000,0.001)
```



2 10 5

Standard approach: -0.5635083268962915 -4.436491673103708

And standard approach we get back: 8.881784197001252e-16 -7.105427357601002e-15

Alternate approach: -0.5635083268962916 -4.436491673103709

And alternate approach we get back: 0.0 0.0

Looks nice

0.001 1000 0.001

Standard approach: -9.999894245993346e-07 -999999.999999

And standard approach we get back: 1.0575401665491313e-08 7.247924804689582e-08

Alternate approach: -1.000000000001e-06 -1000010.5755125057

And alternate approach we get back: 0.0 10575.62534720993

Eeeeek!

$x = [-b \pm \sqrt{b^2 - 4ac}]/(2a)$.

Let $\sqrt{b^2 - 4ac} = s$

Then $x = [-b \pm s]/[2a]$

Multiply x by 1 = $[-b \pm s]/[-b \pm s]$ to get

$x = [b^2 \pm bs \mp bs - s^2][-2ab \pm 2as]$

$x = [b^2 - s^2][-2ab \pm 2as]$

But $s^2 = b^2 - 4ac$ so

$x = [b^2 - b^2 + 4ac]/[-2ab \pm 2as]$

$x = [4ac]/[2ab \pm 2as] = [2c]/[-b \pm s]$

What happens if b is large?

Standard solution has a value $(-b)$ being added to another value (the stuff in square root). If b is large then the square root $\sim b$, and we are computing $-b + b$! Large errors!

Quadratic equations example

```
quad(2,10,5)
quad(0.001,1000,0.001)
```



2 10 5

Standard approach: -0.5635083268962915 -4.436491673103708

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Multiply x by 1 = $[-b \pm s]/[-b \pm s]$ to get

$x = [b^2 \pm bs \mp bs - s^2][-2ab \pm 2as]$

$x = [b^2 - s^2][-2ab \pm 2as]$

But $s^2 = b^2 - 4ac$ so

$x = [b^2 - b^2 + 4ac]/[-2ab \pm 2as]$

$x = [4ac]/[2ab \pm 2as] = [2c]/[-b \pm s]$

What happens if b is large? Standard solution has a value $(-b)$ being added to another value (the stuff in square root).

If b is large then the square root $\sim b$, and we are computing $-b + b$! Large errors! Prefer to add two negative values or two positive values

Quadratic equations example



```
from math import sqrt
def quad(a,b,c):
    print("")
    print(a,b,c)
    squareroot = sqrt(b*b-4*a*c)
    if (b > 0.0):
        sol1 = 2*c/(-b-squareroot)
        sol2 = (-b - squareroot)/(2*a)
    else:
        sol1 = 2*c/(-b+squareroot)
        sol2 = (-b + squareroot)/(2*a)
    print("New approach:",sol1,sol2)
    print("And new approach we get back:",a*sol1*sol1+b*sol1+c,a*sol2*sol2+b*sol2+c)

quad(2,10,5)
quad(0.001,1000,0.001)
quad(2,-10,5)
quad(0.001,-1000,0.001)
```



```
2 10 5
New approach: -0.5635083268962916 -4.436491673103708
And new approach we get back: 0.0 -7.105427357601002e-15
```

```
0.001 1000 0.001
New approach: -1.000000000001e-06 -999999.999999
And new approach we get back: 0.0 7.247924804689582e-08
```

```
2 -10 5
New approach: 0.5635083268962916 4.436491673103708
And new approach we get back: 0.0 -7.105427357601002e-15

0.001 -1000 0.001
New approach: 1.000000000001e-06 999999.999999
And new approach we get back: 0.0 7.247924804689582e-08
```

Much nicer!

Our code is of course missing one thing. What is it?

Error propagation

Let x and y be two real numbers, and x^* and y^* their floating point approximations.

Now, let the computer calculate $x-y$:

First, x and y are replaced by x^* and y^* .

The final result is then $(x^*-y^*)^*$.

Example: $x=301/2000 \approx .15050000$ and $y=301/2001 \approx .150424787$

The exact result is $x-y=301/4002000 \approx .00007521239$

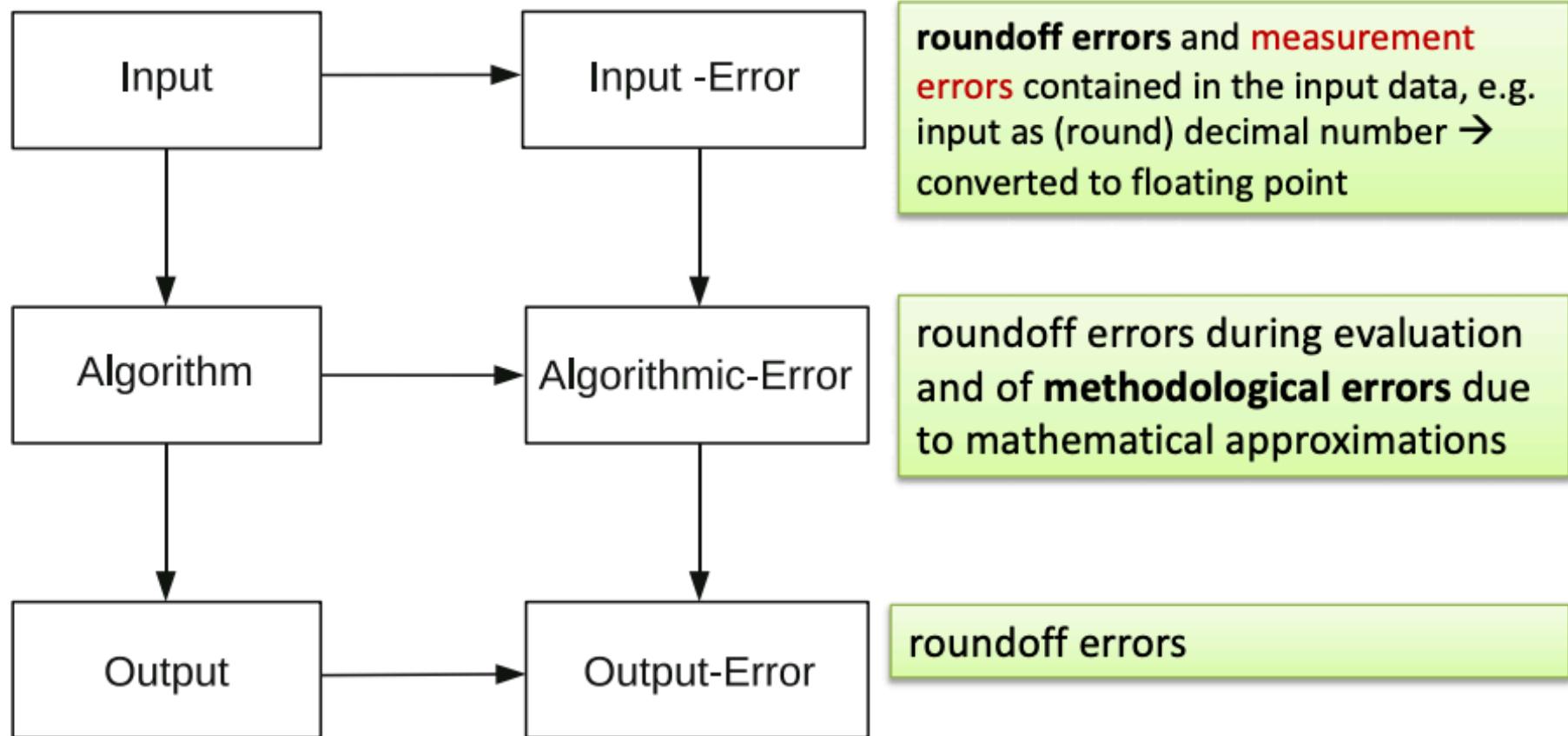
Imagine we have decimal floating point numbers with 4-digit mantissa, i.e., $x^* = .1505$ and $y^* = .1504$ so that $d^* = (x^*-y^*)^* = 0.0001$
Relative error on $x = 0$, Relative error on $y = 10^{-4}$... but relative error on d is 25%!

We will be devising and implementing algorithms this semester. All algorithms are going to be the result of operations on bits, the most basic objects that we have. We have a finite number of these bits, and anytime we move away from integers (which seems pretty likely) we will only have approximations of the solutions.

We want to keep the size of these errors and approximations as small as possible

Errors

Schematic classification of the errors occurring within a numerical procedure



A very nice set of slides from this course last year

Main sources of errors

In any applied numerical computation, there are several key sources of error:

Modeling/Measurement errors

- i. Inexactness of the mathematical model for the underlying physical phenomenon.
- ii. Errors in measurements of parameters entering the model.

These do not concern us too much. The latter is the domain of the experimentalists. I disagree,

Algorithmic errors

but these do not concern us **this semester**

- i. **Roundoff errors** in computer arithmetic (finite numerical precision imposed by the computer).
- ii. **Discretization (methodological) errors**: continuous processes are replaced by discrete ones (e.g., summation to calculate an integral).
- iii. “Termination” errors: infinite algorithms are terminated after a finite number of steps (e.g., iterations like $x_{n+1}=(x_n+a/x_n)/2 \rightarrow \text{sqrt}(a)$, $n \rightarrow \infty$).

See book discussion on program speed!

The latter two are the true domain of numerical analysis and are a consequence of the fact that

- Most systems of equations are too complicated to solve explicitly
- Even with known analytic solution, directly obtaining the precise numerical values may be difficult

A very nice set of slides from this course last year

Methodological Errors

Result from replacement of mathematical expressions by approximate, simpler ones.

E.g. pendulum period:

$$\tau = 4 \sqrt{\frac{\ell}{g}} \int_0^{\frac{\pi}{2}} \frac{d\alpha}{\sqrt{1 - k^2 \sin^2(\alpha)}} \quad k = \sin(\theta_0/2)$$

$$= 4 \sqrt{\frac{\ell}{g}} K_1(k) .$$

Truncate K_1 series at some N:

$$K_1(k) = \frac{\pi}{2} \sum_{n=0}^{\infty} \left[\frac{(2n)!}{2^{2n}(n!)^2} \right]^2 k^{2n}$$

$$= \frac{\pi}{2} \sum_{n=0}^N \left[\frac{(2n)!}{2^{2n}(n!)^2} \right]^2 k^{2n} + R_N(k)$$

Program speed and also ability to implement!

R_N: truncation error

Or for derivatives (Chapter 2):

$$\left. \frac{d}{dx} f(x) \right|_{x=x_0} = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \approx \frac{f(x_0 + h) - f(x_0)}{h}$$

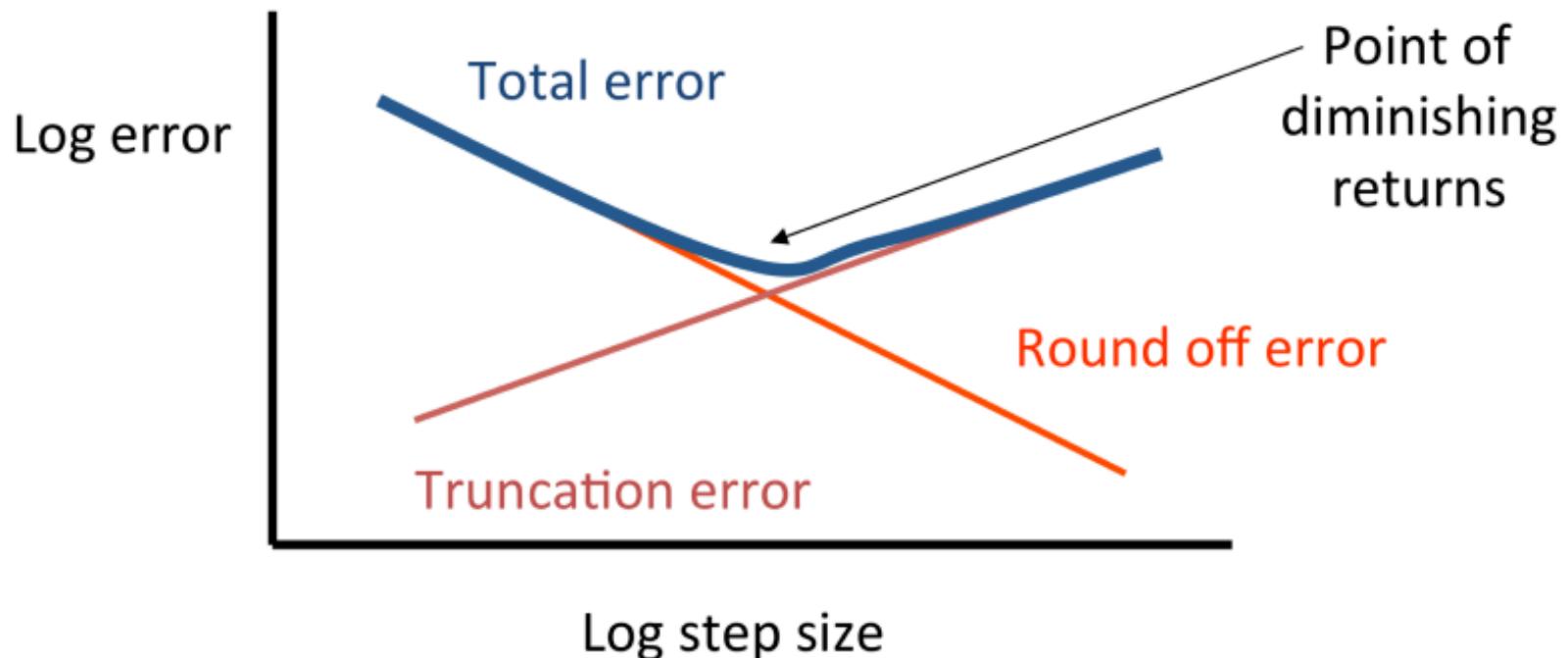
For some finite h → finite difference (relative error can be minimal for some finite! h)

A very nice set of slides from this course last year

Error trade-off

$$f'(x_0) = \left. \frac{d}{dx} f(x) \right|_{x=x_0} \approx \frac{f(x_0 + h) - f(x_0)}{h}$$

- Using a smaller step size reduces truncation error.
- However, it increases the round-off error.
- Trade off/diminishing returns occurs: Always think and test!



A very nice set of slides from this course last year

From the textbook:

```
def f(x):
    return x*(x-1)

def derivative(x,epsilon):
    return (f(x+epsilon)-f(x))/epsilon

x=1
for epsilon in (1e-4,1e-6,1e-8,1e-10,1e-12,1e-14):
    print (epsilon, derivative(x,epsilon))
```

```
/Users/jahreda/Desktop python3 derivative.py
0.0001 1.000099999998899  OK
1e-06 1.000000999177333  Better
1e-08 1.000000039225287  Even better
1e-10 1.000000082840371  Only better again
1e-12 1.0000889005833413  Getting worse
1e-14 0.9992007221626509  Pretty bad
```

True value of $f'(x) = 2*x-1$, $f'(1) = 1$

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Stability

An algorithm, equation or, even more general, a problem is referred to as unstable or ill-conditioned if small changes in the input cause a large change in the output.

Example 1: $x + y = 2.0,$
 $x + 1.01y = 2.01$

Solution: $x=1.0, y=1.0$

Let us suppose we make a small error in the rhs of the second equation:

$$\begin{aligned}x + y &= 2.0, \\x + 1.01y &= 2.02\end{aligned}$$

Solution: **x=0.0, y=2.0**

I.e. a 0.05% input error resulted in a 100% wrong result!

Furthermore, if the y-coefficient 1.01 in the original equation would have a 1% error and be 1.0, the equation **would be unsolvable** altogether!

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Example 2:

$$\begin{cases} \ddot{y} - 10\dot{y} - 11y = 0, \\ y(0) = 1, \quad \dot{y}(0) = -1 \end{cases}$$

General solution: $y = A \exp(-x) + B \exp(11x)$

With initial conditions: $y = \exp(-x)$

Adding a small input error in initial conditions: $y(0) = 1 + \delta$ and $\dot{y}(0) = -1 + \epsilon$

Yields: $\bar{y} = \left(1 + \frac{11\delta}{12} - \frac{\epsilon}{12}\right) \exp(-x) + \left(\frac{\delta}{12} + \frac{\epsilon}{12}\right) \exp(11x)$

i.e. the relative error is $\epsilon_r = \left| \frac{y - \bar{y}}{y} \right|$

$$= \left(\frac{11\delta}{12} - \frac{\epsilon}{12}\right) + \left(\frac{\delta}{12} + \frac{\epsilon}{12}\right) \exp(12x)$$

→ problem is ill-conditioned: for large values of x the second term overrules the first one

See Book for another example for induced instability and the relation to Chaos Theory!

Despite what you may learn in all your intro courses, the vast majority of your undergraduate courses and even a good fraction of your graduate courses, it's rare these days that we can completely solve useful physics problems with pen and paper

Most problems don't have analytical solutions, and must be solved with numerical, approximate methods. And even those problems with analytical solutions likely can't be done by you with your pencils. In addition, visualization of solutions is becoming an ever-critical part of physics, and thus of computational physics

What sorts of problems will we need to solve? Integrals, derivatives, eigenvalue/vector problems, Fourier analysis, differential equations, systems of equations, simulation, etc

Another simple example

```
>>> from math import pow
>>> from numpy import arange
>>> exponent = arange(1,20)
>>> exponent
array([ 1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19])
>>> for val in exponent:
...     epsilon = pow(10,-val)
...     x = 1 - epsilon
...     y = 1 - x ### should be epsilon
...     print(epsilon,y)
...
0.1 0.0999999999999998
0.01 0.010000000000000009
0.001 0.001000000000000009
0.0001 9.99999999998899e-05
1e-05 9.9999999995449e-06
1e-06 1.000000000287557e-06
1e-07 9.9999994736442e-08
1e-08 1.000000050247593e-08
1e-09 9.99999717180685e-10
1e-10 1.000000082740371e-10
1e-11 1.000000082740371e-11
1e-12 9.999778782798785e-13
1e-13 1.000310945187266e-13
1e-14 9.992007221626409e-15
1e-15 9.992007221626409e-16
1e-16 1.1102230246251565e-16
1e-17 0.0
1e-18 0.0
1e-19 0.0
>>>
```

Let's take a close look at this

Homework #1

- 1) Exercise 4.4 in the textbook. You may want to use:

```
import time  
tic = time.perf_counter()  
...  
toc = time.perf_counter()  
time_in_seconds = toc - tic
```

- 2) Give the standard (IEEE) single precision binary representation of the machine approximation for $-1/3$. Don't find an algorithm to do this, but write it out yourself
- 3) Find the smallest positive integer that is not exact in single precision. Show your work
- 4) What are the two's complement representations for these decimal numbers? Use 12 bits: 9, 455, 1021, -12, -1022, -1023.
- 5) What are the octal, binary and hexadecimal representations of the decimal numbers 13, 133, 1333 and 13333? Show your work