

```
In [1]: from keras.utils import np_utils
from keras.datasets import mnist
import seaborn as sns
from keras.initializers import he_normal
import matplotlib.pyplot as plt
import numpy as np
import time
from keras.layers.normalization import BatchNormalization
from keras.optimizers import Adam
from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV
from keras.models import Sequential
from keras.layers import Dense, Activation,Dropout
#from pactools.grid_search import GridSearchCVProgressBar

Using TensorFlow backend.
```

```
In [0]: # https://gist.github.com/greydanus/f6eee59eaf1d90fcb3b534a25362cea4

def plt_dynamic(x, vy, ty, ax, colors=['b']):
    ax.plot(x, vy, 'b', label="Validation Loss")
    ax.plot(x, ty, 'r', label="Train Loss")
    plt.legend()
    plt.grid(linestyle='-')
    fig.canvas.draw()
```

**Observations:** This gives a dynamic plot of values specified.

## Loading the data

```
In [4]: # the data, shuffled and split between train and test sets
(x_train, y_train), (x_test, y_test) = mnist.load_data()

Downloading data from https://s3.amazonaws.com/img-datasets/mnist.npz (https://s3.amazonaws.com/img-datasets/mnist.npz)
11493376/11490434 [=====] - 2s 0us/step

Observations: Getting the trainand test data of MNIST dataset.
```

```
In [0]: print('Total amount of train data is {} and shape of each image is ({},{})'.format(x_train.shape[0],x_train.shape[1],x_train.shape[2]))

Total amount of train data is 60000 and shape of each image is (28,28).
```

```
In [5]: x_train = x_train.reshape(x_train.shape[0], x_train.shape[1]*x_train.shape[2])
x_test = x_test.reshape(x_test.shape[0], x_test.shape[1]*x_test.shape[2])
print('Total amount of train data is {} and shape of each image is {}'.format(x_train.shape[0],x_train.shape[1]))

Total amount of train data is 60000 and shape of each image is 784.
```

```
In [0]: x_train = x_train/255 #Apply data normalization. X => (X - Xmin)/(Xmax-Xmin) = X/255
x_test = x_test/255

Observations: Since the value of pixels lie between 0-255, data needs to be normalized.
```

```
In [7]: # here we are having a class number for each image
print("Class label of 49th image :", y_train[49])
#These need to be converted into a vector.

y_train_cat = np_utils.to_categorical(y_train, 10)
y_test_cat = np_utils.to_categorical(y_test, 10)

print("After converting the output into a vector : ",y_train_cat[49])

Class label of 49th image : 3
After converting the output into a vector :  [0. 0. 0. 1. 0. 0. 0. 0. 0. 0.]

Observations: Convert each label to a vector
```

## 2 Hidden layers

```
In [0]: output_dim = 10
input_dim = x_train.shape[1]
batch_size = 120
nb_epoch = 20
```

### With Batch normalization, Dropouts

```
In [0]: from keras.optimizers import Adam,RMSprop,SGD
def model_keras(l1,l2):

    model = Sequential()
    model.add(Dense(l1, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal())) #Uses Relu activation and he normalization as kernel initializer
    model.add(BatchNormalization()) #Perform Batch normalization
    model.add(Dropout(0.5)) #Add dropouts

    model.add(Dense(l2, activation='relu', kernel_initializer=he_normal()) )
    model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(output_dim, activation='softmax'))

    model.compile(loss='categorical_crossentropy', metrics=['accuracy'], optimizer='adam')

    return model
```

```
In [13]: from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV

model = KerasClassifier(build_fn=model_keras, epochs=nb_epoch, batch_size=batch_size, verbose=0)
param_grid = {'l1': [256,328,512], 'l2': [32,64,128]}

gsearch = GridSearchCV(estimator=model, param_grid=param_grid)
gresult = gsearch.fit(x_train, y_train_cat)

print("Best Accuracy obtained is {} when number of units in layer 1 and 2 are {}".format(gresult.best_score_, gresult.best_params_))
scores = gresult.cv_results_['mean_test_score']
params = gresult.cv_results_['params']
for score, param in zip(scores, params):
    print('Accuracy of {} is obtained for number of units in hidden layer 1 and 2 as {}'.format(score,param))
```

Best Accuracy obtained is 0.9782833398580552 when number of units in layer 1 and 2 are {'l1': 512, 'l2': 128}.

Accuracy of 0.9741666691303253 is obtained for number of units in hidden layer 1 and 2 as {'l1': 256, 'l2': 32}  
Accuracy of 0.9760500049591064 is obtained for number of units in hidden layer 1 and 2 as {'l1': 256, 'l2': 64}  
Accuracy of 0.9762000054121017 is obtained for number of units in hidden layer 1 and 2 as {'l1': 256, 'l2': 128}  
Accuracy of 0.9747833367586136 is obtained for number of units in hidden layer 1 and 2 as {'l1': 328, 'l2': 32}  
Accuracy of 0.9759500049750011 is obtained for number of units in hidden layer 1 and 2 as {'l1': 328, 'l2': 64}  
Accuracy of 0.9774500041007995 is obtained for number of units in hidden layer 1 and 2 as {'l1': 328, 'l2': 128}  
Accuracy of 0.9775000061988831 is obtained for number of units in hidden layer 1 and 2 as {'l1': 512, 'l2': 32}  
Accuracy of 0.9777333381573359 is obtained for number of units in hidden layer 1 and 2 as {'l1': 512, 'l2': 64}  
Accuracy of 0.9782833398580552 is obtained for number of units in hidden layer 1 and 2 as {'l1': 512, 'l2': 128}

```
In [14]: best_model = model_keras(gresult.best_params_['l1'],gresult.best_params_['l2'])
model_fit = best_model.fit(x_train, y_train_cat, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(x_test, y_test_cat))
```

Train on 60000 samples, validate on 10000 samples  
Epoch 1/20  
60000/60000 [=====] - 12s 193us/step - loss: 0.4246 - acc: 0.8727 - val\_loss: 0.1415 - val\_acc: 0.9550  
Epoch 2/20  
60000/60000 [=====] - 6s 107us/step - loss: 0.2038 - acc: 0.9393 - val\_loss: 0.0999 - val\_acc: 0.9679  
Epoch 3/20  
60000/60000 [=====] - 6s 107us/step - loss: 0.1586 - acc: 0.9521 - val\_loss: 0.0844 - val\_acc: 0.9741  
Epoch 4/20  
60000/60000 [=====] - 6s 108us/step - loss: 0.1350 - acc: 0.9592 - val\_loss: 0.0812 - val\_acc: 0.9741  
Epoch 5/20  
60000/60000 [=====] - 6s 108us/step - loss: 0.1191 - acc: 0.9642 - val\_loss: 0.0751 - val\_acc: 0.9742  
Epoch 6/20  
60000/60000 [=====] - 6s 106us/step - loss: 0.1083 - acc: 0.9662 - val\_loss: 0.0695 - val\_acc: 0.9766  
Epoch 7/20  
60000/60000 [=====] - 6s 107us/step - loss: 0.0990 - acc: 0.9704 - val\_loss: 0.0650 - val\_acc: 0.9784  
Epoch 8/20  
60000/60000 [=====] - 6s 107us/step - loss: 0.0935 - acc: 0.9715 - val\_loss: 0.0700 - val\_acc: 0.9786  
Epoch 9/20  
60000/60000 [=====] - 6s 108us/step - loss: 0.0900 - acc: 0.9719 - val\_loss: 0.0625 - val\_acc: 0.9794  
Epoch 10/20  
60000/60000 [=====] - 6s 108us/step - loss: 0.0855 - acc: 0.9728 - val\_loss: 0.0595 - val\_acc: 0.9806  
Epoch 11/20  
60000/60000 [=====] - 6s 108us/step - loss: 0.0800 - acc: 0.9754 - val\_loss: 0.0602 - val\_acc: 0.9815  
Epoch 12/20  
60000/60000 [=====] - 6s 107us/step - loss: 0.0744 - acc: 0.9765 - val\_loss: 0.0577 - val\_acc: 0.9825  
Epoch 13/20  
60000/60000 [=====] - 6s 107us/step - loss: 0.0717 - acc: 0.9777 - val\_loss: 0.0574 - val\_acc: 0.9826  
Epoch 14/20  
60000/60000 [=====] - 6s 106us/step - loss: 0.0685 - acc: 0.9786 - val\_loss: 0.0586 - val\_acc: 0.9827  
Epoch 15/20  
60000/60000 [=====] - 6s 107us/step - loss: 0.0622 - acc: 0.9799 - val\_loss: 0.0569 - val\_acc: 0.9821  
Epoch 16/20  
60000/60000 [=====] - 6s 107us/step - loss: 0.0615 - acc: 0.9801 - val\_loss: 0.0558 - val\_acc: 0.9822  
Epoch 17/20  
60000/60000 [=====] - 6s 107us/step - loss: 0.0621 - acc: 0.9797 - val\_loss: 0.0549 - val\_acc: 0.9840  
Epoch 18/20  
60000/60000 [=====] - 6s 107us/step - loss: 0.0586 - acc: 0.9813 - val\_loss: 0.0549 - val\_acc: 0.9843  
Epoch 19/20  
60000/60000 [=====] - 6s 108us/step - loss: 0.0556 - acc: 0.9824 - val\_loss: 0.0555 - val\_acc: 0.9829  
Epoch 20/20  
60000/60000 [=====] - 6s 107us/step - loss: 0.0535 - acc: 0.9831 - val\_loss: 0.0576 - val\_acc: 0.9833

```
In [19]: import matplotlib.pyplot as plt
model_scores = best_model.evaluate(x_test, y_test_cat, verbose=0)
print('Test score:', model_scores[0])
print('Test accuracy:', model_scores[1])

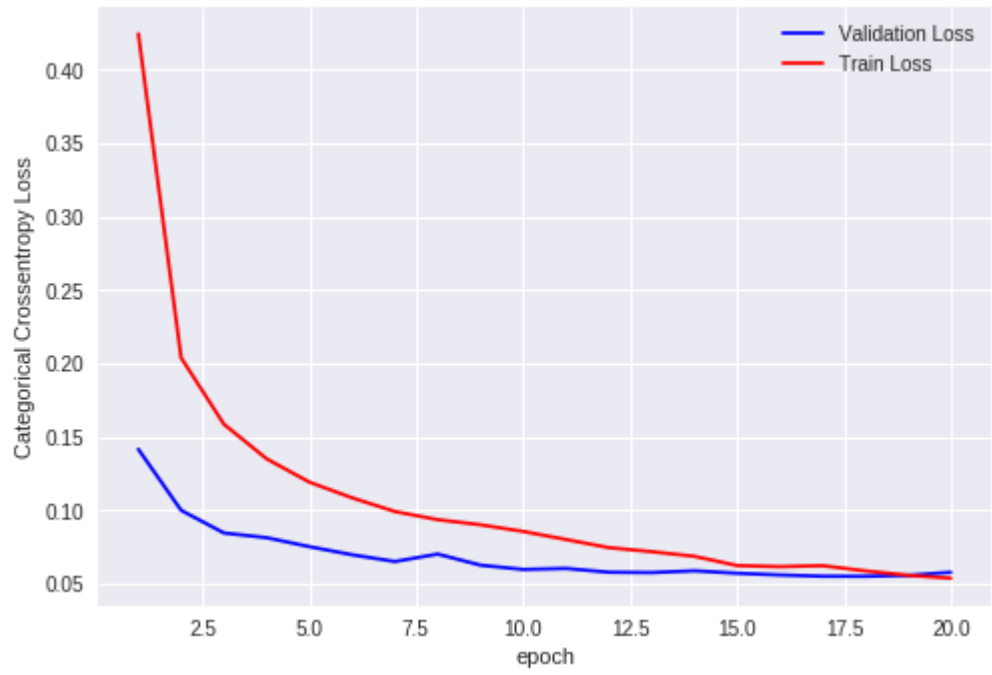
fig,ax = plt.subplots(1,1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy = model_fit.history['val_loss']
ty = model_fit.history['loss']

plt_dynamic(x, vy, ty, ax)
```

Test score: 0.05760298303969903  
Test accuracy: 0.9833



**Observations:** This plot seems do okay but might overfit as epochs are increased.

```
In [16]: import matplotlib.pyplot as plt
import seaborn as sns
w_after = best_model.get_weights()

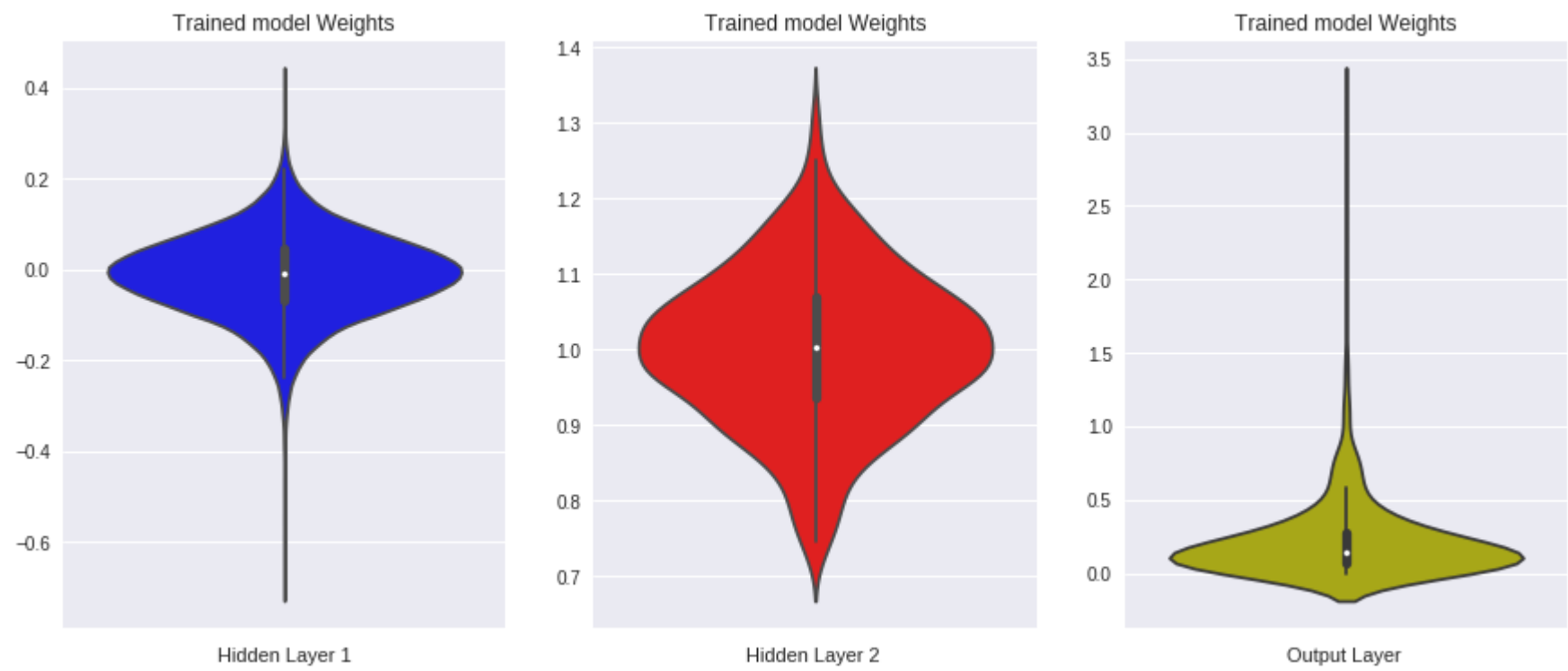
h1_w = w_after[0].flatten().reshape(-1,1)
h2_w = w_after[2].flatten().reshape(-1,1)
out_w = w_after[4].flatten().reshape(-1,1)

fig = plt.figure(figsize = (15,6))
plt.title("Weight matrices after model trained")
plt.subplot(1, 3, 1)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h1_w,color='b')
plt.xlabel('Hidden Layer 1')

plt.subplot(1, 3, 2)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h2_w, color='r')
plt.xlabel('Hidden Layer 2 ')

plt.subplot(1, 3, 3)
plt.title("Trained model Weights")
ax = sns.violinplot(y=out_w,color='y')
plt.xlabel('Output Layer ')
plt.show()
```

/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:588: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
kde\_data = remove\_na(group\_data)  
/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:816: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
violin\_data = remove\_na(group\_data)



Without Dropouts

```
In [0]: from keras.optimizers import Adam,RMSprop,SGD
def model_keras(l1,l2):

    model = Sequential()
    model.add(Dense(l1, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal()))
    model.add(BatchNormalization())
    #model.add(Dropout(0.5))

    model.add(Dense(l2, activation='relu', kernel_initializer=he_normal()) )
    model.add(BatchNormalization())
    #model.add(Dropout(0.5))

    model.add(Dense(output_dim, activation='softmax'))

    model.compile(loss='categorical_crossentropy', metrics=['accuracy'], optimizer='adam')

    return model
    #Taking Longer time without dropouts than without batch normalization
```

```
In [0]: from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV

model = KerasClassifier(build_fn=model_keras, epochs=nb_epoch, batch_size=batch_size, verbose=0)
param_grid = {'l1': [256,328,512], 'l2': [32,64,128]}

gsearch = GridSearchCV(estimator=model, param_grid=param_grid)
gresult = gsearch.fit(x_train, y_train_cat)

print("Best Accuracy obtained is {} when number of units in layer 1 and 2 are {}".format(gresult.best_score_, gresult.best_params_))
scores = gresult.cv_results_['mean_test_score']
params = gresult.cv_results_['params']
for score, param in zip(scores, params):
    print('Accuracy of {} is obtained for number of units in hidden layer 1 and 2 as {}'.format(score,param))
```

Best Accuracy obtained is 0.9772333382368088 when number of units in layer 1 and 2 are {'l1': 512, 'l2': 64}.

Accuracy of 0.9727000021934509 is obtained for number of units in hidden layer 1 and 2 as {'l1': 256, 'l2': 32}  
Accuracy of 0.9740666691462199 is obtained for number of units in hidden layer 1 and 2 as {'l1': 256, 'l2': 64}  
Accuracy of 0.9738666696548461 is obtained for number of units in hidden layer 1 and 2 as {'l1': 256, 'l2': 128}  
Accuracy of 0.9722666690746943 is obtained for number of units in hidden layer 1 and 2 as {'l1': 328, 'l2': 32}  
Accuracy of 0.9757666704654694 is obtained for number of units in hidden layer 1 and 2 as {'l1': 328, 'l2': 64}  
Accuracy of 0.9749000025192897 is obtained for number of units in hidden layer 1 and 2 as {'l1': 328, 'l2': 128}  
Accuracy of 0.9744166692892711 is obtained for number of units in hidden layer 1 and 2 as {'l1': 512, 'l2': 32}  
Accuracy of 0.9772333382368088 is obtained for number of units in hidden layer 1 and 2 as {'l1': 512, 'l2': 64}  
Accuracy of 0.9760333374738693 is obtained for number of units in hidden layer 1 and 2 as {'l1': 512, 'l2': 128}

**Observations:** Best accuracy seems to be obtained for 512 units in hidden layer 1 and 64 units in hidden layer 2.

```
In [0]: best_model = model_keras(gresult.best_params_['l1'],gresult.best_params_['l2']) #Perform on the best number of units for respective Layers
model_fit = best_model.fit(x_train, y_train_cat, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(x_test, y_test_cat))
```

Train on 60000 samples, validate on 10000 samples  
Epoch 1/20  
60000/60000 [=====] - 12s 206us/step - loss: 0.2054 - acc: 0.9408 - val\_loss: 0.1002 - val\_acc: 0.9687  
Epoch 2/20  
60000/60000 [=====] - 7s 115us/step - loss: 0.0752 - acc: 0.9771 - val\_loss: 0.0844 - val\_acc: 0.9722  
Epoch 3/20  
60000/60000 [=====] - 7s 117us/step - loss: 0.0497 - acc: 0.9853 - val\_loss: 0.0761 - val\_acc: 0.9771  
Epoch 4/20  
60000/60000 [=====] - 7s 117us/step - loss: 0.0375 - acc: 0.9882 - val\_loss: 0.0735 - val\_acc: 0.9777  
Epoch 5/20  
60000/60000 [=====] - 7s 118us/step - loss: 0.0276 - acc: 0.9914 - val\_loss: 0.0765 - val\_acc: 0.9787  
Epoch 6/20  
60000/60000 [=====] - 7s 119us/step - loss: 0.0255 - acc: 0.9917 - val\_loss: 0.0860 - val\_acc: 0.9761  
Epoch 7/20  
60000/60000 [=====] - 7s 119us/step - loss: 0.0203 - acc: 0.9936 - val\_loss: 0.0786 - val\_acc: 0.9773  
Epoch 8/20  
60000/60000 [=====] - 7s 117us/step - loss: 0.0166 - acc: 0.9945 - val\_loss: 0.0936 - val\_acc: 0.9743  
Epoch 9/20  
60000/60000 [=====] - 7s 118us/step - loss: 0.0159 - acc: 0.9949 - val\_loss: 0.0753 - val\_acc: 0.9807  
Epoch 10/20  
60000/60000 [=====] - 7s 117us/step - loss: 0.0154 - acc: 0.9951 - val\_loss: 0.0843 - val\_acc: 0.9792  
Epoch 11/20  
60000/60000 [=====] - 7s 117us/step - loss: 0.0120 - acc: 0.9961 - val\_loss: 0.0739 - val\_acc: 0.9788  
Epoch 12/20  
60000/60000 [=====] - 7s 117us/step - loss: 0.0124 - acc: 0.9961 - val\_loss: 0.0742 - val\_acc: 0.9805  
Epoch 13/20  
60000/60000 [=====] - 7s 121us/step - loss: 0.0104 - acc: 0.9965 - val\_loss: 0.0813 - val\_acc: 0.9791  
Epoch 14/20  
60000/60000 [=====] - 7s 121us/step - loss: 0.0083 - acc: 0.9972 - val\_loss: 0.0692 - val\_acc: 0.9826  
Epoch 15/20  
60000/60000 [=====] - 7s 120us/step - loss: 0.0091 - acc: 0.9971 - val\_loss: 0.0924 - val\_acc: 0.9764  
Epoch 16/20  
60000/60000 [=====] - 7s 120us/step - loss: 0.0111 - acc: 0.9962 - val\_loss: 0.0857 - val\_acc: 0.9782  
Epoch 17/20  
60000/60000 [=====] - 7s 122us/step - loss: 0.0095 - acc: 0.9966 - val\_loss: 0.0862 - val\_acc: 0.9784  
Epoch 18/20  
60000/60000 [=====] - 7s 121us/step - loss: 0.0104 - acc: 0.9965 - val\_loss: 0.0748 - val\_acc: 0.9806  
Epoch 19/20  
60000/60000 [=====] - 7s 121us/step - loss: 0.0060 - acc: 0.9982 - val\_loss: 0.0672 - val\_acc: 0.9835  
Epoch 20/20  
60000/60000 [=====] - 7s 120us/step - loss: 0.0062 - acc: 0.9979 - val\_loss: 0.0698 - val\_acc: 0.9820

```
In [0]: import matplotlib.pyplot as plt
model_scores = best_model.evaluate(x_test, y_test_cat, verbose=0)
print('Test score:', model_scores[0])
print('Test accuracy:', model_scores[1])

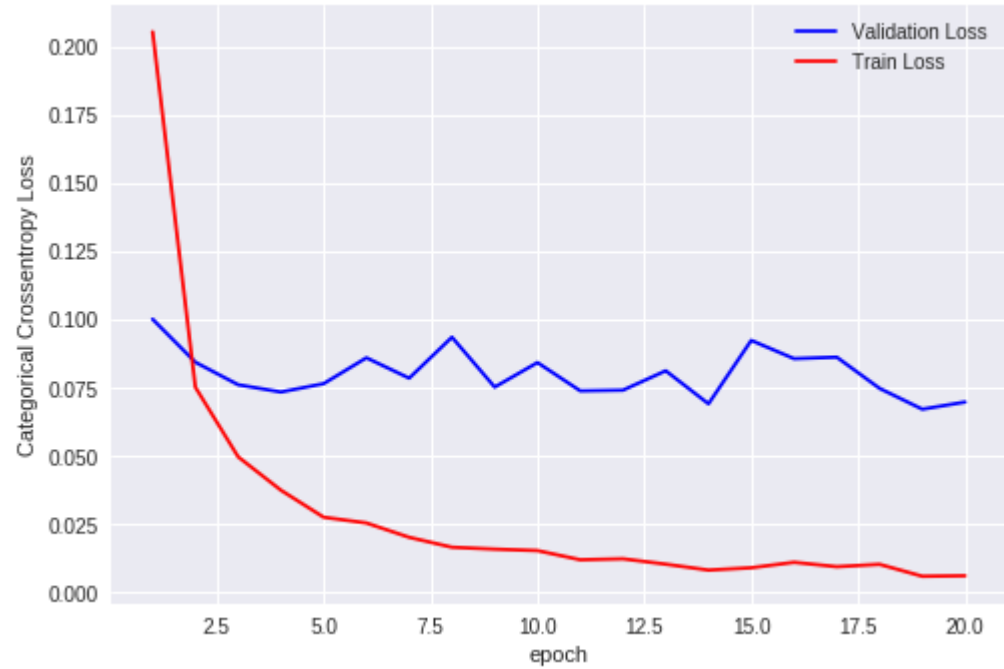
fig,ax = plt.subplots(1,1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy = model_fit.history['val_loss']
ty = model_fit.history['loss']

plt_dynamic(x, vy, ty, ax)
```

Test score: 0.06981187216847902  
Test accuracy: 0.982



**Observations:** The models looks to be severely overfit.



```
In [0]: import matplotlib.pyplot as plt
import seaborn as sns
w_after = best_model.get_weights()

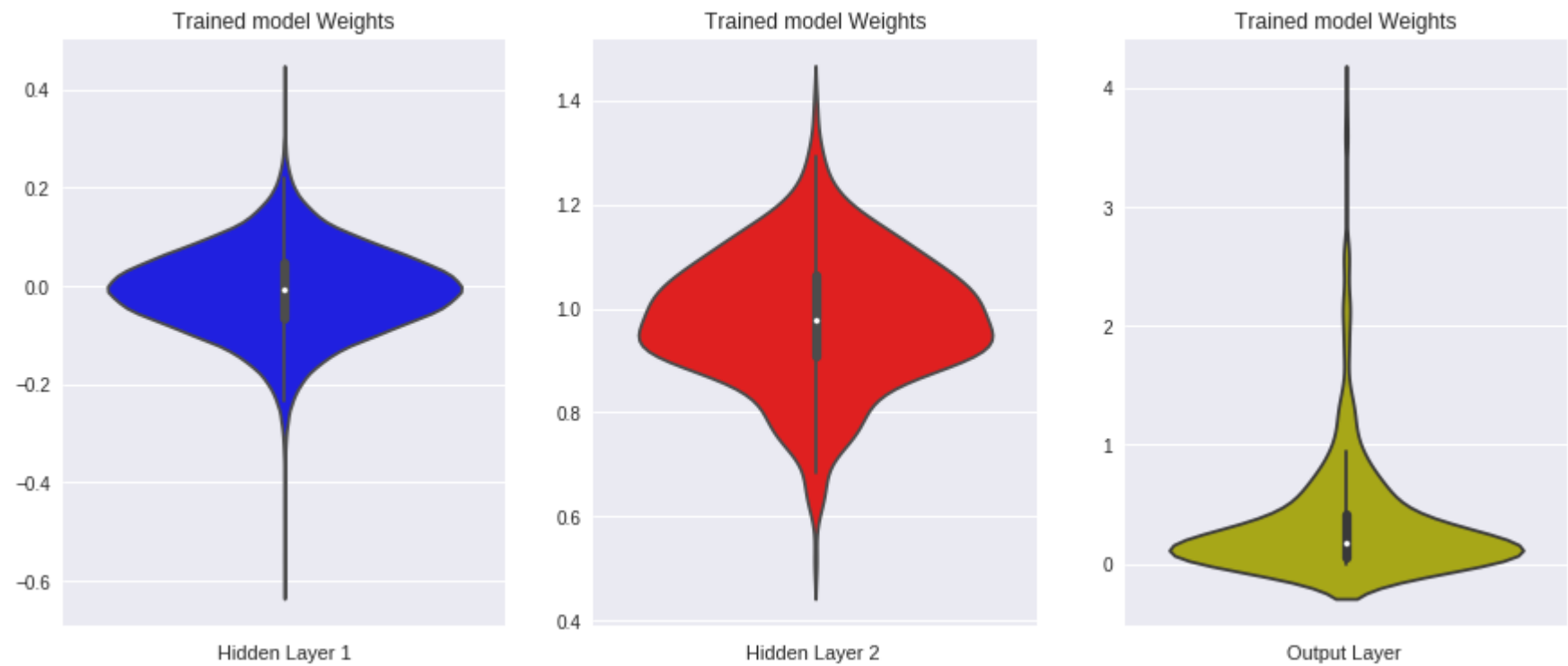
h1_w = w_after[0].flatten().reshape(-1,1)
h2_w = w_after[2].flatten().reshape(-1,1)
out_w = w_after[4].flatten().reshape(-1,1)

fig = plt.figure(figsize = (15,6))
plt.title("Weight matrices after model trained")
plt.subplot(1, 3, 1)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h1_w,color='b')
plt.xlabel('Hidden Layer 1')

plt.subplot(1, 3, 2)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h2_w, color='r')
plt.xlabel('Hidden Layer 2 ')

plt.subplot(1, 3, 3)
plt.title("Trained model Weights")
ax = sns.violinplot(y=out_w,color='y')
plt.xlabel('Output Layer ')
plt.show()
```

/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:588: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
kde\_data = remove\_na(group\_data)  
/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:816: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
violin\_data = remove\_na(group\_data)



**Observations:** The weights seem to be evenly spread.

## Without Batch Normalization

```
In [0]: from keras.optimizers import Adam,RMSprop,SGD
def model_keras(l1,l2):

    model = Sequential()
    model.add(Dense(l1, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal()))
    #model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(l2, activation='relu', kernel_initializer=he_normal()) )
    #model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(output_dim, activation='softmax'))

    model.compile(loss='categorical_crossentropy', metrics=['accuracy'], optimizer='adam')

    return model
```

```
In [0]: from keras.models import Sequential
from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV
from keras.optimizers import Adam,RMSprop,SGD
from keras.layers.normalization import BatchNormalization
from keras.optimizers import Adam
from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV
from keras.models import Sequential
from keras.layers import Dense, Activation,Dropout

model = KerasClassifier(build_fn=model_keras, epochs=nb_epoch, batch_size=batch_size, verbose=0)
param_grid = {'l1': [256,328,512], 'l2': [32,64,128]}

gsearch = GridSearchCV(estimator=model, param_grid=param_grid)
gresult = gsearch.fit(x_train, y_train_cat)

print("Best Accuracy obtained is {} when number of units in layer 1 and 2 are {}.\n".format(gresult.best_score_, gresult.best_params_))
scores = gresult.cv_results_['mean_test_score']
params = gresult.cv_results_['params']
for score, param in zip(scores, params):
    print('Accuracy of {} is obtained for number of units in hidden layer 1 and 2 as {}'.format(score,param))
```

Best Accuracy obtained is 0.9779500048160553 when number of units in layer 1 and 2 are {'l1': 512, 'l2': 128}.

Accuracy of 0.9717666691541672 is obtained for number of units in hidden layer 1 and 2 as {'l1': 256, 'l2': 32}  
Accuracy of 0.9739000020027161 is obtained for number of units in hidden layer 1 and 2 as {'l1': 256, 'l2': 64}  
Accuracy of 0.9762000044584275 is obtained for number of units in hidden layer 1 and 2 as {'l1': 256, 'l2': 128}  
Accuracy of 0.974183337132136 is obtained for number of units in hidden layer 1 and 2 as {'l1': 328, 'l2': 32}  
Accuracy of 0.9759166709184647 is obtained for number of units in hidden layer 1 and 2 as {'l1': 328, 'l2': 64}  
Accuracy of 0.9770166708628336 is obtained for number of units in hidden layer 1 and 2 as {'l1': 328, 'l2': 128}  
Accuracy of 0.9751333359479905 is obtained for number of units in hidden layer 1 and 2 as {'l1': 512, 'l2': 32}  
Accuracy of 0.9766833366552988 is obtained for number of units in hidden layer 1 and 2 as {'l1': 512, 'l2': 64}  
Accuracy of 0.9779500048160553 is obtained for number of units in hidden layer 1 and 2 as {'l1': 512, 'l2': 128}

```
In [0]: best_model = model_keras(gresult.best_params_['11'],gresult.best_params_['12'])
model_fit = best_model.fit(x_train, y_train_cat, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(x_test, y_test_cat))

Train on 60000 samples, validate on 10000 samples
Epoch 1/20
60000/60000 [=====] - 13s 209us/step - loss: 0.4562 - acc: 0.8592 - val_loss: 0.1478 - val_acc: 0.9520
Epoch 2/20
60000/60000 [=====] - 7s 113us/step - loss: 0.2076 - acc: 0.9388 - val_loss: 0.1034 - val_acc: 0.9689
Epoch 3/20
60000/60000 [=====] - 6s 98us/step - loss: 0.1567 - acc: 0.9543 - val_loss: 0.0872 - val_acc: 0.9731
Epoch 4/20
60000/60000 [=====] - 7s 112us/step - loss: 0.1356 - acc: 0.9609 - val_loss: 0.0796 - val_acc: 0.9760
Epoch 5/20
60000/60000 [=====] - 8s 134us/step - loss: 0.1159 - acc: 0.9657 - val_loss: 0.0761 - val_acc: 0.9772
Epoch 6/20
60000/60000 [=====] - 8s 128us/step - loss: 0.1063 - acc: 0.9683 - val_loss: 0.0746 - val_acc: 0.9782
Epoch 7/20
60000/60000 [=====] - 8s 135us/step - loss: 0.0961 - acc: 0.9715 - val_loss: 0.0715 - val_acc: 0.9784
Epoch 8/20
60000/60000 [=====] - 8s 133us/step - loss: 0.0881 - acc: 0.9735 - val_loss: 0.0671 - val_acc: 0.9803
Epoch 9/20
60000/60000 [=====] - 8s 131us/step - loss: 0.0810 - acc: 0.9759 - val_loss: 0.0689 - val_acc: 0.9797
Epoch 10/20
60000/60000 [=====] - 8s 128us/step - loss: 0.0778 - acc: 0.9763 - val_loss: 0.0651 - val_acc: 0.9814
Epoch 11/20
60000/60000 [=====] - 8s 129us/step - loss: 0.0746 - acc: 0.9769 - val_loss: 0.0624 - val_acc: 0.9827
Epoch 12/20
60000/60000 [=====] - 8s 130us/step - loss: 0.0697 - acc: 0.9787 - val_loss: 0.0646 - val_acc: 0.9814
Epoch 13/20
60000/60000 [=====] - 8s 133us/step - loss: 0.0651 - acc: 0.9801 - val_loss: 0.0618 - val_acc: 0.9812
Epoch 14/20
60000/60000 [=====] - 8s 131us/step - loss: 0.0625 - acc: 0.9806 - val_loss: 0.0621 - val_acc: 0.9826
Epoch 15/20
60000/60000 [=====] - 8s 135us/step - loss: 0.0616 - acc: 0.9813 - val_loss: 0.0632 - val_acc: 0.9831
Epoch 16/20
60000/60000 [=====] - 8s 132us/step - loss: 0.0585 - acc: 0.9822 - val_loss: 0.0615 - val_acc: 0.9836
Epoch 17/20
60000/60000 [=====] - 8s 130us/step - loss: 0.0569 - acc: 0.9816 - val_loss: 0.0626 - val_acc: 0.9824
Epoch 18/20
60000/60000 [=====] - 8s 129us/step - loss: 0.0546 - acc: 0.9829 - val_loss: 0.0660 - val_acc: 0.9829
Epoch 19/20
60000/60000 [=====] - 8s 132us/step - loss: 0.0551 - acc: 0.9832 - val_loss: 0.0648 - val_acc: 0.9830
Epoch 20/20
60000/60000 [=====] - 8s 131us/step - loss: 0.0504 - acc: 0.9839 - val_loss: 0.0664 - val_acc: 0.9831
```

```
In [0]: import matplotlib.pyplot as plt
model_scores = best_model.evaluate(x_test, y_test_cat, verbose=0)
print('Test score:', model_scores[0])
print('Test accuracy:', model_scores[1])

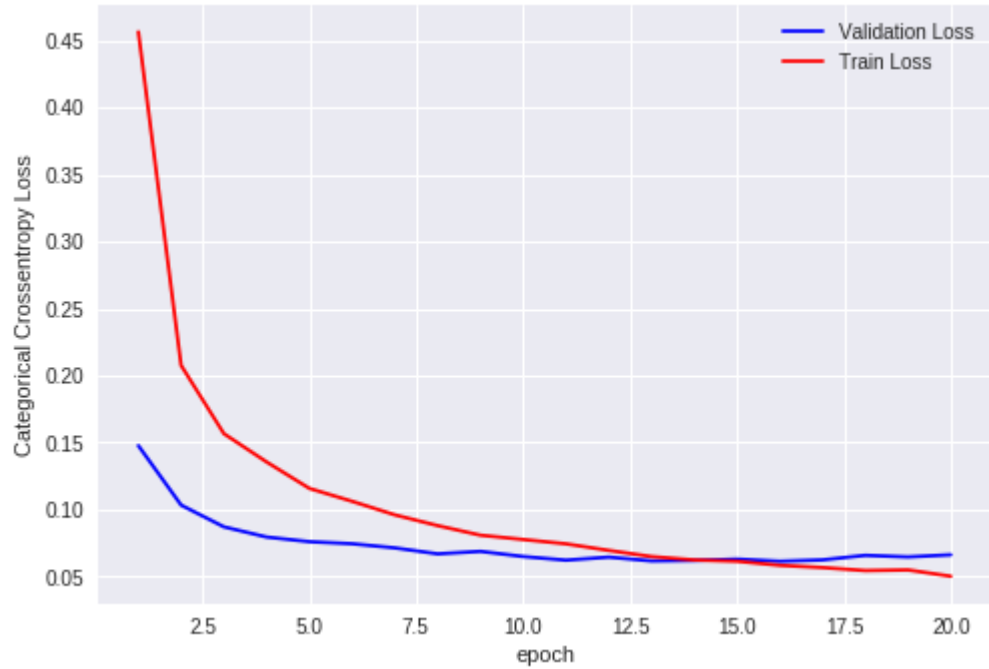
fig,ax = plt.subplots(1,1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy = model_fit.history['val_loss']
ty = model_fit.history['loss']

plt_dynamic(x, vy, ty, ax)
```

Test score: 0.06639465064615924  
Test accuracy: 0.9831



**Observations:** The model seems to be doing okay. As validation error seems to be increasing in the end, there is a chance for overfitting as the number of epochs increase.

```
In [0]: best_model = model_keras(gresult.best_params_['11'],gresult.best_params_['12'])
model_fit = best_model.fit(x_train, y_train_cat, batch_size=batch_size, epochs=30, verbose=0, validation_data=(x_test, y_test_cat)) #Performing on 30 epochs
```

```
In [0]: import matplotlib.pyplot as plt
model_scores = best_model.evaluate(x_test, y_test_cat, verbose=0)
print('Test score:', model_scores[0])
print('Test accuracy:', model_scores[1])

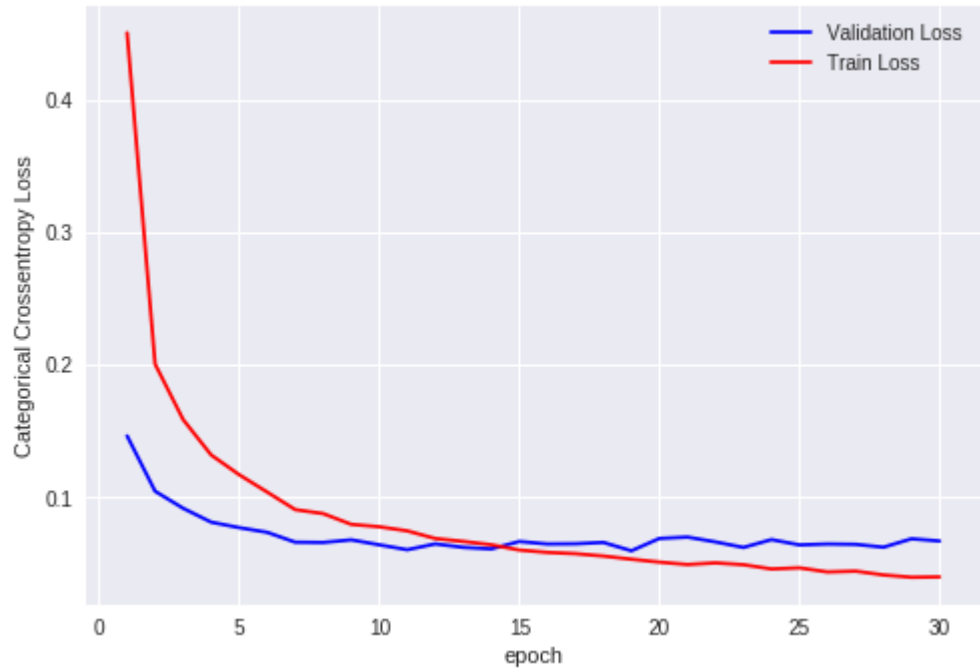
fig,ax = plt.subplots(1,1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,31))

vy = model_fit.history['val_loss']
ty = model_fit.history['loss']

plt_dynamic(x, vy, ty, ax)
```

Test score: 0.06680203758674061  
Test accuracy: 0.9845



Observations: Performing on 30 epochs confirmed that the model is overfitting.

```
In [0]: import matplotlib.pyplot as plt
import seaborn as sns
w_after = best_model.get_weights()

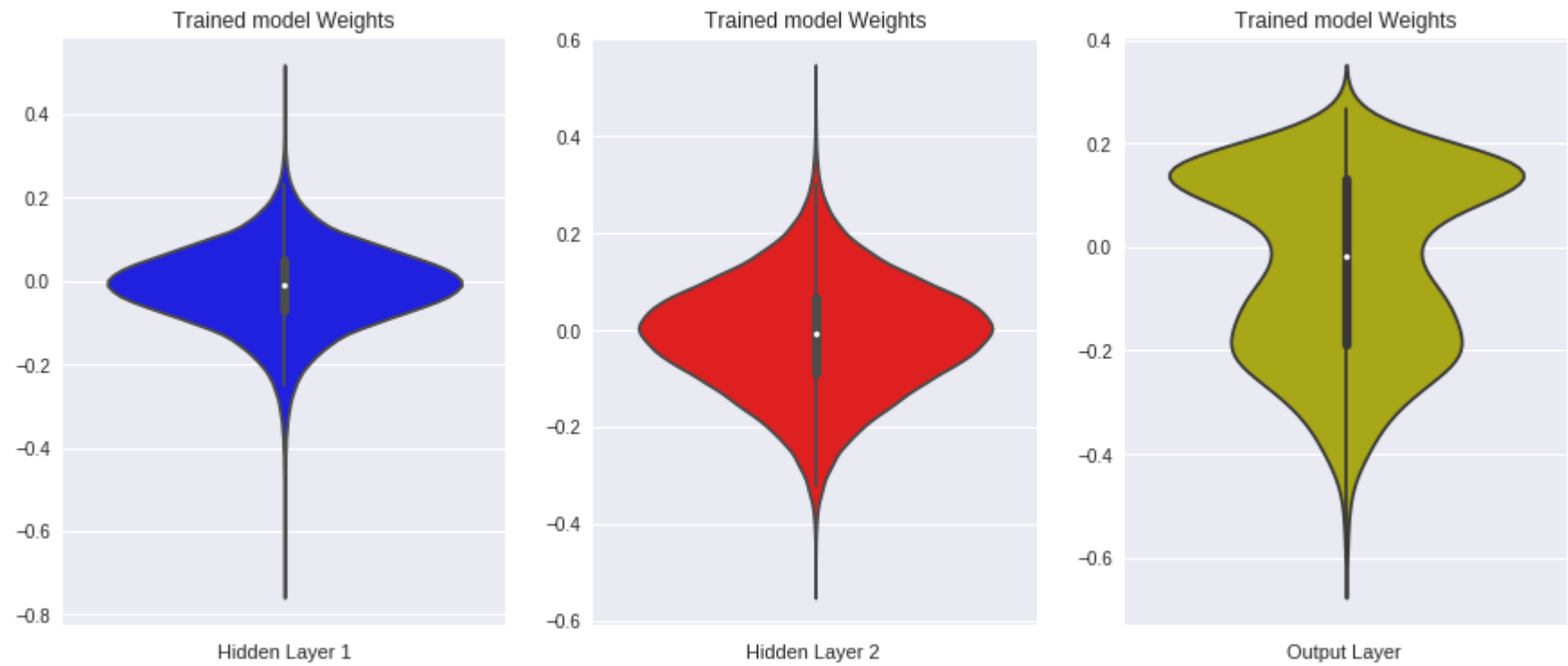
h1_w = w_after[0].flatten().reshape(-1,1)
h2_w = w_after[2].flatten().reshape(-1,1)
out_w = w_after[4].flatten().reshape(-1,1)

fig = plt.figure(figsize = (15,6))
plt.title("Weight matrices after model trained")
plt.subplot(1, 3, 1)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h1_w,color='b')
plt.xlabel('Hidden Layer 1')

plt.subplot(1, 3, 2)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h2_w, color='r')
plt.xlabel('Hidden Layer 2 ')

plt.subplot(1, 3, 3)
plt.title("Trained model Weights")
ax = sns.violinplot(y=out_w,color='y')
plt.xlabel('Output Layer ')
plt.show()
```

/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:588: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
kde\_data = remove\_na(group\_data)  
/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:816: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
violin\_data = remove\_na(group\_data)



Observations: No abnormal distribution in weights observed.

### 3 Hidden layers

#### With Batch Normalization and Dropouts



```
In [0]: from keras.optimizers import Adam,RMSprop,SGD
from keras.layers.normalization import BatchNormalization
from keras.optimizers import Adam
from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV
from keras.models import Sequential
from keras.layers import Dense, Activation,Dropout

def model_keras(l1,l2,l3):

    model = Sequential()
    model.add(Dense(l1, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal()))
    model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(l2, activation='relu', kernel_initializer=he_normal()) )
    model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(l3, activation='relu', kernel_initializer=he_normal()) )
    model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(output_dim, activation='softmax'))

    model.compile(loss='categorical_crossentropy', metrics=['accuracy'], optimizer='adam')

    return model
```

```
In [0]: from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV

model = KerasClassifier(build_fn=model_keras, epochs=nb_epoch, batch_size=batch_size, verbose=0)
param_grid = {'l1': [256,328,512], 'l2': [32,64,128], 'l3': [8,16]}

gsearch = GridSearchCV(estimator=model, param_grid=param_grid)
gresult = gsearch.fit(x_train, y_train_cat)

print("Best Accuracy obtained is {} when number of units in layer 1, 2, 3 are {}".format(gresult.best_score_, gresult.best_params_))
scores = gresult.cv_results_['mean_test_score']
params = gresult.cv_results_['params']
for score, param in zip(scores, params):
    print('Accuracy of {} is obtained for number of units in hidden layer 1, 2, 3 as {}'.format(score,param))
```

Best Accuracy obtained is 0.9760500040054322 when number of units in layer 1, 2, 3 are {'l1': 512, 'l2': 128, 'l3': 16}.

Accuracy of 0.9683333340883254 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 32, 'l3': 8}  
Accuracy of 0.9704833352565765 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 32, 'l3': 16}  
Accuracy of 0.9689166665077209 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 64, 'l3': 8}  
Accuracy of 0.9718500011364619 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 64, 'l3': 16}  
Accuracy of 0.9710500012636185 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 128, 'l3': 8}  
Accuracy of 0.973583336353302 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 128, 'l3': 16}  
Accuracy of 0.9686333343982697 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 32, 'l3': 8}  
Accuracy of 0.9702000017563502 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 32, 'l3': 16}  
Accuracy of 0.9702833341360092 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 64, 'l3': 8}  
Accuracy of 0.9740666699409485 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 64, 'l3': 16}  
Accuracy of 0.9718000016212464 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 128, 'l3': 8}  
Accuracy of 0.9749500033458074 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 128, 'l3': 16}  
Accuracy of 0.9708500022093455 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 32, 'l3': 8}  
Accuracy of 0.9740833373069763 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 32, 'l3': 16}  
Accuracy of 0.9716000024080277 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 64, 'l3': 8}  
Accuracy of 0.9749166713158289 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 64, 'l3': 16}  
Accuracy of 0.9742666703859965 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 128, 'l3': 8}  
Accuracy of 0.9760500040054322 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 128, 'l3': 16}

```
In [0]: best_model = model_keras(gresult.best_params_['l1'],gresult.best_params_['l2'],gresult.best_params_['l3'])
model_fit = best_model.fit(x_train, y_train_cat, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(x_test, y_test_cat))
```

Train on 60000 samples, validate on 10000 samples  
Epoch 1/20  
60000/60000 [=====] - 35s 585us/step - loss: 0.9511 - acc: 0.7090 - val\_loss: 0.2132 - val\_acc: 0.9409  
Epoch 2/20  
60000/60000 [=====] - 15s 253us/step - loss: 0.4561 - acc: 0.8713 - val\_loss: 0.1498 - val\_acc: 0.9560  
Epoch 3/20  
60000/60000 [=====] - 15s 254us/step - loss: 0.3475 - acc: 0.9026 - val\_loss: 0.1218 - val\_acc: 0.9652  
Epoch 4/20  
60000/60000 [=====] - 16s 264us/step - loss: 0.3050 - acc: 0.9158 - val\_loss: 0.1171 - val\_acc: 0.9669  
Epoch 5/20  
60000/60000 [=====] - 15s 253us/step - loss: 0.2710 - acc: 0.9267 - val\_loss: 0.1090 - val\_acc: 0.9693  
Epoch 6/20  
60000/60000 [=====] - 15s 251us/step - loss: 0.2528 - acc: 0.9301 - val\_loss: 0.0901 - val\_acc: 0.9750  
Epoch 7/20  
60000/60000 [=====] - 15s 251us/step - loss: 0.2311 - acc: 0.9358 - val\_loss: 0.0902 - val\_acc: 0.9745  
Epoch 8/20  
60000/60000 [=====] - 15s 253us/step - loss: 0.2143 - acc: 0.9404 - val\_loss: 0.0923 - val\_acc: 0.9743  
Epoch 9/20  
60000/60000 [=====] - 15s 251us/step - loss: 0.2018 - acc: 0.9433 - val\_loss: 0.0831 - val\_acc: 0.9765  
Epoch 10/20  
60000/60000 [=====] - 15s 250us/step - loss: 0.1933 - acc: 0.9452 - val\_loss: 0.0836 - val\_acc: 0.9777  
Epoch 11/20  
60000/60000 [=====] - 15s 253us/step - loss: 0.1853 - acc: 0.9483 - val\_loss: 0.0863 - val\_acc: 0.9789  
Epoch 12/20  
60000/60000 [=====] - 15s 253us/step - loss: 0.1764 - acc: 0.9502 - val\_loss: 0.0784 - val\_acc: 0.9805  
Epoch 13/20  
60000/60000 [=====] - 15s 250us/step - loss: 0.1751 - acc: 0.9504 - val\_loss: 0.0832 - val\_acc: 0.9792  
Epoch 14/20  
60000/60000 [=====] - 15s 251us/step - loss: 0.1665 - acc: 0.9527 - val\_loss: 0.0854 - val\_acc: 0.9793  
Epoch 15/20  
60000/60000 [=====] - 15s 252us/step - loss: 0.1554 - acc: 0.9555 - val\_loss: 0.0810 - val\_acc: 0.9797  
Epoch 16/20  
60000/60000 [=====] - 15s 253us/step - loss: 0.1587 - acc: 0.9529 - val\_loss: 0.0747 - val\_acc: 0.9807  
Epoch 17/20  
60000/60000 [=====] - 15s 248us/step - loss: 0.1512 - acc: 0.9564 - val\_loss: 0.0729 - val\_acc: 0.9819  
Epoch 18/20  
60000/60000 [=====] - 14s 233us/step - loss: 0.1445 - acc: 0.9569 - val\_loss: 0.0718 - val\_acc: 0.9815  
Epoch 19/20  
60000/60000 [=====] - 14s 232us/step - loss: 0.1448 - acc: 0.9573 - val\_loss: 0.0704 - val\_acc: 0.9826  
Epoch 20/20  
60000/60000 [=====] - 15s 254us/step - loss: 0.1338 - acc: 0.9601 - val\_loss: 0.0735 - val\_acc: 0.9821



```
In [0]: import matplotlib.pyplot as plt
model_scores = best_model.evaluate(x_test, y_test_cat, verbose=0)
print('Test score:', model_scores[0])
print('Test accuracy:', model_scores[1])

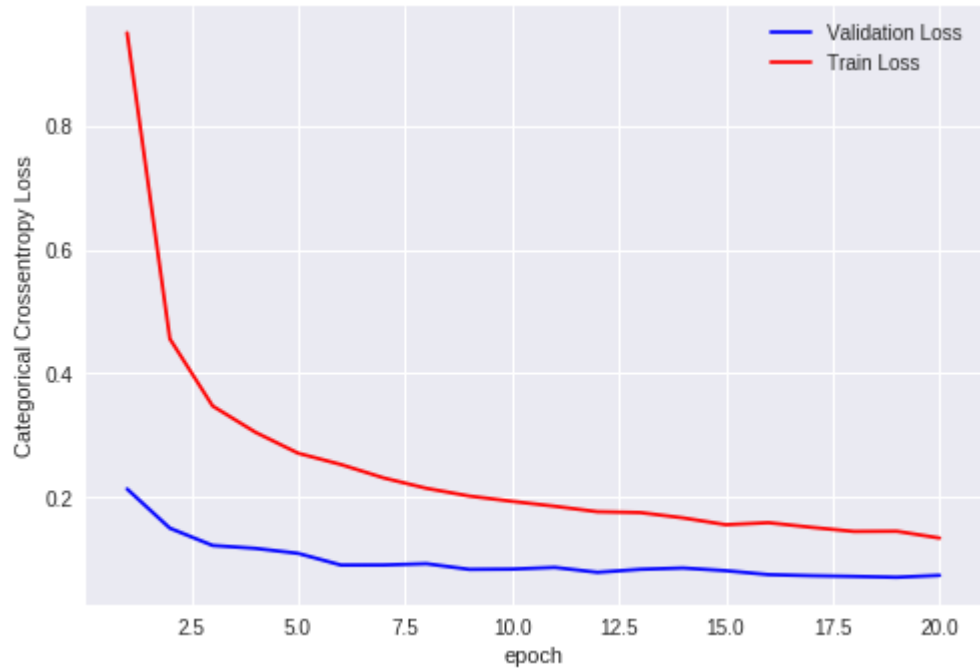
fig,ax = plt.subplots(1,1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy = model_fit.history['val_loss']
ty = model_fit.history['loss']

plt_dynamic(x, vy, ty, ax)
```

Test score: 0.07347509580666083  
Test accuracy: 0.9821



**Observations:** The model performed well, no traces of overfit.

```
In [10]: import matplotlib.pyplot as plt
import seaborn as sns
#w_after = best_model.get_weights()

h1_w = w_after[0].flatten().reshape(-1,1)
h2_w = w_after[2].flatten().reshape(-1,1)
h3_w = w_after[4].flatten().reshape(-1,1)
out_w = w_after[6].flatten().reshape(-1,1)

fig = plt.figure(figsize = (15,6))
plt.title("Weight matrices after model trained")
plt.subplot(1, 4, 1)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h1_w,color='b')
plt.xlabel('Hidden Layer 1')

plt.subplot(1, 4, 2)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h2_w, color='r')
plt.xlabel('Hidden Layer 2 ')

plt.subplot(1, 4, 3)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h3_w, color='g')
plt.xlabel('Hidden Layer 3 ')

plt.subplot(1, 4, 4)
plt.title("Trained model Weights")
ax = sns.violinplot(y=out_w,color='y')
plt.xlabel('Output Layer ')
plt.show()
```

/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:588: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
kde\_data = remove\_na(group\_data)  
/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:816: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
violin\_data = remove\_na(group\_data)



**Without Droputs**

```
In [0]: from keras.optimizers import Adam,RMSprop,SGD
from keras.layers.normalization import BatchNormalization
from keras.optimizers import Adam
from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV
from keras.models import Sequential
from keras.layers import Dense, Activation,Dropout

def model_keras(l1,l2,l3):

    model = Sequential()
    model.add(Dense(l1, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal()))
    model.add(BatchNormalization())
    #model.add(Dropout(0.5))

    model.add(Dense(l2, activation='relu', kernel_initializer=he_normal()) )
    model.add(BatchNormalization())
    #model.add(Dropout(0.5))

    model.add(Dense(l3, activation='relu', kernel_initializer=he_normal()) )
    model.add(BatchNormalization())
    #model.add(Dropout(0.5))

    model.add(Dense(output_dim, activation='softmax'))

    model.compile(loss='categorical_crossentropy', metrics=['accuracy'], optimizer='adam')

    return model
```

```
In [0]: from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV

start = time.time()
model = KerasClassifier(build_fn=model_keras, epochs=nb_epoch, batch_size=batch_size, verbose=0)
param_grid = {'l1': [256,328,512], 'l2': [32,64,128], 'l3': [8,16]}

gsearch = GridSearchCV(estimator=model, param_grid=param_grid)
gresult = gsearch.fit(x_train, y_train_cat)

print("Best Accuracy obtained is {} when number of units in layer 1, 2, 3 are {}".format(gresult.best_score_, gresult.best_params_))
scores = gresult.cv_results_['mean_test_score']
params = gresult.cv_results_['params']
for score, param in zip(scores, params):
    print('Accuracy of {} is obtained for number of units in hidden layer 1, 2, 3 as {}'.format(score,param))
print('\nTotal time taken for execution is',time.time()-start)
```

Best Accuracy obtained is 0.9762000038623809 when number of units in layer 1, 2, 3 are {'l1': 328, 'l2': 128, 'l3': 16}.

Accuracy of 0.9725333353678386 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 32, 'l3': 8}  
Accuracy of 0.974766693925857 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 32, 'l3': 16}  
Accuracy of 0.9741500035524369 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 64, 'l3': 8}  
Accuracy of 0.9741000036795934 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 64, 'l3': 16}  
Accuracy of 0.975166669289271 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 128, 'l3': 8}  
Accuracy of 0.9740833355585734 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 128, 'l3': 16}  
Accuracy of 0.9720166695515314 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 32, 'l3': 8}  
Accuracy of 0.9746500035127004 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 32, 'l3': 16}  
Accuracy of 0.9755833387374878 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 64, 'l3': 8}  
Accuracy of 0.975200003027916 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 64, 'l3': 16}  
Accuracy of 0.973683336853981 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 128, 'l3': 8}  
Accuracy of 0.9762000038623809 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 128, 'l3': 16}  
Accuracy of 0.9722000019550323 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 32, 'l3': 8}  
Accuracy of 0.97525000278155 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 32, 'l3': 16}  
Accuracy of 0.9759333364963532 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 64, 'l3': 8}  
Accuracy of 0.9761166708866755 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 64, 'l3': 16}  
Accuracy of 0.9759833382368088 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 128, 'l3': 8}  
Accuracy of 0.9760666706562042 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 128, 'l3': 16}

Total time taken for execution is 10505.010143756866

```
In [0]: best_model = model_keras(gresult.best_params_['l1'],gresult.best_params_['l2'],gresult.best_params_['l3'])
model_fit = best_model.fit(x_train, y_train_cat, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(x_test, y_test_cat))
```

Train on 60000 samples, validate on 10000 samples  
Epoch 1/20  
60000/60000 [=====] - 35s 582us/step - loss: 0.3122 - acc: 0.9250 - val\_loss: 0.1247 - val\_acc: 0.9658  
Epoch 2/20  
60000/60000 [=====] - 11s 177us/step - loss: 0.0943 - acc: 0.9732 - val\_loss: 0.0932 - val\_acc: 0.9720  
Epoch 3/20  
60000/60000 [=====] - 11s 175us/step - loss: 0.0608 - acc: 0.9816 - val\_loss: 0.0863 - val\_acc: 0.9731  
Epoch 4/20  
60000/60000 [=====] - 10s 175us/step - loss: 0.0455 - acc: 0.9859 - val\_loss: 0.0842 - val\_acc: 0.9743  
Epoch 5/20  
60000/60000 [=====] - 11s 176us/step - loss: 0.0346 - acc: 0.9895 - val\_loss: 0.0822 - val\_acc: 0.9748  
Epoch 6/20  
60000/60000 [=====] - 11s 177us/step - loss: 0.0295 - acc: 0.9907 - val\_loss: 0.0762 - val\_acc: 0.9768  
Epoch 7/20  
60000/60000 [=====] - 11s 178us/step - loss: 0.0224 - acc: 0.9930 - val\_loss: 0.0817 - val\_acc: 0.9754  
Epoch 8/20  
60000/60000 [=====] - 11s 179us/step - loss: 0.0207 - acc: 0.9935 - val\_loss: 0.0803 - val\_acc: 0.9756  
Epoch 9/20  
60000/60000 [=====] - 11s 179us/step - loss: 0.0184 - acc: 0.9940 - val\_loss: 0.0779 - val\_acc: 0.9782  
Epoch 10/20  
60000/60000 [=====] - 11s 176us/step - loss: 0.0168 - acc: 0.9944 - val\_loss: 0.0811 - val\_acc: 0.9802  
Epoch 11/20  
60000/60000 [=====] - 11s 177us/step - loss: 0.0143 - acc: 0.9952 - val\_loss: 0.0894 - val\_acc: 0.9764  
Epoch 12/20  
60000/60000 [=====] - 11s 180us/step - loss: 0.0168 - acc: 0.9943 - val\_loss: 0.0771 - val\_acc: 0.9793  
Epoch 13/20  
60000/60000 [=====] - 11s 178us/step - loss: 0.0144 - acc: 0.9956 - val\_loss: 0.0754 - val\_acc: 0.9799  
Epoch 14/20  
60000/60000 [=====] - 11s 180us/step - loss: 0.0111 - acc: 0.9964 - val\_loss: 0.0850 - val\_acc: 0.9780  
Epoch 15/20  
60000/60000 [=====] - 11s 181us/step - loss: 0.0128 - acc: 0.9955 - val\_loss: 0.0942 - val\_acc: 0.9783  
Epoch 16/20  
60000/60000 [=====] - 11s 185us/step - loss: 0.0113 - acc: 0.9962 - val\_loss: 0.0782 - val\_acc: 0.9805  
Epoch 17/20  
60000/60000 [=====] - 11s 179us/step - loss: 0.0103 - acc: 0.9967 - val\_loss: 0.0792 - val\_acc: 0.9790  
Epoch 18/20  
60000/60000 [=====] - 11s 180us/step - loss: 0.0084 - acc: 0.9976 - val\_loss: 0.0820 - val\_acc: 0.9803  
Epoch 19/20  
60000/60000 [=====] - 11s 177us/step - loss: 0.0106 - acc: 0.9964 - val\_loss: 0.0877 - val\_acc: 0.9802  
Epoch 20/20  
60000/60000 [=====] - 11s 178us/step - loss: 0.0097 - acc: 0.9971 - val\_loss: 0.0873 - val\_acc: 0.9793

```
In [0]: import matplotlib.pyplot as plt
model_scores = best_model.evaluate(x_test, y_test_cat, verbose=0)
print('Test score:', model_scores[0])
print('Test accuracy:', model_scores[1])

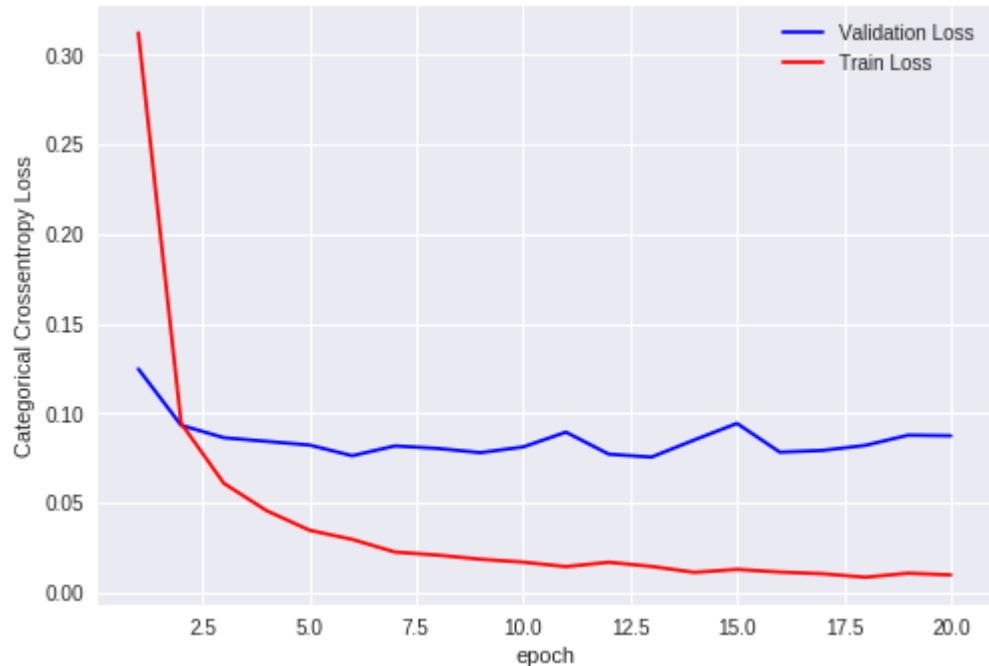
fig,ax = plt.subplots(1,1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy = model_fit.history['val_loss']
ty = model_fit.history['loss']

plt_dynamic(x, vy, ty, ax)
```

Test score: 0.08734963990088727  
Test accuracy: 0.9793



**Observations:** The model seems to be severely overfit without dropouts.

```
In [0]: import matplotlib.pyplot as plt
import seaborn as sns
w_after = best_model.get_weights()

h1_w = w_after[0].flatten().reshape(-1,1)
h2_w = w_after[2].flatten().reshape(-1,1)
h3_w = w_after[4].flatten().reshape(-1,1)
out_w = w_after[6].flatten().reshape(-1,1)

fig = plt.figure(figsize = (15,6))
plt.title("Weight matrices after model trained")
plt.subplot(1, 4, 1)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h1_w,color='b')
plt.xlabel('Hidden Layer 1')

plt.subplot(1, 4, 2)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h2_w, color='r')
plt.xlabel('Hidden Layer 2 ')

plt.subplot(1, 4, 3)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h3_w, color='g')
plt.xlabel('Hidden Layer 3 ')

plt.subplot(1, 4, 4)
plt.title("Trained model Weights")
ax = sns.violinplot(y=out_w,color='y')
plt.xlabel('Output Layer ')
plt.show()
```

/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:588: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
kde\_data = remove\_na(group\_data)  
/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:816: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
violin\_data = remove\_na(group\_data)



Without Batch Normalization



```
In [0]: from keras.optimizers import Adam,RMSprop,SGD
from keras.layers.normalization import BatchNormalization
from keras.optimizers import Adam
from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV
from keras.models import Sequential
from keras.layers import Dense, Activation,Dropout

def model_keras(l1,l2,l3):

    model = Sequential()
    model.add(Dense(l1, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal()))
    #model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(l2, activation='relu', kernel_initializer=he_normal()) )
    #model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(l3, activation='relu', kernel_initializer=he_normal()) )
    #model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(output_dim, activation='softmax'))

    model.compile(loss='categorical_crossentropy', metrics=['accuracy'], optimizer='adam')

    return model
```

```
In [0]: from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV

start = time.time()
model = KerasClassifier(build_fn=model_keras, epochs=nb_epoch, batch_size=batch_size, verbose=0)
param_grid = {'l1': [256,328,512], 'l2': [32,64,128], 'l3': [8,16]}

gsearch = GridSearchCV(estimator=model, param_grid=param_grid)
gresult = gsearch.fit(x_train, y_train_cat)

print("Best Accuracy obtained is {} when number of units in layer 1, 2, 3 are {}".format(gresult.best_score_, gresult.best_params_))
scores = gresult.cv_results_['mean_test_score']
params = gresult.cv_results_['params']
for score, param in zip(scores, params):
    print('Accuracy of {} is obtained for number of units in hidden layer 1, 2, 3 as {}'.format(score,param))
print('\nTotal time taken for execution is',time.time()-start)
```

Best Accuracy obtained is 0.973083335518837 when number of units in layer 1, 2, 3 are {'l1': 512, 'l2': 128, 'l3': 16}.

Accuracy of 0.9522499965826671 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 32, 'l3': 8}  
Accuracy of 0.9621166644493738 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 32, 'l3': 16}  
Accuracy of 0.9578666624625524 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 64, 'l3': 8}  
Accuracy of 0.966099994277955 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 64, 'l3': 16}  
Accuracy of 0.9581833295027415 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 128, 'l3': 8}  
Accuracy of 0.9679333331584931 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 256, 'l2': 128, 'l3': 16}  
Accuracy of 0.9565499978462855 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 32, 'l3': 8}  
Accuracy of 0.965849991893769 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 32, 'l3': 16}  
Accuracy of 0.960866652441025 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 64, 'l3': 8}  
Accuracy of 0.968833338896433 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 64, 'l3': 16}  
Accuracy of 0.9629833317200343 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 128, 'l3': 8}  
Accuracy of 0.9712166682481765 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 328, 'l2': 128, 'l3': 16}  
Accuracy of 0.9568333318630854 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 32, 'l3': 8}  
Accuracy of 0.9662499976158142 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 32, 'l3': 16}  
Accuracy of 0.9632666642665864 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 64, 'l3': 8}  
Accuracy of 0.9718000034093857 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 64, 'l3': 16}  
Accuracy of 0.9640666653315226 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 128, 'l3': 8}  
Accuracy of 0.973083335518837 is obtained for number of units in hidden layer 1, 2, 3 as {'l1': 512, 'l2': 128, 'l3': 16}

Total time taken for execution is 6602.196067333221

```
In [0]: best_model = model_keras(gresult.best_params_['l1'],gresult.best_params_['l2'],gresult.best_params_['l3'])
model_fit = best_model.fit(x_train, y_train_cat, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(x_test, y_test_cat))
```

Train on 60000 samples, validate on 10000 samples  
Epoch 1/20  
60000/60000 [=====] - 30s 492us/step - loss: 1.1250 - acc: 0.6032 - val\_loss: 0.2463 - val\_acc: 0.9375  
Epoch 2/20  
60000/60000 [=====] - 10s 164us/step - loss: 0.6235 - acc: 0.7917 - val\_loss: 0.1766 - val\_acc: 0.9536  
Epoch 3/20  
60000/60000 [=====] - 10s 166us/step - loss: 0.5140 - acc: 0.8265 - val\_loss: 0.1598 - val\_acc: 0.9600  
Epoch 4/20  
60000/60000 [=====] - 10s 170us/step - loss: 0.4707 - acc: 0.8403 - val\_loss: 0.1392 - val\_acc: 0.9665  
Epoch 5/20  
60000/60000 [=====] - 10s 163us/step - loss: 0.4292 - acc: 0.8559 - val\_loss: 0.1325 - val\_acc: 0.9677  
Epoch 6/20  
60000/60000 [=====] - 9s 158us/step - loss: 0.4025 - acc: 0.8670 - val\_loss: 0.1385 - val\_acc: 0.9696  
Epoch 7/20  
60000/60000 [=====] - 9s 158us/step - loss: 0.3796 - acc: 0.8746 - val\_loss: 0.1185 - val\_acc: 0.9733  
Epoch 8/20  
60000/60000 [=====] - 10s 163us/step - loss: 0.3643 - acc: 0.8786 - val\_loss: 0.1201 - val\_acc: 0.9736  
Epoch 9/20  
60000/60000 [=====] - 10s 159us/step - loss: 0.3593 - acc: 0.8817 - val\_loss: 0.1200 - val\_acc: 0.9742  
Epoch 10/20  
60000/60000 [=====] - 10s 160us/step - loss: 0.3476 - acc: 0.8858 - val\_loss: 0.1289 - val\_acc: 0.9729  
Epoch 11/20  
60000/60000 [=====] - 10s 162us/step - loss: 0.3325 - acc: 0.8922 - val\_loss: 0.1185 - val\_acc: 0.9743  
Epoch 12/20  
60000/60000 [=====] - 11s 184us/step - loss: 0.3234 - acc: 0.8947 - val\_loss: 0.1198 - val\_acc: 0.9759  
Epoch 13/20  
60000/60000 [=====] - 11s 182us/step - loss: 0.3138 - acc: 0.8967 - val\_loss: 0.1165 - val\_acc: 0.9762  
Epoch 14/20  
60000/60000 [=====] - 11s 187us/step - loss: 0.3098 - acc: 0.9010 - val\_loss: 0.1113 - val\_acc: 0.9759  
Epoch 15/20  
60000/60000 [=====] - 12s 196us/step - loss: 0.3046 - acc: 0.9019 - val\_loss: 0.1092 - val\_acc: 0.9775  
Epoch 16/20  
60000/60000 [=====] - 11s 177us/step - loss: 0.2960 - acc: 0.9054 - val\_loss: 0.1179 - val\_acc: 0.9772  
Epoch 17/20  
60000/60000 [=====] - 10s 174us/step - loss: 0.2943 - acc: 0.9057 - val\_loss: 0.1145 - val\_acc: 0.9784  
Epoch 18/20  
60000/60000 [=====] - 10s 161us/step - loss: 0.2856 - acc: 0.9090 - val\_loss: 0.1131 - val\_acc: 0.9791  
Epoch 19/20  
60000/60000 [=====] - 10s 164us/step - loss: 0.2824 - acc: 0.9097 - val\_loss: 0.1106 - val\_acc: 0.9790  
Epoch 20/20  
60000/60000 [=====] - 10s 162us/step - loss: 0.2743 - acc: 0.9125 - val\_loss: 0.1191 - val\_acc: 0.9793



```
In [0]: import matplotlib.pyplot as plt
model_scores = best_model.evaluate(x_test, y_test_cat, verbose=0)
print('Test score:', model_scores[0])
print('Test accuracy:', model_scores[1])

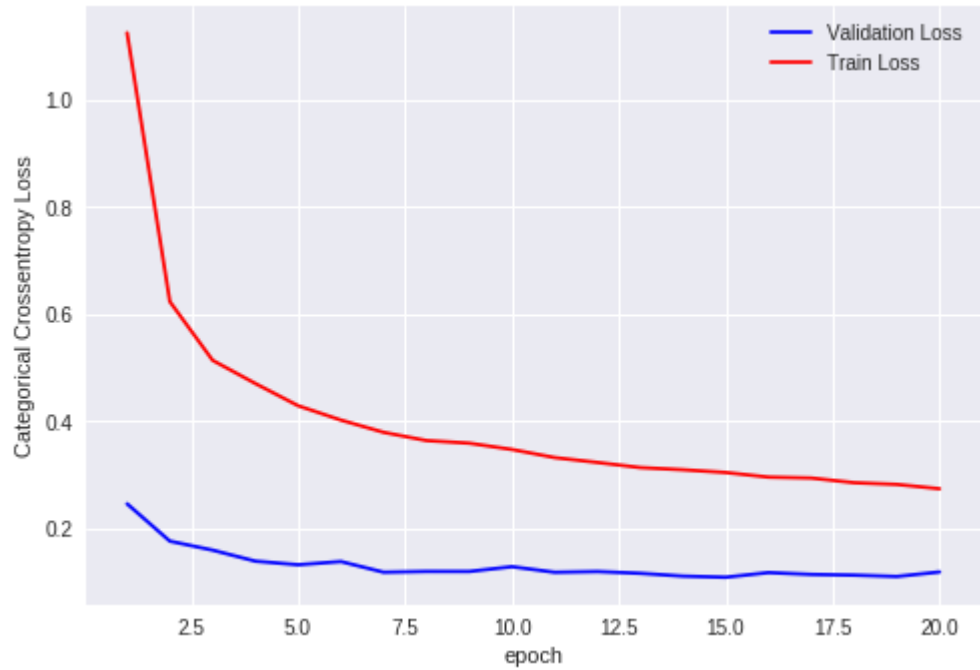
fig,ax = plt.subplots(1,1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy = model_fit.history['val_loss']
ty = model_fit.history['loss']

plt_dynamic(x, vy, ty, ax)
```

Test score: 0.11908461541022407  
Test accuracy: 0.9793



**Observations:** The model seemed to perform well without batch normalization. But there is chance for overfit as the number of epochs increase to a greater number.

```
In [0]: import matplotlib.pyplot as plt
import seaborn as sns
w_after = best_model.get_weights()

h1_w = w_after[0].flatten().reshape(-1,1)
h2_w = w_after[2].flatten().reshape(-1,1)
h3_w = w_after[4].flatten().reshape(-1,1)
out_w = w_after[6].flatten().reshape(-1,1)

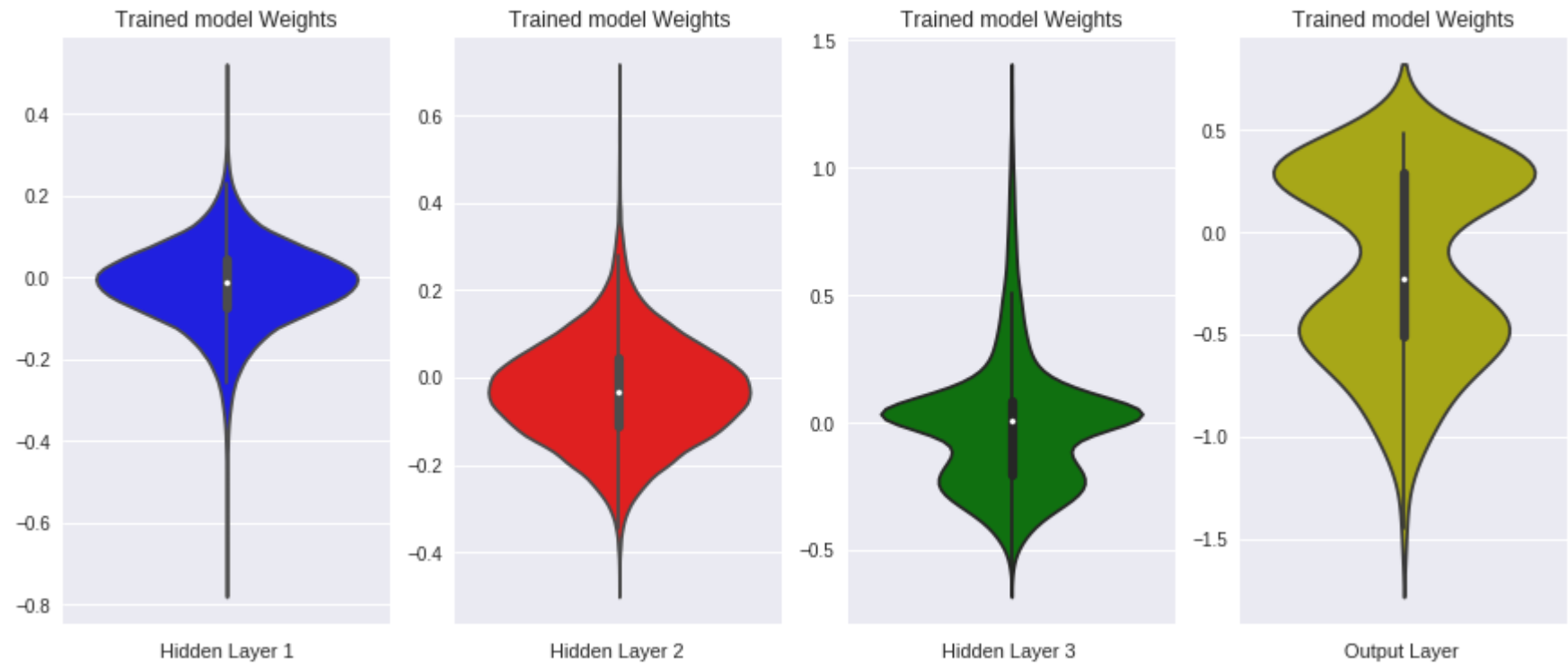
fig = plt.figure(figsize = (15,6))
plt.title("Weight matrices after model trained")
plt.subplot(1, 4, 1)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h1_w,color='b')
plt.xlabel('Hidden Layer 1')

plt.subplot(1, 4, 2)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h2_w, color='r')
plt.xlabel('Hidden Layer 2 ')

plt.subplot(1, 4, 3)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h3_w, color='g')
plt.xlabel('Hidden Layer 3 ')

plt.subplot(1, 4, 4)
plt.title("Trained model Weights")
ax = sns.violinplot(y=out_w,color='y')
plt.xlabel('Output Layer ')
plt.show()
```

/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:588: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
kde\_data = remove\_na(group\_data)  
/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:816: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
violin\_data = remove\_na(group\_data)



## 5 Hidden layers

```
In [0]: from keras.optimizers import Adam,RMSprop,SGD
from keras.layers.normalization import BatchNormalization
from keras.optimizers import Adam
from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV
from keras.models import Sequential
from keras.layers import Dense, Activation,Dropout

def model_keras(l1,l2,l3,l4,l5):

    model = Sequential()
    model.add(Dense(l1, activation='relu', input_shape=(input_dim,), kernel_initializer=he_normal()))
    model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(l2, activation='relu', kernel_initializer=he_normal()) )
    model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(l3, activation='relu', kernel_initializer=he_normal()) )
    model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(l4, activation='relu', kernel_initializer=he_normal()) )
    model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(l5, activation='relu', kernel_initializer=he_normal()) )
    model.add(BatchNormalization())
    model.add(Dropout(0.5))

    model.add(Dense(output_dim, activation='softmax'))

    model.compile(loss='categorical_crossentropy', metrics=['accuracy'], optimizer='adam')

    return model
```

```
In [12]: from keras.wrappers.scikit_learn import KerasClassifier
from sklearn.model_selection import GridSearchCV

model = KerasClassifier(build_fn=model_keras, epochs=nb_epoch, batch_size=batch_size, verbose=0)
param_grid = {'l1': [328,512], 'l2': [128,256], 'l3':[64,96], 'l4': [16,32], 'l5':[4,8]}

gsearch = GridSearchCV(estimator=model, param_grid=param_grid)
gresult = gsearch.fit(x_train, y_train_cat)

print("Best Accuracy obtained is {} when number of units in layer 1, 2, 3, 4, 5 are {}.\n".format(gresult.best_score_, gresult.best_params_))
scores = gresult.cv_results_['mean_test_score']
params = gresult.cv_results_['params']
for score, param in zip(scores, params):
    print('Accuracy of {} is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {}'.format(score,param))
```

Best Accuracy obtained is 0.9714500022331873 when number of units in layer 1, 2, 3, 4, 5 are {'l1': 512, 'l2': 256, 'l3': 96, 'l4': 32, 'l5': 8}.

Accuracy of 0.8723500003814697 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 128, 'l3': 64, 'l4': 16, 'l5': 4}  
Accuracy of 0.9492666640679042 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 128, 'l3': 64, 'l4': 16, 'l5': 8}  
Accuracy of 0.9092166651884714 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 128, 'l3': 64, 'l4': 32, 'l5': 4}  
Accuracy of 0.9665999997854233 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 128, 'l3': 64, 'l4': 32, 'l5': 8}  
Accuracy of 0.8388833321332931 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 128, 'l3': 96, 'l4': 16, 'l5': 4}  
Accuracy of 0.9170999986728032 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 128, 'l3': 96, 'l4': 16, 'l5': 8}  
Accuracy of 0.9092666656573614 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 128, 'l3': 96, 'l4': 32, 'l5': 4}  
Accuracy of 0.9421833352645238 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 128, 'l3': 96, 'l4': 32, 'l5': 8}  
Accuracy of 0.8751333342393239 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 256, 'l3': 64, 'l4': 16, 'l5': 4}  
Accuracy of 0.9070499982833863 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 256, 'l3': 64, 'l4': 16, 'l5': 8}  
Accuracy of 0.9064999980926514 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 256, 'l3': 64, 'l4': 32, 'l5': 4}  
Accuracy of 0.9667333323955536 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 256, 'l3': 64, 'l4': 32, 'l5': 8}  
Accuracy of 0.9024166668653488 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 256, 'l3': 96, 'l4': 16, 'l5': 4}  
Accuracy of 0.920200002749761 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 256, 'l3': 96, 'l4': 16, 'l5': 8}  
Accuracy of 0.9550833304723104 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 256, 'l3': 96, 'l4': 32, 'l5': 4}  
Accuracy of 0.9682166674137116 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 328, 'l2': 256, 'l3': 96, 'l4': 32, 'l5': 8}  
Accuracy of 0.8089833333889643 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 128, 'l3': 64, 'l4': 16, 'l5': 4}  
Accuracy of 0.9427833336591721 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 128, 'l3': 64, 'l4': 16, 'l5': 8}  
Accuracy of 0.9392166657447815 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 128, 'l3': 64, 'l4': 32, 'l5': 4}  
Accuracy of 0.9693500012159347 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 128, 'l3': 64, 'l4': 32, 'l5': 8}  
Accuracy of 0.8837166656255722 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 128, 'l3': 96, 'l4': 16, 'l5': 4}  
Accuracy of 0.9689166666269302 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 128, 'l3': 96, 'l4': 16, 'l5': 8}  
Accuracy of 0.9365833317836125 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 128, 'l3': 96, 'l4': 32, 'l5': 4}  
Accuracy of 0.9692666684786478 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 128, 'l3': 96, 'l4': 32, 'l5': 8}  
Accuracy of 0.9168333315849304 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 256, 'l3': 64, 'l4': 16, 'l5': 4}  
Accuracy of 0.9662999989986419 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 256, 'l3': 64, 'l4': 16, 'l5': 8}  
Accuracy of 0.920599999944369 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 256, 'l3': 64, 'l4': 32, 'l5': 4}  
Accuracy of 0.968966665784518 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 256, 'l3': 64, 'l4': 32, 'l5': 8}  
Accuracy of 0.9193333326180776 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 256, 'l3': 96, 'l4': 16, 'l5': 4}  
Accuracy of 0.9669833319187164 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 256, 'l3': 96, 'l4': 16, 'l5': 8}  
Accuracy of 0.9057666656970977 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 256, 'l3': 96, 'l4': 32, 'l5': 4}  
Accuracy of 0.9714500022331873 is obtained for number of units in hidden layer 1, 2, 3, 4, 5 as {'l1': 512, 'l2': 256, 'l3': 96, 'l4': 32, 'l5': 8}

```
In [13]: best_model = model_keras(gresult.best_params_['11'],gresult.best_params_['12'],gresult.best_params_['13'],gresult.best_params_['14'],gresult.best_params_['15'])
model_fit = best_model.fit(x_train, y_train_cat, batch_size=batch_size, epochs=nb_epoch, verbose=1, validation_data=(x_test, y_test_cat))

Train on 60000 samples, validate on 10000 samples
Epoch 1/20
60000/60000 [=====] - 88s 1ms/step - loss: 1.7494 - acc: 0.3738 - val_loss: 0.7504 - val_acc: 0.8776
Epoch 2/20
60000/60000 [=====] - 23s 392us/step - loss: 1.1585 - acc: 0.5784 - val_loss: 0.3925 - val_acc: 0.9314
Epoch 3/20
60000/60000 [=====] - 24s 398us/step - loss: 0.9365 - acc: 0.6592 - val_loss: 0.2622 - val_acc: 0.9462
Epoch 4/20
60000/60000 [=====] - 24s 393us/step - loss: 0.8257 - acc: 0.6995 - val_loss: 0.1902 - val_acc: 0.9583
Epoch 5/20
60000/60000 [=====] - 23s 389us/step - loss: 0.7558 - acc: 0.7203 - val_loss: 0.1615 - val_acc: 0.9622
Epoch 6/20
60000/60000 [=====] - 23s 391us/step - loss: 0.7131 - acc: 0.7328 - val_loss: 0.1517 - val_acc: 0.9648
Epoch 7/20
60000/60000 [=====] - 24s 397us/step - loss: 0.6825 - acc: 0.7429 - val_loss: 0.1432 - val_acc: 0.9665
Epoch 8/20
60000/60000 [=====] - 24s 403us/step - loss: 0.6585 - acc: 0.7520 - val_loss: 0.1412 - val_acc: 0.9676
Epoch 9/20
60000/60000 [=====] - 24s 402us/step - loss: 0.6469 - acc: 0.7563 - val_loss: 0.1310 - val_acc: 0.9697
Epoch 10/20
60000/60000 [=====] - 24s 403us/step - loss: 0.6273 - acc: 0.7604 - val_loss: 0.1166 - val_acc: 0.9729
Epoch 11/20
60000/60000 [=====] - 24s 400us/step - loss: 0.6184 - acc: 0.7662 - val_loss: 0.1199 - val_acc: 0.9716
Epoch 12/20
60000/60000 [=====] - 24s 405us/step - loss: 0.6112 - acc: 0.7711 - val_loss: 0.1112 - val_acc: 0.9730
Epoch 13/20
60000/60000 [=====] - 24s 401us/step - loss: 0.5987 - acc: 0.7748 - val_loss: 0.1071 - val_acc: 0.9744
Epoch 14/20
60000/60000 [=====] - 24s 403us/step - loss: 0.5838 - acc: 0.7801 - val_loss: 0.1080 - val_acc: 0.9756
Epoch 15/20
60000/60000 [=====] - 24s 404us/step - loss: 0.5773 - acc: 0.7828 - val_loss: 0.1015 - val_acc: 0.9771
Epoch 16/20
60000/60000 [=====] - 25s 409us/step - loss: 0.5659 - acc: 0.7865 - val_loss: 0.0970 - val_acc: 0.9777
Epoch 17/20
60000/60000 [=====] - 24s 405us/step - loss: 0.5627 - acc: 0.7896 - val_loss: 0.1030 - val_acc: 0.9772
Epoch 18/20
60000/60000 [=====] - 24s 400us/step - loss: 0.5378 - acc: 0.7991 - val_loss: 0.1035 - val_acc: 0.9779
Epoch 19/20
60000/60000 [=====] - 24s 395us/step - loss: 0.5560 - acc: 0.7941 - val_loss: 0.1045 - val_acc: 0.9782
Epoch 20/20
60000/60000 [=====] - 24s 398us/step - loss: 0.5305 - acc: 0.8000 - val_loss: 0.1057 - val_acc: 0.9779
```

```
In [14]: import matplotlib.pyplot as plt
model_scores = best_model.evaluate(x_test, y_test_cat, verbose=0)
print('Test score:', model_scores[0])
print('Test accuracy:', model_scores[1])

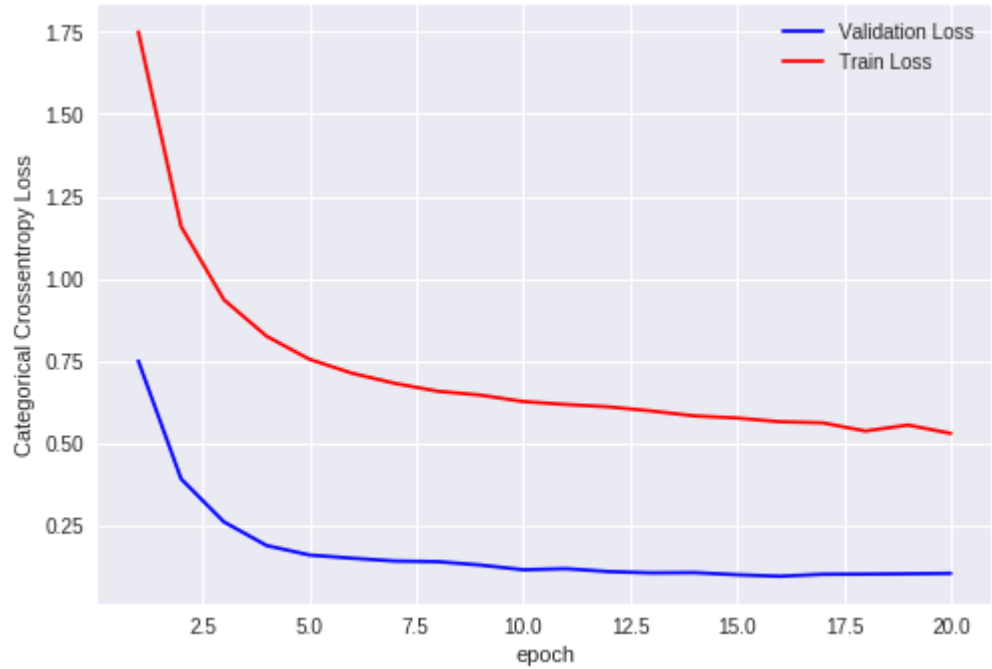
fig,ax = plt.subplots(1,1)
ax.set_xlabel('epoch') ; ax.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy = model_fit.history['val_loss']
ty = model_fit.history['loss']

plt_dynamic(x, vy, ty, ax)
```

Test score: 0.10571024847328662  
Test accuracy: 0.9779



**Observations:** The model performed well on test data.



```
In [15]: import matplotlib.pyplot as plt
import seaborn as sns
w_after = best_model.get_weights()

h1_w = w_after[0].flatten().reshape(-1,1)
h2_w = w_after[2].flatten().reshape(-1,1)
h3_w = w_after[4].flatten().reshape(-1,1)
h4_w = w_after[6].flatten().reshape(-1,1)
h5_w = w_after[8].flatten().reshape(-1,1)
out_w = w_after[10].flatten().reshape(-1,1)

fig = plt.figure(figsize = (15,6))
plt.title("Weight matrices after model trained")
plt.subplot(1, 6, 1)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h1_w,color='b')
plt.xlabel('Hidden Layer 1')

plt.subplot(1, 6, 2)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h2_w, color='r')
plt.xlabel('Hidden Layer 2 ')

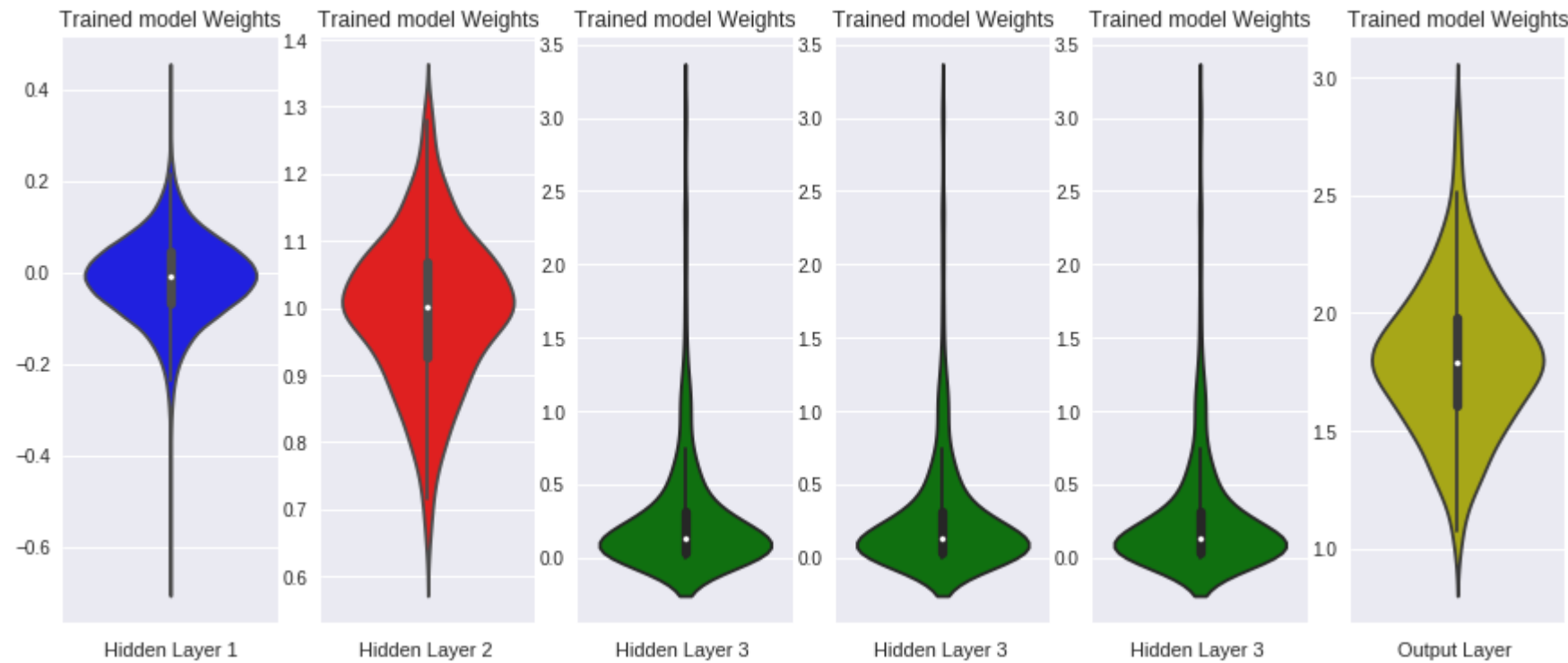
plt.subplot(1, 6, 3)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h3_w, color='g')
plt.xlabel('Hidden Layer 3 ')

plt.subplot(1, 6, 4)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h3_w, color='g')
plt.xlabel('Hidden Layer 3 ')

plt.subplot(1, 6, 5)
plt.title("Trained model Weights")
ax = sns.violinplot(y=h3_w, color='g')
plt.xlabel('Hidden Layer 3 ')

plt.subplot(1, 6, 6)
plt.title("Trained model Weights")
ax = sns.violinplot(y=out_w,color='y')
plt.xlabel('Output Layer ')
plt.show()
```

/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:588: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
kde\_data = remove\_na(group\_data)  
/usr/local/lib/python3.6/dist-packages/seaborn/categorical.py:816: FutureWarning: remove\_na is deprecated and is a private function. Do not use.  
violin\_data = remove\_na(group\_data)



## Conclusions

- 1) Models **with Batch normalization and dropouts** seems to **perform better than others**.
- 2) Models **without dropouts** seem to **perform terribly** as they become **overfit**.
- 3) Models **without batch normalization** seems to **perform okay** but **chances of overfitting** as number of epochs increase.
- 4) Models with **only Batch normalization** takes a **lot of time** to compute compared to models with **only Dropouts**.
- 5) Model seems to **perform best** when number of **hidden layers is 3** with 512, 128, 16 units in layer 1, 2, 3 respectively with an **accuracy of 98.21%**.

```
In [7]: from prettytable import PrettyTable

x = PrettyTable()
x.field_names = ["Number of hidden layers","Optimizations", "Accuracy", "Train error", "Test error", "Units in respective layers", "Performance"]
x.add_row([2, "With BN and Dropouts", "97.33%", "0.0535", "0.0576", "512, 128", "Marginal performance"])
x.add_row(["", "Without Dropouts", "98.20%", "0.0062", "0.0698", "512, 64", "Severely overfit"])
x.add_row(["", "Without BN", "98.31%", "0.0504", "0.0664", "512, 128","Chances of overfit as epochs increase"])
x.add_row(["", "", "", "", "", "", ""])
x.add_row([3, "With BN and Dropouts", "98.21%", "0.1338", "0.0735", "512, 128, 16", "Good performance"])
x.add_row(["", "Without Dropouts", "97.93%", "0.0097", "0.0873", "328, 128, 16", "Severely overfit"])
x.add_row(["", "Without BN", "97.93%", "0.2743", "0.1191", "512, 128, 16", "Good Performance"])
x.add_row(["", "", "", "", "", "", ""])
x.add_row([5, "With BN and Dropouts", "97.79%", "0.5305", "0.1057", "512, 256, 96, 32, 8", "Good Performance"])
print(x.get_string())
```

Number of hidden layers	Optimizations	Accuracy	Train error	Test error	Units in respective layers	Performance
2	With BN and Dropouts	97.33%	0.0535	0.0576	512, 128	Marginal performance
	Without Dropouts	98.20%	0.0062	0.0698	512, 64	Severely overfit
	Without BN	98.31%	0.0504	0.0664	512, 128	Chances of overfit as epochs increase
3	With BN and Dropouts	98.21%	0.1338	0.0735	512, 128, 16	Good performance
	Without Dropouts	97.93%	0.0097	0.0873	328, 128, 16	Severely overfit
	Without BN	97.93%	0.2743	0.1191	512, 128, 16	Good Performance
5	With BN and Dropouts	97.79%	0.5305	0.1057	512, 256, 96, 32, 8	Good Performance