Analysis of RILAs

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1 Introduction

Registered index-linked annuities (RILAs) are a relatively new financial instrument. Originating in the 1990s, they offer the potential for increased returns over traditional fixed-income annuities while shielding the owner from some of the volatility of directly participating in the equity market. The key component is the link to an index; the income provided by the annuity is a function of the performance of a financial index over the crediting period. This function is called the credited rate formula. However, it is not always obvious how these functions will affect the overall distribution of returns. For that reason, I have been contracted to analyze the annuities offered by Allianz Life in their Index Advantage+ series of products.

1.1 Index Advantage+

Index Advantage+ is a series of variable annuities offered by Allianz Life [2]. Within the series, there are several products, each with a different credited rate formula. Allianz offers several different indices to which to link a chosen annuity, but this analysis only focuses on the S&P 500 index.

1.1.1 Terminology

- Buffer — An amount of protection from loss. If the underlying index has negative performance, but the loss is less than the size of the buffer, the owner loses nothing. If the loss exceeds the size of the buffer, the owner only loses the value beyond the size of the buffer. For example, if the buffer is 10% and the index drops by 14%, the owner loses only 4%.
- Cap — A maximum return. If the performance of the underlying index exceeds this value, the owner is credited only as much as the cap.
- Floor — A maximum amount of loss. If the underlying index has negative performance in excess of the floor, the owner only loses the floor amount.
- Participation Rate — An amount the performance of the underlying index is multiplied by when calculating the credited rate. For example, if the participation rate is 110%, and the index goes up 10%, the owner is credited 11%.
- Trigger Rate — Fixed interest rate credited if the index return is positive.

1.2 Customer Information

The customer for whom I am analyzing the annuities is currently 60 years old and planning to retire in six years. Investing with a principal of $100,000, they are interested in supplementing their retirement income. Due to the proximity to retirement, minimizing downside risk is particularly important. The customer cannot afford to take any significant loss due to market volatility, as they will not be participating in any later market recoveries. Avoiding loss of principal, then, is a major concern for making any recommendation.

2 Data Collection

The data used for the analysis comes from the historical price of the SPY ETF, which tracks the S&P 500 index, obtained from Yahoo Finance. Specifically, the daily closing price from 2003 to 2023 was used for parameter estimation.

3 Methodology

The overall approach is as follows:

1. Model the S&P 500 as Geometric Brownian Motion
2. Estimate GBM parameters from historical data
3. Simulate the S&P 500 via 100,000 random walks
4. Compute credited rates of each product for each walk
5. Compute and compare distribution of returns for each product

3.1 Assumptions

- Credited rates formulas are the same from year to year. In reality, the rates change from month to month, but usually not by large amounts.
- No early withdrawals or performance locks. Essentially, I assume the principal is paid to the account and no further action is taken until the six-year crediting period is over.
- Drift and volatility parameters of GBM will remain constant over the next six years. In reality, these are likely to change at least a little. However, the parameters were estimated with data that included significant economic downturns (the 2008 crisis, COVID 19, etc.) so I believe the estimates are sufficiently conservative.
- For one-year term products, the choice of product does not change from year-to-year. For example, the customer will not use the Index Precision in one year and the Index Guard in the next.

3.2 Estimation of GBM Parameters

Geometric Brownian Motion has the form

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

Drift (μ) and volatility (σ) were estimated from the daily closing price of the SPY ETF from 2003 to 2023 using maximum likelihood estimators [1]. In the following formulas, time 0 is January 1, 2003, and time T is December 31, 2023. S_t refers to the price of SPY at time t.

$$\hat{\mu} = \frac{\ln(S_T) - \ln(S_0)}{21} + \frac{1}{2}\hat{\sigma}^2$$

$$\hat{\sigma}^2 = -\frac{1}{5292}\frac{(\ln(S_T) - \ln(S_0))^2}{21} + \frac{1}{21}\sum_{n=1}^{5292}(\ln(S_{t_n}) - \ln(S_{t_n - 1}))^2$$

Through these formulas, I found that

$$\hat{\mu} = 0.0966$$

$$\hat{\sigma} = 0.1892$$

3.3 Simulation Method

100,000 simulations of GBM were run for 1,512 steps each (6 years of daily movement) via a standard Euler-Maryuama method.

3.4 Calculation of Credited Interest Rates

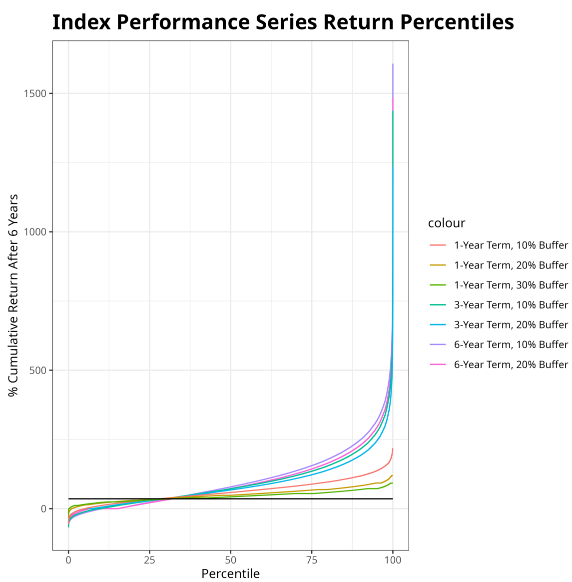
The credited interest rate for each product is calculated via a point-to-point method. In other words, the percent return for the linked index is calculated between the term start date and the term end date. For a six-year term, this means the percent return from the term start date to the beginning of the withdrawal period. For terms less than six years, the rate is compounded. In other words, the amount earned from one term is used as principal in the next. The exact formulas used are listed in Appendix A.

4 Analysis

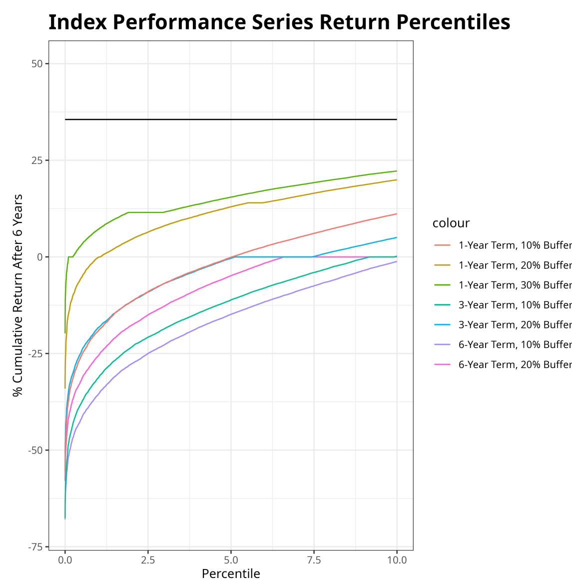
I examined the percentiles of returns for each product to find which one offers the best risk-reward tradeoff. As mentioned earlier, the customer is nearing retirement, so an option that minimizes downside risk should be chosen. However, some risk will need to be accepted to get a larger rate of return. In all graphs, the performances of the products are compared against a fixed annuity which offers a 5.2% return per year (35.55% cumulative return over 6 years), represented by the black horizontal line. This is to give a sense of how likely one is to beat the safest option offered by Allianz by choosing a riskier product.

4.1 Index Performance Series

Within the Index Performance series of products, there are several different options, so I dedicated part of my analysis just to this series.



(a) Percentiles of the cumulative returns for the Index Performance series



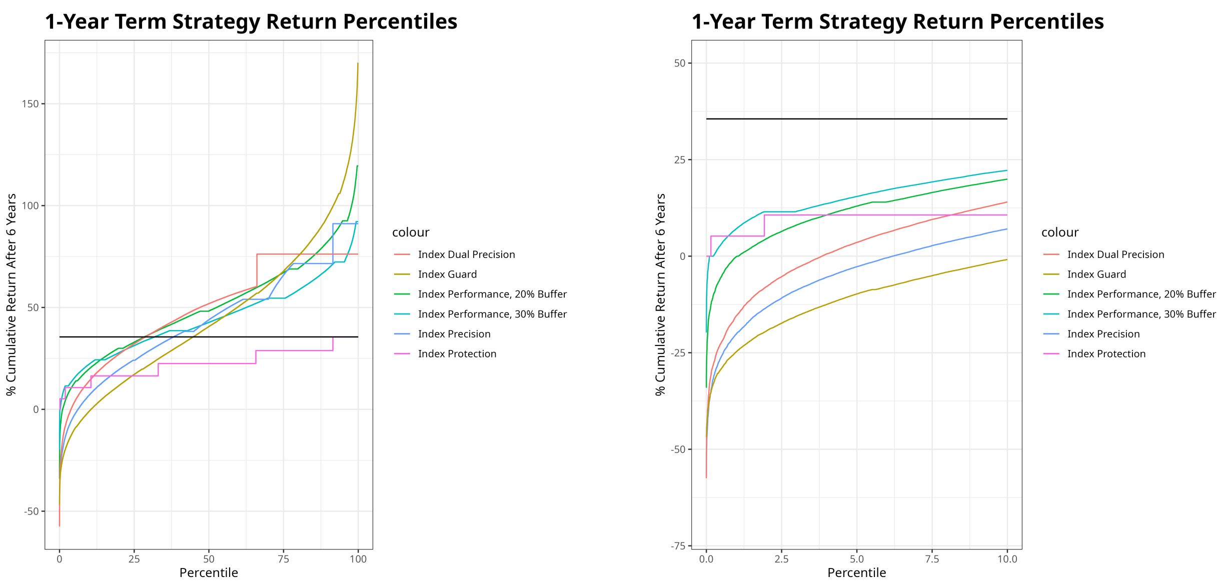
(b) The bottom 10% of cumulative returns for the Index Performance series

As shown in the graphs, the multi-year terms for the Index Performance series can have some spectacular returns at the high end of the distribution. However, these terms also come with great downside risk, making them unsuitable for a customer nearing retirement. Furthermore, consecutive three-year terms and consecutive one-year terms with 10% buffer do not offer substantial downside protection, but they do cut off the high end of returns to some degree. The cumulative returns over 6 years of these products are summarized in the table below.

Annuity	0.1% Low	1% Low	10% Low	Median	Chance to Beat Fixed Annuity
6-Year Index Performance, 10% Buffer	-51.7%	-35.5%	-1.1%	78.5%	69.2%
6-Year Index Performance, 20% Buffer	-41.7%	-25.5%	0.0%	72.5%	67.8%
3-Year Index Performance, 10% Buffer	-48.5%	-31.3%	0.1%	71.9%	70.6%
3-Year Index Performance, 20% Buffer	-34.9%	-18.3%	5.0%	67.5%	70.6%
1-Year Index Performance, 10% Buffer	-37.3%	-19.2%	11.2%	58.4%	72.0%
1-Year Index Performance, 20% Buffer	-14.8%	-0.1%	19.9%	48.3%	71.6%
1-Year Index Performance, 30% Buffer	-0.2%	7.1%	22.2%	42.7%	68.2%

4.2 One-Year Term Products

This analysis includes all the one-year term products except for the one-year Index Performance with 10% Buffer, since I determined its risk to be too great to be worth considering.



(a) Percentiles of the cumulative returns for products with a 1-year term

(b) The bottom 10% of cumulative returns for products with a 1-year term

Similar to the Index Performance series, the products offering the highest possible returns also come with the greatest possible downside risk. Some products offer better tradeoffs than others; in particular, Index Dual Precision is almost strictly better than Index Precision, with Index Precision only offering an minor advantage in the very best of circumstances. If one was willing to accept the downside risk of Index Guard, they would be much better off choosing a six-year Index Performance term with a 20% buffer. Index Dual Precision and Index Performance with 20% buffer are somewhat comparable, however the latter offers much better downside risk and has higher returns in the best case scenarios. The cumulative returns of the one-year terms are summarized in the table below.

Annuity	0.1% Low	1% Low	10% Low	Median	Chance to Beat Fixed Annuity
1-Year Index Performance, 10% Buffer	-37.3%	-19.2%	11.2%	58.4%	72.0%
1-Year Index Performance, 20% Buffer	-14.8%	-0.1%	19.9%	48.3%	71.6%
1-Year Index Performance, 30% Buffer	-0.2%	7.1%	22.2%	42.7%	68.2%
Index Precision	-37.5%	-20.1%	7.0%	44.0%	61.8%
Index Dual Precision	-33.5%	-15.1%	14.0%	51.9%	71.3%
Index Guard	-37.1%	-24.7%	-0.8%	40.6%	55.3%

5 Conclusion and Recommendation

Overall, the most compelling product for the customer is the one-year Index Performance with a 20% buffer. The downside risk is quite low; the realistic worst-case scenario is that the customer just gets their $100,000 back after six years. However, they are most likely to earn a substantial return on investment over that period, with the potential to nearly double their investment if things go well in the market. The one-year Index Performance with a 30% buffer is also appropriate, however I feel it cuts off too much of the high-end of the distribution to be worth it. That said, if the customer wants to turn a low chance of loss into a negligible chance, it is not a bad option. For future work, it would be interesting to see how the distribution of returns change when linking to different indices. Furthermore, Allianz offers many other annuities besides the Index Advantage+ series. A complete analysis should also include these other options, since it's possible that they present a better risk-reward tradeoff than those discussed here. Finally, it would be worth performing the simulations with a more sophisticated model of the S&P 500 returns than GBM. While GBM is not a bad model per se, the reality of the market is that volatility is not constant; it would be interesting to see if the results change when taking this into consideration.

Appendix A: Credited Rate Formulas

These rates are accurate as of April 2024. $S_{i,i+1}$ is the relative change of the S&P 500 index from the start of the term to the end of the term. c_{r_i} is the credited return for that term.

5.1 Index Performance 6 Year Length, 10% Buffer

$$c_{r_i} = \begin{cases} 1.3 \cdot S_{i,i+1} & S_{i,i+1} > 0 \\ min(S_{i,i+1} + 0.10, 0) & \text{otherwise} \end{cases}$$

5.2 Index Performance 6 Year Length, 20% Buffer

$$c_{r_i} = \begin{cases} 1.2 \cdot S_{i,i+1} & S_{i,i+1} > 0 \\ min(S_{i,i+1} + 0.20, 0) & \text{otherwise} \end{cases}$$

5.3 Index Performance 3 Year Length, 10% Buffer

$$c_{r_i} = \begin{cases} 1.1 \cdot S_{i,i+1} & S_{i,i+1} > 0 \\ min(S_{i,i+1} + 0.10, 0) & \text{otherwise} \end{cases}$$

5.4 Index Performance 3 Year Length, 20% Buffer

$$c_{r_i} = \begin{cases} S_{i,i+1} & S_{i,i+1} > 0 \\ min(S_{i,i+1} + 0.20, 0) & \text{otherwise} \end{cases}$$

5.5 Index Performance 1 Year Length, 10% Buffer

$$c_{r_i} = \begin{cases} min(S_{i,i+1}, 0.2125) & S_{i,i+1} > 0 \\ min(S_{i,i+1} + 0.10, 0) & \text{otherwise} \end{cases}$$

5.6 Index Performance 1 Year Length, 20% Buffer

$$c_{r_i} = \begin{cases} min(S_{i,i+1}, 0.1400) & S_{i,i+1} > 0 \\ min(S_{i,i+1} + 0.20, 0) & \text{otherwise} \end{cases}$$

5.7 Index Performance 1 Year Length, 30% Buffer

$$c_{r_i} = \begin{cases} min(S_{i,i+1}, 0.1150) & S_{i,i+1} > 0 \\ min(S_{i,i+1} + 0.30, 0) & \text{otherwise} \end{cases}$$

5.8 Index Precision Strategy

$$c_{r_i} = \begin{cases} 0.1140 & S_{i,i+1} > 0 \\ min(S_{i,i+1} + 0.10, 0) & \text{otherwise} \end{cases}$$

5.9 Index Dual Precision Strategy

$$c_{r_i} = \begin{cases} 0.0990 & S_{i,i+1} > -0.10 \\ S_{i,i+1} + 0.10 & \text{otherwise} \end{cases}$$

5.10 Index Guard Strategy

$$c_{r_i} = \begin{cases} min(S_{i,i+1}, 0.1800) & S_{i,i+1} > 0 \\ max(S_{i,i+1}, -0.1) & \text{otherwise} \end{cases}$$

5.11 Index Protection Strategy

$$c_{r_i} = \begin{cases} 0.052 & S_{i,i+1} > 0 \\ 0 & \text{otherwise} \end{cases}$$

References

[1] Parsiad Azimzadeh. *Maximum likelihood estimation of geometric Brownian motion parameters*. URL: https://parsiad.ca/blog/2020/maximum_likelihood_estimation_of_geometric_brownian_motion_parameters/. (accessed: 05.09.2024).

[2] Allianz Life. *Current rates for Allianz Index Advantage+ Income® Variable Annuity*. URL: https://www.allianzlife.com/what-we-offer/annuities/registered-index-linked-annuities/index-advantage-plus-income/rates. (accessed: 04.12.2024).