Homework

16.1 A useful feature of the conditional logit model with a specification including a full set of alternative specific constants is

$$s_j = \frac{1}{I}\sum_{i=1}^{I} \frac{\exp(\nu_{ij})}{\sum_{k=1}^{J}\exp(\nu_{ik})}, \qquad j = 1, ..., J, \tag{1}$$

where s_j is the proportion of times in the data that alternative j is selected, and the right-hand side is the average predicted probability for alternative j. Using the log-likelihood function in (16.13) and the specification $\nu_{ij} = \delta_j + \beta X_{ij}$ for $j = 1, ..., J-1$ and $\nu_{ij} = \beta X_{iJ}$ for $j = J$, show that the first-order conditions with respect to $\delta_1, ..., \delta_{J-1}$ for maximum likelihood estimation lead to (1) for all alternatives.

16.2 For the specification in exercise 16.1, derive expressions for the elasticities of the probability of alternative j with respect to a change in X_{ij} and a change in X_{ik}, for k not equal to j.

16.3 Suppose you have estimated a model that includes only the installation and operating cost variables. These results are

$$\nu_{ij} = -0.6231 \times ic_{ij} - 0.4580 \times oc_{ij}, \quad j = 1, ..., 5.$$

In addition, the data for three of the observations are as follows:

	IC					OC				
ID	j=1	j=2	j=3	j=4	j=5	j=1	j=2	j=3	j=4	j=5
3	5.9948	7.8305	7.1986	9.0011	10.483	1.6558	1.378	4.3906	4.0474	1.7147
14	5.6844	6.1743	6.8718	7.1872	7.7554	1.1945	1.1186	3.6592	3.9514	1.7353
21	6.5754	8.9216	7.8807	7.9952	10.831	1.7615	1.4893	4.4099	2.8629	2.0387

(a) The negative of the coefficient on oc_{ij} can be interpreted as the marginal utility of a dollar of annual income. Given this, interpret the ratio of the coefficient on ic_{ij} over the coefficient on oc_{ij}.

(b) Using a spreadsheet, compute the probability of choice for each alternative/person combination.

(c) Using a spreadsheet and the formula derived in exercise 16.2, compute for each person the elasticity of the probability of alternative 1, with respect to the installation cost for each of the five alternatives (i.e. compute one own- and four cross-cost elasticities).

(d) Once again using the negative of the coefficient on oc_{ij} as the marginal utility of annual income, use a spreadsheet to compute the one-time WTP for a 15% reduction in the installation cost of alternative $j = 5$, for each of the three people (recall that the cost variables are measured in 100s of USD, and adjust accordingly).

16.4 Using *home_heating* data, complete the following.

a. Estimate a model that includes only the installation and operating cost variables, and check that you are able to replicate the estimates shown in exercise 16.3.

b. Estimate a model that includes the operating and installation cost variables, as well as the full set of $J-1$ alternative specific constants (ASCs). Comment on the differences in the estimates relative to part (a). How does inclusion of the ASCs affect the estimate of the marginal willingness to pay for a change in installation cost? What is the likely explanation for the differences?

c. Now consider the role of income in explaining choices. Specify a model that allows you to test whether higher income households are more or less likely to install a central (as opposed to room) heating system (i.e., more likely to select option $j = 1, 2, 3$). Run your model with and without ASCs, and comment on the differences.

d. Consider a situation in which a 15 % of installation cost rebate is being offered when a heat pump is selected. Using the model with oc_{ij}, ic_{ij}, and the full set of ASCs, predict how the average probability of a household selecting the heat pump changes due to the rebate.

e. Using the same model as in (d), calculate the sample average willingness to pay for the rebate program. Compare your predictions for observations 3, 14, and 21 to what you derived in exercise 16.3. Which of the two calculations you find most plausible? Explain your reasoning.