Project Proposal: Adaptive Synchronization in Biological Networks

Project Title: Modeling Hebbian Learning and Synaptic Plasticity using a Memristive FitzHugh-Nagumo System

Student Names: Narasimha and Vanshika **Course:** Systems Biology

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1. Abstract

This project explores the design principles of adaptive biological networks by modeling the phenomenon of Synaptic Plasticity. While traditional neural models assume static connection weights, real biological systems utilize dynamic, history-dependent connections to enable learning and memory. This project aims to replicate and extend the findings of **Shatnawi et al. (2023)**, who demonstrated complex dynamics in memristive neuron models. We implement a continuous-time differential equation model to investigate how memristive coupling—acting as a synthetic synapse—facilitates robust synchronization (Hebbian Learning) between excitable cells. This provides a concrete mathematical framework for understanding how biological networks self-organize and maintain robustness in dynamic environments.

2. Objectives

1. **Develop a Dynamic Model:** Implement a coupled system of Differential Equations representing two excitable neurons (Pre-synaptic and Post-synaptic) using the FitzHugh-Nagumo formalism.
2. **Simulate Plasticity:** Replace static coupling constants with a **Memristive State Variable (M)** that evolves according to the potential difference (flux) between neurons, replicating the "learning" dynamics observed in plastic networks.
3. **Analyze System Behavior:** Demonstrate the transition from an "Unconnected" state to a "Synchronized" state, mapping this behavior to biological **Design Principles** of adaptation and robustness.

3. Alignment with Course Syllabus

This project has been specifically designed to cover all five units of the Systems Biology curriculum:

Course Unit	Project Implementation & Relevance
Unit 1: Network Organization (Motifs)	We model a **2-Node Feed-Forward Motif** (Teacher $\rightarrow$ Student). Instead of a static edge, we model a **dynamic edge**, analyzing how simple network motifs can exhibit complex emergent behaviors like "Associative Learning."
Unit 2: Design Principles	The project demonstrates the principle of **Adaptation**. The system does not just react to inputs; it changes its internal structure (synaptic weight) to optimize signal transmission, mimicking the biological imperative of energy-efficient communication.
Unit 3: Dynamic Modelling	The core methodology relies on solving a system of coupled **Non-linear Ordinary Differential Equations (ODEs)**. We analyze the phase-space trajectories and the time-evolution of the state variables (v, w, M).
Unit 4: Switches & Clocks	The FitzHugh-Nagumo model is the canonical mathematical description of a **Biological Switch** (Excitable Medium). We explore how the memristor acts as a secondary "Conductance Switch," locking the system into a synchronized state.
Unit 5: Robustness & Noise	We investigate **Structural Stability**. The memristive coupling acts as a buffer, allowing the system to achieve robust synchronization even when the individual neurons have mismatched parameters or initial conditions.

4. Methodology

4.1 The Mathematical Model

We utilize the FitzHugh-Nagumo equations to represent neuronal excitability. We adapt the memristive coupling concept from **Shatnawi et al. (2023)** to model the synapse as a plastic element in continuous time.

The Neuron (Plant):

$$\frac{dv}{dt} = v - \frac{v^3}{3} - w + I_{synaptic}$$

$$\tau \frac{dw}{dt} = v + a - bw$$

The Memristor (Controller): To simulate the synapse, we introduce a state variable M (Memory Trace) that follows a flux-controlled plasticity rule:

$$\frac{dM}{dt} = \alpha(\Delta v)^2(1 - M) - \beta M$$

- **Growth (α):** If the potential difference (Δv) is significant, the weight increases (Learning).
- **Decay (β):** In the absence of activity, the weight decays (Forgetting).

4.2 Simulation Tools

The model will be implemented in **Python** using numerical integration (Euler Method) to visualize the time-series evolution of the membrane potentials and the synaptic weight.

5. Expected Outcomes

1. **Visual Demonstration of Learning:** A plot showing the Synaptic Weight (M) rising from 0 to 1 solely due to system activity (Self-Organization).
2. **Synchronization:** Evidence that the "Student" neuron begins to fire in phase with the "Teacher" neuron only *after* the learning phase is complete.
3. **Phase Plane Analysis:** A Nullcline plot illustrating the limit cycle behavior of the coupled system.

6. References

1. **Alon, U.** (2019). *An Introduction to Systems Biology: Design Principles of Biological Circuits*. Chapman & Hall.
2. **Shatnawi, M. T., et al.** (2023). "A Multistable Discrete Memristor and Its Application to Discrete-Time FitzHugh–Nagumo Model." *Electronics*, 12(13), 2929.