1.I = \x.x S = \x.\y.\z.x z(y z) l= S I 求 l mn

解:

l m n

= (\x.\y.\z.x z(y z)) (\x.x) m n

=[(\x.x)/x][m/y][n/z] (x z(y z))

= (\x.x)n(mn)

= n(m n)

2. ZERO = \f.\x.x SUCC = \n.\f.\x.f (n f x) 求 SUCC (SUCC ZERO)

解:

SUCC (SUCC ZERO)

= SUCC([ZERO /n]\f.\x.f (n f x))

=SUCC(\f.\x.f (ZERO f x))

=SUCC(\f.\x.f ((\f.\x.x) f x))

=SUCC(\f.\x.f ([f/f][x/x] x))

=SUCC(\f.\x.f x)

= (\n.\f.\x.f (n f x)) (\f.\x.f x)

= [\f.\x.f x /n]( \f.\x.f (n f x))

= \f.\x.f ((\f.\x.f x) f x )

= \f.\x.f (f x)

3. POW = \b.\e.e b 求 POW TWO THREE

解:

POW TWO THREE

= (THREE)(TWO)

=(\f.\x.f (f(f x)))(TWO)

=[TWO/f]( \x.f (f(f x)))

=\x. (\f1.\x1.f1 (f1 x1))( (\f1.\x1.f1 (f1 x1))(( \f1.\x1.f1 (f1 x1)) x))

= \x. (\f1.\x1.f1 (f1 x1))( (\f1.\x1.f1 (f1 x1))( \x1.x (x x1))

= \x. (\f1.\x1.f1 (f1 x1))([ \x1.x (x x1)/f1] (\f1.\x1.f1 (f1 x1))

= \x. (\f1.\x1.f1 (f1 x1))( \x1. (\x1.x (x x1)) ((\x1.x (x x1)) x1))

= \x. (\f1.\x1.f1 (f1 x1)) ( \x1. (\x1.x (x x1)) (x (x x1)) )

= \x. \x1. ( \x1. (\x1.x (x x1)) (x (x x1) )) (( \x1. (\x1.x (x x1)) (x (x x1)) ) x1)

= \x. \x1. ( \x1. (\x1.x (x x1)) (x (x x1) )) (\x1.x (x x1)) (x (x x1)) )

= \x. \x1. ( \x1. (x (x (x (x x1) ))))( x (x (x (x x1)))) = \x. \x1. (x (x (x (x ( x (x (x (x x1)))) )))

=\f.\x.f(f(f(f(f(f(f(f x))))))))

4. TRUE = \x.\y.x FALSE = \x.\y.y AND = \p.\q.p q p OR = \p.\q.p p q NOT = \p.\a.\b.p b a IF = \p.\a.\b.p a b 求 NOT (NOT TRUE)

求 IF (OR FALSE FALSE) a b

解:

NOT (NOT TRUE)

=NOT((\p.\a.\b.p b a)(\x.\y.x))

=NOT(\a.\b. (\x.\y.x) b a)

=(\p.\a1.\b1.p b1 a1)(\a.\b. (\x.\y.x) b a)

=\a1.\b1. ((\a.\b. (\x.\y.x) b a) b1 a1)

=\a1.\b1. ((\a.\b.b b1 a1)

=\a1.\b1. (a1)

=TRUE

IF (OR FALSE FALSE) a b

= IF (FALSE FALSE FALSE)a b

= (\p.\a.\b.p a b)(FALSE FALSE FALSE) a b

= ((FALSE FALSE) FALSE)ab

=((\x.\y.y \x.\y.y) \x.\y.y)a b

=(\y.y \x.\y.y)ab

=\x.\y.y ab

=b

5 LEQ = \m.\n.ISZERO (SUB m n) 求大于等于 GEQ

解 GEQ = \m.\n.ISZERO (SUB n m)

6.Lambda> FACT1 = \f.\n.IF (ISZERO n) ONE (MULT n (f f (PRED n))) Lambda> FACT = FACT1 FACT1 求 FACT THREE

解:FACT THREE

= FACT1 FACT1 THREE

=[ FACT1/f][ THREE /n] IF (ISZERO n) ONE (MULT n (f f (PRED n)))

= IF (ISZERO THREE) ONE (MULT THREE (FACT1 FACT1 (PRED THREE)))

= MULT THREE (FACT1 FACT1 (PRED THREE))

= MULT THREE (FACT1 FACT1 TWO)

= MULT THREE ([ FACT1/f][ TWO /n]IF (ISZERO n) ONE (MULT n (f f (PRED n))))

= MULT THREE (MULT TWO (FACT1 FACT1 (PRED TWO)))

= MULT THREE (MULT TWO (FACT1 FACT1 ONE))

= MULT THREE (MULT TWO ([ FACT1/f][ ONE /n]IF (ISZERO n) ONE (MULT n (f f (PRED n)))))

= MULT THREE (MULT TWO (IF (ISZERO ONE) ONE (MULT ONE (FACT1 FACT1 (PRED ONE)))))

= MULT THREE (MULT TWO (MULT ONE (FACT1 FACT1 ZERO)))

= MULT THREE (MULT TWO (MULT ONE ONE))

=SIX

=\f.\x.f(f(f(f(f(f x)))))

7.Lambda> ADD = W (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) 求 ADD TWO FOUR

解: ADD TWO FOUR

=(\x.x x) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) TWO FOUR

=[(\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)))/x] (x x) TWO FOUR

= (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) TWO FOUR

= [(\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)))/f][n/TWO][M/FOUR] IF (ISZERO m) n (f f (SUCC n) (PRED m))

=(\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)))(THREE)(THREE)

=[(\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)))/f] (f f (SUCC THREE) (PRED THREE))

=(\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)))(FOUR)(TWO)

= [(\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)))/f] (f f (SUCC FOUR) (PRED TWO))

=(\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)))(FIVE)(ONE)

=[(\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)))/f] (f f (SUCC FIVE) (PRED ONE))

=[(\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)))/f](f f (SIX)(ZERO))

=(\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (SIX)(ZERO)

= IF (ISZERO ZERO) SIX (f f (SUCC SIX) (PRED ZERO))

=SIX

=\f.\x.f(f(f(f(f(f x)))))

8. Lambda> FACT2 = \f.\n.IF (ISZERO n) ONE (MULT n (f (PRED n))) Lambda> FACTY = Y FACT2

求 FACTY THREE

解:FACTY THREE

= Y FACT2 THREE

= (\g.(\x.g (x x)) \x.g (x x)) FACT2 THREE

=[ FACT2/g]( (\x.g (x x)) \x.g (x x)) THREE

=((\x. FACT2 (x x)) \x. FACT2 (x x) )THREE

=[\x. FACT2 (x x) / x] FACT2 (x x) THREE

=( FACT2((\x. FACT2 (x x)) (\x. FACT2 (x x)))))THREE

= [((\x. FACT2 (x x)) (\x. FACT2 (x x)))/f][ THREE/n]IF (ISZERO n) ONE (MULT n (f (PRED n))))

= MULT THREE (((\x. FACT2 (x x)) (\x. FACT2 (x x))) (PRED THREE))

= MULT THREE ((\x. FACT2 (x x)) (\x. FACT2 (x x)) (TWO))

= MULT THREE ((\x. FACT2 (x x)) (\x. FACT2 (x x)) (TWO))

= MULT THREE ( FACT2 ((\x. FACT2 (x x)) (\x. FACT2 (x x)))) (TWO)

= MULT THREE (MULT TWO (((\x. FACT2 (x x)) (\x. FACT2 (x x))) (ONE))

= MULT THREE (MULT TWO ((FACT2 ((\x. FACT2 (x x)) (\x. FACT2 (x x))))) (ONE)

= MULT THREE (MULT TWO (MULT ONE ((\x. FACT2 (x x)) (\x. FACT2 (x x))) (ZERO)))

= MULT THREE (MULT TWO (MULT ONE (FACT2 ((\x. FACT2 (x x)) (\x. FACT2 (x x)))) (ZERO)))

= MULT THREE (MULT TWO (MULT ONE (MULT ONE)))

=SIX

=\f.\x.f(f(f(f(f(f x)))))

9. Lambda> CONS a (CONS b (CONS c NIL)) 求 CAR (CDR (CONS a (CONS b (CONS c NIL))))

解:CAR (CDR (CONS a (CONS b (CONS c NIL))))

= CAR (CDR (CONS a (CONS b (\x.\y.\f. f x y c NIL))))

= CAR (CDR (CONS a (CONS b (\f. f c NIL))))

= CAR (CDR (\f. f a (\f. f b (\f. f c NIL))))

= CAR ((\f. f a (\f. f b (\f. f c NIL))) FALSE)

=CAR((\f. f b (\f. f c NIL)))

=(\f. f b (\f. f c NIL))TRUE

=b

10. 求 有序对的 LENGTH

解:

Y = \g.(\x.g (x x)) \x.g (x x)

FACT1=(\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) ZERO

LENGTH = Y FACT1 = Y (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) ZERO

LENGTH (CONS a (CONS b (CONS c NIL)))

= (\g.(\x.g (x x)) \x.g (x x)) (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) ZERO (CONS a (CONS b (CONS c NIL)))

=( (\x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1)) \x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1)) ZERO (CONS a (CONS b (CONS c NIL)))

=((\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (\x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1) \x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1))) ZERO (CONS a (CONS b (CONS c NIL)))

=[(\x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1) \x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1))/g][ ZERO/c][ (CONS a (CONS b (CONS c NIL)))/x] \g.\c.\x. NULL x c (g (SUCC c) (CDR x))

=\p.p (\x.\y.FALSE) (CONS a (CONS b (CONS c NIL))) ZERO ((\x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1) \x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1)) (SUCC ZERO) (CDR CONS a (CONS b (CONS c NIL))))

=\p.p (\x.\y.FALSE) (CONS a (CONS b (CONS c NIL))) ZERO ((\x1. (\g.\c.\x. NULL x c (g (SUCC c)

(CDR x))) (x1 x1) \x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1)) (ONE) (\f. f b (\f. f c NIL))

= (\f. f a (\f. f b (\f. f c NIL))) (\x.\y.FALSE) ZERO ((\x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1) \x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1)) (ONE) (\f. f b (\f. f c NIL))

= FALSE ZERO ((\x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1) \x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1)) (ONE) (\f. f b (\f. f c NIL))

= ((\x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1) \x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1)) (ONE) (\f. f b (\f. f c NIL))

=((\x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1) \x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1)) (TWO) (\f. f c NIL)

=((\x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1) \x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1)) (THREE) (NIL)

=(( (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (\x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1) \x1. (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) (x1 x1))) (THREE) (NIL)

=THREE

=\f.\x.f(f(f x))