# 离散直方图

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

- 输入图像灰度级$r_k$的概率近似为

$$p_r(r_k) = \frac{n_k}{MN} \qquad k = 0, 1, 2, \ldots, L-1$$

- 离散变换函数（直方图均衡）

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, 2, \ldots, L-1$$

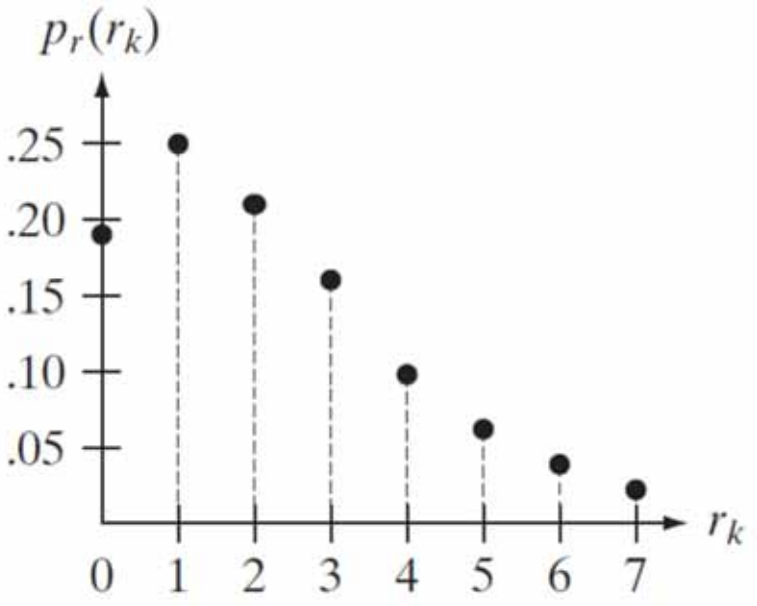
  - 反变换

$$r_k = T^{-1}(s_k) \qquad k = 0, 1, 2, \ldots, L-1$$

# 举例

- 3比特数字图像的灰度分布和直方图值

| $r_k$ | $n_k$ | $p_r(r_k) = n_k/MN$ |
|---|---|---|
| $r_0 = 0$ | 790 | 0.19 |
| $r_1 = 1$ | 1023 | 0.25 |
| $r_2 = 2$ | 850 | 0.21 |
| $r_3 = 3$ | 656 | 0.16 |
| $r_4 = 4$ | 329 | 0.08 |
| $r_5 = 5$ | 245 | 0.06 |
| $r_6 = 6$ | 122 | 0.03 |
| $r_7 = 7$ | 81 | 0.02 |



# 举例

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

- 变换函数

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

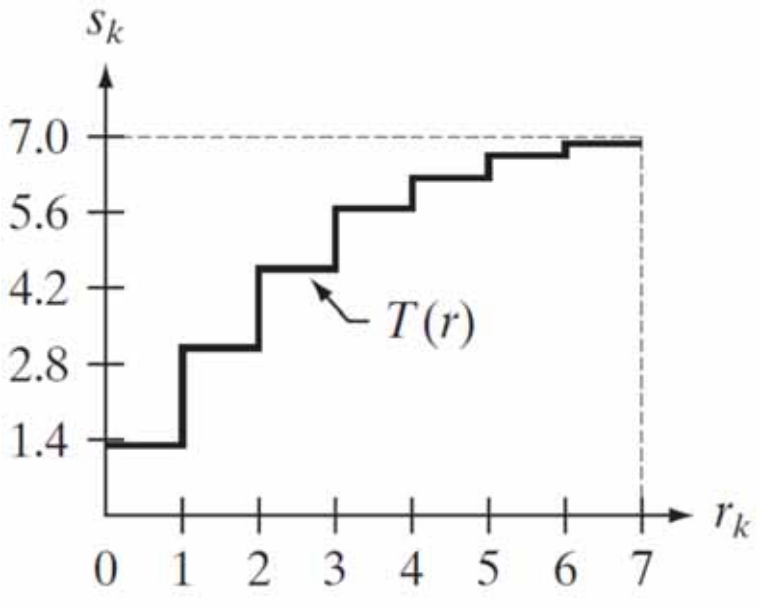
$$s_2 = 4.55, s_3 = 5.67, s_4 = 6.23$$

$$s_5 = 6.65, s_6 = 6.86, s_7 = 7.00$$

# 举例

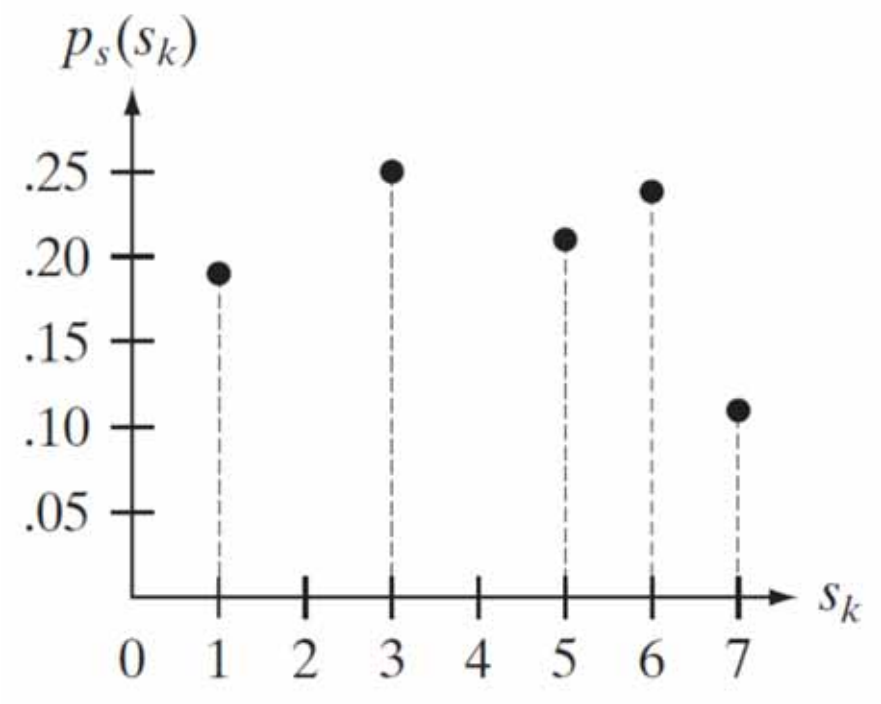
- 变换函数

$$s_0 = 1.33 \rightarrow 1 \qquad s_4 = 6.23 \rightarrow 6$$
$$s_1 = 3.08 \rightarrow 3 \qquad s_5 = 6.65 \rightarrow 7$$
$$s_2 = 4.55 \rightarrow 5 \qquad s_6 = 6.86 \rightarrow 7$$
$$s_3 = 5.67 \rightarrow 6 \qquad s_7 = 7.00 \rightarrow 7$$

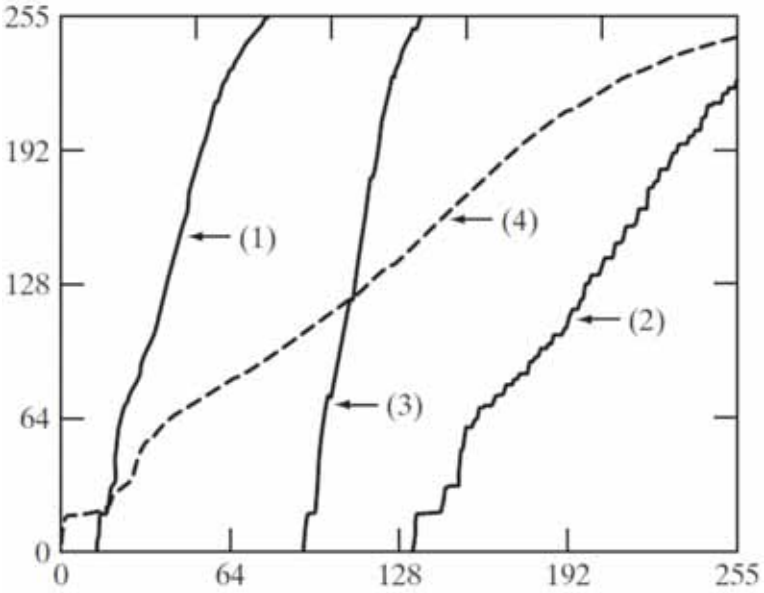


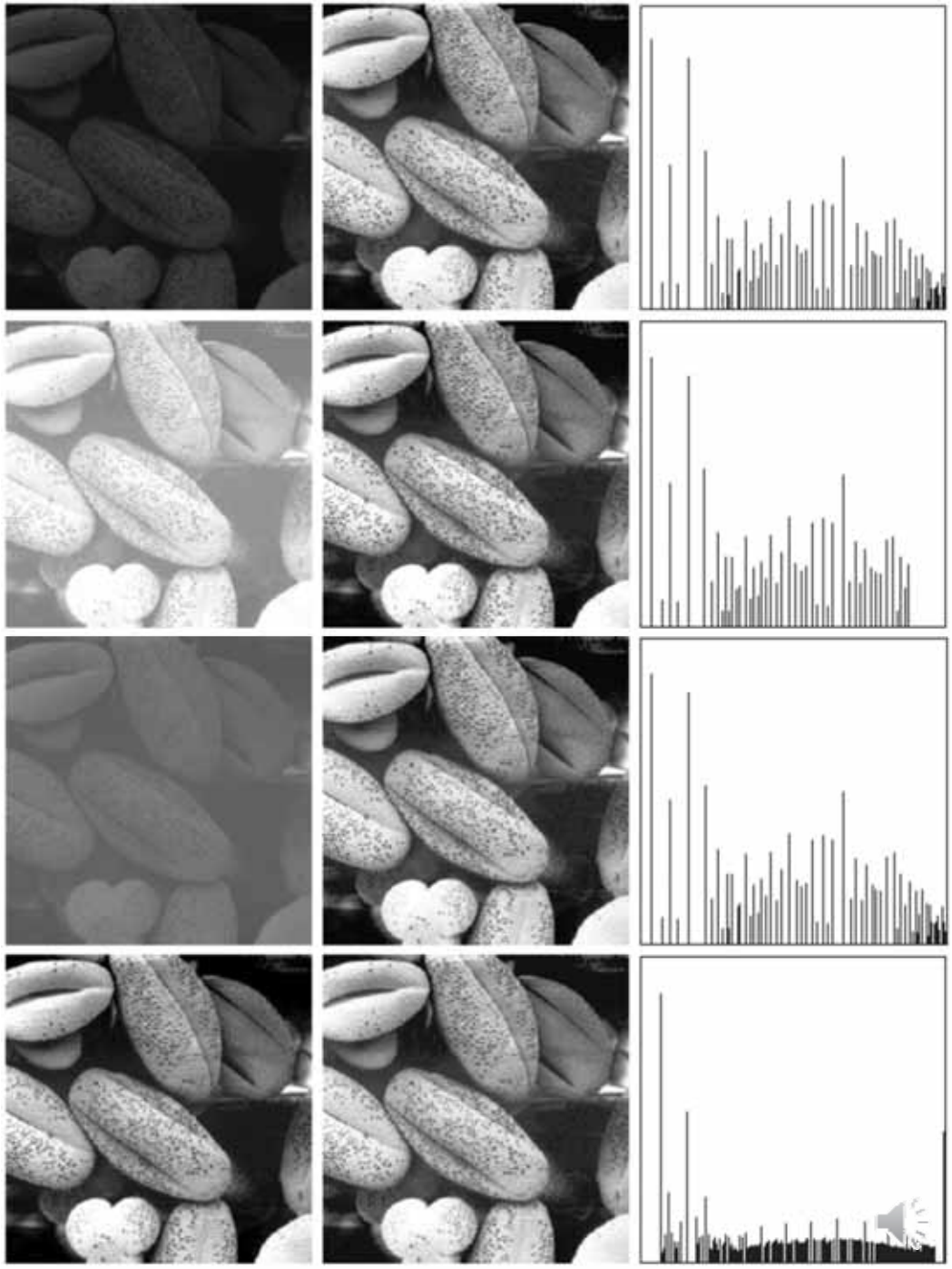
# 举例

- 均衡后的直方图



# 举例



# 更明显的例子

原图

直方图均衡

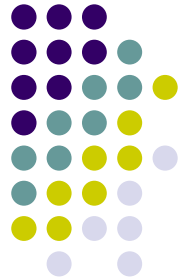
# 直方图匹配（规定化）

- 对某些应用，均匀直方图不是最好的方法
- 有时候希望输出图像具有指定的直方图

输出直方图要求为某个特定分布

输入  灰度变换函数$T(r)$  输出

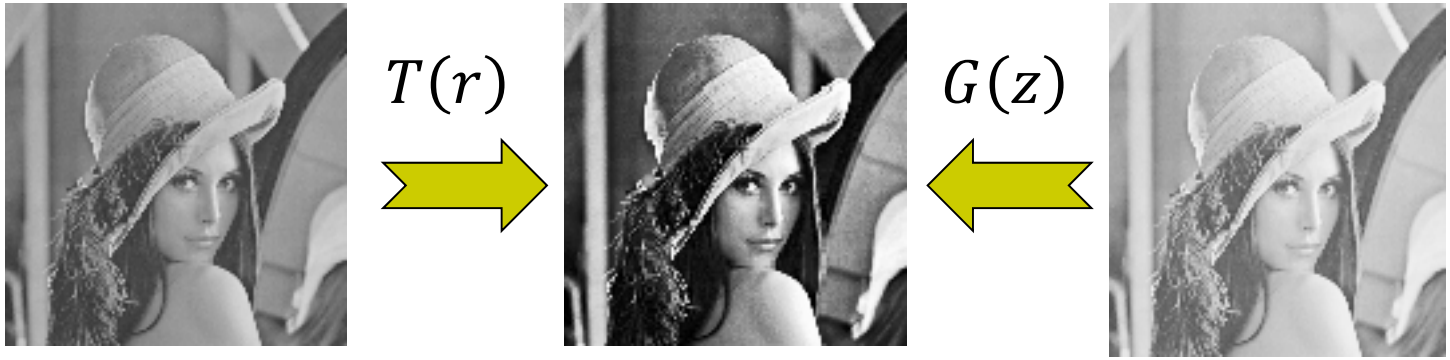
# 直方图匹配



- 输入图像灰度值概率密度$p_r(r)$
- 指定灰度值概率密度$p_z(z)$
- 如何设计变换函数使得输出图像概率密度为$p_z(z)$？

# 核心思想

- 以均衡化直方图图像为桥梁

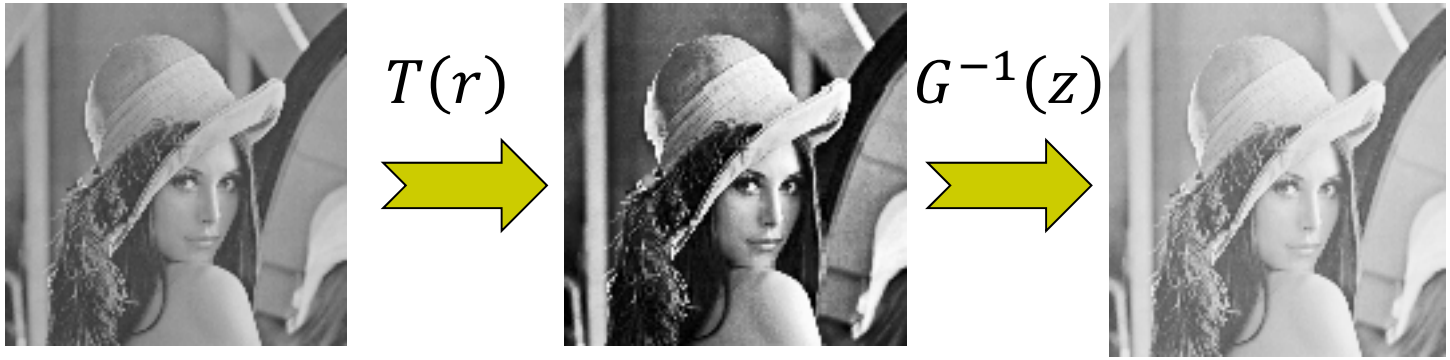



A　　均衡化图像B　　C

- 直方图均衡：A到B
- 直方图均衡：C到B

# 核心思想

- 以均衡化直方图图像为桥梁




A　　均衡化图像B　　C

- 直方图均衡：A到B
- 直方图均衡的反函数：B到C

# 实现方式

- 输入图像灰度值概率密度$p_r(r)$

$$s = T(r) = (L-1)\int_0^r p_r(w)\,\mathrm{d}w$$

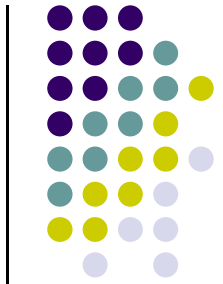
- 指定灰度值概率密度$p_z(z)$

$$G(z) = (L-1)\int_0^z p_z(t)\,\mathrm{d}t = s$$

- 反函数唯一

$$z = G^{-1}(s) = G^{-1}\big(T(r)\big)$$

# 具体步骤



1. 由输入图像计算$p_r(r)$
2. 根据下面的公式计算$s$

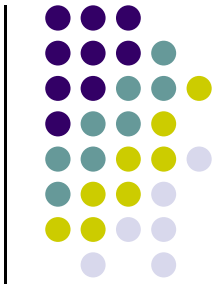
$$s = T(r) = (L-1)\int_0^r p_r(w)\,\mathrm{d}w$$

3. 根据$p_z(z)$，计算变换函数$G(z)$

$$G(z) = (L-1)\int_0^z p_z(t)\,\mathrm{d}t$$

4. 计算反变换函数$z = G^{-1}(s)$
5. 将反变换函数作用到所有的$s$

# 举例



- 输入概率密度$p_r(r) = 2r/(L-1)^2$

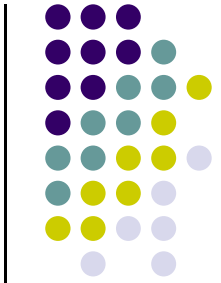
$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw = \frac{2}{(L-1)}\int_0^r w\,dw = \frac{r^2}{(L-1)}$$

- 指定概率密度$p_z(z) = 3z^2/(L-1)^3$

$$G(z) = (L-1)\int_0^z p_z(w)\,dw = \frac{3}{(L-1)^2}\int_0^z w^2\,dw = \frac{z^3}{(L-1)^2}$$

$$z = \left[(L-1)^2 s\right]^{1/3}$$

# 离散直方图



- 离散情况更加简单
- 输入离散直方图$p_r(r_k)$

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, 2, \ldots, L-1$$

- 指定离散直方图$p_z(z_q)$

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) = s_k$$

  - 查表实现

$$z_q = G^{-1}(s_k)$$

# 具体步骤

1. 计算输入图像直方图$p_r(r)$，并计算$s_k$，并四舍五入
2. 依据给定直方图$p_z(z)$，计算变化函数$G$的所有值，并四舍五入，存储表中
3. 对于每一个$s_k$，通过查表，找到对应的$z_q$
   - $G(z_q)$最接近$s_k$
   - 结果不唯一时，选择最小的$z_q$

# 举例

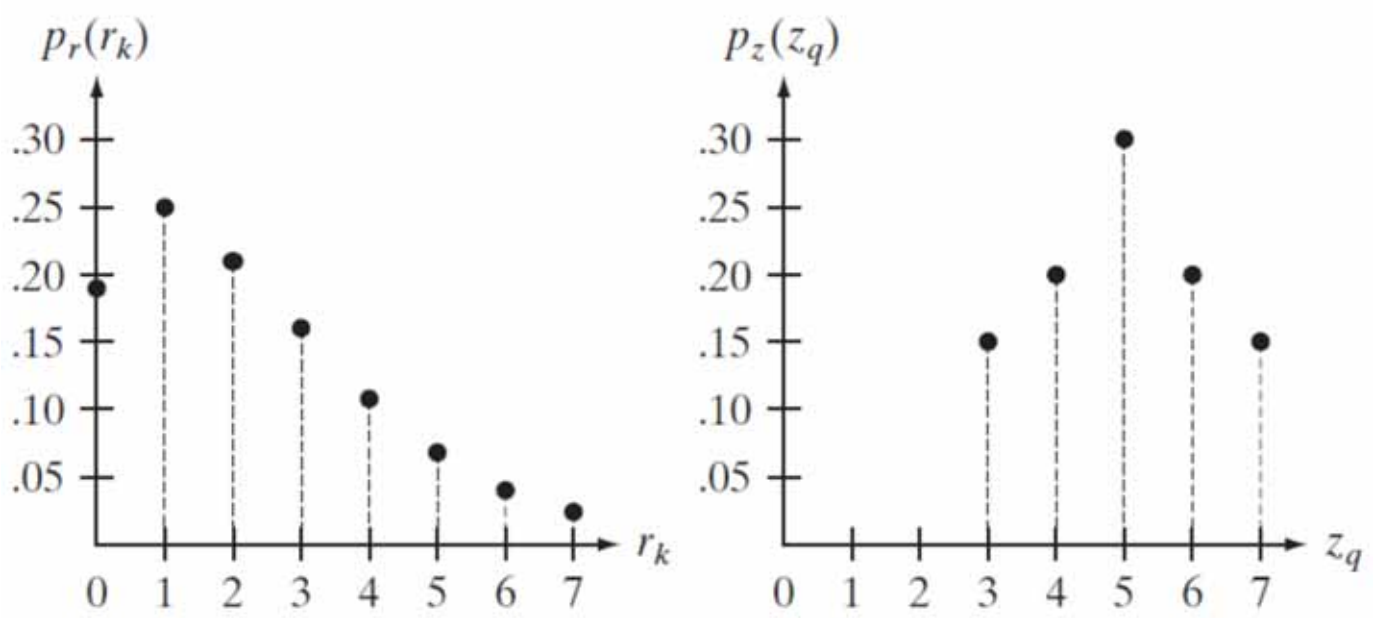
- 3比特数字图像、指定直方图

| $r_k$ | $n_k$ | $p_r(r_k) = n_k/MN$ |
|---|---|---|
| $r_0 = 0$ | 790 | 0.19 |
| $r_1 = 1$ | 1023 | 0.25 |
| $r_2 = 2$ | 850 | 0.21 |
| $r_3 = 3$ | 656 | 0.16 |
| $r_4 = 4$ | 329 | 0.08 |
| $r_5 = 5$ | 245 | 0.06 |
| $r_6 = 6$ | 122 | 0.03 |
| $r_7 = 7$ | 81 | 0.02 |

| $z_q$ | Specified $p_z(z_q)$ |
|---|---|
| $z_0 = 0$ | 0.00 |
| $z_1 = 1$ | 0.00 |
| $z_2 = 2$ | 0.00 |
| $z_3 = 3$ | 0.15 |
| $z_4 = 4$ | 0.20 |
| $z_5 = 5$ | 0.30 |
| $z_6 = 6$ | 0.20 |
| $z_7 = 7$ | 0.15 |

# 举例

- 3比特数字图像、指定直方图



# 举例

- 对输入图像计算执行直方图均衡

$$s_0 = 1.33 \rightarrow 1 \qquad s_4 = 6.23 \rightarrow 6$$
$$s_1 = 3.08 \rightarrow 3 \qquad s_5 = 6.65 \rightarrow 7$$
$$s_2 = 4.55 \rightarrow 5 \qquad s_6 = 6.86 \rightarrow 7$$
$$s_3 = 5.67 \rightarrow 6 \qquad s_7 = 7.00 \rightarrow 7$$

- 对指定直方图执行直方图均衡

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00$$

$$G(z_1) = 7\sum_{j=0}^{1} p_z(z_j) = 7\left[p(z_0) + p(z_1)\right] = 0.00$$

# 举例

- 对输入图像计算执行直方图均衡

$$s_0 = 1.33 \rightarrow 1 \qquad s_4 = 6.23 \rightarrow 6$$
$$s_1 = 3.08 \rightarrow 3 \qquad s_5 = 6.65 \rightarrow 7$$
$$s_2 = 4.55 \rightarrow 5 \qquad s_6 = 6.86 \rightarrow 7$$
$$s_3 = 5.67 \rightarrow 6 \qquad s_7 = 7.00 \rightarrow 7$$

- 对指定直方图执行直方图均衡

$$G(z_0) = 0.00 \rightarrow 0 \qquad G(z_4) = 2.45 \rightarrow 2$$
$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_5) = 4.55 \rightarrow 5$$
$$G(z_2) = 0.00 \rightarrow 0 \qquad G(z_6) = 5.95 \rightarrow 6$$
$$G(z_3) = 1.05 \rightarrow 1 \qquad G(z_7) = 7.00 \rightarrow 7$$

# 举例

- 获得$G$的逆映射

$$s_0 = 1.33 \rightarrow 1 \qquad s_4 = 6.23 \rightarrow 6$$
$$s_1 = 3.08 \rightarrow 3 \qquad s_5 = 6.65 \rightarrow 7$$
$$s_2 = 4.55 \rightarrow 5 \qquad s_6 = 6.86 \rightarrow 7$$
$$s_3 = 5.67 \rightarrow 6 \qquad s_7 = 7.00 \rightarrow 7$$

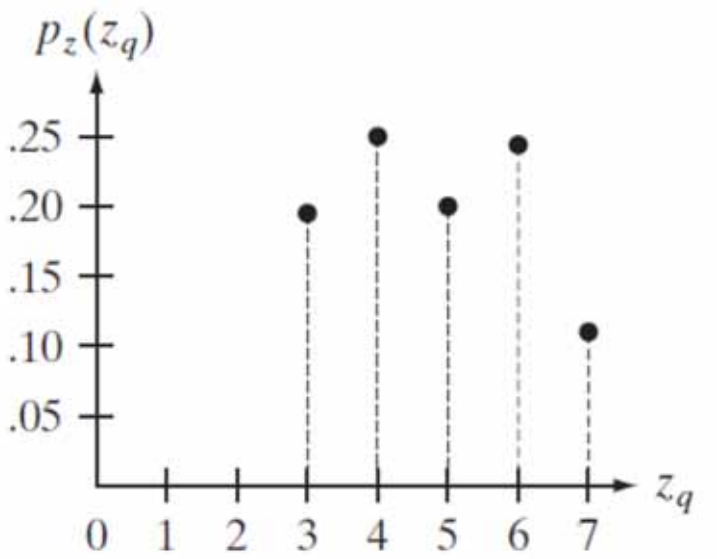
| $z_q$ | $G(z_q)$ |
|---|---|
| $z_0 = 0$ | 0 |
| $z_1 = 1$ | 0 |
| $z_2 = 2$ | 0 |
| $z_3 = 3$ | 1 |
| $z_4 = 4$ | 2 |
| $z_5 = 5$ | 5 |
| $z_6 = 6$ | 6 |
| $z_7 = 7$ | 7 |

| $s_k$ | $\rightarrow$ | $z_q$ |
|---|---|---|
| 1 | $\rightarrow$ | 3 |
| 3 | $\rightarrow$ | 4 |
| 5 | $\rightarrow$ | 5 |
| 6 | $\rightarrow$ | 6 |
| 7 | $\rightarrow$ | 7 |

# 举例

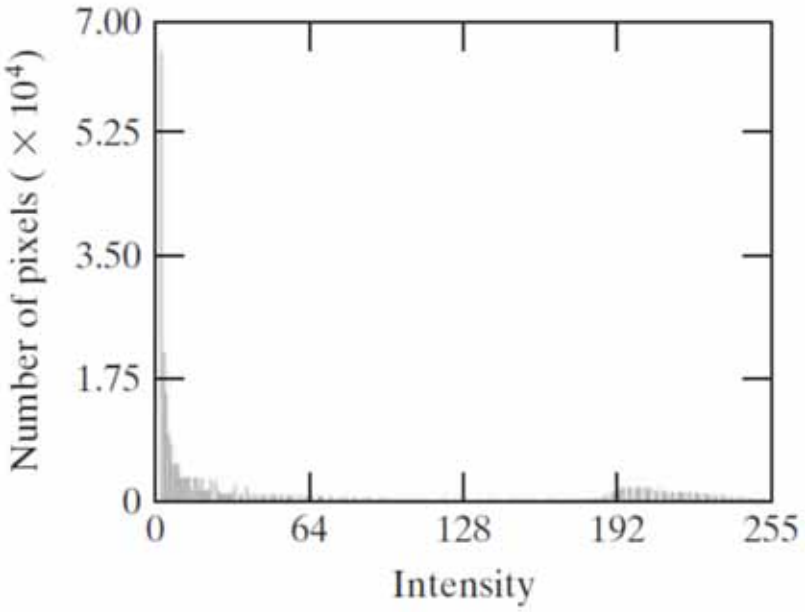
- 最终结果

| $z_q$ | Specified $p_z(z_q)$ | Actual $p_z(z_k)$ |
|---|---|---|
| $z_0 = 0$ | 0.00 | 0.00 |
| $z_1 = 1$ | 0.00 | 0.00 |
| $z_2 = 2$ | 0.00 | 0.00 |
| $z_3 = 3$ | 0.15 | 0.19 |
| $z_4 = 4$ | 0.20 | 0.25 |
| $z_5 = 5$ | 0.30 | 0.21 |
| $z_6 = 6$ | 0.20 | 0.24 |
| $z_7 = 7$ | 0.15 | 0.11 |



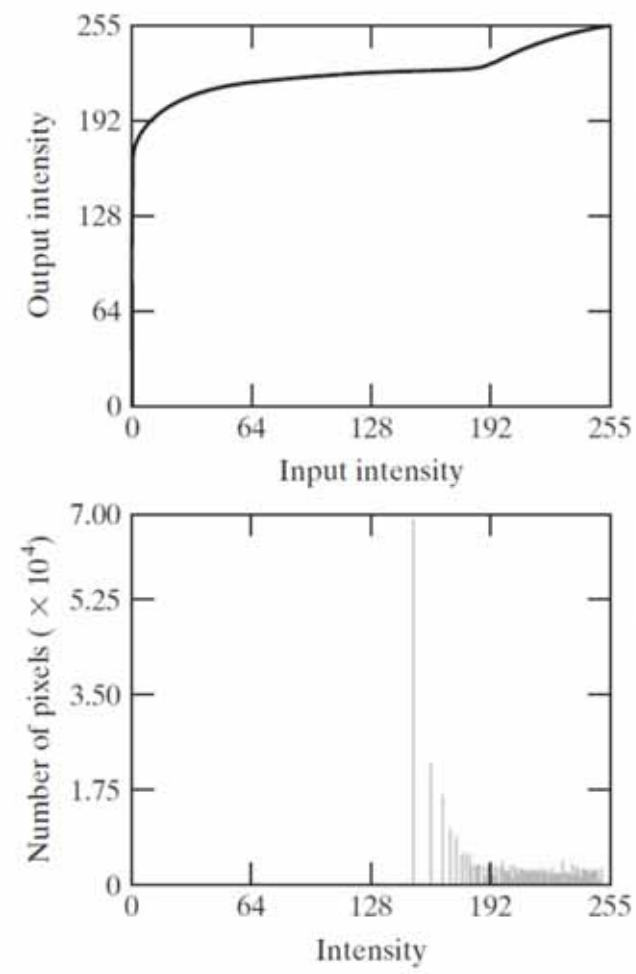
# 举例

- 火星卫星图像


7.00
5.25
3.50
1.75
0
Number of pixels ( × 10⁴)
0
64
128
192
255
Intensity

# 举例

- 直方图均衡

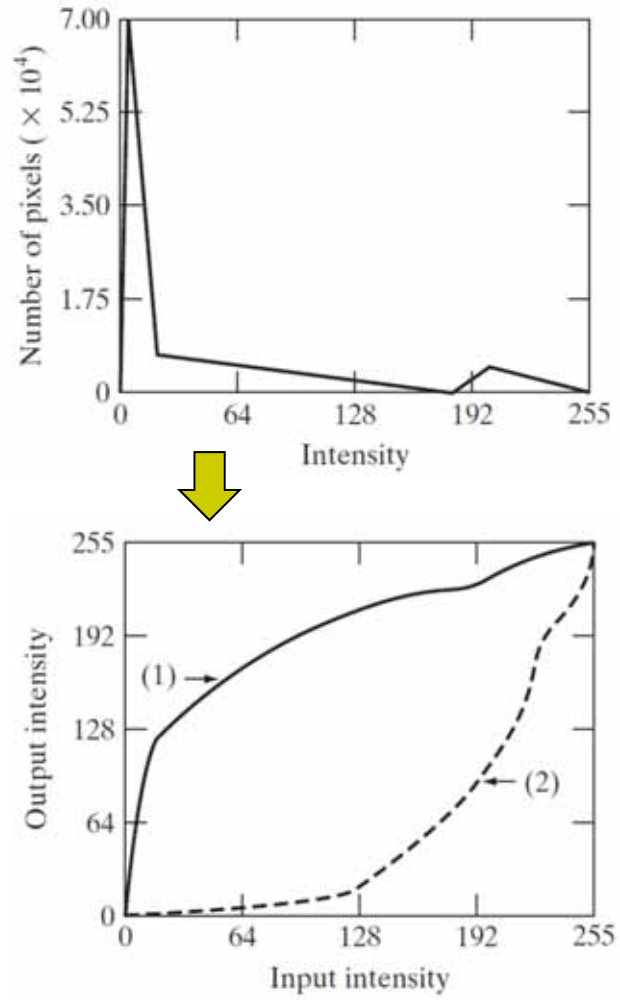


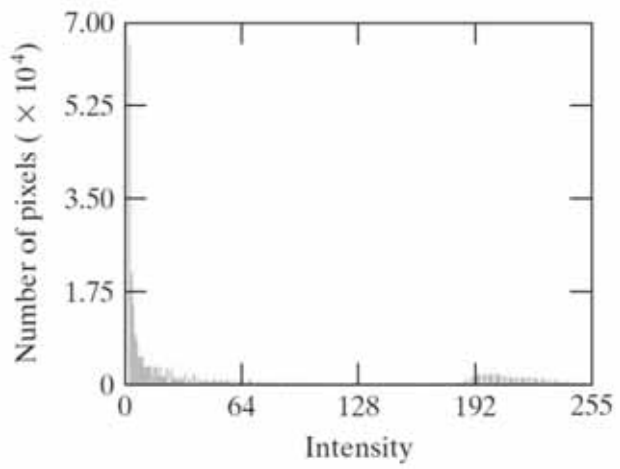

图像变得更亮，但是对比度并没有明显改善！

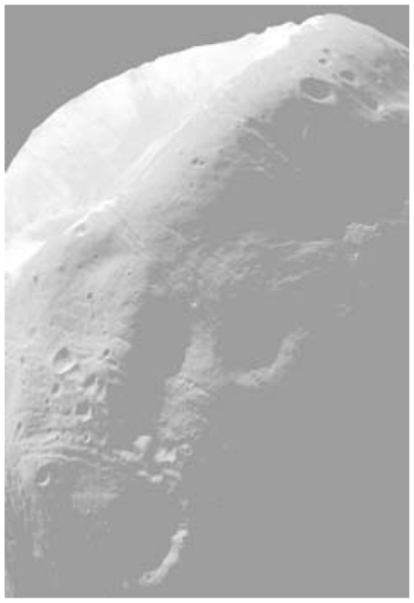
$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

# 举例

- 直方图匹配


7.00
5.25
3.50
1.75
0
Number of pixels ( × 10⁴)
0
64
128
192
255
Intensity
255
192
128
64
0
Output intensity
(1)
(2)
0
64
128
192
255
Input intensity


7.00
5.25
3.50
1.75
0
Number of pixels ( × 10⁴)
0
64
128
192
255
Intensity



# 举例

指定直方图
是关键！

- 直方图匹配