Chapter 7

LTI Discrete-Time Systems in the Transform-Domain

Chapter 7 LTI Discrete-Time Systems in the Transform-Domain

Types of Transfer Functions

- In the case of digital transfer functions with frequency-selective frequency responses, there are two types of classifications
- (1) Classification based on the shape of the magnitude function $|H(e^{j\omega})|$
- (2) Classification based on the the form of the phase function $\theta(\omega)$

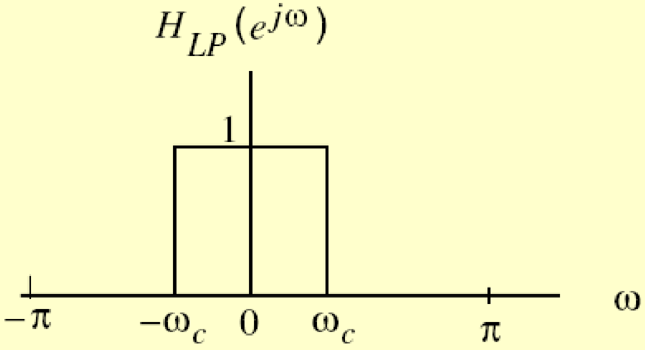
7.1 Transfer Function Classification Based on Magnitude Characteristics

7.1.1 Digital Filters with Ideal Magnitude Responses

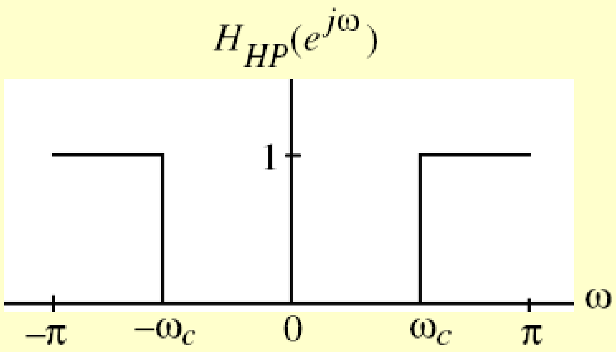
- One common classification is based on an ideal magnitude response
- A digital filter designed to pass signal components of certain frequencies without distortion should have a magnitude response equal to one at these frequencies, and should have a magnitude response equal to zero at all other frequencies

Ideal Filters

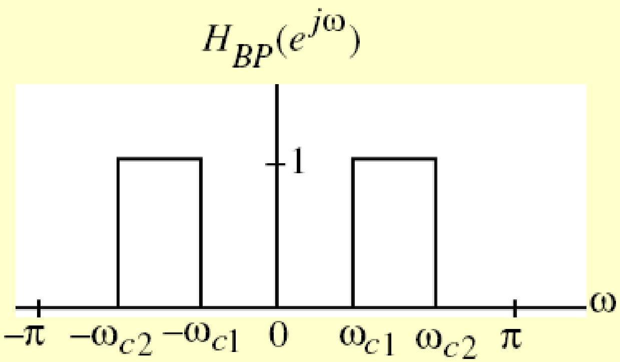
- **Frequency responses of the four popular types of ideal digital filters with real impulse response coefficients:**



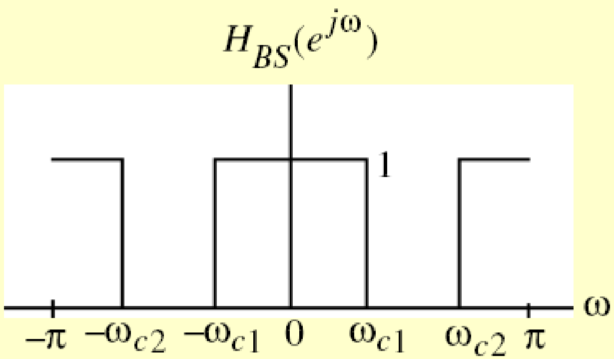
Lowpass



Highpass



Bandpass



Bandstop

1、通、阻带
2、截止频率

Ideal Filters

- The frequencies ω_c , ω_{c1} , and ω_{c2} are called the **cutoff frequencies**
- An ideal filter has a magnitude response equal to one in the passband and zero in the stopband, and has a zero phase everywhere

Ideal Filters

- Earlier in the course we derived the inverse DTFT of the frequency response $H_{LP}(e^{j\omega})$ of the ideal lowpass filter:

$$h_{LP}[n] = \frac{\sin \omega_c n}{\pi n}, \quad -\infty < n < \infty$$

双边的(非因果)
无限长

- We have also shown that the above impulse response is not absolutely summable, and hence, the corresponding transfer function is not BIBO stable

Ideal Filters

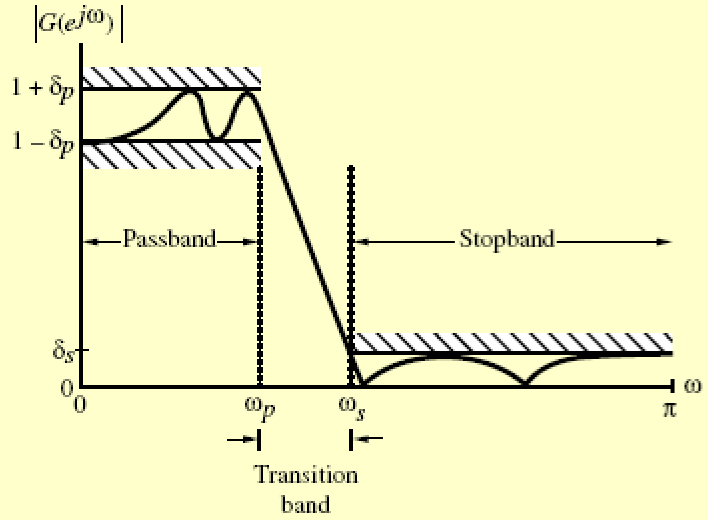
- To develop stable and realizable transfer functions, the ideal frequency response specifications are relaxed by including a **transition band** between the passband and the stopband

过渡带

- This permits the magnitude response to decay slowly from its maximum value in the passband to the zero value in the stopband

Ideal Filters

- Moreover, the magnitude response is allowed to vary by a small amount both in the passband and the stopband
- Typical magnitude response specifications of a lowpass filter are shown below



Chapter 7 LTI Discrete-Time Systems in the Transform-Domain

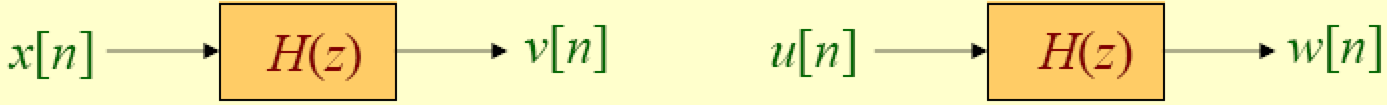
- 7.1 Transfer Function Classification Based on Magnitude Characteristics
- 7.2 Transfer Function Classification Based on Phase Characteristics
- 7.3 Types of Linear-Phase FIR Transfer Functions
- 7.4 Simple Digital Filters
- 7.5 Inverse Systems

7.2 Transfer Function Classification Based on Phase Characteristics

- A second classification of a transfer function is with respect to its phase characteristics
- In many applications, it is necessary that the digital filter designed does not distort the phase of the input signal components with frequencies in the passband

Zero-Phase Transfer Function

- For non-real-time processing of real-valued input signals of finite length, zero-phase filtering can be very simply implemented by relaxing the causality requirement
- One zero-phase filtering scheme is sketched below



$$u[n] = v[-n], \qquad y[n] = w[-n]$$

输入x[n]与输出y[n]之间的频响是实数，无相位信息

Linear-Phase Transfer Function

- In the case of a causal transfer function with a nonzero phase response, the phase distortion can be avoided by ensuring that the transfer function has a unity magnitude and a **linear-phase** characteristic in the frequency band of interest

Linear-Phase Transfer Function

- The most general type of a filter with a linear phase has a frequency response given by

$$H(e^{j\omega}) = e^{-j\omega D}$$

线性相位意味着一个系统的相频特性是频率的线性函数，此时通过这一系统的各频率分量的时延为一相同的常数，即群时延(group delay)

$$\tau(\omega) = D$$

Linear-Phase Transfer Function

- The output $y[n]$ of this filter to an input $x[n] = Ae^{j\omega n}$ is then given by

$$y[n] = Ae^{-j\omega D}e^{j\omega n} = Ae^{j\omega(n-D)}$$

- If $x_a(t)$ and $y_a(t)$ represent the continuous-time signals whose sampled versions, sampled at $t = nT$, are $x[n]$ and $y[n]$ given above, then the delay between $x_a(t)$ and $y_a(t)$ is precisely the group delay of amount D

$$h[n] \Leftrightarrow H(e^{j\omega})$$

$$\Downarrow \qquad \qquad \Downarrow$$

$$\delta[n-D] \qquad \qquad e^{-j\omega D}$$

$$y[n] = x[n] * h[n]$$

$$= Ae^{j\omega n} * \delta[n-D]$$

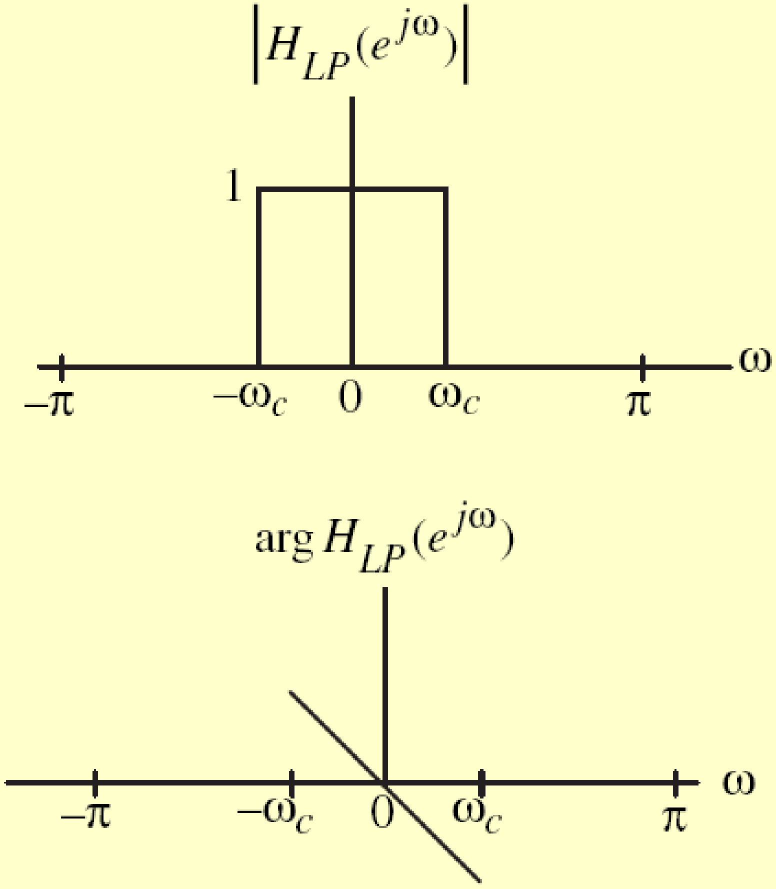
$$= Ae^{j\omega(n-D)}$$

Linear-Phase Transfer Function

- If it is desired to pass input signal components in a certain frequency range undistorted in both magnitude and phase, then the transfer function should exhibit a unity magnitude response and a linear-phase response in the band of interest

Linear-Phase Transfer Function

The frequency response of a lowpass filter with a linear-phase in the passband



Linear-Phase Transfer Function

- Example - Determine the impulse response of an ideal lowpass filter with a linear phase response:

$$H_{LP}(e^{j\omega}) = \begin{cases} e^{-j\omega n_o}, & 0 < |\omega| < \omega_c \\ 0, & \omega_c \le |\omega| \le \pi \end{cases}$$

- The impulse response

线性相位性的加入

$$h_{LP}[n] = \frac{\sin \omega_c (n - n_o)}{\pi (n - n_o)}, \quad -\infty < n < \infty$$

- As before, the above filter is noncausal and of doubly infinite length, and hence, unrealizable

Linear-Phase Transfer Function

- By truncating the impulse response to a finite number of terms, a realizable FIR approximation to the ideal lowpass filter can be developed
- The truncated approximation may or may not exhibit linear phase, depending on the value of n_o chosen

Linear-Phase Transfer Function

- If we choose $n_o = N/2$ with N a positive integer, the truncated and shifted approximation

$$\hat{h}_{LP}[n] = \frac{\sin \omega_c (n - N/2)}{\pi (n - N/2)}, \quad 0 \le n \le N$$

will be a length $N+1$ causal linear-phase FIR filter

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7. 3 Types of Linear-Phase FIR Transfer Functions

Linear-Phase FIR Transfer Functions

- It is impossible to design an IIR transfer function with an exact linear-phase
- It is always possible to design an FIR transfer function with an exact linear-phase response
- We now develop the forms of the linear-phase FIR transfer function $H(z)$ with real impulse response $h[n]$

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n} = \breve{H}\left(\omega\right)e^{j\varphi(\omega)}$$

$\breve{H}\left(\omega\right)$—或正或负的实数

$\varphi(\omega)$—$H(e^{j\omega})$的相频特性

Linear-Phase FIR Transfer Functions

- Let $$H(z) = \sum_{n=0}^{N} h[n] z^{-n}$$
- If $H(z)$ is to have a linear-phase, its frequency response must be of the form
$$H(e^{j\omega}) = e^{j(c\omega+\beta)} \breve{H}(\omega)$$
where c and β are constants, and $\breve{H}(\omega)$, called the amplitude response, also called the zero-phase response, is a real function of ω

Linear-Phase FIR Transfer Functions

- For a real impulse response, the magnitude response $|H(e^{j\omega})|$ is an even function of ω, i.e.,

$$|H(e^{j\omega})| = |H(e^{-j\omega})|$$

- Since $|H(e^{j\omega})| = |\breve{H}(\omega)|$, the amplitude response is then either an even function or an odd function of ω, i.e.

$$\breve{H}(-\omega) = \pm\breve{H}(\omega)$$

Linear-Phase FIR Transfer Functions

- The frequency response satisfies the relation

$$H(e^{j\omega}) = H^*(e^{-j\omega})$$

or, equivalently, the relation

$$e^{j(c\omega+\beta)}\breve{H}(\omega) = e^{-j(-c\omega+\beta)}\breve{H}(-\omega) \qquad (7.44)$$

Linear-Phase FIR Transfer Functions

- If $\breve{H}(\omega)$ is an even function $\breve{H}(-\omega) = \breve{H}(\omega)$, then the above relation leads to

$$e^{j(c\omega+\beta)}\breve{H}(\omega) = e^{-j(-c\omega+\beta)}\breve{H}(-\omega) \qquad (7.44)$$

$$e^{j\beta} = e^{-j\beta} \Longrightarrow \beta = 0, \pi$$

$$H(e^{j\omega}) = e^{j(c\omega+\beta)}\breve{H}(\omega) \qquad (7.42)$$

$$\breve{H}(\omega) = \pm e^{-jc\omega} H(e^{j\omega}) = \pm \sum_{n=0}^{N} h(n) e^{-j\omega(c+n)} \qquad (7.45)$$

Linear-Phase FIR Transfer Functions

$$\breve{H}(\omega) = \pm e^{-jc\omega} H(e^{j\omega}) = \pm \sum_{n=0}^{N} h(n) e^{-j\omega(c+n)} \qquad (7.45)$$

$$\breve{H}(-\omega) = \pm \sum_{n=0}^{N} h(\ell) e^{j\omega(c+\ell)} \qquad (7.46)$$

$$\overset{\ell = N-n}{\Rightarrow} \pm \sum_{n=0}^{N} h(N-n) e^{j\omega(c+N-n)} \qquad (7.47)$$

$$\begin{cases} h[n] = h[N-n], & 0 \le n \le N \\ (c = -N/2) & \end{cases}$$

Linear-Phase FIR Transfer Functions

- If $\breve{H}(\omega)$ is an even function $\breve{H}(-\omega) = \breve{H}(\omega)$, then the above relation leads to

$$h[n] = h[N-n], \qquad 0 \le n \le N$$

$$(c = -N/2)$$

- Thus, the FIR filter with an even amplitude response will have a linear phase if it has a symmetric impulse response

Linear-Phase FIR Transfer Functions

- If $\breve{H}(\omega)$ is an odd function of ω, then

$$h[n] = -h[N-n], \quad 0 \le n \le N$$

$$(c = -N/2)$$

- Therefore, a FIR filter with an odd amplitude response will have linear-phase response if it has an antisymmetric impulse response

线性相位FIR滤波器的条件

- h[n]具有对称性——

  奇对称（$h[n]=-h[N-n]$）

  偶对称 ($h[n]=h[N-n]$)

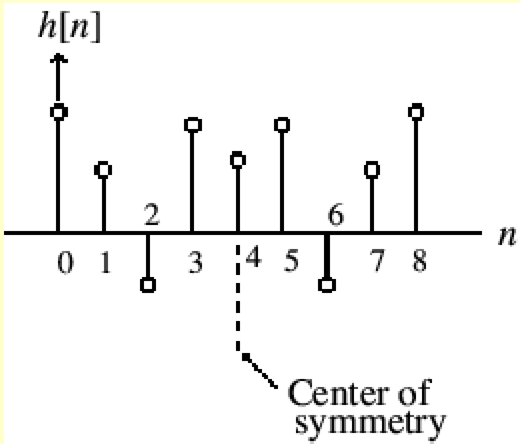
  其中，群时延$c=-N/2$

- 根据$h[n]$的对称性和N的奇偶性可把线性相位FIR滤波器分为四类

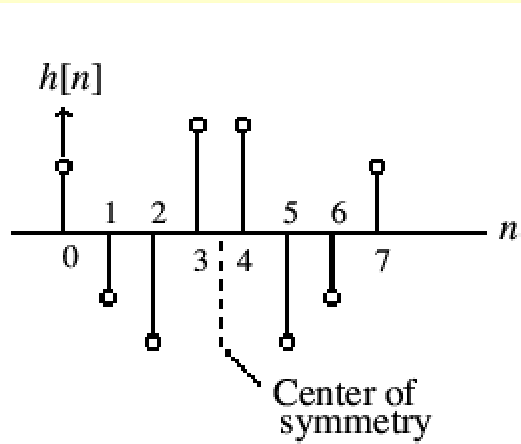
Linear-Phase FIR Transfer Functions

Four types of linear-phase FIR transfer functions:

偶对称

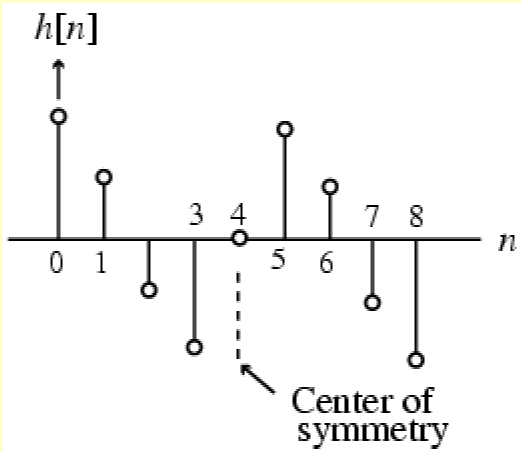


Type 1: $N = 8$

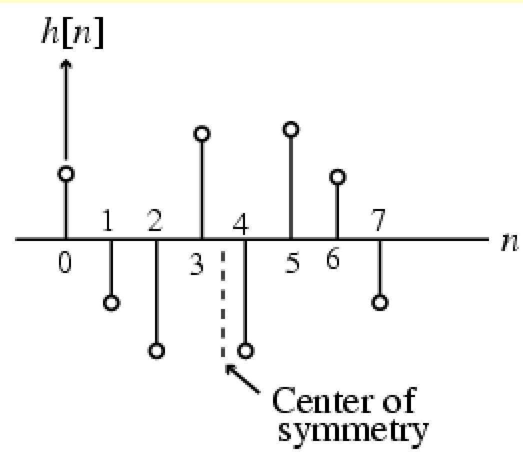


Type 2: $N = 7$

奇对称



Type 3: $N = 8$



Type 4: $N = 7$

Linear-Phase FIR Transfer Functions

Type 1: Symmetric Impulse Response with Odd Length

- In this case, the degree N is even
- The frequency response is of the form

$$H(e^{j\omega}) = e^{-jN\omega/2}\breve{H}(\omega)$$

where the **amplitude response** $\breve{H}(\omega)$ is of the form

$$\breve{H}(\omega) = h[\tfrac{N}{2}] + 2\sum_{n=1}^{N/2} h[\tfrac{N}{2} - n]\cos(\omega n)$$

Linear-Phase FIR Transfer Functions

Type 2: Symmetric Impulse Response with Even Length

- In this case, the degree N is odd
- The expression for the frequency response in the general case for Type 2 FIR filters is of the form

$$H(e^{j\omega}) = e^{-jN\omega/2}\breve{H}(\omega)$$

where the amplitude response is given by

$$\breve{H}(\omega) = 2\sum_{n=1}^{(N+1)/2} h[\tfrac{N+1}{2} - n]\cos(\omega(n - \tfrac{1}{2}))$$

Linear-Phase FIR Transfer Functions

Type 3: Antiymmetric Impulse Response with Odd Length

- In this case, the degree N is even
- In the general case

$$H(e^{j\omega}) = je^{-jN\omega/2}\breve{H}(\omega)$$

where the amplitude response is of the form

$$\breve{H}(\omega) = 2\sum_{n=1}^{N/2} h[\tfrac{N}{2} - n]\sin(\omega n)$$

Linear-Phase FIR Transfer Functions

Type 4: Antiymmetric Impulse Response with Even Length

- In this case, the degree N is even
- In the general case we have

$$H(e^{j\omega}) = je^{-jN\omega/2}\breve{H}(\omega)$$

where now the amplitude response is of the form

$$\breve{H}(\omega) = 2\sum_{n=1}^{(N+1)/2} h[\tfrac{N+1}{2} - n]\sin(\omega(n - \tfrac{1}{2}))$$

Linear-Phase FIR Transfer Functions

General Form of Frequency Response

- In each of the four types of linear-phase FIR filters, the frequency response is of the form

$$H(e^{j\omega}) = e^{-jN\omega/2} e^{j\beta} \breve{H}(\omega)$$

- The amplitude response $\breve{H}(\omega)$ for each of the four types of linear-phase FIR filters can become negative over certain frequency ranges, typically in the stopband

Linear-Phase FIR Transfer Functions

- **Example** – Consider the causal Type 1 FIR transfer function

$$H_1(z) = -1 + 2z^{-1} - 3z^{-2} + 6z^{-3} - 3z^{-4} + 2z^{-5} - z^{-6}$$

- Its amplitude and phase responses are given by

$$\breve{H}_1(\omega) = 6 - 6\cos(\omega) + 4\cos(2\omega) - 2\cos(3\omega)$$

$$\theta_1(\omega) = -3\omega$$

Linear-Phase FIR Transfer Functions

- Next, consider the causal Type 1 FIR transfer function

$$H_2(z) = 1 - 2z^{-1} + 3z^{-2} - 6z^{-3} + 3z^{-4} - 2z^{-5} + z^{-6}$$

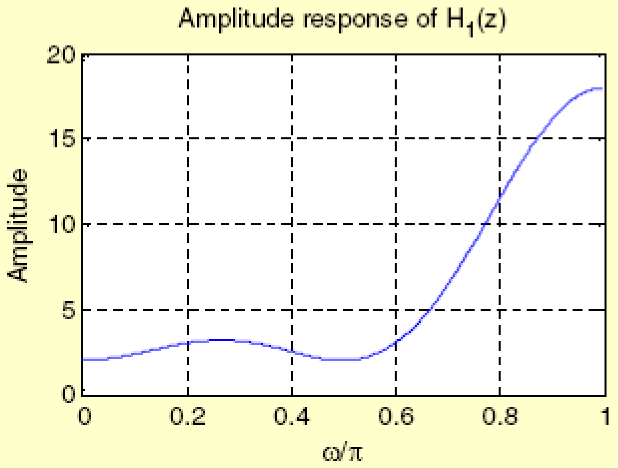
- Its amplitude and phase responses are given by

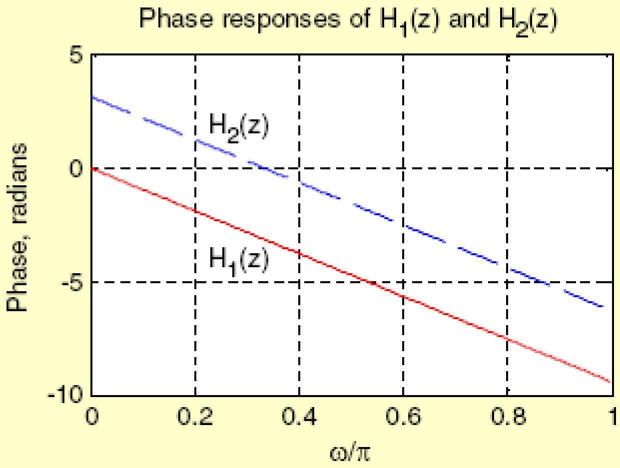
$$\breve{H}_2(\omega) = -\breve{H}_1(\omega)$$
$$\theta_2(\omega) = -3\omega + \pi$$

- Note: $|H_1(e^{j\omega})| = |H_2(e^{j\omega})|$

Linear-Phase FIR Transfer Functions

- Hence, $H_1(z)$ and $H_2(z)$ have identical magnitude responses but phase responses differing by π as shown below





四种线性相位FIR数字滤波器小结

1、相位特性只取决于h(n)的对称性，而与h(n)的值无关。

2、幅度特性取决于h(n)。

3、设计FIR数字滤波器时，在保证h(n)对称的条件下，只要完成幅度特性的逼近即可。

注意：当H(ω)用$|H(\omega)|$表示时，当H(ω)为奇对称时，其相频特性中还应加一个固定相移π

Zero Locations of Linear-Phase FIR Transfer Functions

- Consider first an FIR filter with a symmetric impulse response: $h[n] = h[N-n]$
- Its transfer function can be written as

$$H(z) = \sum_{n=0}^{N} h[n] z^{-n} = \sum_{n=0}^{N} h[N-n] z^{-n}$$

变量代换

$$\overset{m=N-n}{\Leftrightarrow} \sum_{m=0}^{N} h(m) z^{m-N} = z^{-N} \sum_{m=0}^{N} h(m) \left(z^{-1}\right)^{-m} = z^{-N} H(z^{-1})$$

Zero Locations of Linear-Phase FIR Transfer Functions

- Hence for an FIR filter with a symmetric impulse response of length $N+1$ we have

$$H(z) = z^{-N} H(z^{-1})$$

- A real-coefficient polynomial $H(z)$ satisfying the above condition is called a **mirror-image polynomial** (MIP)

Zero Locations of Linear-Phase FIR Transfer Functions

- Now consider first an FIR filter with an antisymmetric impulse response:

$$h[n] = -h[N-n]$$

- Hence, the transfer function $H(z)$

$$H(z) = -z^{-N} H(z^{-1})$$

- A real-coefficient polynomial $H(z)$ satisfying the above condition is called a **antimirror-image polynomial** (AIP)

线性相位FIR滤波器的零点特性

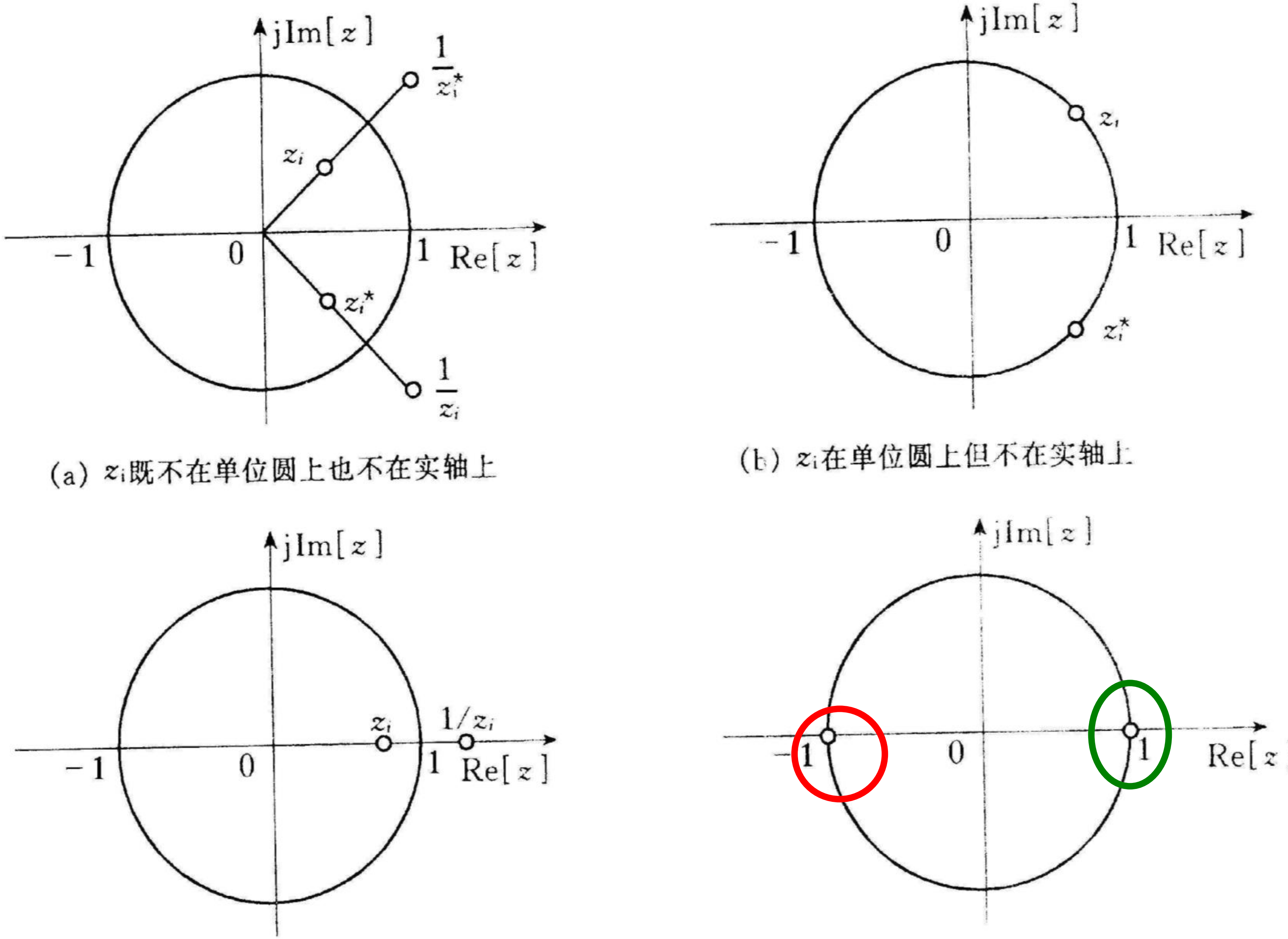
$$h(n) = \pm h(N-n)$$

↓

$$H(z) = \pm z^{-N} H(z^{-1})$$

若z= z_{0i}是H(z)的零点，则z= ${z_{0i}}^{-1}$也一定是H(z)的零点；由于h(n)是实数，H(z)的零点还必须共轭成对。

结论：零点必须是互为倒数的共轭对



(a) z_i既不在单位圆上也不在实轴上

(b) z_i在单位圆上但不在实轴上

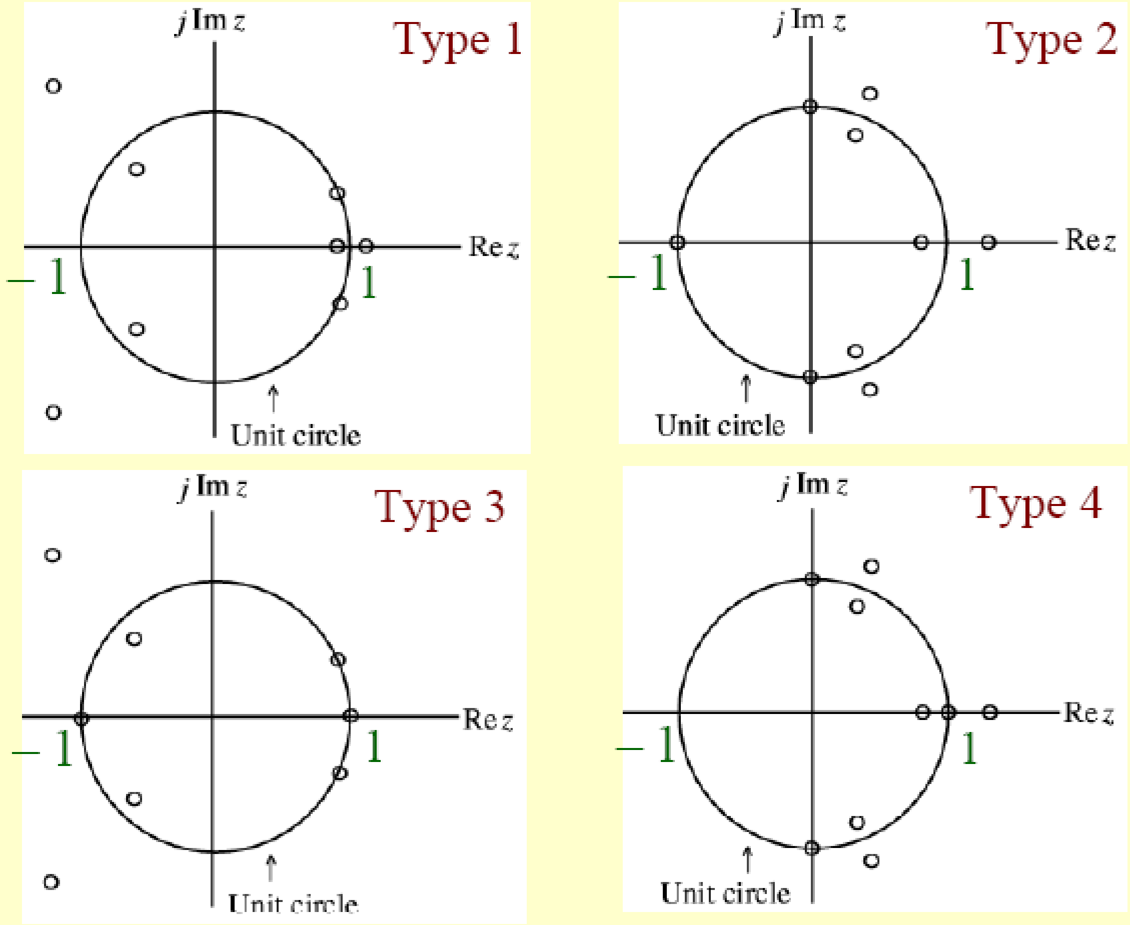
(c) z_i在实轴上但不在单位圆上

(d) z_i既在单位圆上又在实轴上

图 4.2　线性相位 FIR 滤波器的四种不同零点结构

Zero Locations of Linear-Phase FIR Transfer Functions

- Typical zero locations shown below



Zero Locations of Linear-Phase FIR Transfer Functions

Type 1	Type 2	Type 3	Type 4
No restriction Can design any type	Cannot design highpass and bandstop Zero at $\omega = \pi$	Cannot design lowpass, highpass, and bandstop Zero at $\omega = 0$ and $\omega = \pi$	Cannot design lowpass, and bandstop Zero at $\omega = 0$

线性相位滤波器是FIR滤波器中最重要的一种，应用最广。实际使用时应根据需用选择其合适类型，并在设计时遵循其约束条件。

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Simple FIR Digital Filters

Lowpass FIR Digital Filters

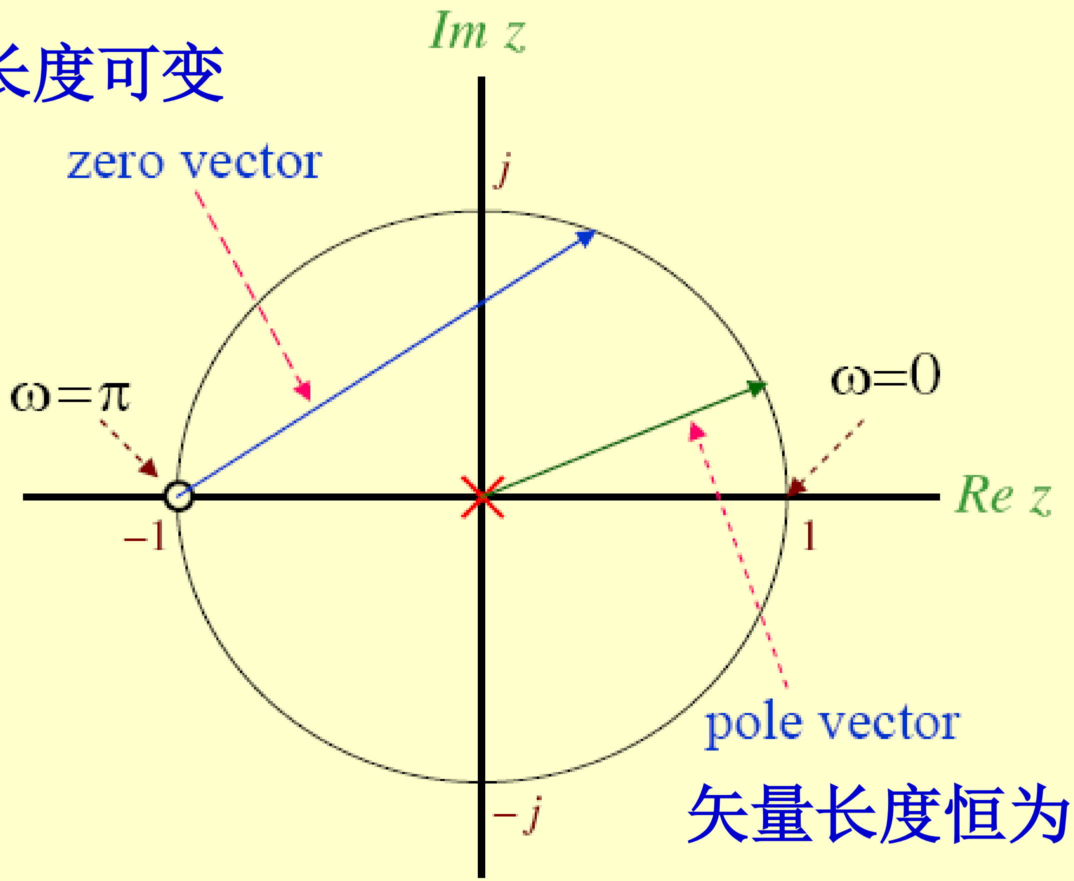
- The simplest lowpass FIR digital filter is the 2-point moving-average filter given by

$$H_0(z) = \tfrac{1}{2}(1 + z^{-1}) = \frac{z+1}{2z}$$

- The above transfer function has a zero at $z = -1$ and a pole at $z = 0$
- Note that here the pole vector has a unity magnitude for all values of ω

Simple FIR Digital Filters

矢量长度可变



矢量长度恒为1

幅度谱值＝零点矢量长度 / 极点矢量长度

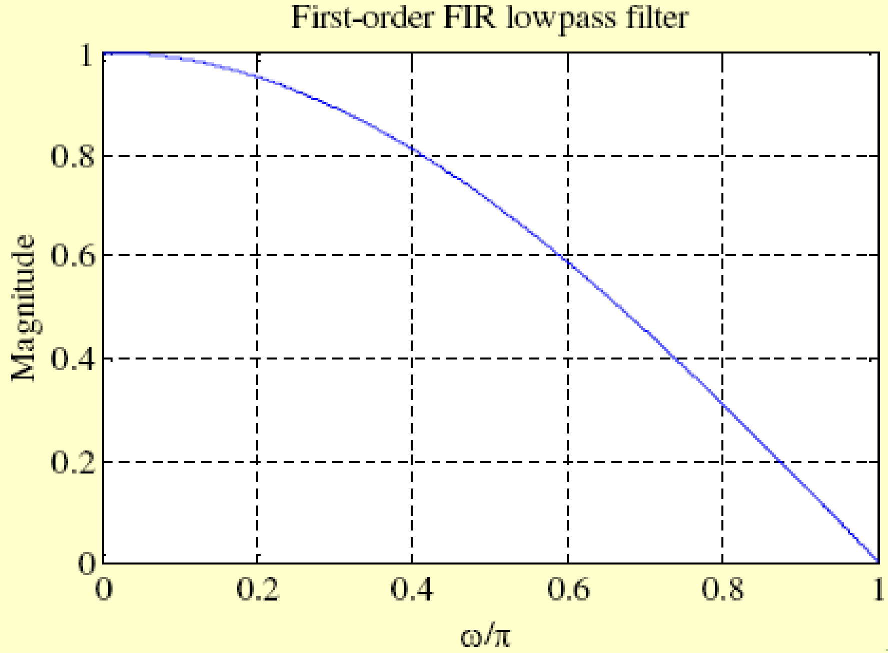
Simple FIR Digital Filters

- The frequency response

$$H_0(e^{j\omega}) = e^{-j\omega/2}\cos(\omega/2)$$

- The magnitude response

$$|H_0(e^{j\omega})| = \cos(\omega/2)$$



Simple FIR Digital Filters

- The frequency $\omega = \omega_c$ at which

$$\left|H_0(e^{j\omega_c})\right| = \frac{1}{\sqrt{2}}\left|H_0(e^{j0})\right|$$

is of practical interest since here the gain $\mathcal{G}(\omega_c)$ in dB is given by

$$\mathcal{G}(\omega_c) = 20\log_{10}\left|H(e^{j\omega_c})\right|$$

$$= 20\log_{10}\left|H(e^{j0})\right| - 20\log_{10}\sqrt{2} \cong -3 \text{ dB}$$

since the dc gain $\mathcal{G}(0) = 20\log_{10}\left|H(e^{j0})\right| = 0$

Simple FIR Digital Filters

- Thus, the gain $\mathcal{G}(\omega)$ at $\omega = \omega_c$ is approximately 3 dB less than the gain at $\omega = 0$
- As a result, ω_c is called the **3-dB cutoff frequency**
- To determine the value of ω_c we set

$$|H_0(e^{j\omega_c})|^2 = \cos^2(\omega_c/2) = \tfrac{1}{2}$$

which yields $\omega_c = \pi/2$

Simple FIR Digital Filters

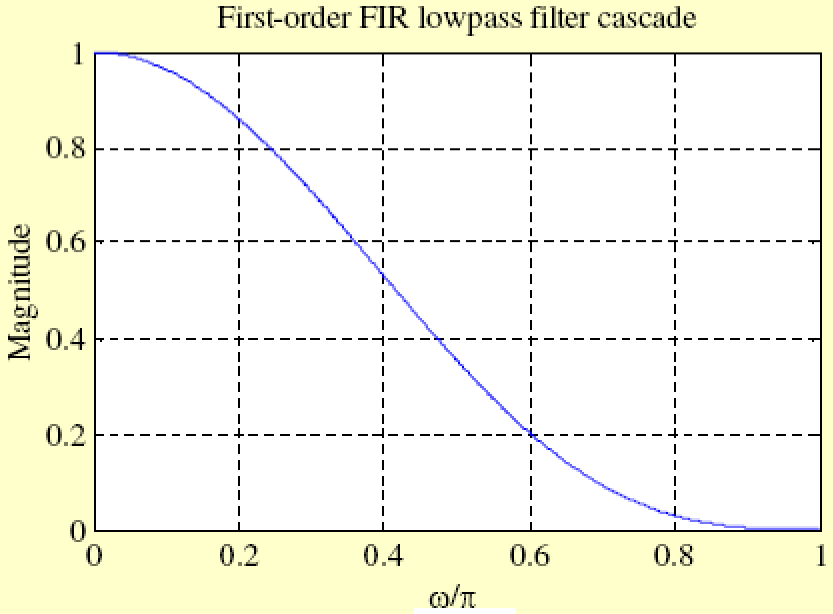
- The 3-dB cutoff frequency ω_c can be considered as the passband edge frequency
- As a result, for the filter $H_0(z)$ the passband width is approximately $\pi/2$
- The stopband is from $\pi/2$ to π

Simple FIR Digital Filters

- A cascade of the simple FIR filter

$$H_0(z) = \tfrac{1}{2}(1 + z^{-1})$$

results in an improved lowpass frequency response as illustrated below for a cascade of 3 sections



Simple FIR Digital Filters

- The 3-dB cutoff frequency of a cascade of M sections is given by

$$\omega_c = 2\cos^{-1}(2^{-1/2M})$$

- For $M = 3$, the above yields $\omega_c = 0.302\pi$
- Thus, the cascade of first-order sections yields a sharper magnitude response but at the expense of a decrease in the width of the passband
- A better approximation to the ideal lowpass filter is given by a higher-order moving-average filter

Simple FIR Digital Filters

Highpass FIR Digital Filters

- The simplest highpass FIR filter is obtained from the simplest lowpass FIR filter by replacing z with $-z$
- This results in

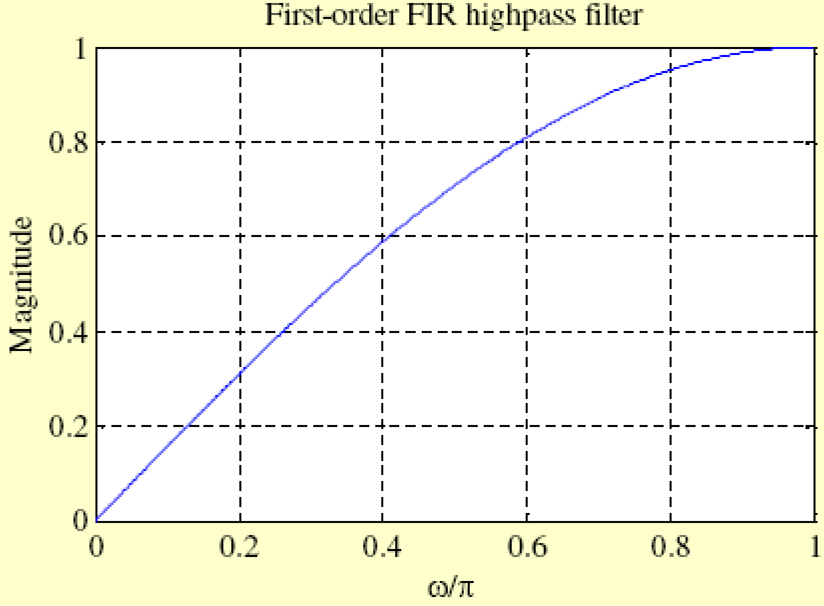
$$H_1(z) = \frac{1}{2}(1 - z^{-1})$$

Simple FIR Digital Filters

- Corresponding frequency response is given by

$$H_1(e^{j\omega}) = j\,e^{-j\omega/2}\sin(\omega/2)$$

whose magnitude response is plotted below



Simple IIR Digital Filters

Lowpass IIR Digital Filters

- We have shown earlier that the first-order causal IIR transfer function

$$H(z) = \frac{K}{1 - \alpha z^{-1}}, \quad 0 < \alpha < 1$$

has a lowpass magnitude response for $\alpha > 0$

Simple IIR Digital Filters

- An improved lowpass magnitude response is obtained by adding a factor $(1+z^{-1})$ to the numerator of transfer function

$$H(z) = \frac{K(1+z^{-1})}{1-\alpha z^{-1}}, \quad \boxed{0 < \alpha < 1}$$

- This forces the magnitude response to have a zero at $\omega = \pi$ in the stopband of the filter

Simple IIR Digital Filters

- On the other hand, the first-order causal IIR transfer function

$$H(z) = \frac{K}{1 - \alpha z^{-1}}, \quad \boxed{-1 < \alpha < 0}$$

has a highpass magnitude response for $\alpha < 0$

Simple IIR Digital Filters

- However, the modified transfer function obtained with the addition of a factor $(1+z^{-1})$ to the numerator

$$H(z) = \frac{K(1+z^{-1})}{1-\alpha z^{-1}}, \quad -1 < \alpha < 0$$

exhibits a lowpass magnitude response

Simple IIR Digital Filters

- The modified first-order lowpass transfer function for both positive and negative values of α is then given by

$$H_{LP}(z) = \frac{K(1+z^{-1})}{1-\alpha z^{-1}}, \quad 0 < |\alpha| < 1$$

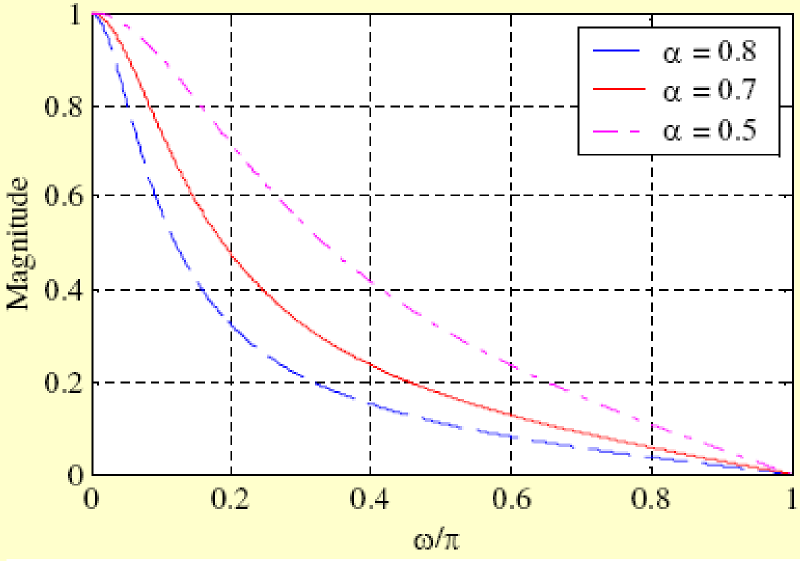
- $|H_{LP}(e^{j\omega})|$ is a monotonically decreasing function of ω from $\omega = 0$ to $\omega = \pi$

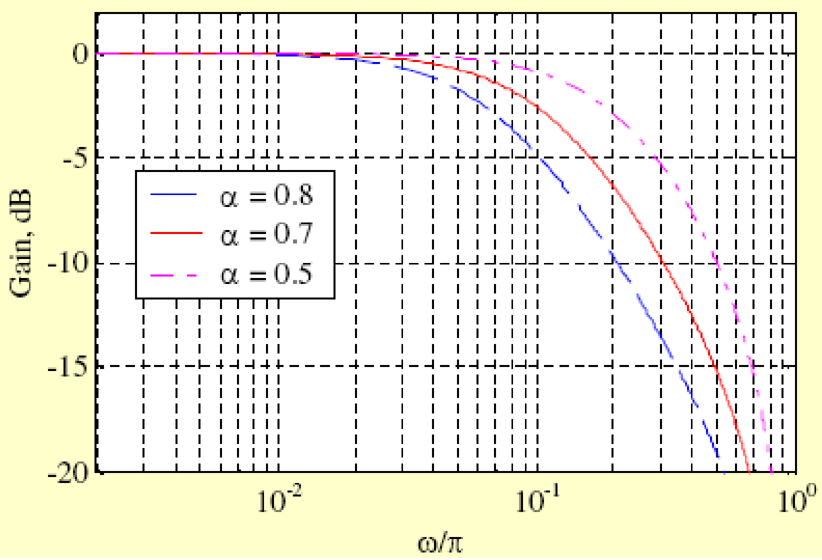
Simple IIR Digital Filters

- To this end, we choose $K = (1-\alpha)/2$ resulting in the first-order IIR lowpass transfer function

$$H_{LP}(z) = \frac{1-\alpha}{2}\left(\frac{1+z^{-1}}{1-\alpha z^{-1}}\right), \quad 0 < |\alpha| < 1$$

- $|H_{LP}(e^{j\omega})|$





Simple IIR Digital Filters

- To determine the 3-dB cutoff frequency we set

$$|H_{LP}(e^{j\omega_c})|^2 = \frac{1}{2}$$

- The solution resulting in a stable transfer function $H_{LP}(z)$ is given by

$$\alpha = \frac{1 - \sin\omega_c}{\cos\omega_c}$$

(Thus, we can design a first-order lowpass IIR digital filter with a specified 3-dB cutoff frequency.)

Simple IIR Digital Filters

Highpass IIR Digital Filters

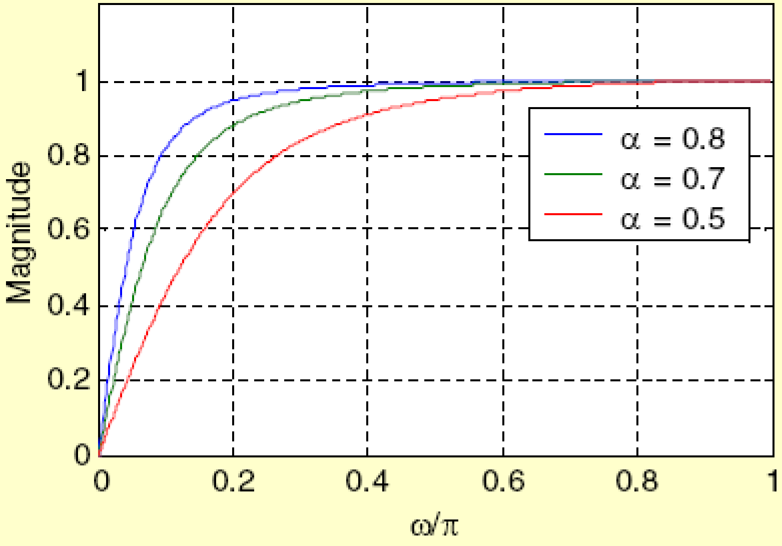
- A first-order causal highpass IIR digital filter has a transfer function given by

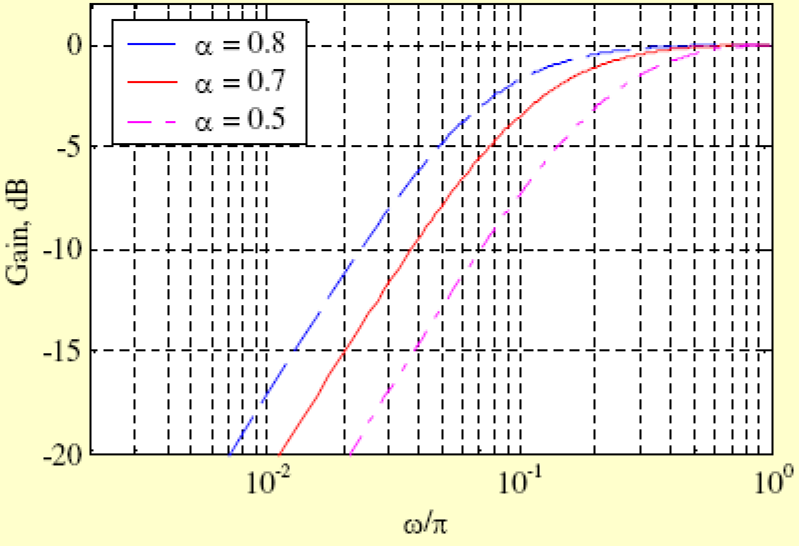
$$H_{HP}(z) = \frac{1+\alpha}{2}\left(\frac{1-z^{-1}}{1-\alpha z^{-1}}\right)$$

where $|\alpha| < 1$ for stability

Simple IIR Digital Filters

- Magnitude and gain responses of $H_{HP}(z)$ are shown below





Simple IIR Digital Filters

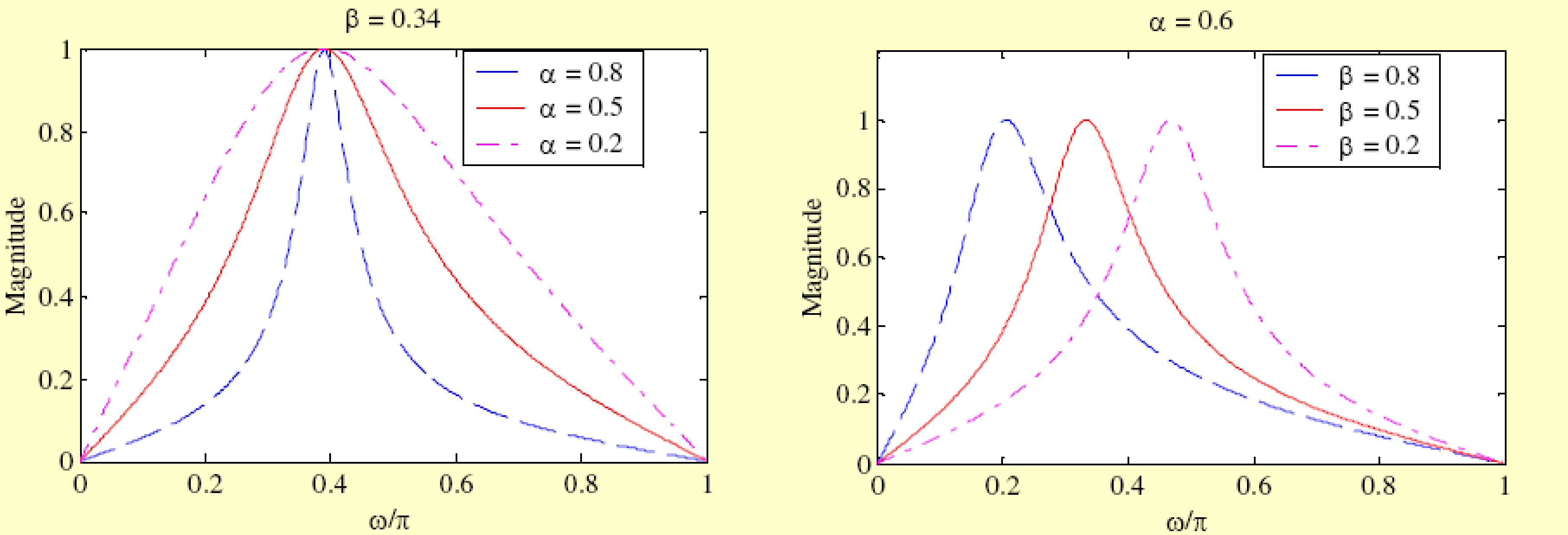
Bandpass IIR Digital Filters

- A 2nd-order bandpass digital transfer function is given by

$$H_{BP}(z) = \frac{1-\alpha}{2}\left(\frac{1-z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}\right)$$

Simple IIR Digital Filters

- Plots of $|H_{BP}(e^{j\omega})|$ are shown below



Simple IIR Digital Filters

- It assumes a maximum value of 1 at $\omega = \omega_o$, called the **center frequency** of the bandpass filter
- The frequencies ω_{c1} and ω_{c2} where $|H_{BP}(e^{j\omega})|^2$ becomes 1/2 are called the **3-dB cutoff frequencies**
- The difference between the two cutoff frequencies, assuming $\omega_{c2} > \omega_{c1}$ is called the **3-dB bandwidth**

Simple IIR Digital Filters

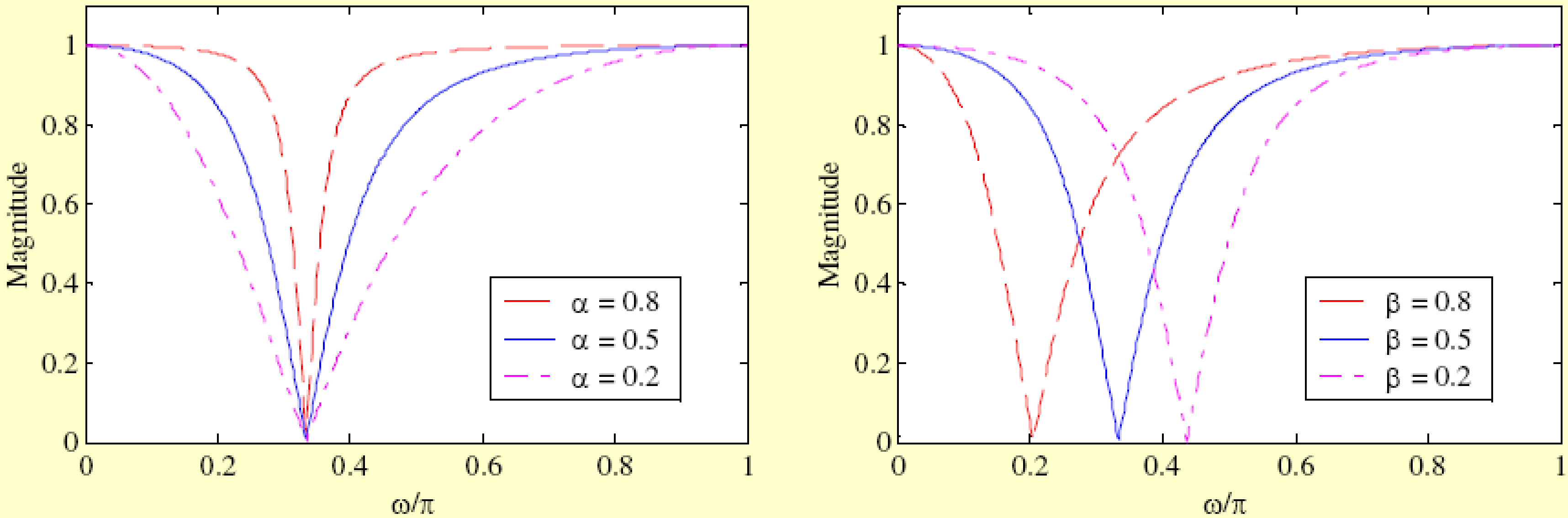
Bandstop IIR Digital Filters

- A 2nd-order bandstop digital filter has a transfer function given by

$$H_{BS}(z) = \frac{1+\alpha}{2}\left(\frac{1-2\beta z^{-1}+z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}\right)$$

Simple IIR Digital Filters

- Its magnitude response is plotted below



- It goes to 0 at $\omega = \omega_o$, where ω_o , called the **notch frequency**
- The digital transfer function $H_{BS}(z)$ is more commonly called a **notch filter**

Homework

- **7.7 (证明带通)，7.8（画滤波器组的频响）**
- **7.39、7.45 (linear phase FIR filter)**
- **7.55(simple filter and cutoff)**