Chapter 8

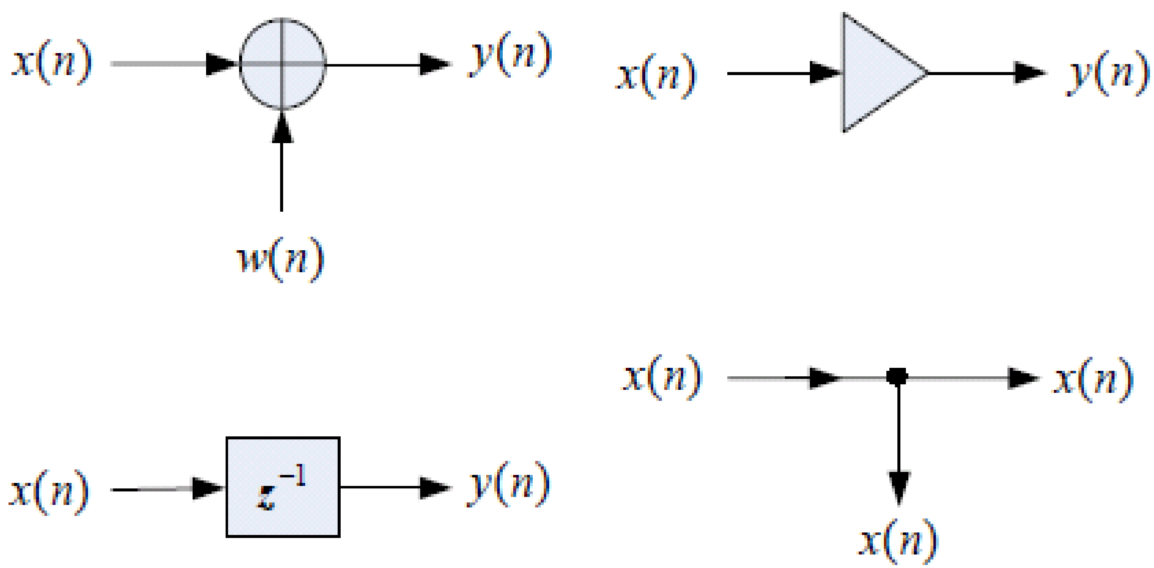
Digital Filter Structures

Digital Filter Structures

- **8.1 Block Diagram Representation**
- **8.2 Equivalent Structures**
- **8.3 Basic FIR Digital Filter Structures**
- **8.4 Basic IIR Digital Filter Structures**
- **8.5 Realization of Basic Structures Using MATLAB**
- **8.6 Allpass Filters**
- **8.7 IIR Tapped Cascaded Lattice Structures**
- **8.8 FIR Tapped Cascaded Lattice Structures**

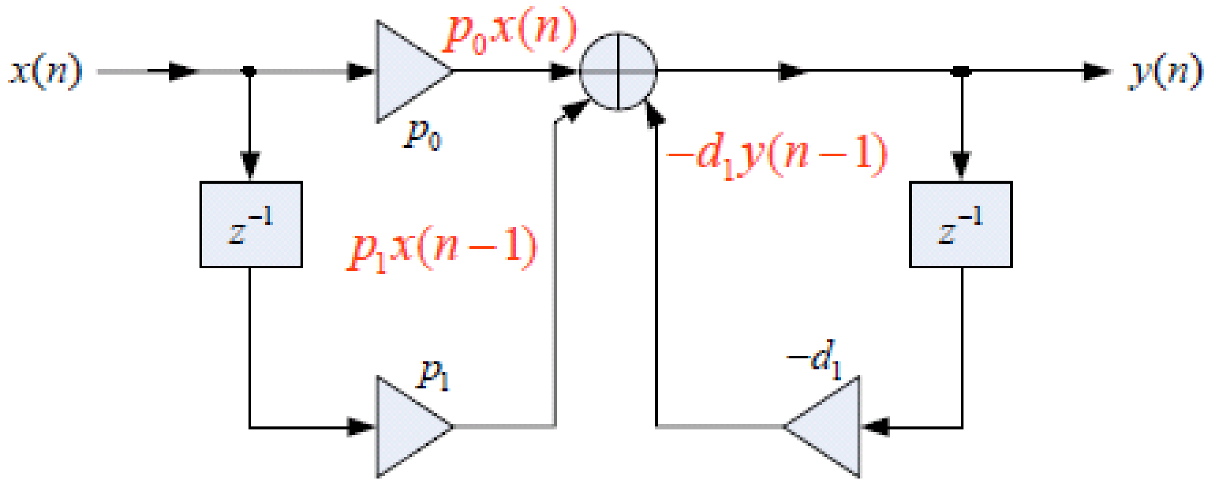
8.1 Block Diagram Representation

- The computational algorithm of an LTI digital filter can be conveniently represented in block diagram form using the basic building blocks shown below



8.1 Block Diagram Representation

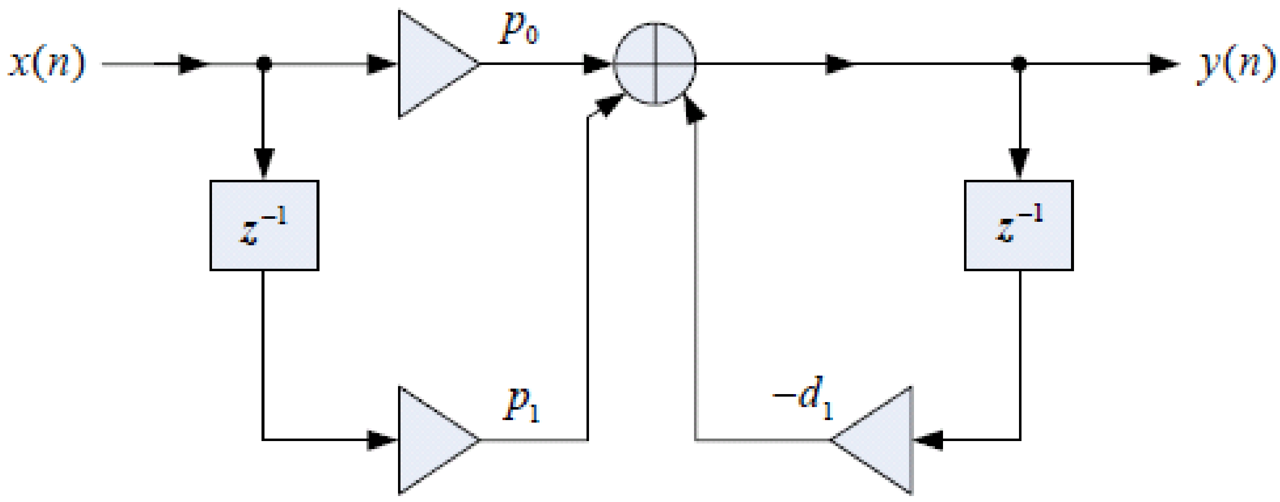
- To illustrate what we mean by a computational algorithm, consider the causal first-order LTI digital filter shown below



$$y(n) = -d_1 y(n-1) + p_0 x(n) + p_1 x(n-1)$$

8.1 Block Diagram Representation

$$y(n) = -d_1 y(n-1) + p_0 x(n) + p_1 x(n-1)$$



- Noncanonic: two delays to realize a first-order difference equation

8.3 Basic FIR Digital Filter Structures

- **Direct Form**
- **Cascade Form**
- **Linear-phase Structure**

8.3 Basic FIR Digital Filter Structures

- A causal FIR filter of order $N-1$ is characterized by a transfer function $H(z)$ given by

$$H(z) = \sum_{k=0}^{N-1} h(k) z^{-k}$$

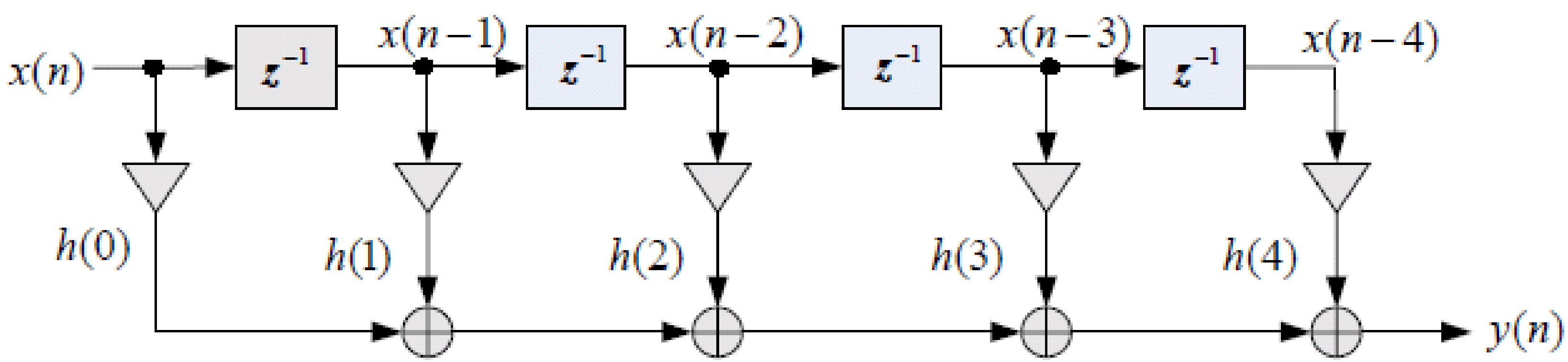
which is a polynomial in z^{-1}

- In the time-domain the input-output relation of the above FIR filter is given by

$$y(n) = \sum_{k=0}^{N-1} h(k) x(n-k)$$

8.3 Basic FIR Digital Filter Structures

- A direct form realization of an FIR filter can be readily developed from the convolution sum description as indicated below for $N = 5$



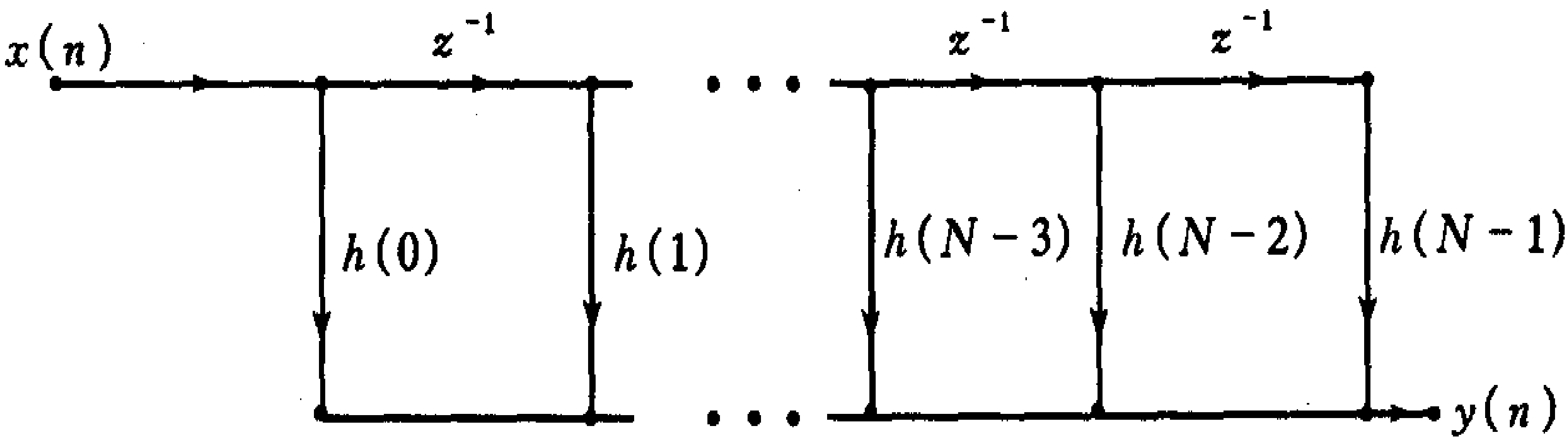
8.3 Basic FIR Digital Filter Structures

- An analysis of this structure yields

$$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2)$$
$$+h(3)x(n-3) + h(4)x(n-4)$$

which is precisely of the form of the convolution sum description

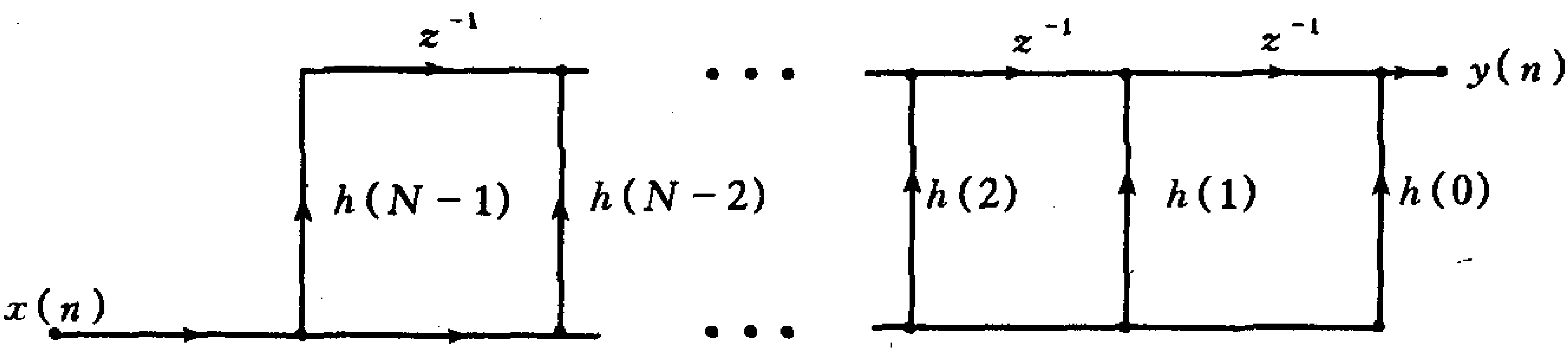
直接由差分方程可画出对应的网络结构：



FIR 滤波器的横截型结构

直接型的转置:

- The direct form structure shown on the previous slide is also known as a tapped delay line or a transversal filter.



横截型的转置结构

8.3 Basic FIR Digital Filter Structures

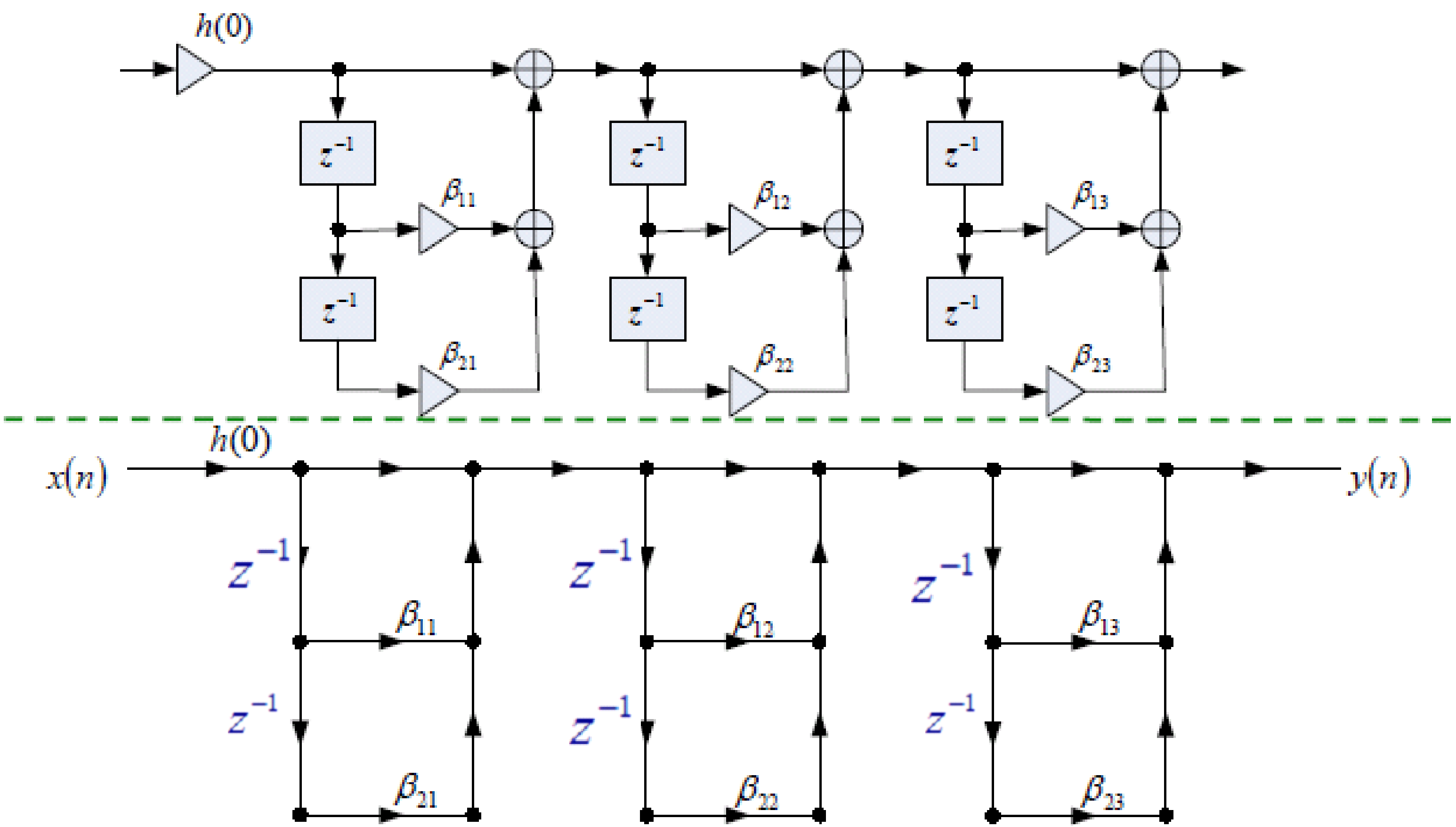
- A higher-order FIR transfer function can also be realized as a cascade of second order FIR sections and possibly a first-order section
- To this end we express $H(z)$ as

$$H(z) = h(0)\prod_{k=1}^{K}\left(1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}\right)$$

where $k = N/2$ if N is even, and $k = (N+1)/2$ if N is odd, with $\beta_{2k} = 0$

8.3 Basic FIR Digital Filter Structures

- A cascade realization for $N = 6$ is shown below



8.3 Basic FIR Digital Filter Structures

- **Linear-phase FIR filter of order N is characterized by the symmetric impulse response**

$$h[n]=h[N-n]$$

- **An antisymmetric impulse response condition**

$$h[n]=-h[N-n]$$

results in a constant group delay and "almost linear-phase" property

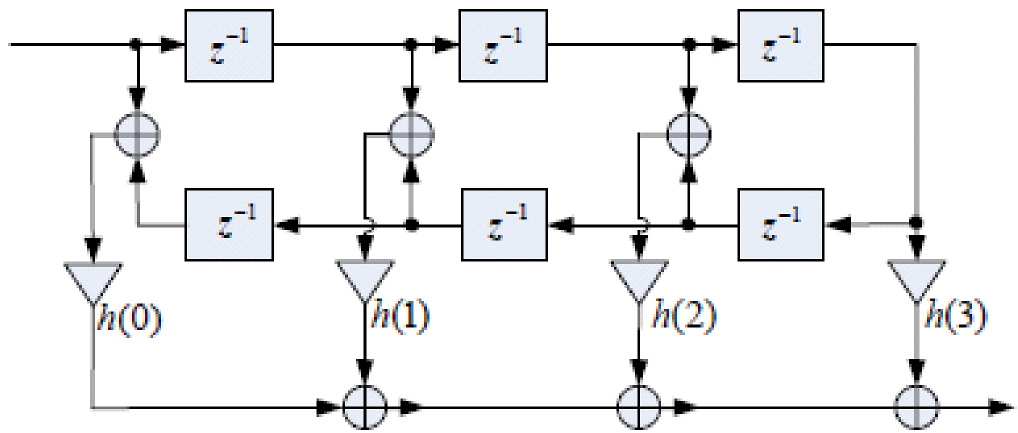
- **Symmetry of the impulse response coefficients can be used to reduce the number of multiplications**

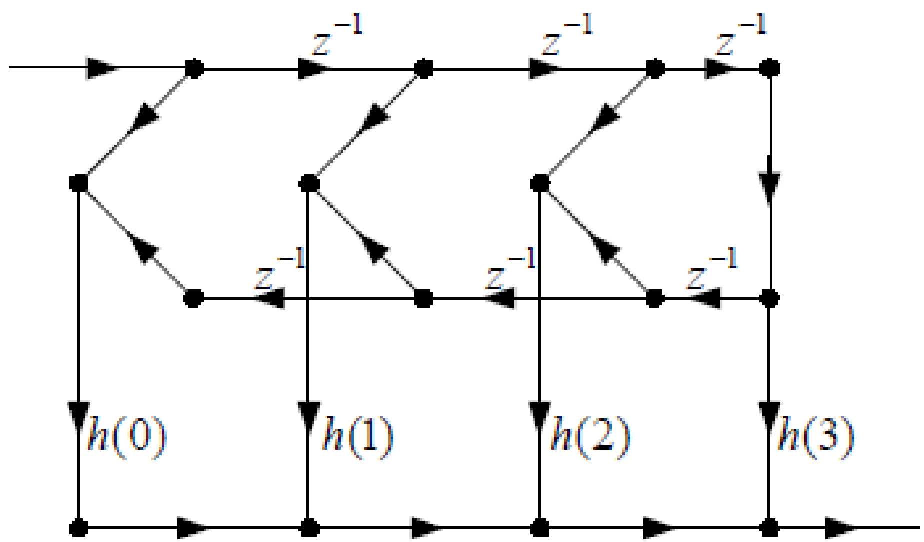
8.3 Basic FIR Digital Filter Structures

- Length M is odd (M=7) **Order N＝M－1**

$$H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3}$$
$$+h(2)z^{-4} + h(1)z^{-5} + h(0)z^{-6}$$
$$= h(0)(1+z^{-6}) + h(1)(z^{-1}+z^{-5})$$
$$+h(2)(z^{-2}+z^{-4}) + h(3)z^{-3}$$

8.3 Basic FIR Digital Filter Structures





8.3 Basic FIR Digital Filter Structures

- Length M is even (M=8) **Order N＝M－1**

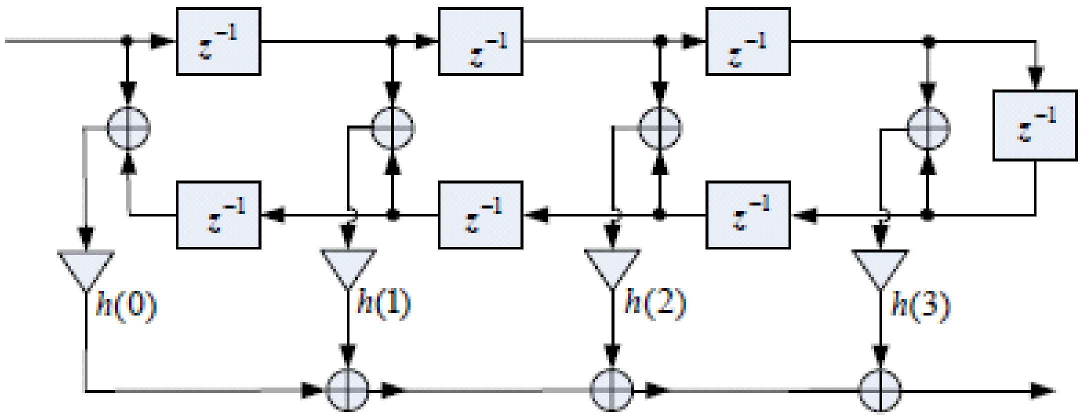
$$H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3}$$

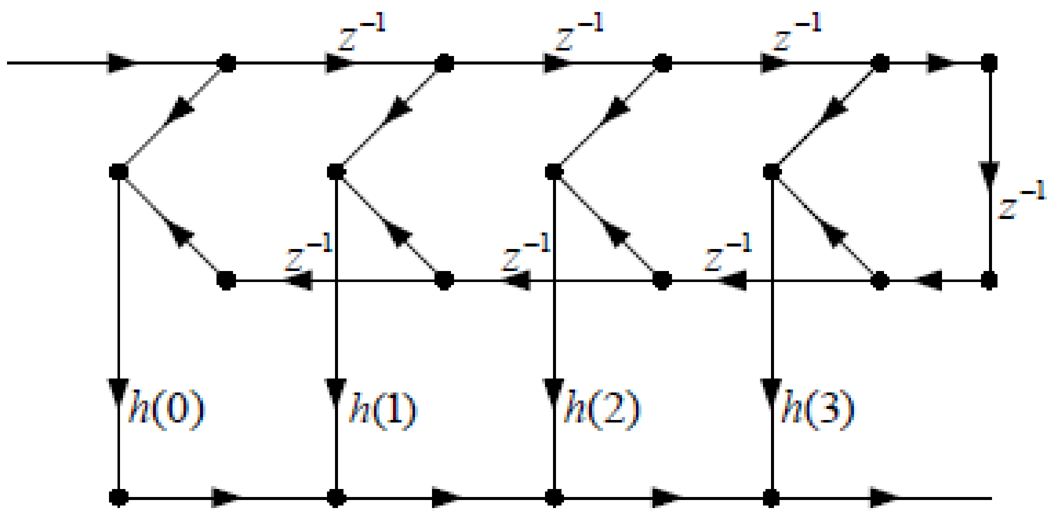
$$+h(3)z^{-4} + h(2)z^{-5} + h(1)z^{-6} + h(0)z^{-7}$$

$$= h(0)(1 + z^{-7}) + h(1)(z^{-1} + z^{-6})$$

$$+h(2)(z^{-2} + z^{-5}) + h(3)(z^{-3} + z^{-4})$$

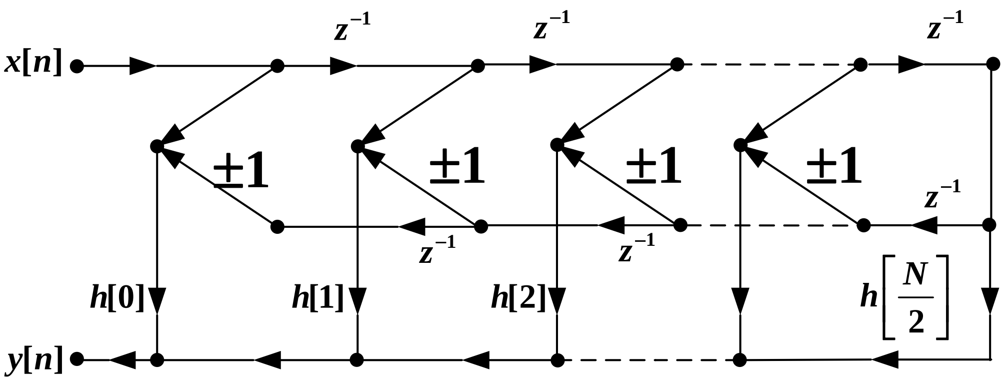
8.3 Basic FIR Digital Filter Structures





3、线性相位型

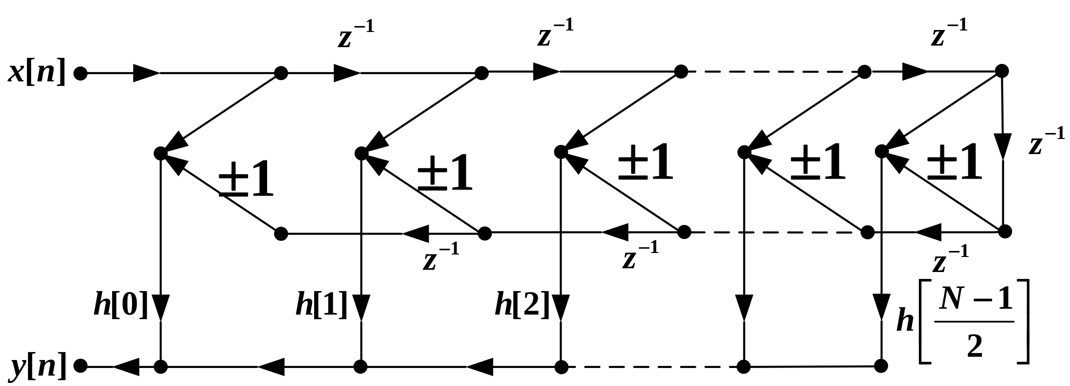
Type 1 and 3



（N/2+1）乘法器

直接型（N+1）个乘法器

Type 2 and 4



(N+1)/2乘法器

8.4 Basic IIR Digital Filter Structures

- **Direct Form**
- **Cascade Form**
- **Parallel Form**

8.4 Basic IIR Digital Filter Structures

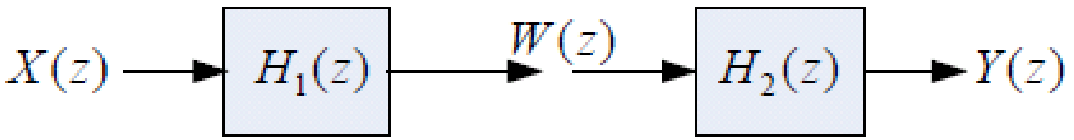
- **The causal IIR digital filters we are concerned with in this course are characterized by a real rational transfer function of or, equivalently by a constant coefficient difference equation.**
- **From the difference equation representation, it can be seen that the realization of the causal IIR digital filters requires some form of feedback.**

8.4 Basic IIR Digital Filter Structures

- Direct forms -- Coefficients are directly the transfer function coefficients
- Consider for simplicity a 3rd-order IIR filter with a transfer function (assuming $d_0 = 1$)

$$H(z) = \frac{P(z)}{D(z)} = \frac{p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3}}{1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3}}$$

- We can implement $H(z)$ as a cascade of two filter sections as shown below

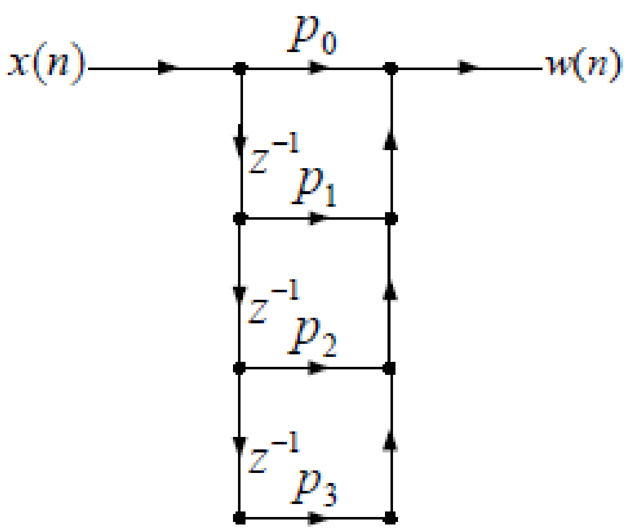


8.4 Basic IIR Digital Filter Structures

- where $H_1(z) = P(z) = p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3}$

  $H_2(z) = 1 / D(z)$

- The filter section $H_1(z)$ can be seen to be an FIR filter and can be realized as shown below

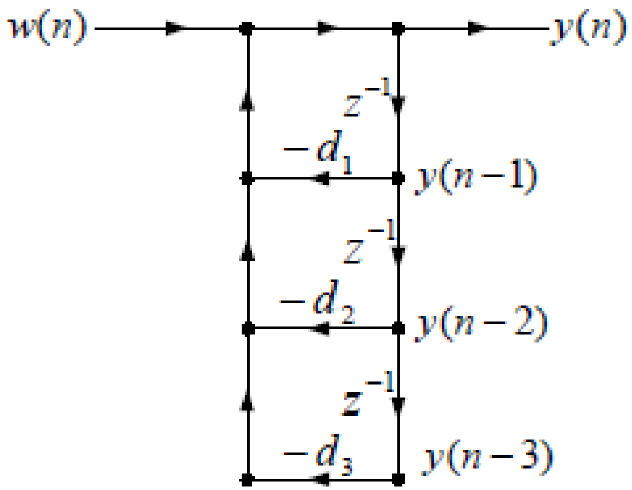


8.4 Basic IIR Digital Filter Structures

- The time-domain representation of $H_2(z)$ is given by

$$y(n) = w(n) - d_1 y(n-1) - d_2 y(n-2) - d_3 y(n-3)$$

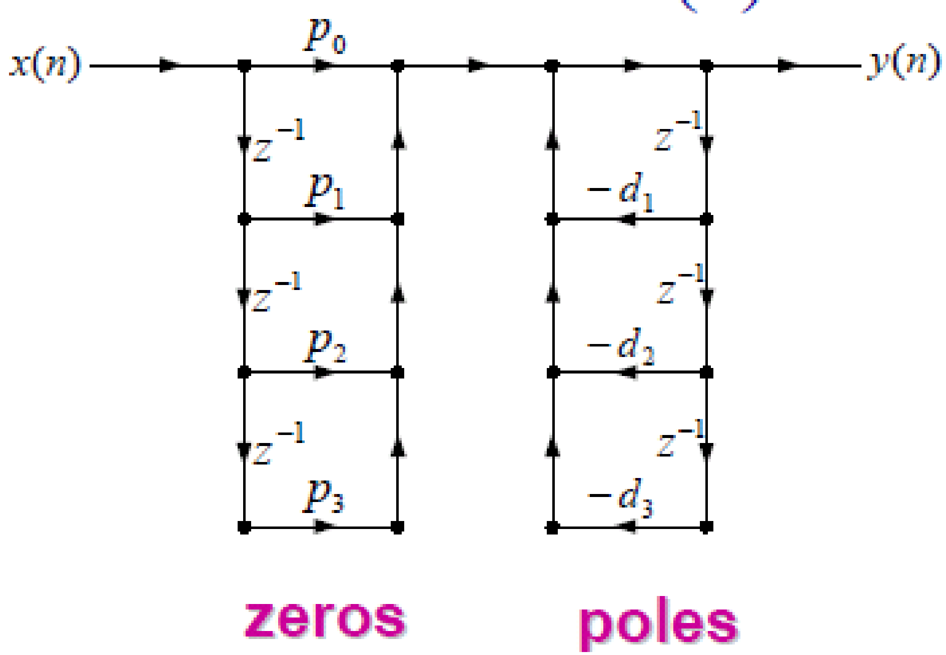
- Realization of $H_2(z)$ follows from the above equation and is shown below



8.4 Basic IIR Digital Filter Structures

- Considering the basic cascade realization results in *Direct form* I :

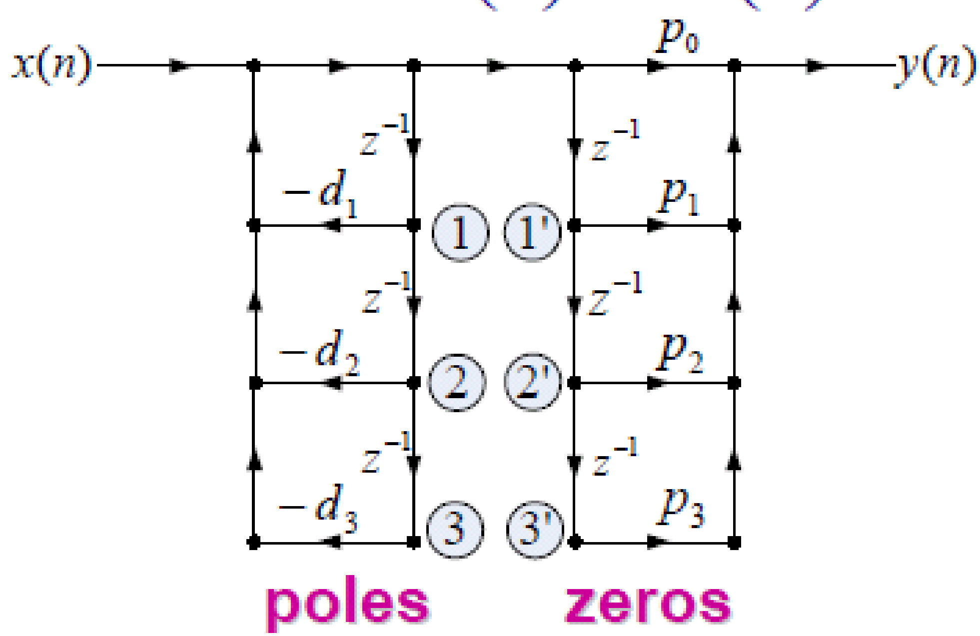
$$H(z) = P(z) \cdot \frac{1}{D(z)}$$



8.4 Basic IIR Digital Filter Structures

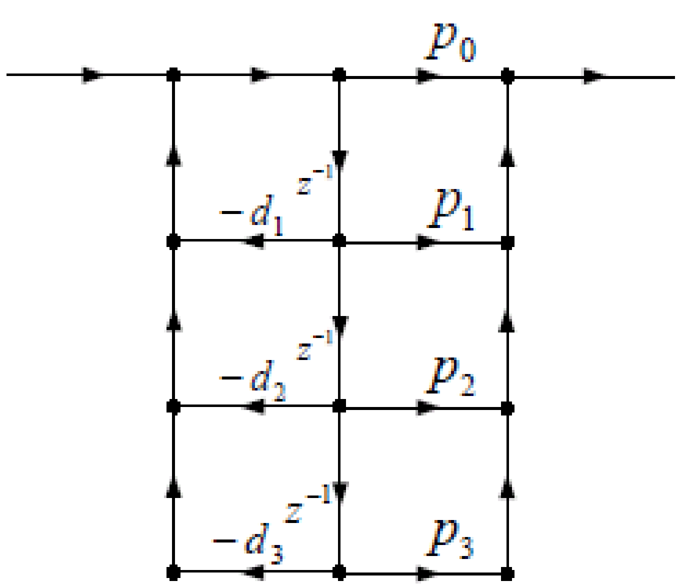
- Changing the order of blocks in cascade results in *Direct form* II :

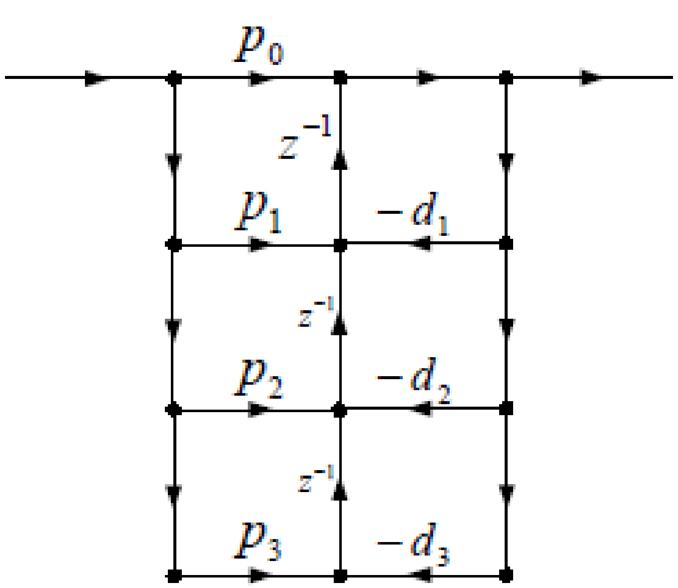
$$H(z) = P(z) \cdot \frac{1}{D(z)} = \frac{1}{D(z)} \cdot P(z)$$



8.4 Basic IIR Digital Filter Structures

- Sharing of all delays reduces the total number of delays to 3 resulting in a canonic realization shown below along with its transpose structure.





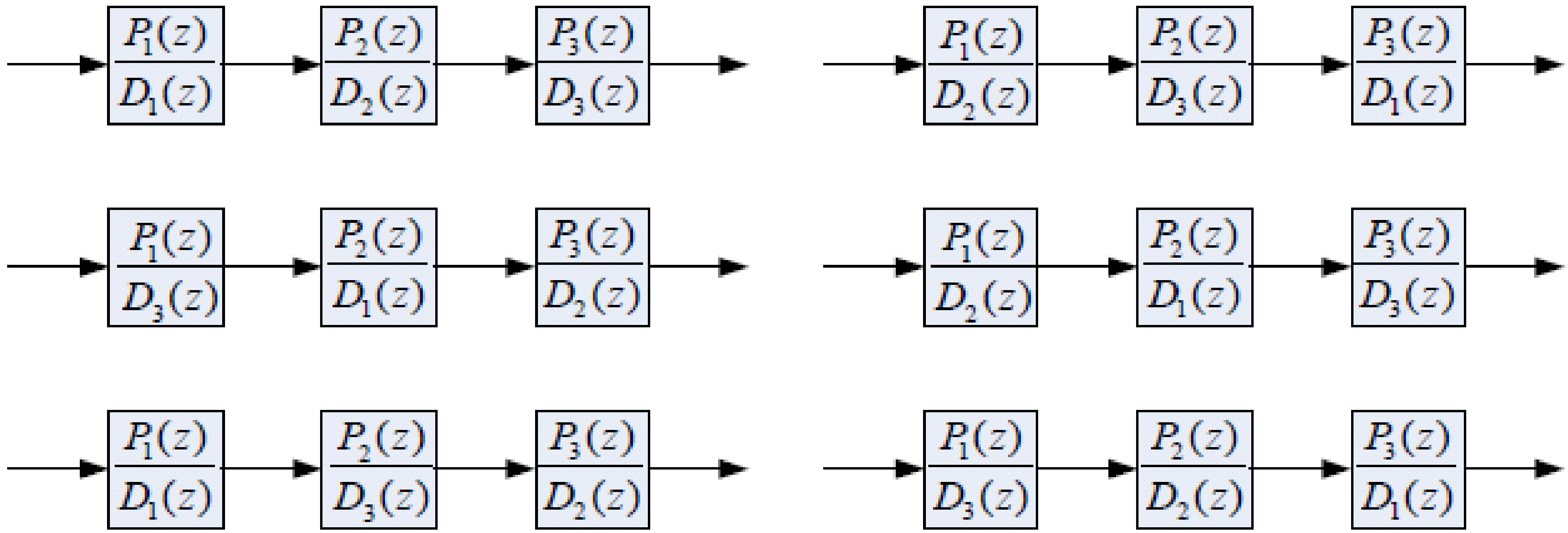
8.4 Basic IIR Digital Filter Structures

- By expressing the numerator and the denominator polynomials of the transfer function as a product of polynomials of lower degree, a digital filter can be realized as a cascade of low-order filter sections
- Consider, for example, $H(z)=P(z)/D(z)$ expressed as

$$H(z)=\frac{P(z)}{D(z)}=\frac{P_1(z)P_2(z)P_3(z)}{D_1(z)D_2(z)D_3(z)}$$

8.4 Basic IIR Digital Filter Structures

- Examples of cascade realizations obtained by different pole-zero pairings are shown below



8.4 Basic IIR Digital Filter Structures

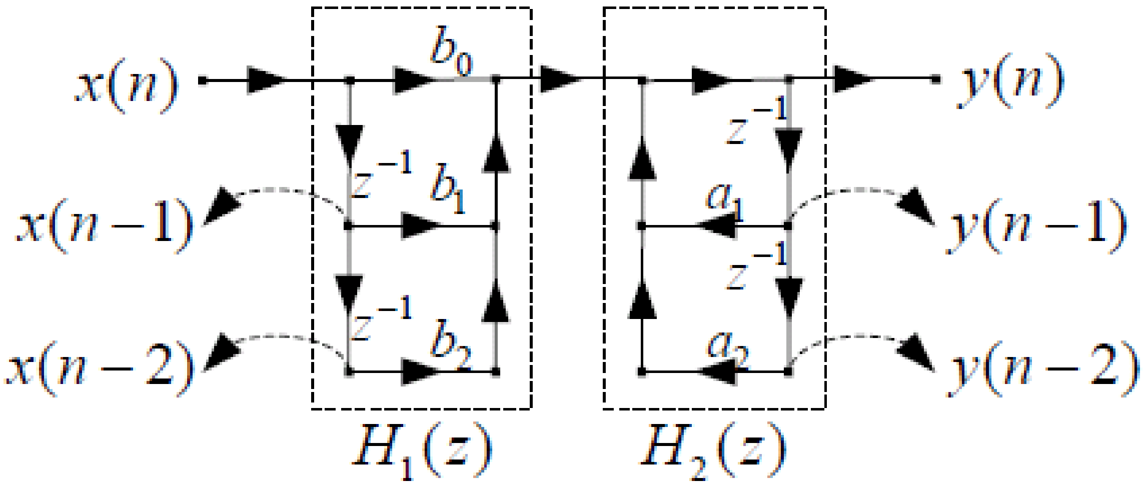
- There are altogether a total of 36 $(P_3^2 \cdot P_3^2)$ different cascade realizations of

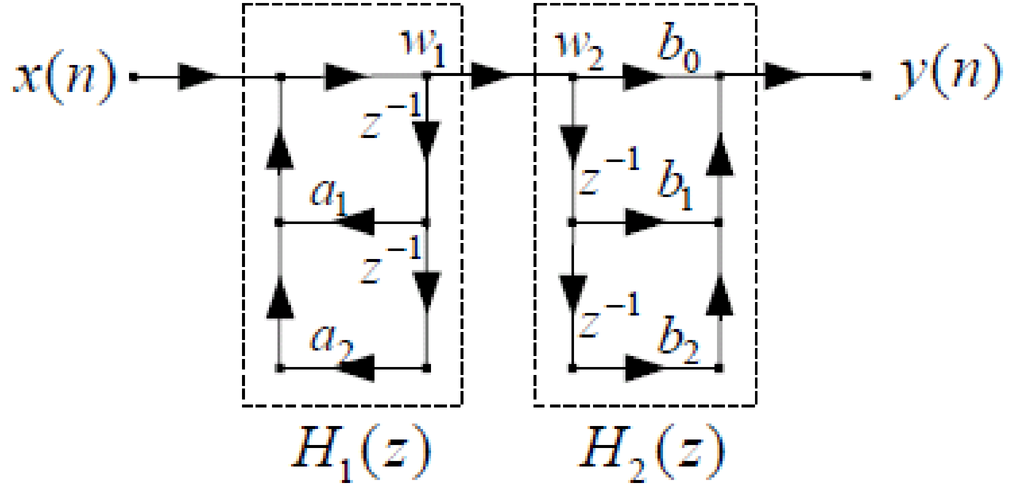
$$H(z) = \frac{P(z)}{D(z)} = \frac{P_1(z)P_2(z)P_3(z)}{D_1(z)D_2(z)D_3(z)}$$

based on pole-zero-pairings and ordering

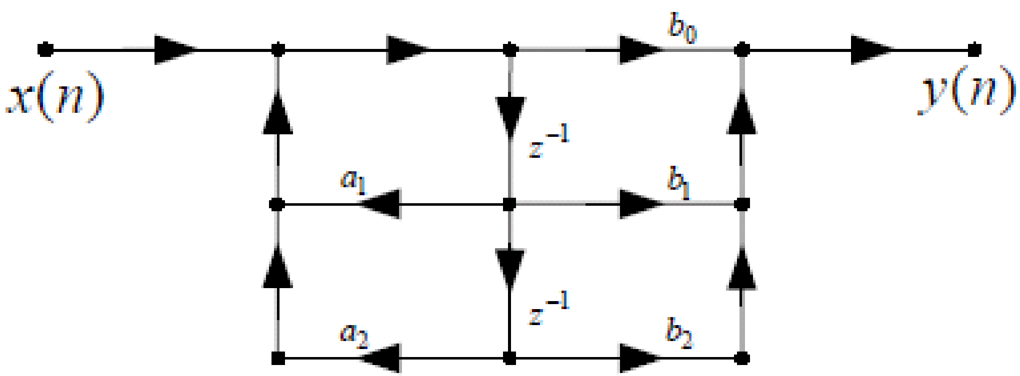
- Due to finite wordlength effects, each such cascade realization behaves differently from Others

8.4 Basic IIR Digital Filter Structures





8.4 Basic IIR Digital Filter Structures



- Usually, the polynomials are factored into a product of 1st-order and 2nd-order polynomials:

$$H(z) = p_0 \prod_k \left(\frac{1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}{1 + \alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}} \right)$$

for a first-order factor $\alpha_{2k} = \beta_{2k} = 0$

8.4 Basic IIR Digital Filter Structures

- Realizing complex conjugate poles and zeros with second order blocks results in real coefficients

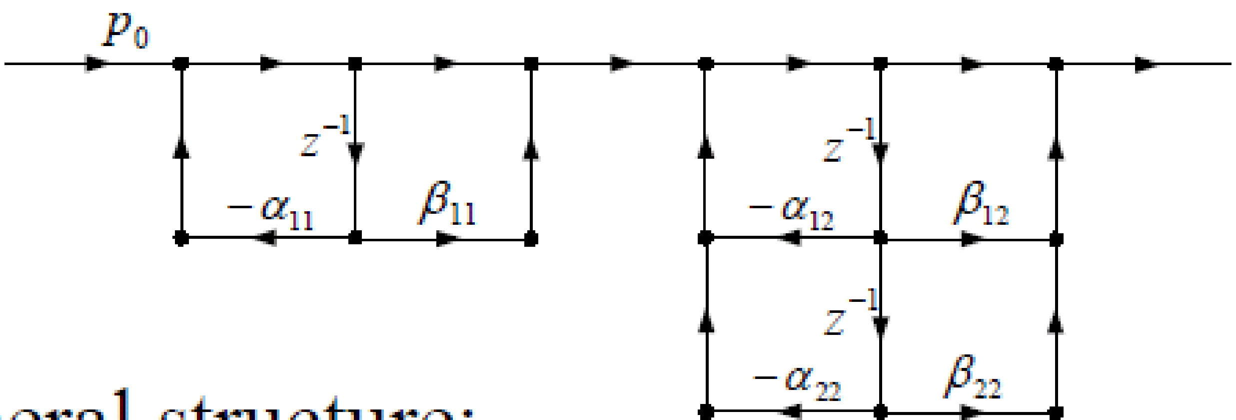
Example

- Third order transfer function

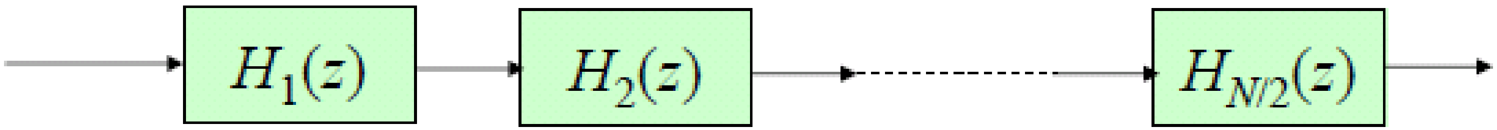
$$H(z) = \frac{P(z)}{D(z)} = p_0 \left(\frac{1 + \beta_{11} z^{-1}}{1 + \alpha_{11} z^{-1}} \right) \left(\frac{1 + \beta_{12} z^{-1} + \beta_{22} z^{-2}}{1 + \alpha_{12} z^{-1} + \alpha_{22} z^{-2}} \right)$$

8.4 Basic IIR Digital Filter Structures

- One possible realization is shown below



- General structure:



8.4 Basic IIR Digital Filter Structures

- Parallel realizations are obtained by making use of the partial fraction expansion of the transfer function

Parallel form I:

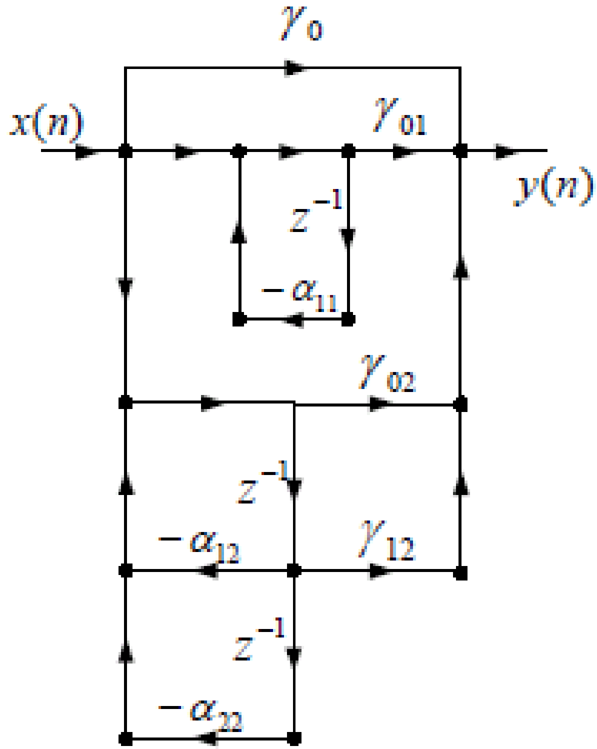
$$H(z) = \gamma_0 + \sum_k \left(\frac{\gamma_{0k} + \gamma_{1k} z^{-1}}{1 + \alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}} \right)$$

Parallel form II:

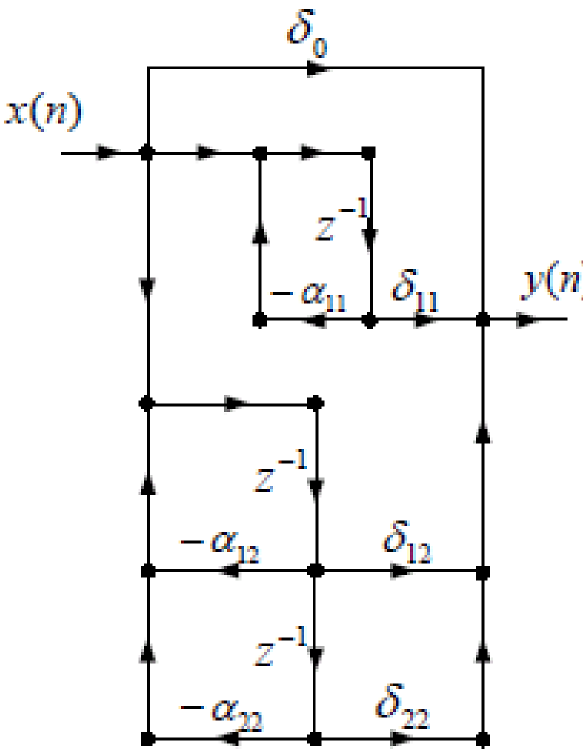
$$H(z) = \delta_0 + \sum_k \left(\frac{\delta_{1k} z^{-1} + \delta_{2k} z^{-2}}{1 + \alpha_{1k} z^{-1} + \alpha_{2k} z^{-2}} \right)$$

8.4 Basic IIR Digital Filter Structures

- The two basic parallel realizations of a 3rd order IIR transfer function are shown below



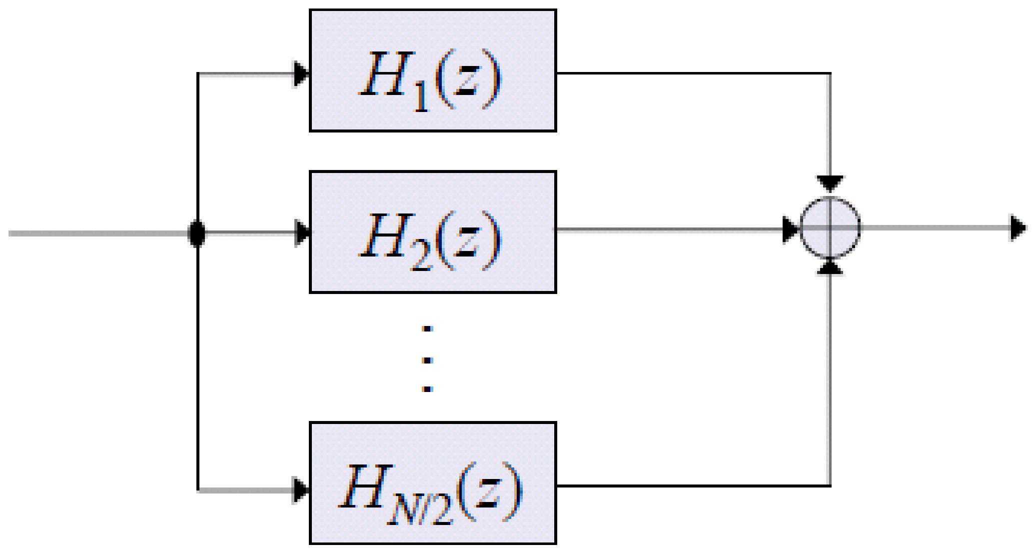
Parallel Form I



Parallel Form II

8.4 Basic IIR Digital Filter Structures

- General structure:



- Easy to realize:

  No choices in section ordering and
  No choices in pole and zero pairing

8.4 Basic IIR Digital Filter Structures

Example

- A partial-fraction expansion of

$$H(z) = \frac{0.44 + 0.362z^{-2} + 0.002z^{-3}}{1 + 0.4z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$

in z^{-1} yields

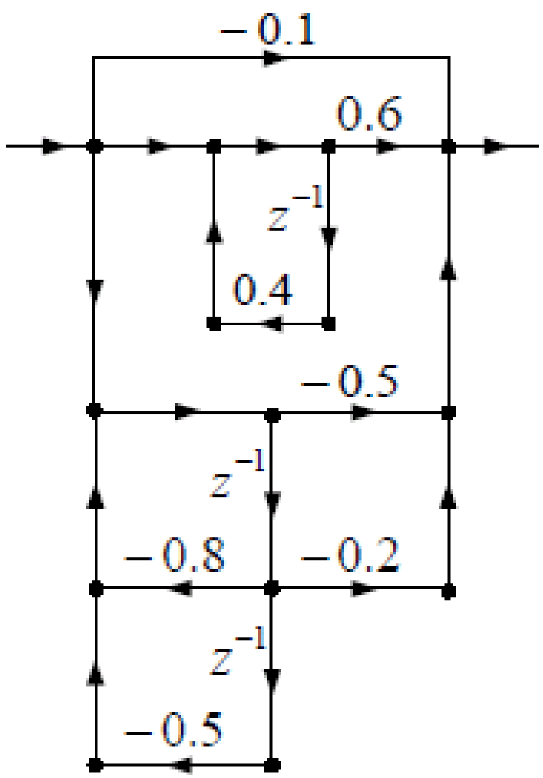
$$H(z) = -0.1 + \frac{0.6}{1 - 0.4z^{-1}} + \frac{-0.5 - 0.2z^{-1}}{1 + 0.8z^{-1} + 0.5z^{-2}}$$

- Likewise, a partial-fraction expansion of $H(z)$ in z yields

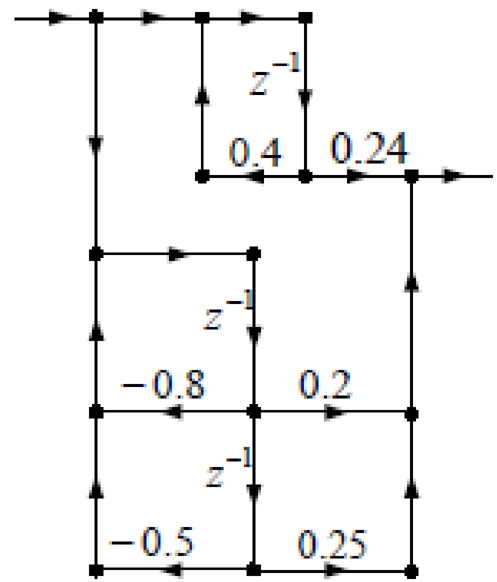
$$H(z) = \frac{0.24z^{-1}}{1 - 0.4z^{-1}} + \frac{0.2z^{-1} + 0.25z^{-2}}{1 + 0.8z^{-1} + 0.5z^{-2}}$$

8.4 Basic IIR Digital Filter Structures

- Their realizations are shown below



Parallel Form I



Parallel Form II

IIR滤波器结构比较

- 哪种结构对系数的量化效应最不敏感？
- 哪种结构运算效率最高？
- 直接I型与直接II型的优缺点比较。

Homework

- **Problems:8.13, 8.24(a), 8.28**
- **Matlab Exercises: M8.2**