# Chapter 9

## IIR Digital Filter Design

# Chapter 9

**9.1 Preliminary Considerations**

**9.2 Bilinear Transformation Method of IIR Filter Design**

**9.3 Design of Lowpass IIR Digital Filters**

**9.4 Design of Highpass, Bandpass and Bandstop IIR Digital Filters**

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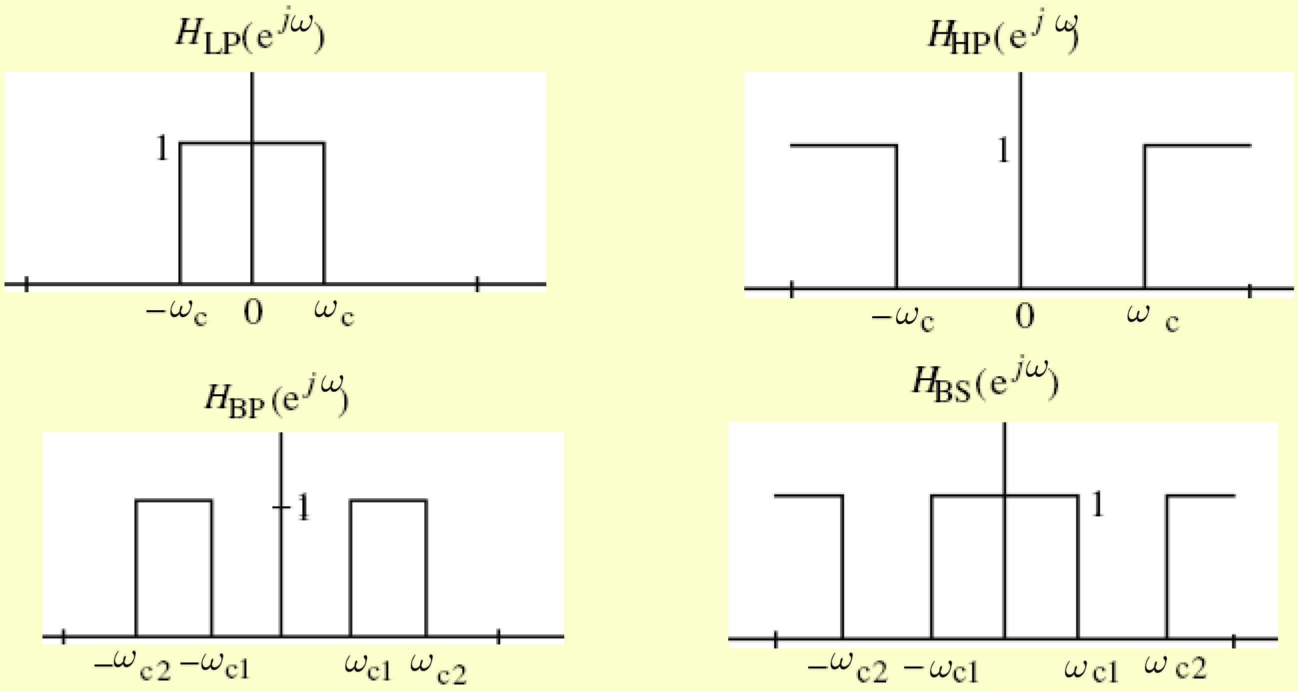
**9.6 IIR Digital Filter Design Using Matlab**

# Digital Filter Design

- Objective - Determination of a realizable transfer function $G(z)$ approximating a given frequency response specification is an important step in the development of a digital filter
- If an IIR filter is desired, $G(z)$ should be a stable real rational function
- Digital filter design is the process of deriving the transfer function $G(z)$

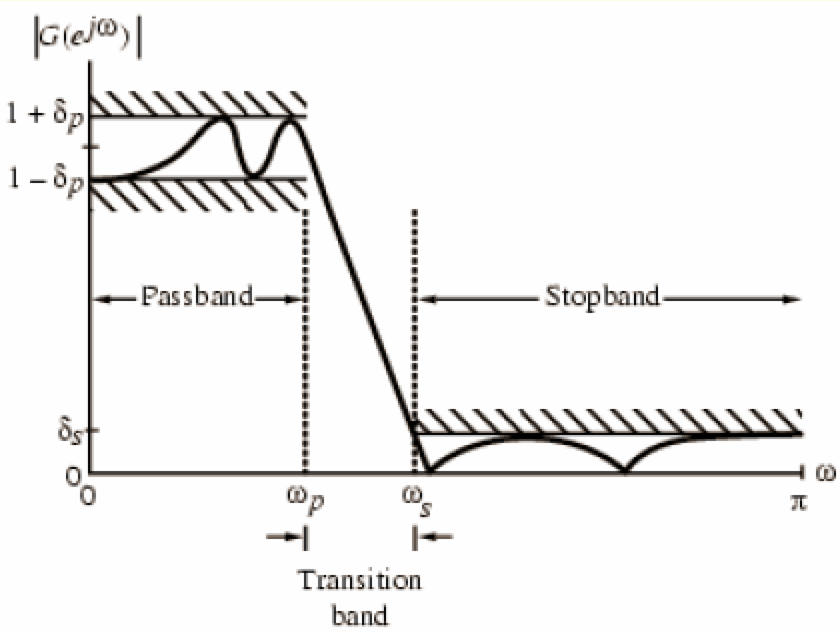
# Digital Filter Specifications

- We discuss in this course only the magnitude approximation problem
- There are four basic types of ideal filters with magnitude responses as shown below



# Digital Filter Specifications

- For example, the magnitude response $\left|G(e^{j\omega})\right|$ of a digital lowpass filter may be given as indicated below



# Digital Filter Specifications

- $\omega_p$ - **passband edge frequency**
- $\omega_s$ - **stopband edge frequency**
- $\delta_p$ - **peak ripple value** in the passband
- $\delta_s$ - **peak ripple value** in the stopband
- Since $G(e^{j\omega})$ is a periodic function of $\omega$, and $|G(e^{j\omega})|$ of a real-coefficient digital filter is an even function of $\omega$
- As a result, filter specifications are given only for the frequency range $0 \leq \omega \leq \pi$

# Digital Filter Specifications

- Specifications are often given in terms of **loss function** $\mathcal{A}(\omega) = -20\log_{10}\left|G(e^{j\omega})\right|$ in dB
- **Peak passband ripple**

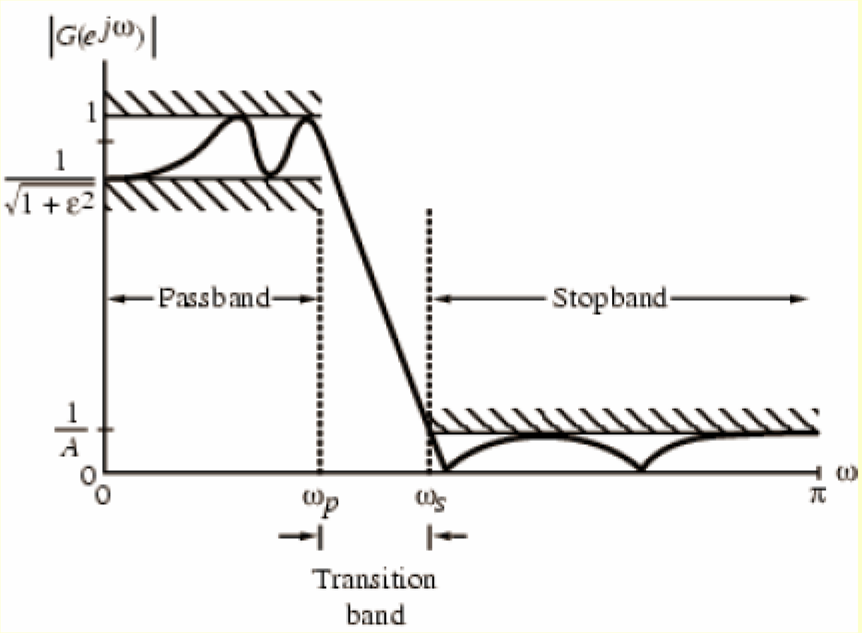
$$\alpha_p = -20\log_{10}(1-\delta_p) \quad \text{dB}$$

- **Minimum stopband attenuation**

$$\alpha_s = -20\log_{10}(\delta_s) \ \text{dB}$$

# Digital Filter Specifications

- Magnitude specifications may alternately be given in a normalized form as indicated below



# Digital Filter Specifications

- Here, the maximum value of the magnitude in the passband is assumed to be unity
- $1/\sqrt{1+\varepsilon^2}$ - Maximum passband deviation, given by the minimum value of the magnitude in the passband
- $\dfrac{1}{A}$ - Maximum stopband magnitude

# Digital Filter Specifications

- For the normalized specification, maximum value of the gain function or the minimum value of the loss function is 0 dB
- **Maximum passband attenuation** -

$$\alpha_{max} = 20\log_{10}\left(\sqrt{1+\varepsilon^2}\right) \text{ dB}$$

- For $\delta_p << 1$, it can be shown that

$$\alpha_{max} \cong -20\log_{10}(1-2\delta_p) \text{ dB}$$

# Digital Filter Specifications

- In practice, passband edge frequency $F_p$ and stopband edge frequency $F_s$ are specified in Hz
- For digital filter design, normalized bandedge frequencies need to be computed from specifications in Hz using

$$\omega_p = \frac{\Omega_p}{F_T} = \frac{2\pi F_p}{F_T} = 2\pi F_p T$$

$$\omega_s = \frac{\Omega_s}{F_T} = \frac{2\pi F_s}{F_T} = 2\pi F_s T$$

# Digital Filter Specifications

- Example - Let $F_p = 7$ kHz, $F_s = 3$ kHz, and $F_T = 25$ kHz
- Then

$$\omega_p = \frac{2\pi(7\times 10^3)}{25\times 10^3} = 0.56\pi$$

$$\omega_s = \frac{2\pi(3\times 10^3)}{25\times 10^3} = 0.24\pi$$

# Selection of Filter Type

- The transfer function $H(z)$ meeting the frequency response specifications should be a causal transfer function
- For IIR digital filter design, the IIR transfer function is a real rational function of $z^{-1}$:

$$H(z)=\frac{p_0+p_1z^{-1}+p_2z^{-2}+\cdots+p_Mz^{-M}}{d_0+d_1z^{-1}+d_2z^{-2}+\cdots+d_Nz^{-N}}, \quad M\le N$$

- $H(z)$ must be a stable transfer function and must be of lowest order $N$ for reduced computational complexity

# Selection of Filter Type

- For FIR digital filter design, the FIR transfer function is a polynomial in $z^{-1}$ with real coefficients:

$$H(z) = \sum_{n=0}^{N} h[n] z^{-n}$$

- For reduced computational complexity, degree $N$ of $H(z)$ must be as small as possible
- If a linear phase is desired, the filter coefficients must satisfy the constraint:

$$h[n] = \pm h[N-n]$$

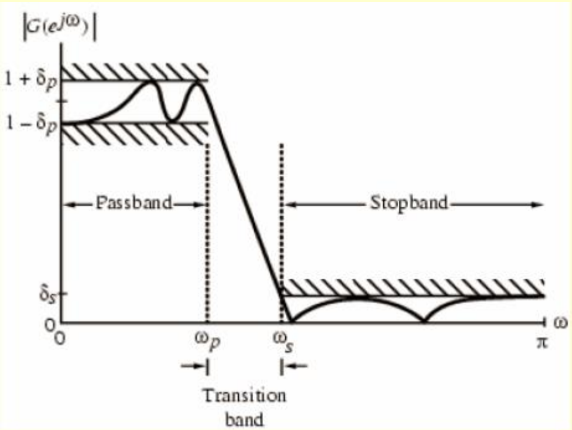
# Selection of Filter Type

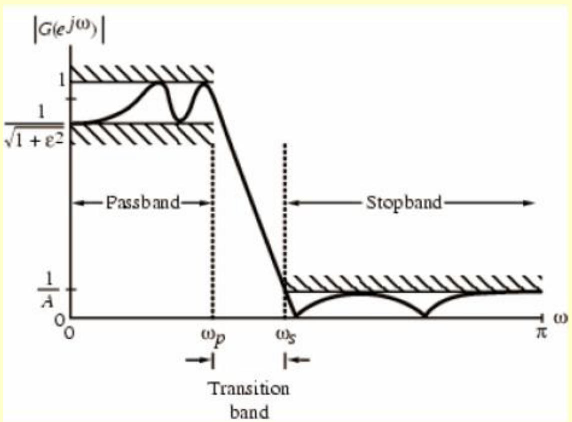
- Advantages in using an FIR filter -

  (1) Can be designed with exact linear phase,

  (2) Filter structure always stable with quantized coefficients

- Disadvantages in using an FIR filter - Order of an FIR filter, in most cases, is considerably higher than the order of an equivalent IIR filter meeting the same specifications, and FIR filter has thus higher computational complexity




# 滤波器设计

传递函数G(Z)

准备工作

准备工作

## 滤波器的设计参数

$$\omega_p, \omega_s, \delta_p, \delta_s$$

$$\omega_p, \omega_s, \frac{1}{\sqrt{1+\varepsilon^2}}, \frac{1}{A}$$

横坐标频率的归一化处理

纵坐标求损失函数

## 滤波器的类型

FIR or IIR

IIR

FIR

双线性变换法

窗口设计法

# Digital Filter Design: Basic Approaches

- Most common approach to IIR filter design - (1) Convert the digital filter specifications into an analog prototype lowpass filter specifications
- (2) Determine the analog lowpass filter transfer function $H_a(s)$
- (3) Transform $H_a(s)$ into the desired digital transfer function $G(z)$

# 通过模拟滤波器来设计IIR数字滤波器

$$H_a(s) \xrightarrow{\text{某种变换}} G(z)$$

$$s \xleftrightarrow{\text{变换}} z$$

$s \xleftrightarrow{\text{变换}} z$ 的映射关系应满足

(1)数字滤波器频响应能模仿模拟滤波器频响.

$$H_a(s)\big|_{s=j\Omega} \xleftrightarrow{\text{互为映射关系}} G(z)\big|_{z=e^{j\omega}}$$

(2)因果稳定的模拟系统变换为数字系统仍为因果稳定的.

$$H_a(s)\big|_{\mathrm{Re}[s]<0} \xleftrightarrow{\text{互为映射关系}} G(z)\big|_{|z|<1}$$

# Digital Filter Design: Basic Approaches

- Most common approach to IIR filter design - (1) Convert the digital filter specifications into an analog prototype lowpass filter specifications
- (2) Determine the analog lowpass filter transfer function $H_a(s)$
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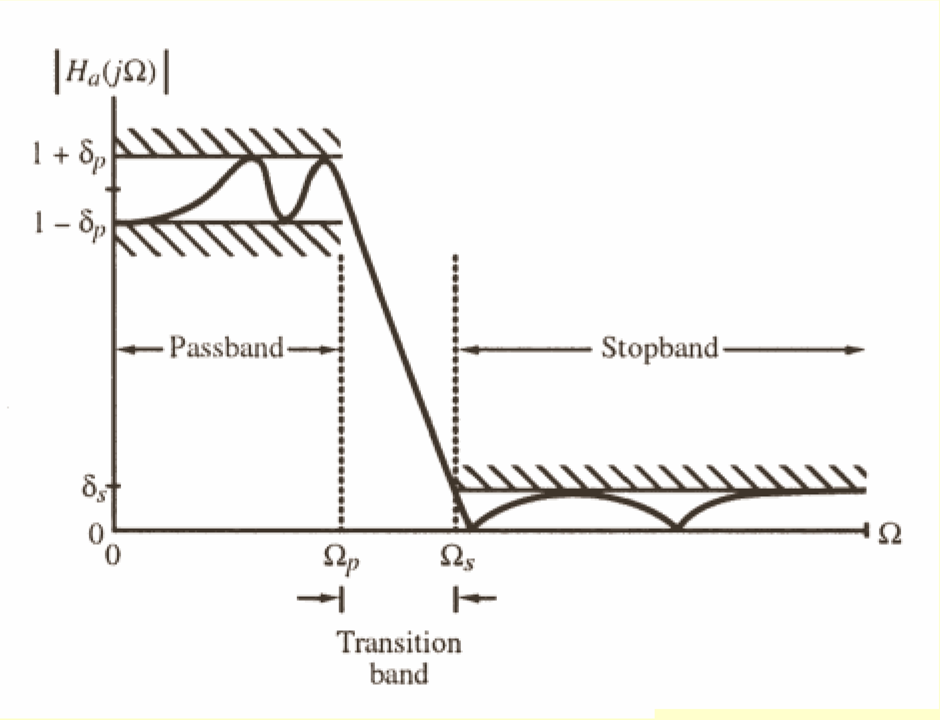
# 环节1——模拟低通原型滤波器的设计

# 环节1——模拟低通原型滤波器的设计

- 什么是模拟低通原型滤波器？
- 模拟低通原型滤波器有哪些设计参数？
- 有哪些模拟低通原型滤波器？
- 如何设计？

# Analog Lowpass Filter Specifications

- Typical magnitude response $|H_a(j\Omega)|$ of an analog lowpass filter may be given as indicated below



# Analog Lowpass Filter Specifications

- In the **passband**, defined by $0 \le \Omega \le \Omega_p$, we require

$$1-\delta_p \le |H_a(j\Omega)| \le 1+\delta_p, \quad |\Omega| \le \Omega_p$$

i.e., $|H_a(j\Omega)|$ approximates unity within an error of $\pm\delta_p$

- In the **stopband**, defined by $\Omega_s \le \Omega < \infty$, we require

$$|H_a(j\Omega)| \le \delta_s, \quad \Omega_s \le |\Omega| < \infty$$

i.e., $|H_a(j\Omega)|$ approximates zero within an error of $\delta_s$

# Analog Lowpass Filter Specifications

- $\Omega_p$ - **passband edge frequency**
- $\Omega_s$ - **stopband edge frequency**
- $\delta_p$ - **peak ripple value** in the passband
- $\delta_s$ - **peak ripple value** in the stopband
- **Peak passband ripple**

$$\alpha_p = -20\log_{10}(1-\delta_p) \text{ dB}$$

- **Minimum stopband attenuation**

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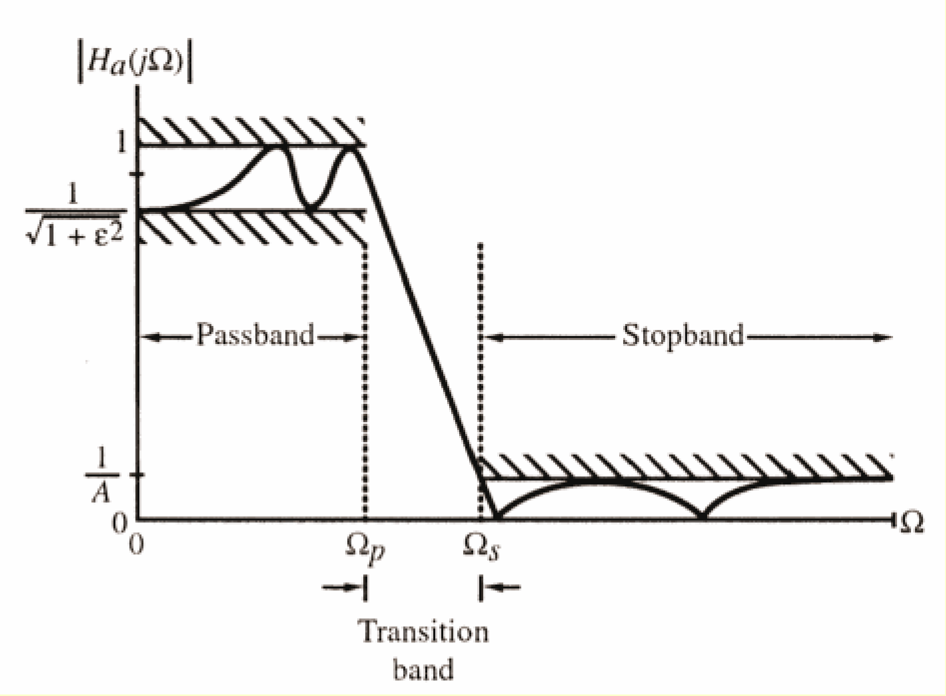
# Analog Lowpass Filter Specifications

- Magnitude specifications may alternately be given in a normalized form as indicated below



# Analog Lowpass Filter Specifications

- Here, the maximum value of the magnitude in the passband assumed to be unity

- $1/\sqrt{1+\varepsilon^2}$ - Maximum passband deviation, given by the minimum value of the magnitude in the passband

- $\dfrac{1}{A}$ - Maximum stopband magnitude

# Analog Lowpass Filter Design

- Two additional parameters are defined -

(1) **Transition ratio** $k = \dfrac{\Omega_p}{\Omega_s}$

For a lowpass filter $k < 1$

(2) **Discrimination parameter** $k_1 = \dfrac{\varepsilon}{\sqrt{A^2 - 1}}$

Usually $k_1 << 1$

# 环节1——模拟低通原型滤波器的设计

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- 有哪些模拟低通原型滤波器？
- 如何设计巴特沃兹模拟低通原型滤波器？

# 模拟低通原型滤波器

- 巴特沃兹滤波器（Butterworth）
- 切比雪夫滤波器（Chebeshev）
- 椭圆滤波器（Episoide）

一般说来，相同指标下，椭圆滤波器阶次最低，切比雪夫次之，巴特沃兹最高，参数的灵敏度则恰恰相反。

# 环节1——模拟低通原型滤波器的设计

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**核心——求** $\Omega_c$ and $N$

# Butterworth模拟原型滤波器阶数N的求解

——详见Page 577 附录A.2

以及Page434 9.3节

# Butterworth Approximation

- The magnitude-square response of an $N$-th order analog lowpass **Butterworth filter** is given by

$$|H_a(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}}$$

- First $2N-1$ derivatives of $|H_a(j\Omega)|^2$ at $\Omega = 0$ are equal to zero
- The Butterworth lowpass filter thus is said to have a **maximally-flat magnitude** at $\Omega = 0$

# Butterworth Approximation

- Gain in dB is $\mathcal{G}(\Omega) = 10\log_{10}|H_a(j\Omega)|^2$

- As $\mathcal{G}(0) = 0$ and

$$\mathcal{G}(\Omega_c) = 10\log_{10}(0.5) = -3.0103 \cong -3 \text{ dB}$$

$\Omega_c$ is called the **3-dB cutoff frequency**

# Butterworth Approximation

- Two parameters completely characterizing a Butterworth lowpass filter are $\Omega_c$ and $N$
- These are determined from the specified bandedges $\Omega_p$ and $\Omega_s$, and minimum passband magnitude $1/\sqrt{1+\varepsilon^2}$, and maximum stopband ripple $1/A$

# Butterworth Approximation

- $\Omega_c$ and $N$ are thus determined from

$$\left|H_a(j\Omega_p)\right|^2 = \frac{1}{1+(\Omega_p/\Omega_c)^{2N}} = \frac{1}{1+\varepsilon^2}$$

$$\left|H_a(j\Omega_s)\right|^2 = \frac{1}{1+(\Omega_s/\Omega_c)^{2N}} = \frac{1}{A^2}$$

- Solving the above we get

$$N = \frac{1}{2}\cdot\frac{\log_{10}[(A^2-1)/\varepsilon^2]}{\log_{10}(\Omega_s/\Omega_p)} = \frac{\log_{10}(1/k_1)}{\log_{10}(1/k)}$$

# Butterworth Approximation

- Since order $N$ must be an integer, value obtained is rounded up to the next highest integer
- This value of $N$ is used next to determine $\Omega_c$ by satisfying either the stopband edge or the passband edge specification exactly
- If the stopband edge specification is satisfied, then the passband edge specification is exceeded providing a safety margin

# Butterworth Approximation

- Example - Determine the lowest order of a Butterworth lowpass filter with a 1-dB cutoff frequency at 1 kHz and a minimum attenuation of 40 dB at 5 kHz
- Now

$$10\log_{10}\left(\frac{1}{1+\varepsilon^2}\right)=-1$$

which yields $\varepsilon^2 = 0.25895$

and

$$10\log_{10}\left(\frac{1}{A^2}\right)=-40$$

which yields $A^2 = 10{,}000$

# Butterworth Approximation

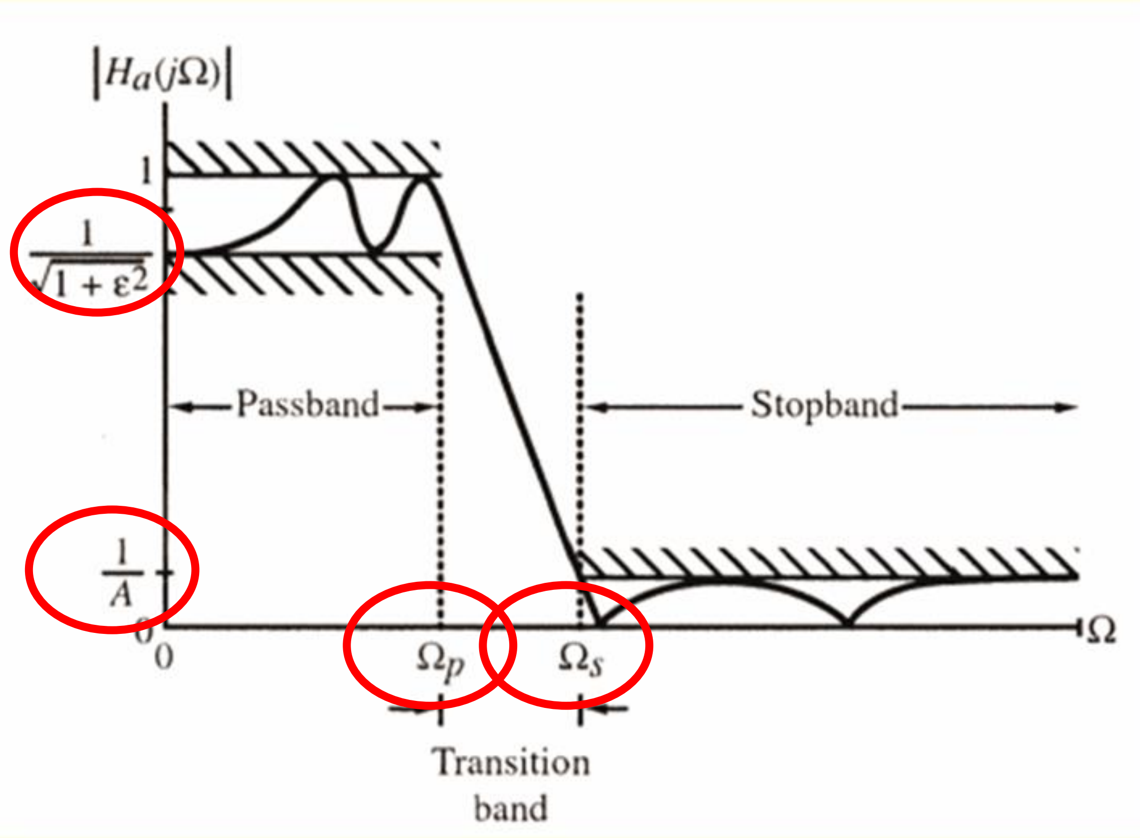
- Therefore $$\frac{1}{k_1} = \frac{\sqrt{A^2 - 1}}{\varepsilon} = 196.51334$$

  and $$\frac{1}{k} = \frac{\Omega_s}{\Omega_p} = 5$$

- Hence

$$N = \frac{\log_{10}(1/k_1)}{\log_{10}(1/k)} = 3.2811$$

- We choose $N = 4$

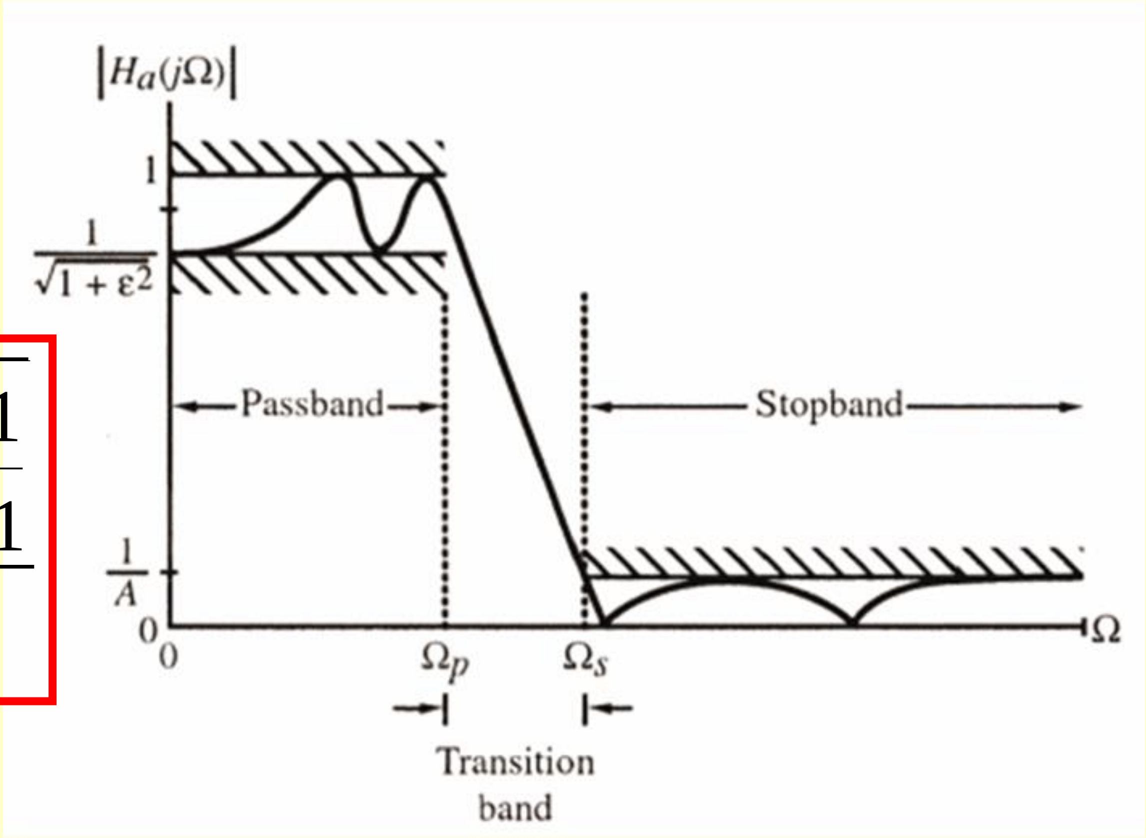



$$N_{exact} = \frac{1}{2}\frac{\log_{10}\left[(A^2-1)/\varepsilon^2\right]}{\log_{10}\left(\Omega_s/\Omega_p\right)}$$

$$\Omega_p \quad \Omega_s \quad \alpha_p \quad \alpha_s$$

↓

$$N_{exact} = \frac{\log_{10}\sqrt{\dfrac{10^{\alpha_s/10}-1}{10^{\alpha_p/10}-1}}}{\log_{10}\left(\Omega_s/\Omega_p\right)}$$




$$\alpha_p = -20\log_{10}\frac{1}{\sqrt{1+\varepsilon^2}} = 10\log_{10}\left(1+\varepsilon^2\right)$$

$$\alpha_s = -20\log_{10}\frac{1}{A} = 10\log_{10}A^2$$

$$\Rightarrow \begin{cases} \varepsilon^2 = 10^{\alpha_p/10}-1 \\ A^2 = 10^{\alpha_s/10} \end{cases}$$

$$\left|H_a(j\Omega_p)\right|^2 = \frac{1}{1+(\Omega_p/\Omega_c)^{2N}} = \frac{1}{1+\varepsilon^2}$$ 1

$$\left|H_a(j\Omega_s)\right|^2 = \frac{1}{1+(\Omega_s/\Omega_c)^{2N}} = \frac{1}{A^2}$$ 2

***Solving the equations***

$$N_{exact} = \frac{1}{2}\frac{\log_{10}\left[(A^2-1)/\varepsilon^2\right]}{\log_{10}\left(\Omega_s/\Omega_p\right)} = \frac{\log_{10}\sqrt{\frac{10^{\alpha_s/10}-1}{10^{\alpha_p/10}-1}}}{\log_{10}\left(\Omega_s/\Omega_p\right)}$$

( $N_{exact}$ may not be an integer)

$$N = [N_{exact}]$$

To determine $\Omega_c$ we use

$$\left|H_a(j\Omega_p)\right|^2 = \frac{1}{1+(\Omega_p/\Omega_c)^{2N}} = \frac{1}{1+\varepsilon^2}$$

说明：亦可利用公式2求截止频率，但由于N通常经过近似取整运算，而非精确值，因此将导致使用不同公式得到的截止频率有少许偏差。

环节2——根据N，查表获得模拟低通原型滤波器的传递函数$H_{an}(s)$，并反归一化，获得模拟滤波器传递函数$H_a(s)$

# Butterworth Approximation

**one way for determining :** $H_a(s)$

- **determine the filter order N by the method above**
- **determine the normalized N-order lowpass Butterworth filter** $H_{an}(s)$ **(** $\Omega_c = 1$ **)by looking up the table in which the impulse response of normalized lowpass Butterworth filters are available (see the next page).**
- **obtain** $H_a(s)$ **by denormalizing** $H_a(s) = H_{an}(s)\Big|_{s=\frac{s}{\Omega_c}}$

# Butterworth Approximation

| N | $H_{an}(s)$ |
|---|---|
| 1 | $\frac{1}{1+s}$ |
| 2 | $\frac{1}{1+1.4142s+s^2}$ |
| 3 | $\frac{1}{1+2s+2s^2+s^3}$ |
| 4 | $\frac{1}{1+2.6131s+3.4142s^2+2.6131s^3+s^4}$ |
| 5 | $\frac{1}{1+3.2361s+5.2361s^2+5.2361s^3+3.2361s^4+s^5}$ |
| 6 | $\frac{1}{1+3.8637s+7.4641s^2+9.1416s^3+7.4641s^4+3.8637s^5+s^6}$ |
| 7 | $\frac{1}{1+4.4940s+10.0978s^2+14.5918s^3+14.5918s^4+10.0978s^5+4.4940s^6+s^7}$ |

# Butterworth Approximation

**Example: Design an analog Butterworth lowpass filter satisfying the following specifications:**

$$\alpha_p = 3dB, \quad \alpha_s = 20dB, \quad \Omega_p = 1, \quad \Omega_s = 3.0777$$

**Solution:**

$$10\lg\left(\frac{1}{1+\varepsilon^2}\right) = -3 \quad \Rightarrow \varepsilon^2 = 1$$

$$10\lg\left(\frac{1}{A^2}\right) = -20 \quad \Rightarrow A^2 = 100$$

$$N = \frac{1}{2}\frac{\lg[(A^2-1)/\varepsilon^2]}{\lg(\Omega_s/\Omega_p)} \doteq 2.0438$$

$$N = 3$$

**since**

$$\frac{1}{1+(\Omega_s/\Omega_c)^{2N}} = \frac{1}{A^2} \quad \Rightarrow \Omega_c = 1.4309$$

$$H_{LP}(s) = \frac{1}{1+2s+2s^2+s^3}\bigg|_{s=s/\Omega_c}$$

Section 9.2

$$H_a(s) \xrightarrow{\text{某种变换}} G(z)$$

Section 9.1

$$s \xleftrightarrow{\text{变换}} z$$

# 环节3——套用映射公式将模拟滤波器映射为数字滤波器

# IIR Digital Filter Design: Bilinear Transformation Method

- Bilinear transformation -

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

- Above transformation maps a single point in the $s$-plane to a unique point in the $z$-plane and vice-versa
- Relation between $G(z)$ and $H_a(s)$ is then given by

$$G(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

# Bilinear Transformation

- Digital filter design consists of 3 steps:

  (1) Develop the specifications of $H_a(s)$ by applying the inverse bilinear transformation to specifications of $G(z)$

  (2) Design $H_a(s)$

  (3) Determine $G(z)$ by applying bilinear transformation to $H_a(s)$

- As a result, the parameter $T$ has no effect on $G(z)$ and $T = 2$ is chosen for convenience

# Bilinear Transformation

- Inverse bilinear transformation for $T = 2$ is

$$z = \frac{1+s}{1-s}$$

- For $s = \sigma_o + j\Omega_o$

$$z = \frac{(1+\sigma_o) + j\Omega_o}{(1-\sigma_o) - j\Omega_o} \quad \Rightarrow |z|^2 = \frac{(1+\sigma_o)^2 + \Omega_o^2}{(1-\sigma_o)^2 + \Omega_o^2}$$

- Thus,

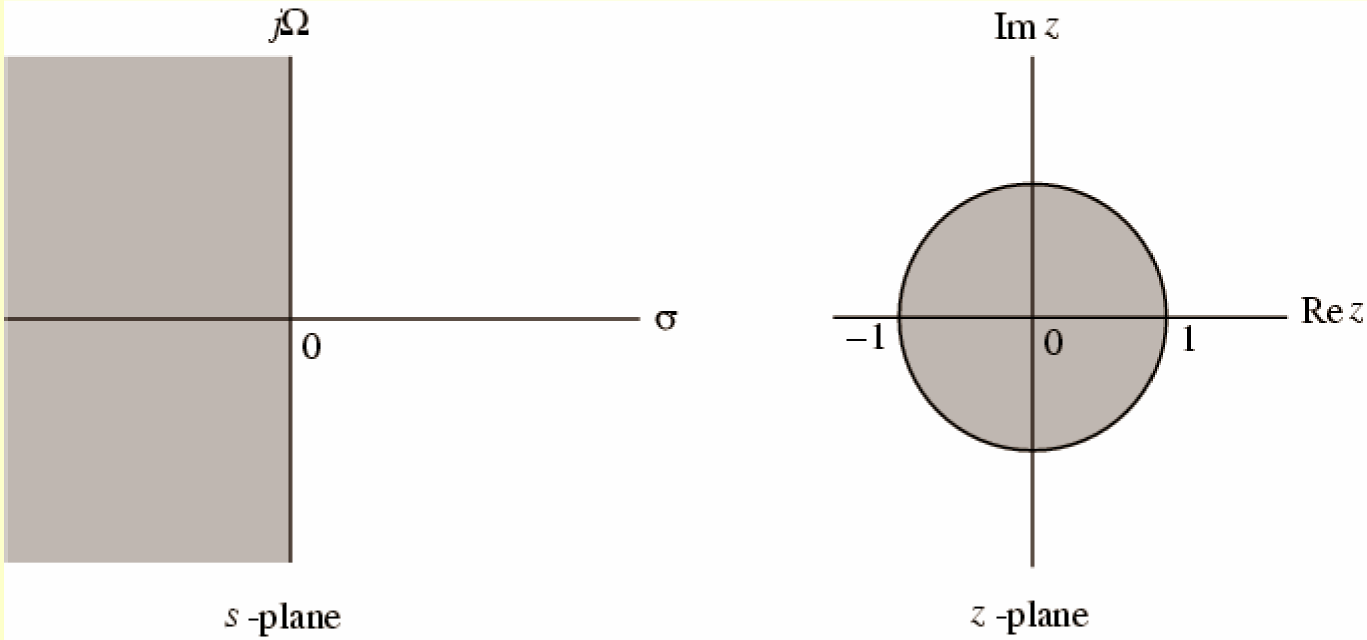
$$\sigma_o = 0 \rightarrow |z| = 1$$

$$\sigma_o < 0 \rightarrow |z| < 1$$

$$\sigma_o > 0 \rightarrow |z| > 1$$

# Bilinear Transformation

- Mapping of $s$-plane into the $z$-plane



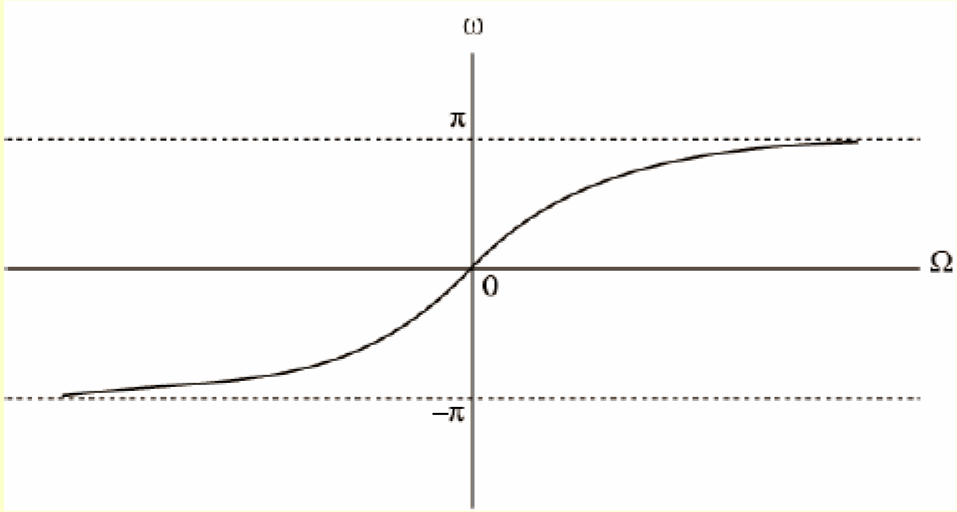
# Bilinear Transformation

- For $z = e^{j\omega}$ with $T = 2$ we have

$$j\Omega = \frac{1 - e^{-j\omega}}{1 + e^{-j\omega}} = \frac{e^{-j\omega/2}(e^{j\omega/2} - e^{-j\omega/2})}{e^{-j\omega/2}(e^{j\omega/2} + e^{-j\omega/2})}$$

$$= \frac{j2\sin(\omega/2)}{2\cos(\omega/2)} = j\tan(\omega/2)$$

or $\Omega = \tan(\omega/2)$



# Bilinear Transformation

- Mapping is highly nonlinear
- Complete negative imaginary axis in the $s$-plane from $\Omega = -\infty$ to $\Omega = 0$ is mapped into the lower half of the unit circle in the $z$-plane from $z = -1$ to $z = 1$
- Complete positive imaginary axis in the $s$-plane from $\Omega = 0$ to $\Omega = \infty$ is mapped into the upper half of the unit circle in the $z$-plane from $z = 1$ to $z = -1$

这种频率间的非线性映射关系将导致——

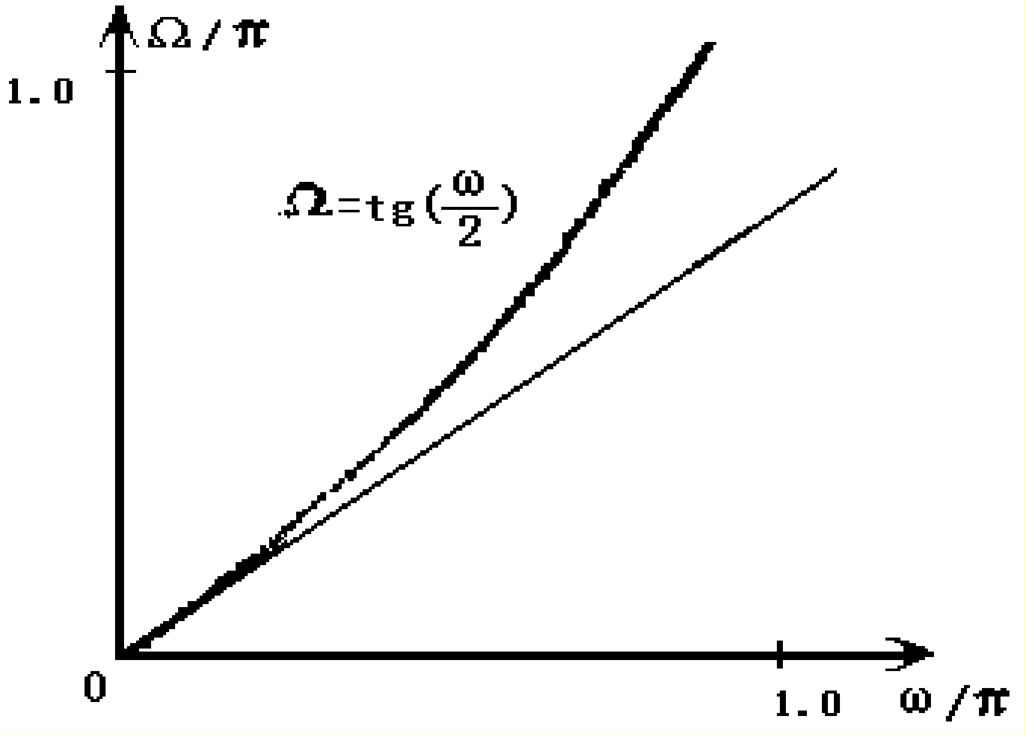
一个考尔型的模拟滤波器$H_a(s)$，双线性变换后，得到的G(z)在通带与阻带内都仍保持与原模拟滤波器相同的等起伏特性，只是通带截止频率、过渡带的边缘频率，以及起伏的峰点、谷点频率等临界频率点发生了非线性变化，即**畸变**。

这种频率点的畸变可以通过**预畸**来加以校正。

预畸变即将模拟滤波器的临界频率事先加以畸变，然后通过双线性变换后正好映射到所需要的频率上

利用关系式：

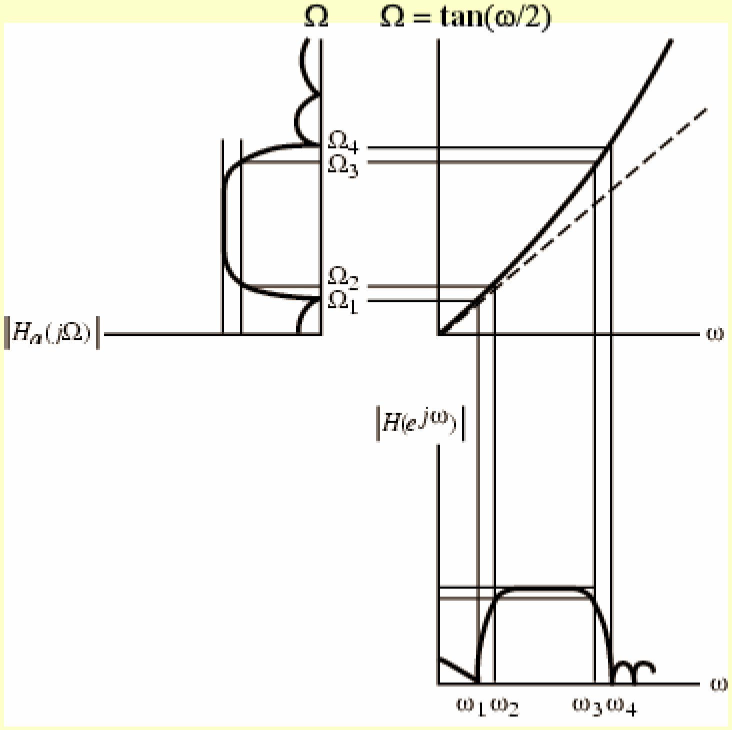
$$\Omega_i = \frac{2}{T} tg\left(\frac{\omega_i}{2}\right)$$




注意：预畸不能在整个频率段消除非线性畸变，只能消除模拟和数字滤波器在特征频率点的畸变

# Bilinear Transformation

- Nonlinear mapping introduces a distortion in the frequency axis called **frequency warping**
- Effect of warping shown below



# IIR Digital Filter Design Using Bilinear Transformation

- Example - Consider

$$H_a(s) = \frac{\Omega_c}{s + \Omega_c}$$

- Applying bilinear transformation to the above we get the transfer function of a first-order digital lowpass Butterworth filter

$$G(z) = H_a(s)\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}} = \frac{\Omega_c(1+z^{-1})}{(1-z^{-1})+\Omega_c(1+z^{-1})}$$

# IIR Digital Filter Design Using Bilinear Transformation

- Rearranging terms we get

$$G(z) = \frac{1-\alpha}{2} \cdot \frac{1+z^{-1}}{1-\alpha z^{-1}}$$

where

$$\alpha = \frac{1-\Omega_c}{1+\Omega_c} = \frac{1-\tan(\omega_c/2)}{1+\tan(\omega_c/2)}$$

# Bilinear Transformation

- Steps in the design of a digital filter -

  (1) Prewarp $(\omega_p, \omega_s)$ to find their analog equivalents $(\Omega_p, \Omega_s)$

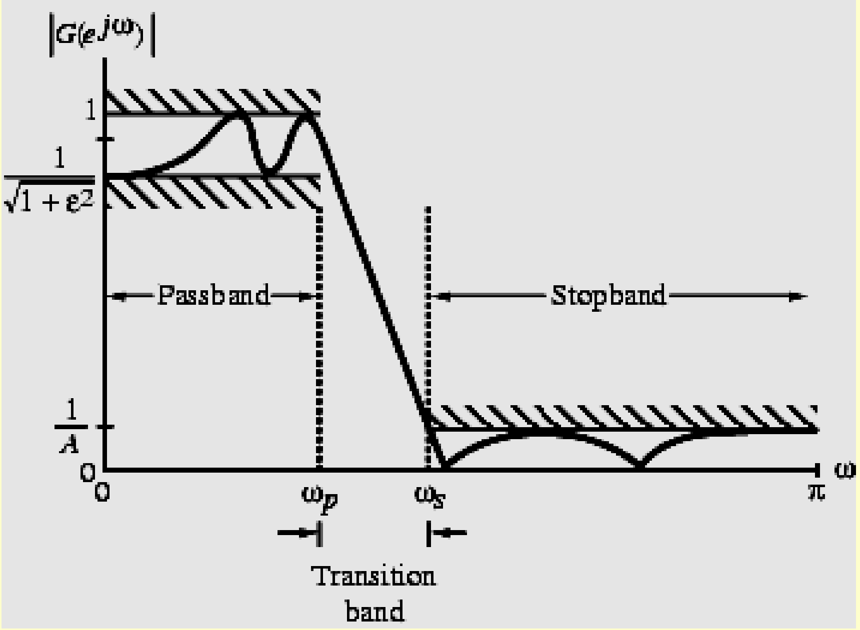
  (2) Design the analog filter $H_a(s)$

  (3) Design the digital filter $G(z)$ by applying bilinear transformation to $H_a(s)$

- Transformation can be used only to design digital filters with prescribed magnitude response with piecewise constant values
- Transformation does not preserve phase response of analog filter

# § 9.3 Design of LP IIR Digital Filter

<u>Example</u> - Design a lowpass Butterworth digital filter with $\omega_p = 0.25\pi$, $\omega_s = 0.55\pi$, $\alpha_p = 0.5$ dB, and $\alpha_s = 15$ dB




**Analysis:**

If $|G(e^{j0})| = 1$ this implies

$$20\log_{10}|G(e^{j0.25\pi})| \geq -0.5$$

$$20\log_{10}|G(e^{j0.55\pi})| \leq -15$$

# § 9.3 Design of LP IIR Digital Filter

- **Solution:**

**（1）Prewarping (T=2)**

$\Omega_p = \tan(\omega_p/2) = \tan(0.25\pi/2) = 0.4142136$

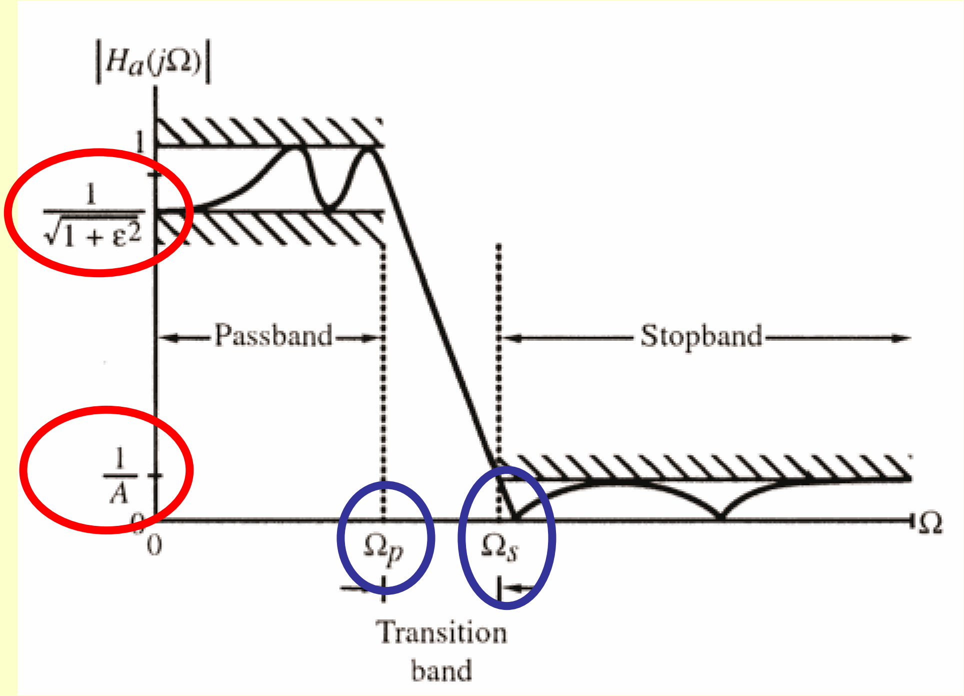
$\Omega_s = \tan(\omega_s/2) = \tan(0.55\pi/2) = 1.1708496$

第一步：参数的预畸

# Analog Lowpass Filter Specifications

第二步：模拟滤波器的设计

- Magnitude specifications may alternately be given in a normalized form as indicated below



- **Analog filter Design**

$$-20\lg\frac{1}{\sqrt{1+\varepsilon^2}}=0.5 \quad\Rightarrow\quad \varepsilon^2=0.1220185$$

$$-20\lg\frac{1}{A}=15 \quad\Rightarrow\quad A^2=31.622777$$

**The inverse transition ratio is**

**$1/k = \Omega_s / \Omega_p = 2.8266809$**

**The inverse discrimination ratio is**

$$1/k_1=\frac{\sqrt{A^2-1}}{\varepsilon}=15.841979$$

Thus $$N=\frac{\log_{10}(1/k_1)}{\log_{10}(1/k)}=2.6586997$$

# 换一种方法，请尝试用以下公式计算N

$$N_{exact} = \frac{\log_{10}\sqrt{\dfrac{10^{\alpha_s/10}-1}{10^{\alpha_p/10}-1}}}{\log_{10}\left(\Omega_s/\Omega_p\right)}$$

# § 9.3 Design of LP IIR Digital Filter

**Choose *N = 3***

To determine $\Omega_c$ we use

$$\left|H_a(j\Omega_p)\right|^2 = \frac{1}{1+(\Omega_p/\Omega_c)^{2N}} = \frac{1}{1+\varepsilon^2}$$

**We then get**

**$\Omega_c$=0.588148**

# Butterworth Approximation

| N | $H_{an}(s)$ |
| --- | --- |
| 1 | $\frac{1}{1+s}$ |
| 2 | $\frac{1}{1+1.4142s+s^2}$ |
| 3 | $\frac{1}{1+2s+2s^2+s^3}$ |
| 4 | $\frac{1}{1+2.6131s+3.4142s^2+2.6131s^3+s^4}$ |
| 5 | $\frac{1}{1+3.2361s+5.2361s^2+5.2361s^3+3.2361s^4+s^5}$ |
| 6 | $\frac{1}{1+3.8637s+7.4641s^2+9.1416s^3+7.4641s^4+3.8637s^5+s^6}$ |
| 7 | $\frac{1}{1+4.4940s+10.0978s^2+14.5918s^3+14.5918s^4+10.0978s^5+4.4940s^6+s^7}$ |

# § 9.3 Design of LP IIR Digital Filter

- **3rd-order lowpass Butterworth transfer function for $\Omega_c$=1 is**

  $H_{an}(s)=1/(s^3+2s^2+2s+1)=1/[(s+1)(s^2+s+1)]$

- **Denormalizing to get**

**we arrive at**

解归一化

$$H_a(s) = H_{an}(\frac{s}{\Omega_c}) = H_{an}(\frac{s}{0.588148})$$

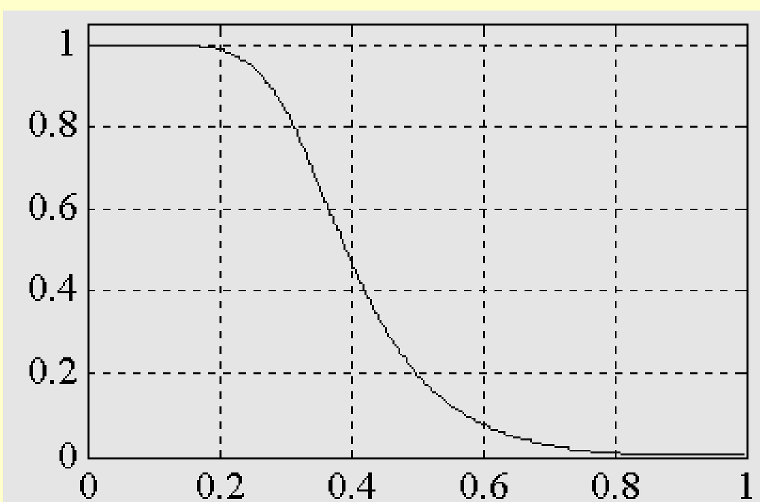
# § 9.3 Design of LP IIR Digital Filter

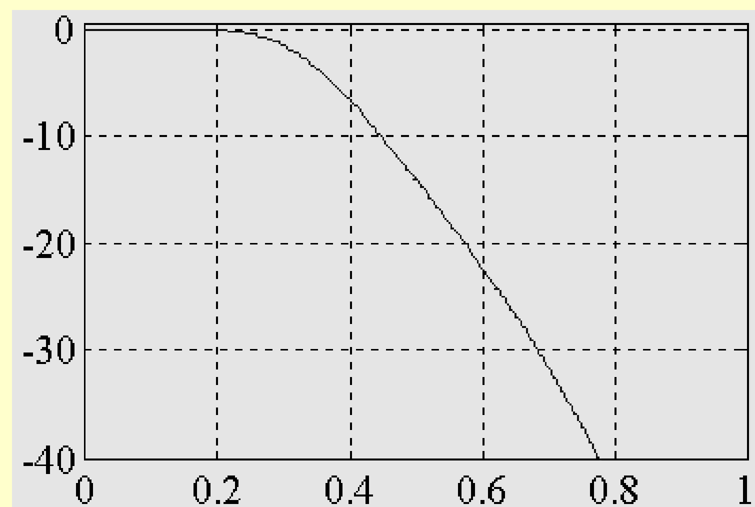
第三步：映射（变量代换）

- **Applying bilinear transformation to $H_a(s)$ we get the desired digital transfer function**

$$G(z) = H_a(s)\big|_{s=\frac{1-z^{-1}}{1+z^{-1}}}$$

**Magnitude and gain responses of $G(z)$ shown below:**



# 第9章重点

- 基本概念：如s－z映射的基本条件；滤波器的主要参数及其表示（ $\alpha_p \alpha_s \Omega_s \Omega_p$ ）
- 双线性变换法的**IIR**数字滤波器设计（重点低通）

① 特征频率的归一化

② 特征频率点处的预畸

③ 求模拟低通原型滤波器的传递函数

④ **S-Z**映射

# Homework

- **Problem: 9.11　　　　　Matlab: M.1**

**补充作业题：Using the bilinear transformation and a lowpass analog Butterworth filter**

$$H_a(s)=\frac{1}{s^2+\sqrt{2}s+1}$$

**Design a second-order lowpass digital filter with 3-dB cutoff frequency 3kHz and operating at a sampling rate of 12kHz.**

**(a) Determine the system function of the desired lowpass digital filter.**

**(b) Draw the canonical realization form of the designed filter.**