Chapter 10

FIR Digital Filter Design

Chapter 10
FIR Digital Filter Design

10.1 Preliminary Considerations

10.2 FIR Filter Design Based on Windowed Fourier Series

10.5 FIR Digital Filter Design Using Matlab

10.1.1 Basic Approaches to FIR Digital Filter Design

$$h(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}d\omega$$

$$H(e^{j\omega}) \longleftrightarrow h(n) \longrightarrow H(z)$$

$$H(e^{j\omega})=\sum_{n=-\infty}^{\infty} h(n)e^{-j\omega n}$$

$$H(z)=\sum_{n=-\infty}^{\infty} h(n)z^{-n}$$

characterize the behavior of filters

coefficients of the structure

filter implementation

10.1.1 Basic Approaches to FIR Digital Filter Design

FIR filter approximation:

Design a causal system which has a finite-length impulse response $h_t(n)$.

$$H_t(e^{j\omega}) \xrightarrow{approximation} H_D(e^{j\omega}) \quad \text{(ideal)}$$

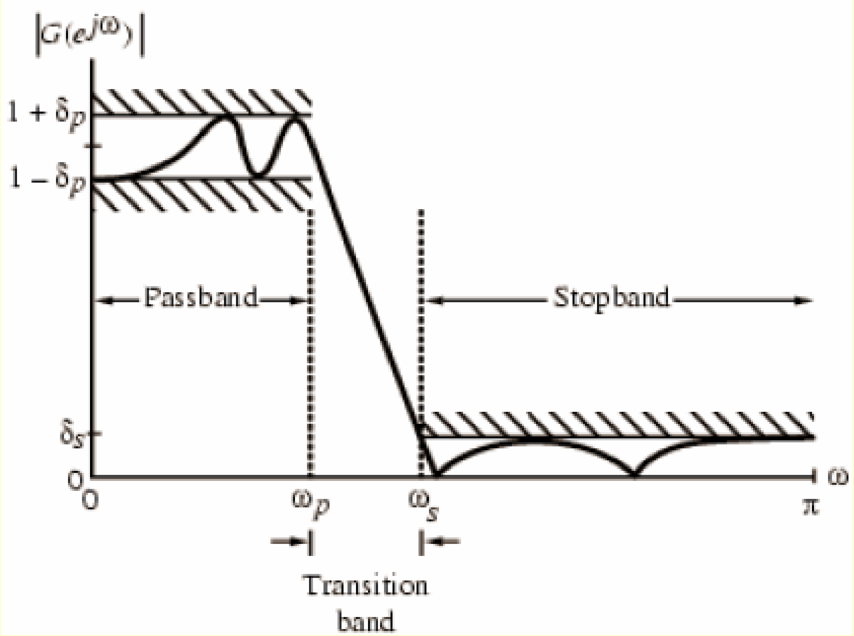
$$h_t(n) \xrightarrow{approximation} h_D(n) \quad \text{(noncausal , infinite-duration)}$$

$$\text{methods} \begin{cases} \text{in frequecy: frequency sampling (Problems 10.31, 10.32)} \\ \text{in time: window function} \\ Computer-based\ optimization\ methods \end{cases}$$

In general, consider the **linear phase** filters.

10.1.2 Estimation of the Filter Order

FIR Digital filter specification (e.g. lowpass filter)



ω_p : passband edge frequency

ω_r : stopband edge frequency

δ_p : ripple in the passband

δ_s : ripple in the stopband

$G_p = 20\lg(1-\delta_p)$: gain in the passband (dB)

$\alpha_p = -G_p$: attenuation in the passband (dB)

$G_s = 20\lg\delta_s$: gain in the stopband (dB)

$\alpha_s = -G_s$: attenuation in the stopband (dB)

10.1.2 Estimation of the Filter Order

Kaiser's Formula

$$N \cong \frac{-20\log_{10}(\sqrt{\delta_p \delta_s}) - 13}{14.6(\omega_s - \omega_p)/2\pi}$$

Bellanger's Formula

$$N \cong -\frac{2\log_{10}(10\delta_p \delta_s)}{3(\omega_s - \omega_p)/2\pi} - 1$$

10.1.2 Estimation of the Filter Order

Hermann's Formula

$$N \cong \frac{D_\infty(\delta_p,\delta_s) - F(\delta_p,\delta_s)[(\omega_s-\omega_p)/2\pi]^2}{(\omega_s-\omega_p)/2\pi}$$

where,

$$D_\infty(\delta_p,\delta_s) = [a_1(\log_{10}\delta_p)^2 + a_2(\log_{10}\delta_p) + a_3]\log_{10}\delta_s$$
$$-[a_4(\log_{10}\delta_p)^2 + a_5(\log_{10}\delta_p) + a_6]$$

$$F(\delta_p,\delta_s) = b_1 + b_2[\log_{10}\delta_p - \log_{10}\delta_s]$$

$$a_1 = 0.005309,\quad a_2 = 0.07114,\quad a_3 = -0.4761,$$

$$a_4 = 0.00266,\quad a_5 = 0.5941,\quad a_6 = 0.4278,$$

$$b_1 = 11.01217,\quad b_2 = 0.51244$$

10.1.2 Estimation of the Filter Order

Table 10.1: Comparison of FIR filter orders

Filter No.	Actual order	Kaiser's Formula	Bellanger's Formula	Hermann's Formula
1	159	158	163	151
2	38	34	37	37
3	14	12	13	12

Chapter 10
FIR Digital Filter Design

10.1 Preliminary Considerations

10.2 FIR Filter Design Based on Windowed Fourier Series

10.5 FIR Digital Filter Design Using Matlab

一、窗口法设计思路

窗口设计法

从单位脉冲响应序列着手，使$h_t(n)$逼近理想的单位脉冲响应序列$h_d(n)$。

$$
\begin{array}{ccc}
H_d(e^{j\omega}) & \Leftrightarrow & h_d(n) \\
\Uparrow & & \Uparrow \\
H_t(e^{j\omega}) & \Leftrightarrow & h_t(n)
\end{array}
$$

- Let $H_d(e^{j\omega})$ denote the desired frequency response
- Since $H_d(e^{j\omega})$ is a periodic function of ω with a period 2π, it can be expressed as a Fourier series

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d[n] e^{-j\omega n}$$

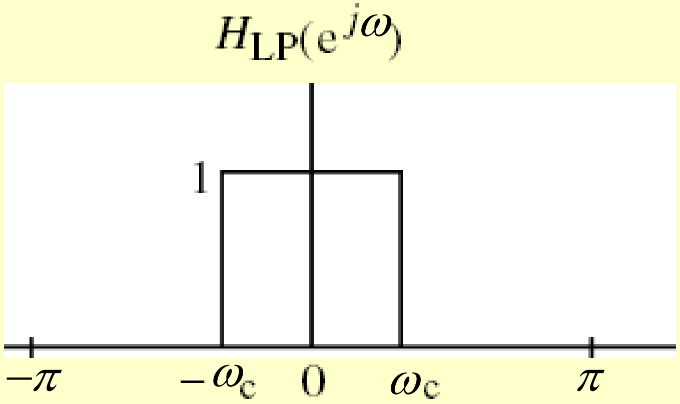
where

$$h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega, \quad -\infty \le n \le \infty$$

- In general, $H_d(e^{j\omega})$ is piecewise constant with sharp transitions between bands
- In which case, $\{h_d[n]\}$ is of infinite length and noncausal

10.2.1 Impulse Responses of Ideal Filters

1. lowpass filters



ideal magnitude response

$$\left|H_{LP}(e^{j\omega})\right| = \begin{cases} 1, & for \quad |\omega| \le \omega_c \\ 0, & for \quad \omega_c < |\omega| \le \pi \end{cases}$$

phase response $\theta(\omega) = 0$

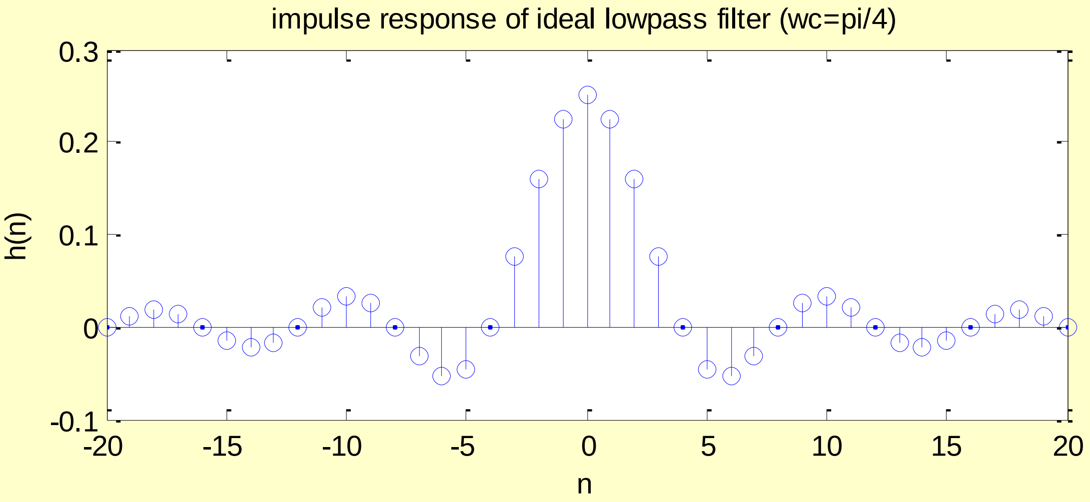
10.2.1 Impulse Responses of Ideal Filters

1. lowpass filters

impulse response

$$h_{LP}(n) = F^{-1}\left[H_{LP}(e^{j\omega})\right] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} H_{LP}(e^{j\omega})e^{j\omega n}d\omega$$

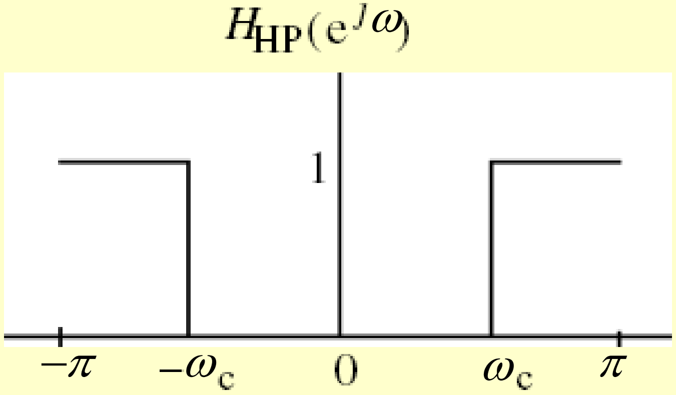
$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}d\omega = \frac{\sin(\omega_c n)}{\pi n}, \qquad -\infty < n < \infty$$



10.2.1 Impulse Responses of Ideal Filters

2. highpass filters

ideal magnitude response



impulse response

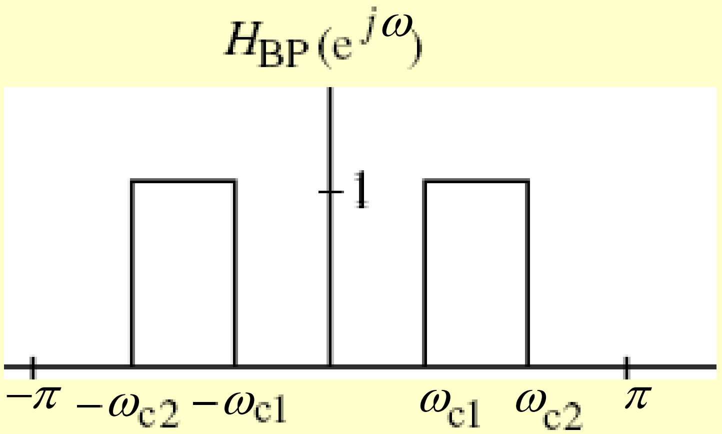
$$h_{HP}(n) = F^{-1}\left[H_{HP}(e^{j\omega})\right] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{HP}(e^{j\omega})e^{j\omega n}d\omega$$

$$= \begin{cases} 1 - \dfrac{\omega_c}{\pi}, & for\ n = 0 \\ -\dfrac{\sin(\omega_c n)}{\pi n}, & for\ n \neq 0 \end{cases}$$

10.2.1 Impulse Responses of Ideal Filters

3. bandpass filters

ideal magnitude response



impulse response

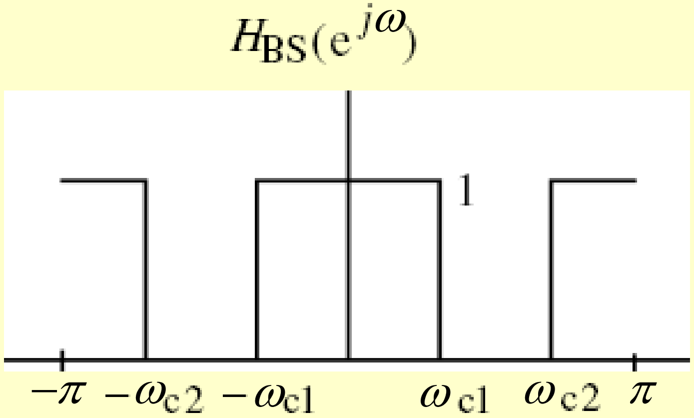
$$h_{BP}(n) = F^{-1}\left[H_{BP}(e^{j\omega})\right] = \frac{1}{\pi n}[\sin(\omega_{c2}n) - \sin(\omega_{c1}n)], \quad -\infty < n < \infty$$

10.2.1 Impulse Responses of Ideal Filters

4. bandstop filters

ideal magnitude response

$$\left|H_{BS}(e^{j\omega})\right| = \begin{cases} 1, & for \quad 0 \le |\omega| \le \omega_{c1} \\ 0, & for \quad \omega_{c1} 0 < |\omega| < \omega_{c2} \\ 1, & for \quad \omega_{c2} \le |\omega| \le \pi \end{cases}$$



impulse response

$$h_{BS}(n) = F^{-1}\left[H_{BS}(e^{j\omega})\right] = \begin{cases} 1 - \dfrac{\omega_{c2} - \omega_{c1}}{\pi}, & for \ n = 0 \\ \dfrac{1}{\pi n}[\sin(\omega_{c1} n) - \sin(\omega_{c2} n)], & for \ n \ne 0 \end{cases}$$

窗口设计法

从单位脉冲响应序列着手，使$h_t(n)$逼近理想的单位脉冲响应序列$h_d(n)$。

但一般来说，理想频响$H_d(e^{j\omega})$是分段恒定，在边界频率处有突变点，所以，通过IDTFT得到的理想单位脉冲响应$h_d(n)$往往都是无限长序列，而且是非因果的。

矛盾产生了

$h_t(n)$有限长、因果 ⟶ $h_d(n)$无限长、非因果

怎样用一个有限长的序列去近似无限长的$h_d(n)$?

最简单的办法是直接截取一段 $h_d(n)$ 代替 $h_t(n)$ 。

Window Method designing FIR filter:

Truncating the infinite, double-sided $h_d(n)$ to a finite length , which is the FIR impulse response approximating the ideal response.

这种截取可以形象地想象为$h_t(n)$是通过一个“窗口”所看到的一段$h_d(n)$.

$$h_t(n)=h_d(n)\,w(n)$$

在这里，窗口函数就是矩形脉冲函数$R_N(n)$，当然以后我们还可看到，为了改善设计滤波器的特性，窗函数还可以有其它的形式，相当于在矩形窗内对$h_d(n)$作一定的加权处理。

二、窗口法设计步骤

Problems concerned：

1、对理想 $h_d(n)$ 截取哪段作为 FIR 滤波器的 $h_t(n)$？

2、截取多长，即 FIR 滤波器的阶数取多少？

3、FIR 频率特性 $H_t(\omega)$ 能在多大程度上近似理想频响 $H_d(\omega)$？近似程度与什么有关？

- Objective - Find a finite-duration $\{h_t[n]\}$ of length $2M+1$ whose DTFT $H_t(e^{j\omega})$ approximates the desired DTFT $H_d(e^{j\omega})$ in some sense

$$H_d(e^{j\omega}) \Leftrightarrow h_d(n)$$

$$\Uparrow$$

$$H_t(e^{j\omega}) \Leftrightarrow h_t(n)$$

Steps:

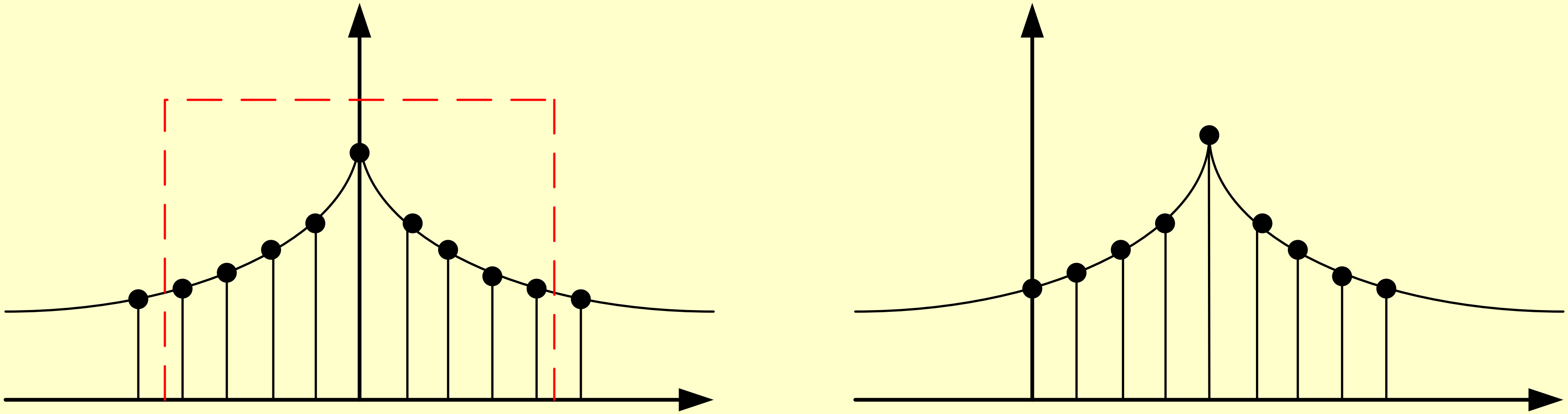
1. Pick an odd length 2*M*+1

2. Calculate the 2*M*+1 coefficients

$$h_d(n) = \int_{-\pi}^{\pi} H_d(\omega) \cdot e^{j\omega k} \frac{d\omega}{2\pi}, \quad -M \le k \le M$$

3. Make them causal by the delay

$$h_t(n) = h_d(n - M) \qquad (n = 0 \sim 2M)$$



Equivalent forms of *h(n)*:

- $h(n) = h_d(n - M)w(n)$

- $h(n) = \hat{h}_d(n - M)$

$$\hat{h}_d(n) = h_d(n)w_0(n) = \begin{cases} h_d(n) & -M \le n \le M \\ 0 & others \end{cases}$$

Example : Determine the <u>length-11</u>, <u>rectangularly windowed</u> impulse response that approximate:

(a) an ideal lowpass filter of cutoff frequency $\omega_c = \pi / 4$ **;**

Solution: M=(11-1)/2=5

(a) $$d(k) = \frac{\sin(\pi k / 4)}{\pi k}, \qquad -5 \le k \le 5$$

$$h(n) = d(n-5) = \frac{\sin(\pi(n-5)/4)}{\pi(n-5)}, \qquad 0 \le n \le 10$$

$$\mathrm{h} = [-\frac{\sqrt{2}}{10\pi}, 0, \frac{\sqrt{2}}{6\pi}, \frac{1}{2\pi}, \frac{\sqrt{2}}{2\pi}, \frac{1}{4}, \frac{\sqrt{2}}{2\pi}, \frac{1}{2\pi}, \frac{\sqrt{2}}{6\pi}, 0, -\frac{\sqrt{2}}{10\pi}]$$

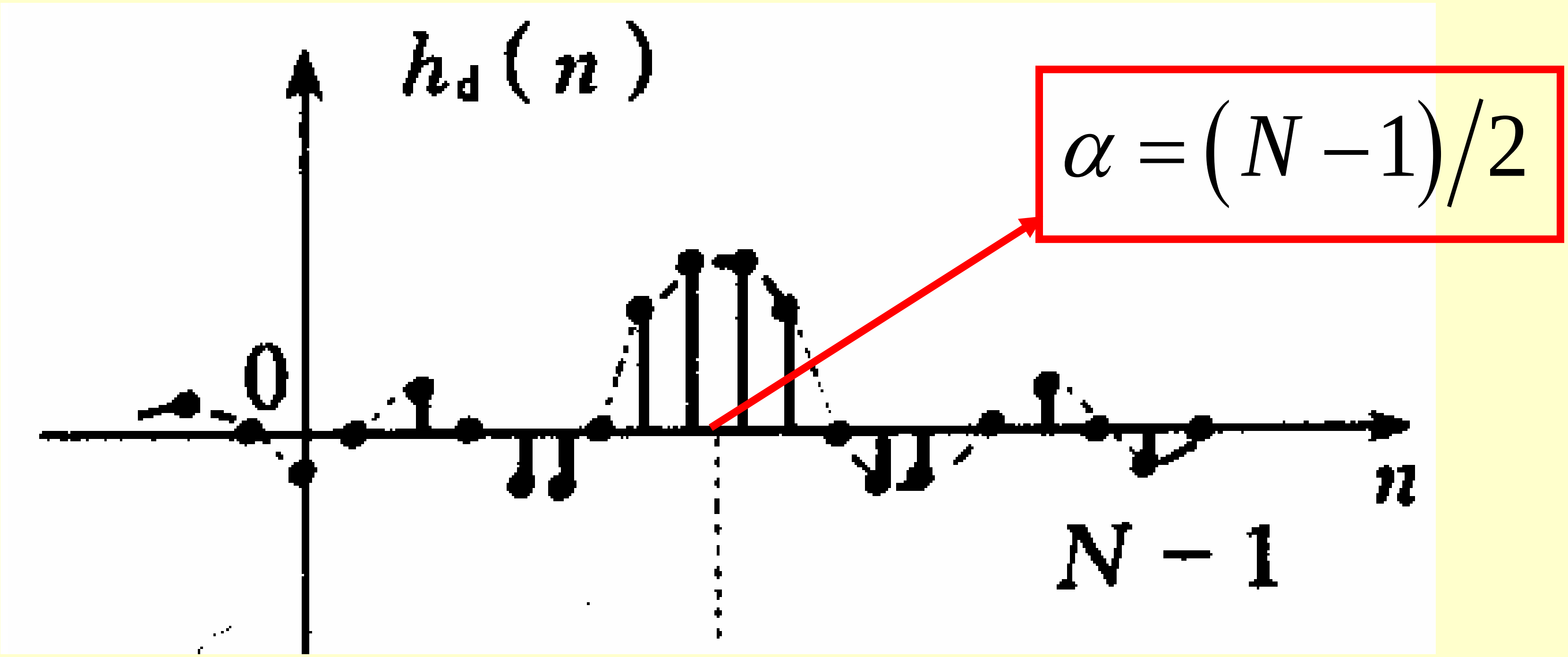
三、窗口法的频域实现过程

窗口法设计FIR滤波器

$$H_d(e^{j\omega}) \Rightarrow h_d(n)$$

a. 对于给定的理想低通滤波器 $H_d(e^{j\omega})$

$$h_d(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d\left(e^{j\omega}\right)e^{j\omega n}d\omega=\frac{\sin(\omega_c(n-\alpha))}{\pi(n-\alpha)}$$



窗口法设计原理

$$H_d(e^{j\omega}) \Rightarrow h_d(n) \Rightarrow h_d(n)w_R(n)$$

$$\boldsymbol{h_t(n)}$$

$$h_t(n) = h_d(n)w_R(n) = \begin{cases} h_d(n) & 0 \le n \le N-1 \\ 0 & n\text{为其它值} \end{cases}$$

窗口法设计FIR滤波器

$$H_d(e^{j\omega}) \Rightarrow h_d(n) \Rightarrow h_d(n)w_R(n)$$

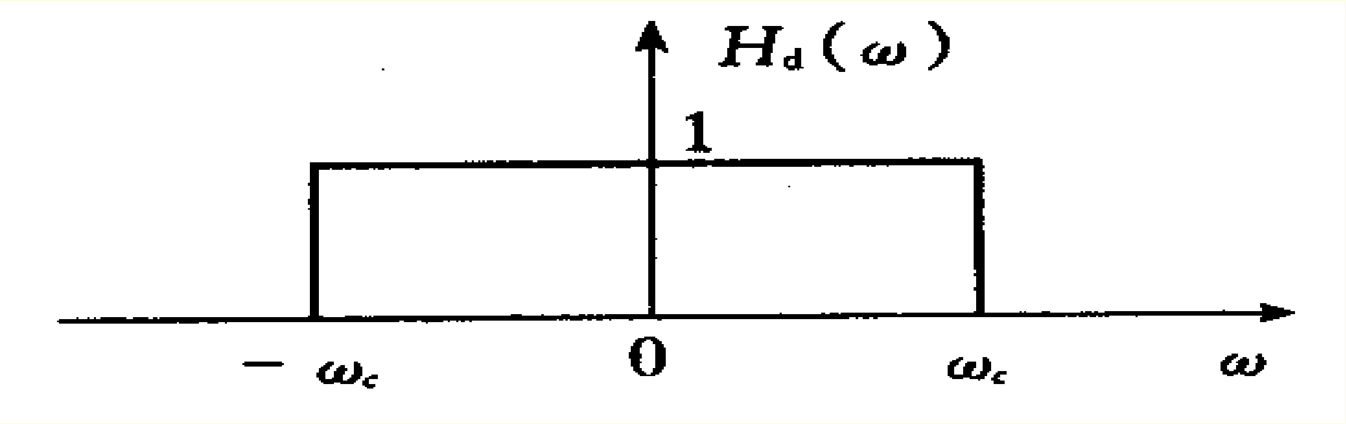
$$\boldsymbol{H_t(e^{j\omega}) \Leftarrow \quad h_t(n)}$$

频域卷积 $H_t(e^{j\omega}) = H_d(e^{j\omega}) * W_R(e^{j\omega})$

$$H_t(e^{j\omega}) = H_d(e^{j\omega}) * W_R(e^{j\omega})$$

$$H_d(e^{j\omega}) = H_d(\omega)e^{-j\omega\alpha}$$

$$H_d(\omega) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \omega_c \le |\omega| \le \pi \end{cases}$$

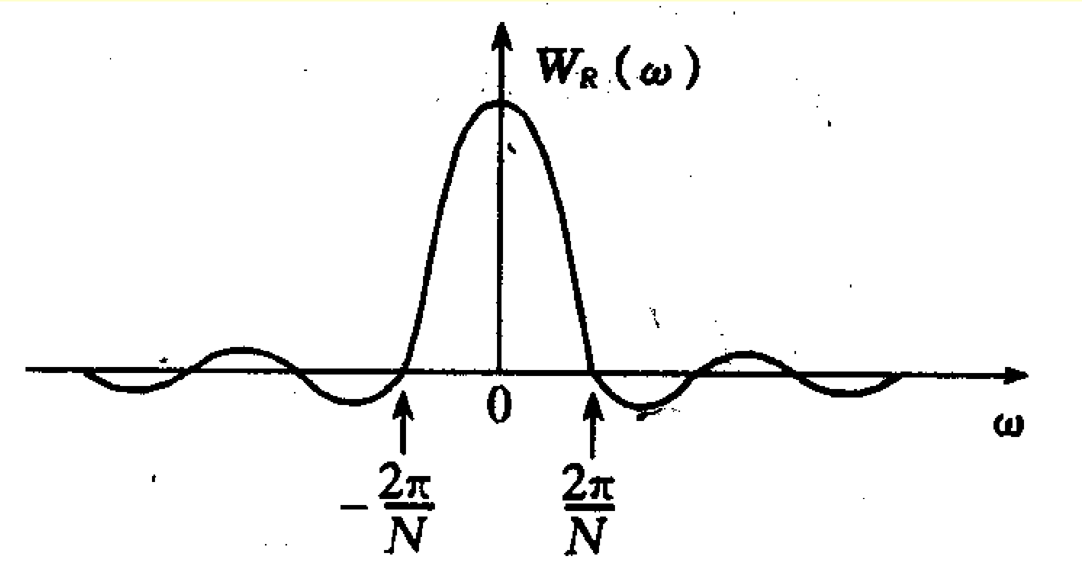


$$H_t(e^{j\omega}) = H_d(e^{j\omega}) * W_R(e^{j\omega})$$

$$W_R(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$

$$= W_R(\omega) e^{-j\omega\alpha}$$

$$W_R(\omega) = \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$



$$H_t(e^{j\omega}) = H_d(e^{j\omega}) * W_R(e^{j\omega})$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\theta}) W_R[e^{j(\omega-\theta)}] d\theta$$

$$= e^{-j\omega\alpha}\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\theta) W_R(\omega-\theta) d\theta\right]$$

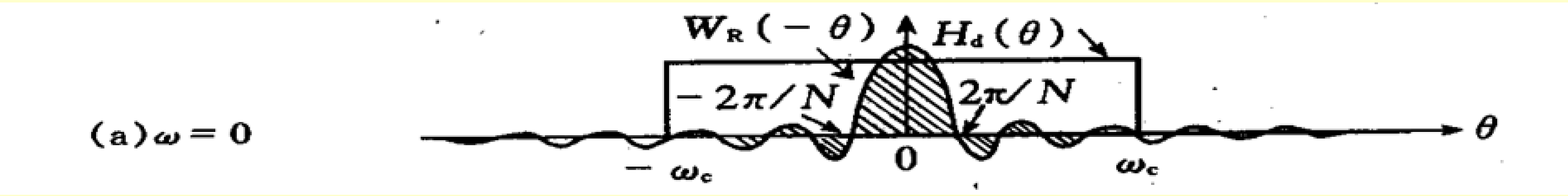
$$H(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\theta) W_R(\omega-\theta) d\theta$$

$$H(\omega)=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d(\theta)W_R(\omega-\theta)d\theta$$

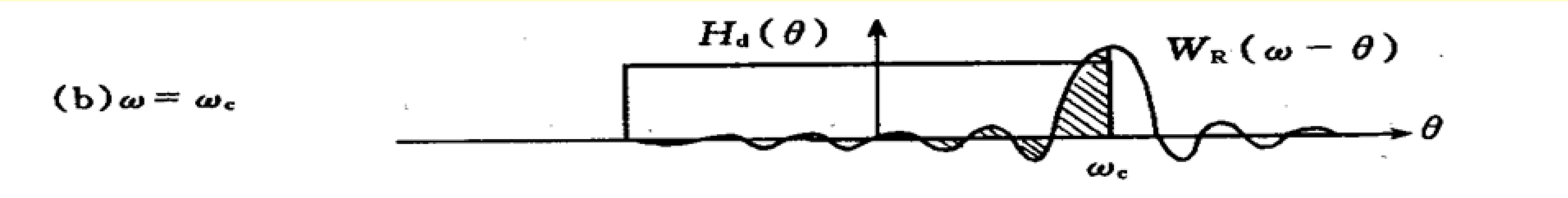
从4个特殊频率点看卷积结果：

（1）ω=0时，H(0)等于 $W_R(\theta)$在$[-\omega_c,\omega_c]$内的积分面积

因一般 $\omega_c >> 2\pi/N$ 故H(0)近似为$W_R(\theta)$ 在[-π, π]内的积分面积

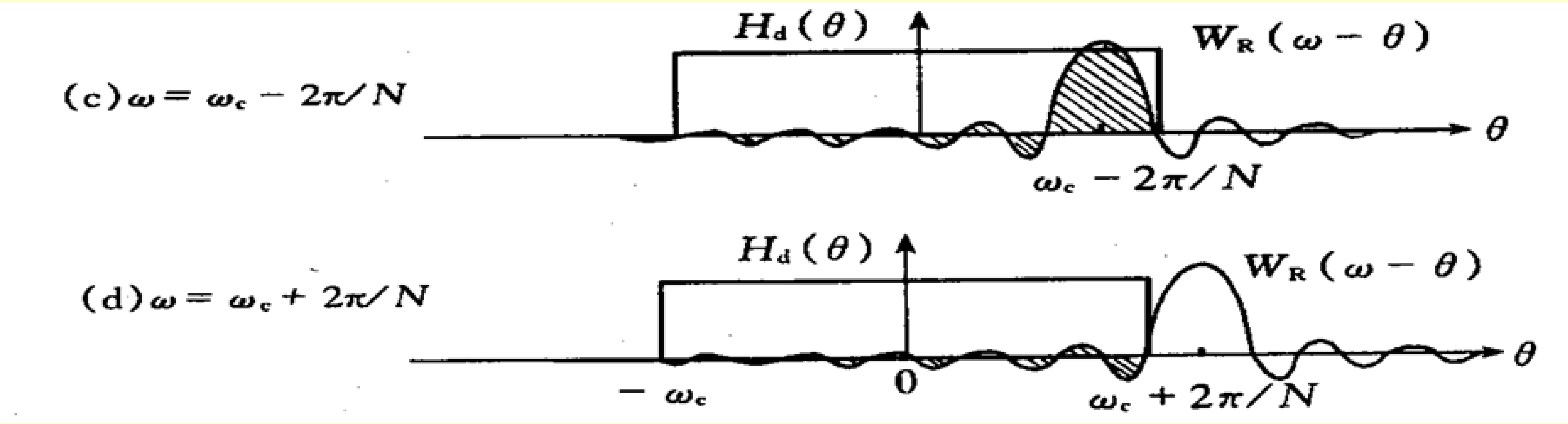


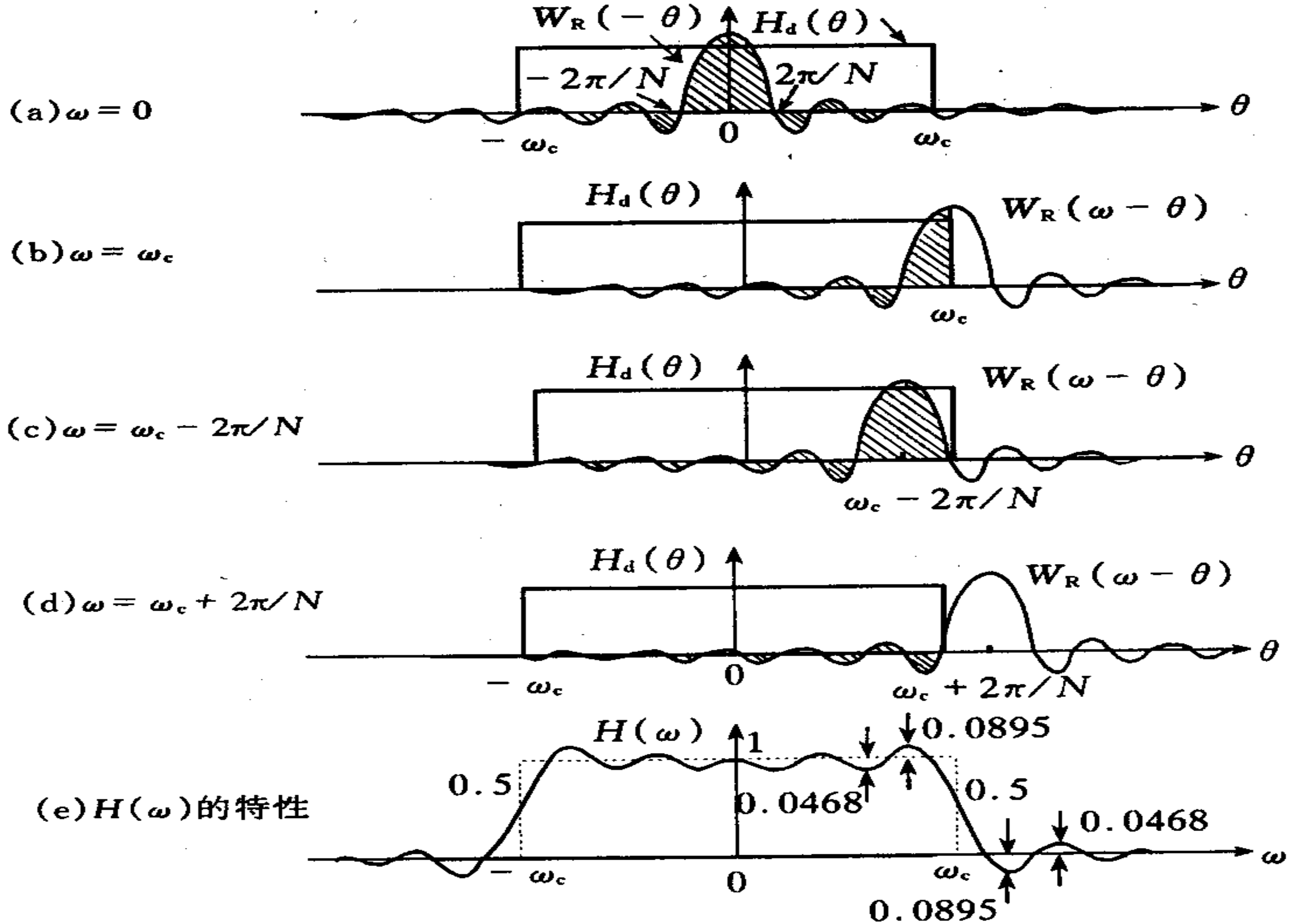
（2）$\omega=\omega_c$时，一半重叠，$H(\omega_c)=0.5\,H(0)$;



（3）$\omega = \omega_c - \frac{2\pi}{N}$ 时，第一旁瓣（负数）在通带外，出现正肩峰；

（4）$\omega = \omega_c + \frac{2\pi}{N}$ 时，第一旁瓣（负数）在通带内，出现负肩峰。





(a) $\omega = 0$
$W_R(-\theta)$
$H_d(\theta)$
$-2\pi/N$
$2\pi/N$
θ
$-\omega_c$
0
ω_c
(b) $\omega = \omega_c$
$H_d(\theta)$
$W_R(\omega-\theta)$
θ
ω_c
(c) $\omega = \omega_c - 2\pi/N$
$H_d(\theta)$
$W_R(\omega-\theta)$
θ
$\omega_c - 2\pi/N$
(d) $\omega = \omega_c + 2\pi/N$
$H_d(\theta)$
$W_R(\omega-\theta)$
θ
$-\omega_c$
0
$\omega_c + 2\pi/N$
(e) $H(\omega)$的特性
$H(\omega)$
1
0.0895
0.5
0.0468
0.5
0.0468
ω
$-\omega_c$
0
ω_c
0.0895

矩形窗的卷积过程

窗口函数对理想特性的影响：

①改变了理想频响的边沿特性，形成过渡带，宽为

$4\pi/N$，等于$W_R(\omega)$的主瓣宽度。（决定于窗长）

②过渡带两旁产生肩峰和余振（带内、带外起伏），取决于 $W_R(\omega)$的旁瓣，旁瓣多，余振多；旁瓣相对值大，肩峰强，与 N无关。（决定于窗口形状）

③N增加, 过渡带宽减小, 肩峰值不变。

因主瓣附近

$$W_R(\omega)=\frac{\sin(\omega N/2)}{\sin(\omega/2)}\approx N\frac{\sin(N\omega/2)}{N\omega/2}=N\frac{\sin x}{x}$$

其中，$x=N\omega/2$，所以N的改变不能改变主瓣与旁瓣的比例关系，只能改变$W_R(\omega)$的绝对值大小和起伏的密度；当N增加时，幅值变大，频率轴变密，而最大肩峰永远为8.95%，这种现象称为吉布斯（Gibbs）效应。

10.2.4 FIR filter approximation with Fixed Window Functions

Rectangular window

The simple truncation of the Fourier series, can be interpreted as the product between the ideal $h_d(n)$ and a rectangular window defined by

$$w_R(n) = \begin{cases} 1, & \text{for } |n| \le M \\ 0, & \text{for } |n| > M \end{cases}$$

§ 10.2.4 Fixed Window Functions

- **Using a tapered window causes the height of the sidelobes to diminish, with a corresponding increase in the main lobe width resulting in a wider transition at the discontinuity**
- **Hann（汉宁窗）:**

$$W[n]=0.5+0.5\cos[2\pi n/(2M+1)],\ -M\leq n\leq M$$

- **Hamming（汉明窗）:**

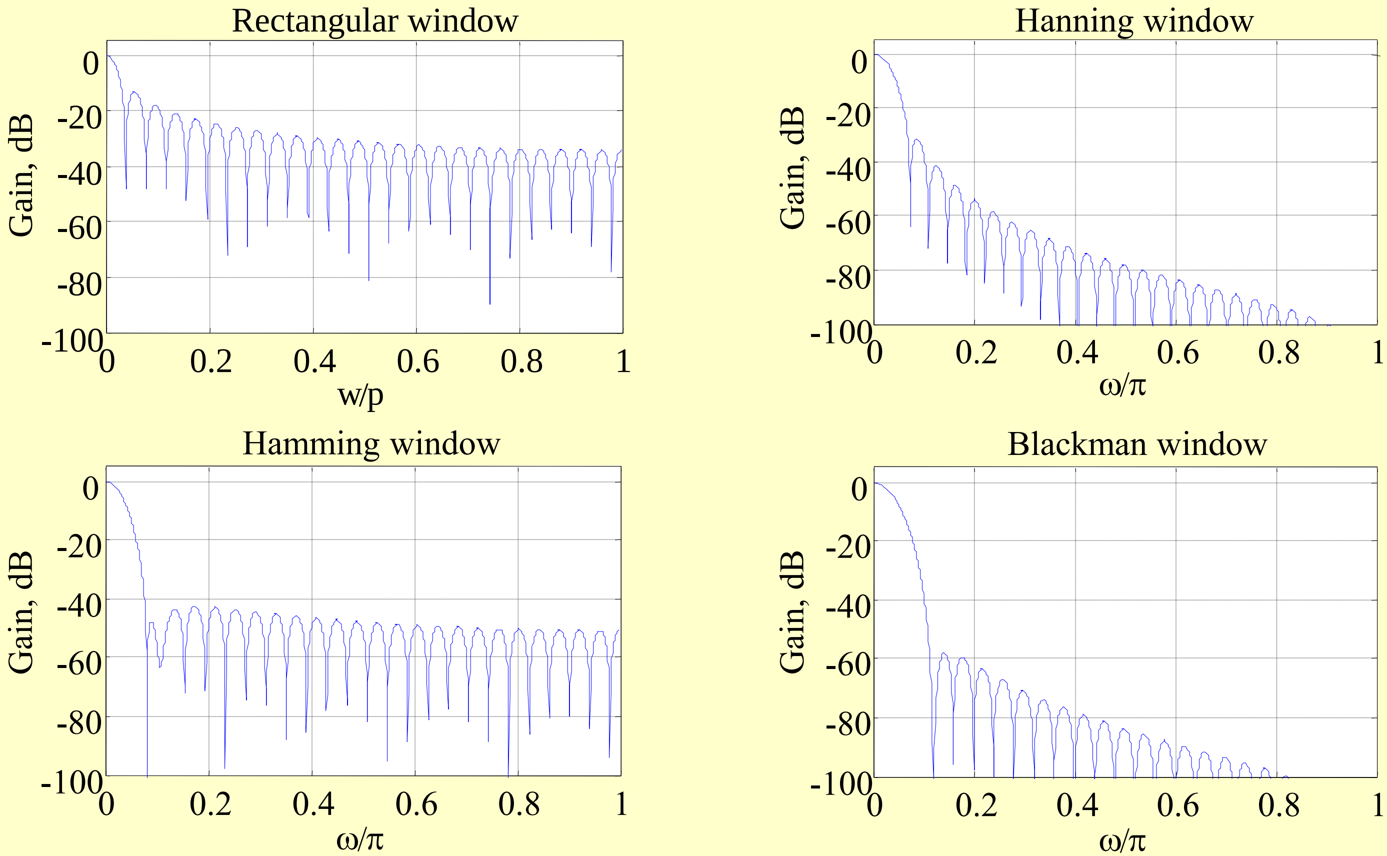
$$W[n]=0.54+0.46\cos[2\pi n/(2M+1)],\ -M\leq n\leq M$$

- **Blackman（布莱克曼窗）:**

$$W[n]=0.42+0.5\cos[2\pi n/(2M+1)]+0.08\cos[4\pi n/(2M+1)]$$

§ 10.2.4 Fixed Window Functions

- **Plots of magnitudes of the DTFTs of these windows for *M* = 25 are shown below**



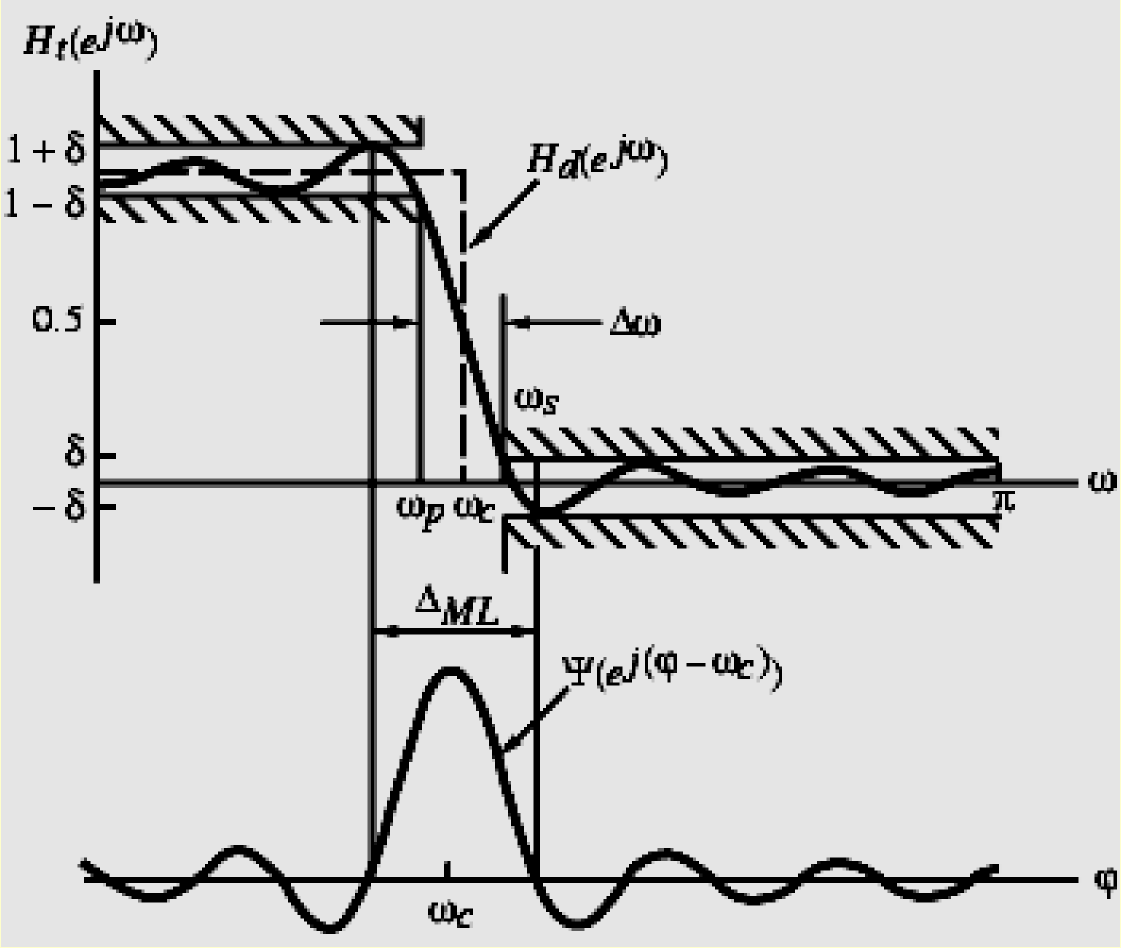
§ 10.2.4 Fixed Window Functions

- **Magnitude spectrum of each window characterized by a main lobe centered at w = 0 followed by a series of sidelobes with decreasing amplitudes**
- **Parameters predicting the performance of a window in filter design are:**
  - **Main lobe width**
  - **Relative sidelobe level**

§ 10.2.4 Fixed Window Functions

- **Main lobe width** Δ_{ML} - given by the distance between zero crossings on both sides of main lobe
- **Relative sidelobe level** A_{sl} - given by the difference in dB between amplitudes of largest sidelobe and main lobe

§ 10.2.4 Fixed Window Functions



- **Passband and stopband ripples are the same**

§ 10.2.4 Fixed Window Functions

- **Distance between the locations of the maximum passband deviation and minimum stopband value $\cong \Delta_{ML}$**

- **Width of transition band**

$$\Delta w = \omega_s - \omega_p < \Delta_{ML}$$

§ 10.2.4 Fixed Window Functions

- To ensure a fast transition from passband to stopband, window should have a very small main lobe width
- To reduce the passband and stopband rippled, the area under the sidelobes should be very small
- Unfortunately, these two requirements are contradictory

肩峰值的大小决定了滤波器通带内的平稳程度和阻带内的衰减，所以对滤波器的性能有很大的影响。

改变窗函数的形状，可改善滤波器的特性，窗函数有许多种，但要满足以下两点要求：

①窗谱主瓣宽度要窄，以获得较陡的过渡带；

②相对于主瓣幅度，旁瓣要尽可能小，使能量尽量集中在主瓣中，这样就 可以减小肩峰和余振，以提高阻带衰减和通带平稳性。

但实际上这两点不能兼得，一般总是通过增加主瓣宽度来换取对旁瓣的抑制。

§ 10.2.4 Fixed Window Functions

- **In the case of rectangular, Hann, Hamming, and Blackman windows, the value of ripple does not depend on filter length or cutoff frequency ω_c , and is essentially constant**
- **In addition,**

$$\Delta w \approx c/M$$

where *c* is a constant for most practical purposes

§ 10.2.4 Fixed Window Functions

- **Filter Design Steps -**

(1) Set $\omega_c = (\omega_p + \omega_s)/2$

(2) Choose window based on specified

(3) Estimate M using

$$\Delta_\omega \approx c / M$$

FIR Filter Approximation

- **The transition bandwidths $\Delta\omega$ depend on the M. When M is constant, the transition bandwidth using rectangular window is the narrowest.**
- **The minimum attenuations in the stopband are constant, and don’t depend on M.**

FIR Filter Approximation

- **How to select the window functions?**
  - **Select the window functions which minimum attenuation in the stopband satisfying the specification required.**
  - **Following the above step, select the window function which transition bandwidth is the narrowest.**

FIR Filter Approximation with Window Functions

Window function	Transition bandwidth $\Delta\omega$	Minimum stopband attenuation
Rectangular	$\frac{0.92\pi}{M}$	20.9dB
Bartlett		
Hanning	$\frac{3.11\pi}{M}$	43.9dB
Hamming	$\frac{3.32\pi}{M}$	54.5dB
Blackman	$\frac{5.56\pi}{M}$	75.3dB

Example:

$\alpha_s = 40dB$? $\qquad$ $\alpha_s = 50dB$?

Hanning window $\qquad$ *Hamming window*

FIR Filter Approximation

The design procedure of FIR filter with window functions

- **According the type of filter, determine the ideal impulse response h(n).**
- **According the specifications given by the application, select the window function and determine the the M.**

- Windowing: $h_t(n) = h_d(n) \cdot w(n)$

  Delay M sample : $h_t^{'}(n) = h_t(n - M)$ (make the filter be causal)

FIR Filter Approximation with Window Functions

Example: Design a lowpass FIR filter satisfying the specification below using the windows function method:

passband edge frequency ω_p : 0.4π

stopband edge frequency ω_s : 0.6π

the stopband attenuation in dB is not less than α_s =50dB

FIR Filter Approximation with Window Functions

SOLUTION:

i) Compute the cutoff frequency of ideal lowpass filter.

$$\omega_c = (\omega_p + \omega_s)/2 = 0.5\pi$$

Determine the impulse response of the ideal lowpass filter.

$$h_d(n) = \int_{-\omega_c}^{\omega_c} e^{j\omega n} d\omega = \begin{cases} 0.5, & for \quad n = 0 \\ \dfrac{1}{\pi \cdot n} \sin(0.5\pi \ n), & for \quad n \neq 0 \end{cases}$$

FIR Filter Approximation with Window Functions

ii) Since the stopband attenuation is not less than 50dB, we select the Hamming window. For Hamming window, the transition bandwidth,

$$\Delta\omega = \frac{3.32\pi}{M} \le (\omega_s - \omega_p)$$

Thus, $M \ge 16.6$. **We choose** $M = 17$

The window function:

$$w_H(n) = 0.54 + 0.46\cos(\frac{2\pi \cdot n}{35}), \quad for\ |n| \le 17$$

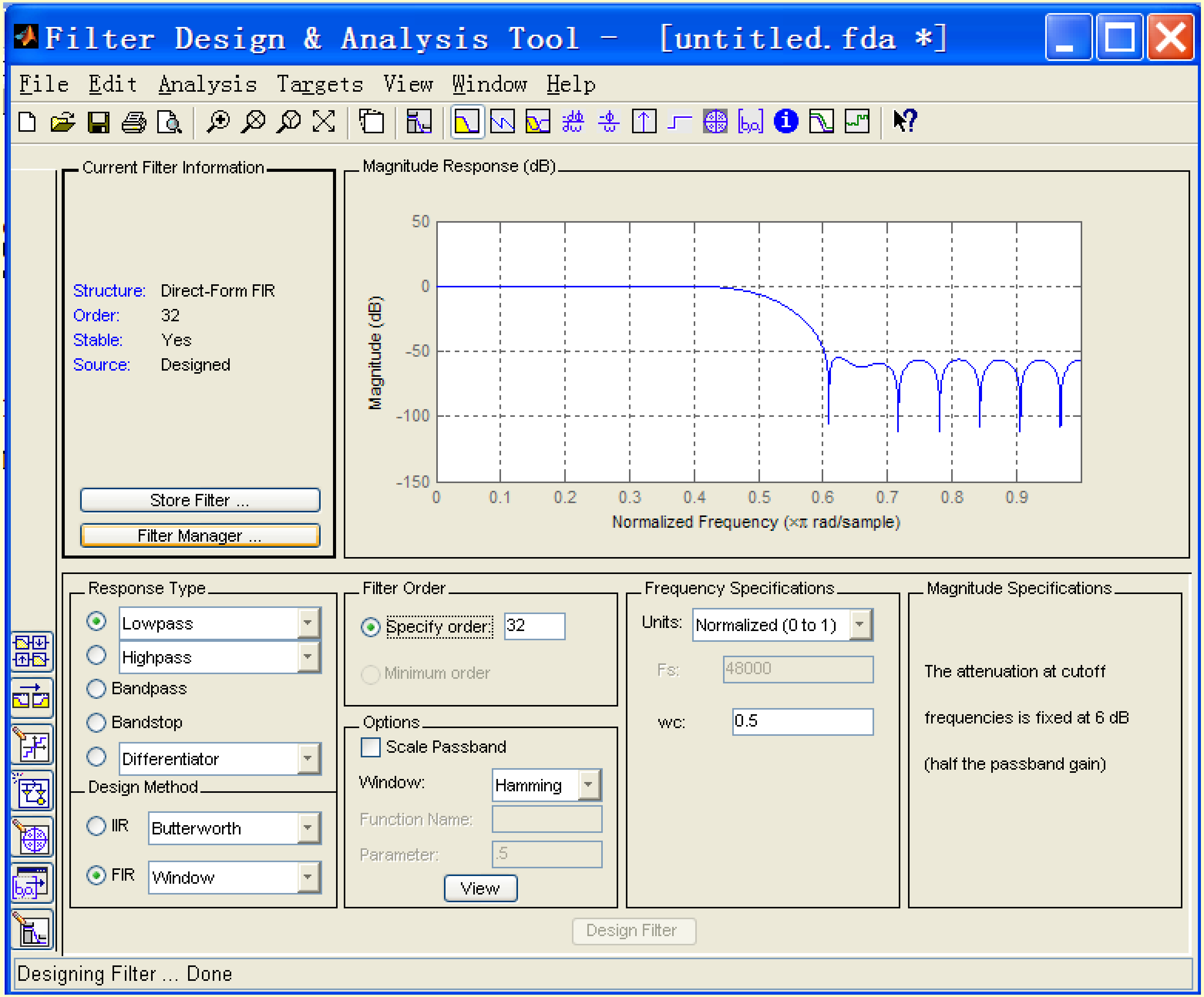
FIR Filter Approximation with Window Functions

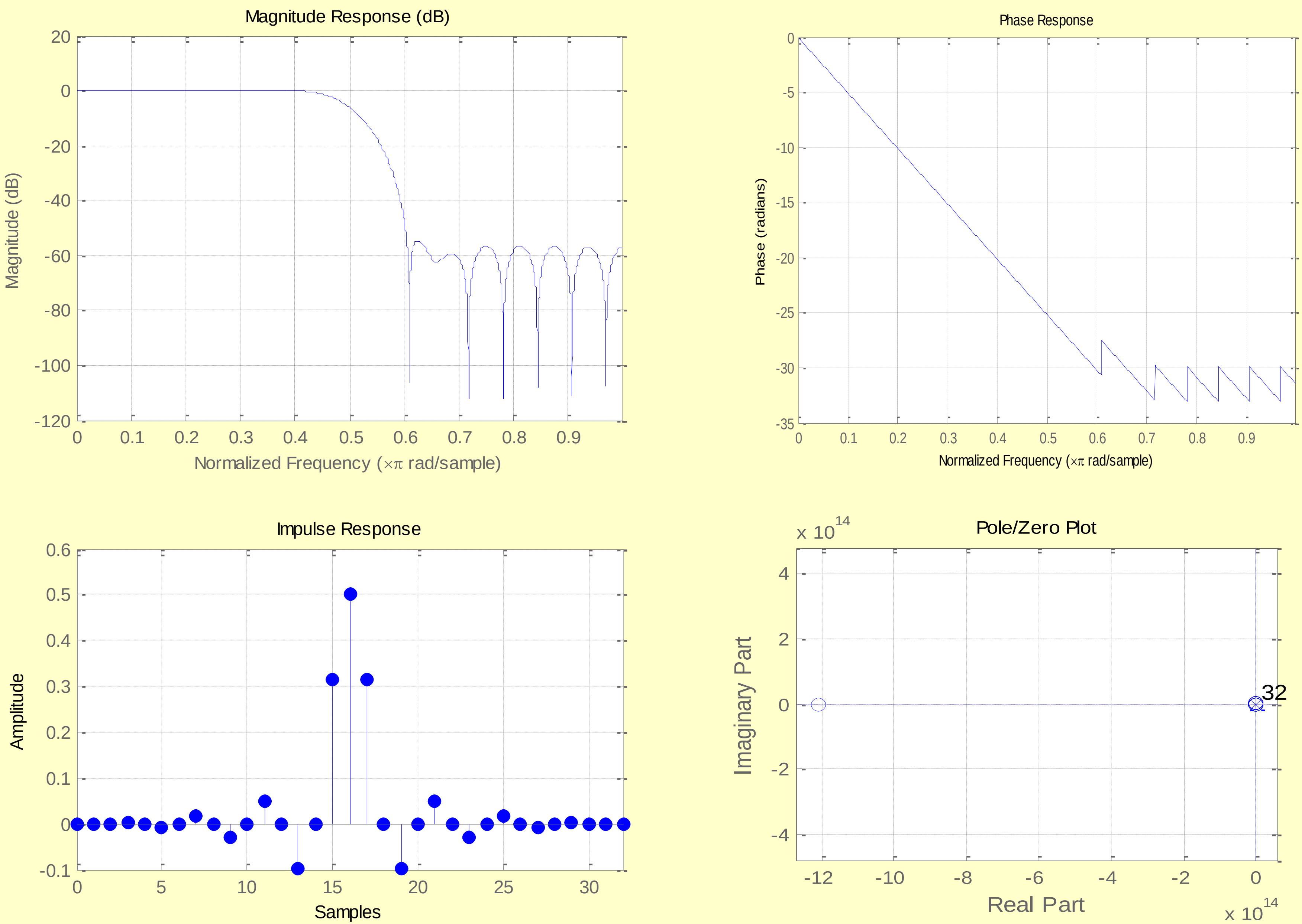
iii) Determine the impulse response of the lowpass FIR filter:

$$h_t(n) = h_d(n) \cdot w_H(n)$$

$$h_t^{'}(n) = h_t(n-M) = \begin{cases} 0.5, \quad for \quad n = 17 \\ \dfrac{1}{\pi(n-17)} \sin[0.5\pi(n-17)] \cdot \{0.54 + 0.46\cos[\dfrac{2\pi(n-17)}{25}]\}, \\ \qquad\qquad\qquad for \quad 0 < |n-17| \leq 17 \end{cases}$$

Simulation


Filter Design & Analysis Tool - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 32
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Magnitude (dB)
50
0
-50
-100
-150
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Normalized Frequency (×π rad/sample)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR Butterworth
FIR Window
Filter Order
Specify order: 32
Minimum order
Options
Scale Passband
Window: Hamming
Function Name:
Parameter: .5
View
Frequency Specifications
Units: Normalized (0 to 1)
Fs: 48000
wc: 0.5
Magnitude Specifications
The attenuation at cutoff
frequencies is fixed at 6 dB
(half the passband gain)
Design Filter
Designing Filter ... Done


Magnitude Response (dB)
Magnitude (dB)
Normalized Frequency (×π rad/sample)
Phase Response
Phase (radians)
Normalized Frequency (×π rad/sample)
Impulse Response
Amplitude
Samples
Pole/Zero Plot
x 10^14
Imaginary Part
Real Part
32

Chapter 10
FIR Digital Filter Design

- **10.1 Preliminary Considerations**
- **10.2 FIR Filter Design Based on Windowed Fourier Series**
- **10.5 FIR Digital Filter Design Using Matlab**

FIR Digital Filter Design Using MATLAB

- Order Estimation -
- Kaiser's Formula:

$$N \cong \frac{-20\log_{10}(\sqrt{\delta_p \delta_s})}{14.6(\omega_s - \omega_p)/2\pi}$$

- Note: Filter order N is inversely proportional to transition band width $(\omega_s - \omega_p)$ and does not depend on actual location of transition band

FIR Digital Filter Design Using MATLAB

- Hermann-Rabiner-Chan's Formula:

$$N \cong \frac{D_\infty(\delta_p,\delta_s) - F(\delta_p,\delta_s)[(\omega_s-\omega_p)/2\pi]^2}{(\omega_s-\omega_p)/2\pi}$$

where

$$D_\infty(\delta_p,\delta_s) = [a_1(\log_{10}\delta_p)^2 + a_2(\log_{10}\delta_p) + a_3]\log_{10}\delta_s + [a_4(\log_{10}\delta_p)^2 + a_5(\log_{10}\delta_p) + a_6]$$

$$F(\delta_p,\delta_s) = b_1 + b_2[\log_{10}\delta_p - \log_{10}\delta_s]$$

with $a_1 = 0.005309,\ a_2 = 0.07114,\ a_3 = -0.4761$

$$a_4 = 0.00266,\ a_5 = 0.5941,\ a_6 = 0.4278$$

$$b_1 = 11.01217,\quad b_2 = 0.51244$$

FIR Digital Filter Design Using MATLAB

- Formula valid for $\delta_p \geq \delta_s$
- For $\delta_p < \delta_s$, formula to be used is obtained by interchanging δ_p and δ_s
- Both formulas provide only an estimate of the required filter order N
- Frequency response of FIR filter designed using this estimated order may or may not meet the given specifications
- If specifications are not met, increase filter order until they are met

FIR Digital Filter Design Using MATLAB

- MATLAB code fragments for estimating filter order using Kaiser's formula

```
num = - 20*log10(sqrt(dp*ds)) - 13;
den = 14.6*(Fs - Fp)/FT;
N = ceil(num/den);
```

- M-file remezord implements Hermann-Rabiner-Chan's order estimation formula

Window-Based FIR Filter Design Using MATLAB

- Window Generation - Code fragments to use

```
w = blackman(L);
w = hamming(L);
w = hanning(L);
w = chebwin(L, Rs);
w = kaiser(L, beta);
```

where window length L is odd

Window-Based FIR Filter Design Using MATLAB

- Example - Kaiser window design for use in a lowpass FIR filter design
- Specifications of lowpass filter: $\omega_p = 0.3\pi$, $\omega_s = 0.4\pi$, $\alpha_s = 50$ dB $\Rightarrow \delta_s = 0.003162$
- Code fragments to use

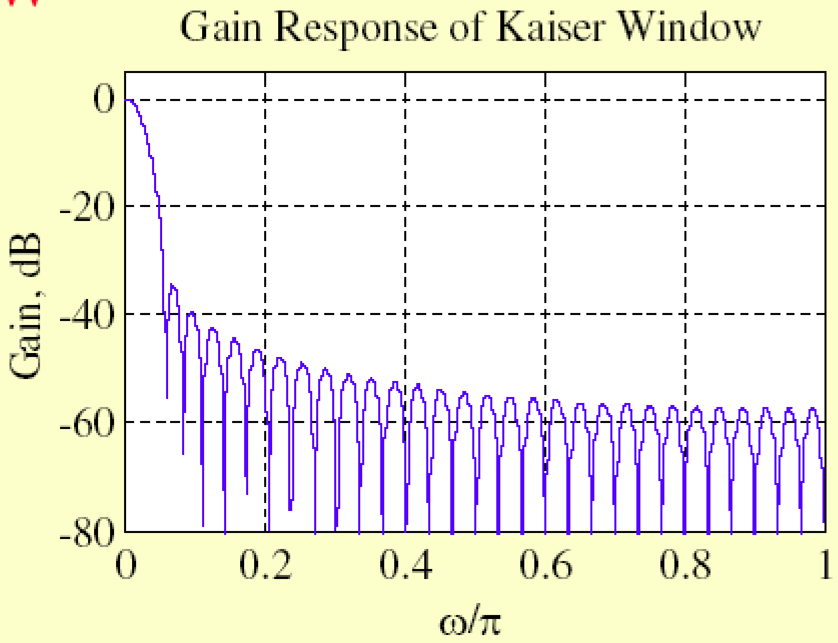
```
[N, Wn, beta, ftype] = kaiserord(fpts, mag,dev);
w = kaiser(N+1, beta);
```

where

```
fpts = [0.3   0.4]
mag = [1   0]
dev = [0.003162   0.003162]
```

Window-Based FIR Filter Design Using MATLAB

- Plot of the gain response of the Kaiser window



Window-Based FIR Filter Design Using MATLAB

- M-files available are fir1 and fir2
- fir1 is used to design conventional lowpass, highpass, bandpass, bandstop and multiband FIR filters
- fir2 is used to design FIR filters with arbitrarily shaped magnitude response
- In fir1, Hamming window is used as a default if no window is specified

Window-Based FIR Filter Design Using MATLAB

- Example - Design using a Hamming window an FIR filter of order 100 with three different constant magnitude levels: 0.3 in the frequency range [0, 0.28], 1.0 in the frequency range [0.3, 0.5], and 0.7 in the frequency range [0.52, 1.0]

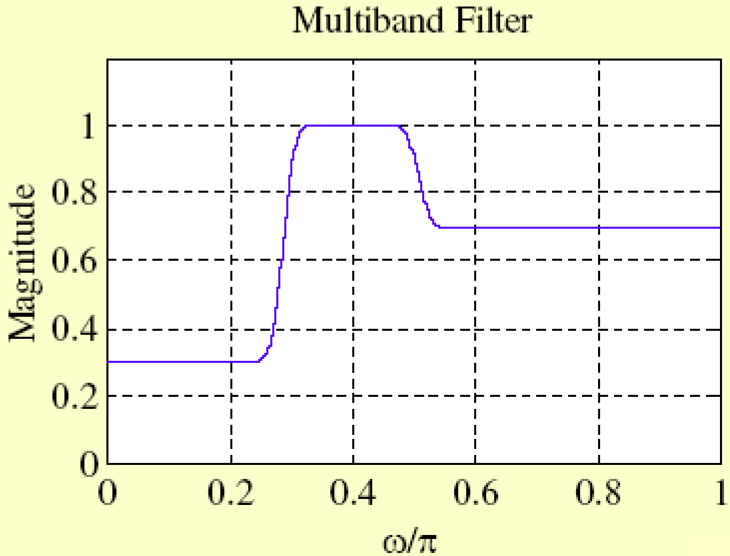
Window-Based FIR Filter Design Using MATLAB

- Code fragment to use

  b = fir2(100, fpts, mval);

  where fpts = [0 0.28 0.3 0.5 0.52 1];

  mval = [0.3 0.3 1.0 1.0 0.7 0.7];



Homework

Problems:10.3（a）, 10.17（a）