# Digital Signal Processing

# 第一章重点

- 模拟信号、数字信号、抽样数据信号、量化阶梯信号之间的区别与联系。
- 数字信号的产生过程；
- 典型的数字信号处理系统框图；

## ■ Digital processing of an analog signal

### Complete block-diagram


前后两个滤波器的类型：模拟低通滤波器

各自的作用：前者抗混叠，后者平滑

滤波器设置的目的、折叠频率等

# 第二章重点

- 序列的分类
- 序列的运算
- 采样及采样定理、混叠现象。

# 序列的分类

- **A discrete-time signal can be classified in various ways**
  - **Length: Finite-length vs. Infinite-length**
  - **Symmetry: Conjugate-symmetric vs. Conjugate-antisymmetric**
  - **Periodic: Periodic vs. Aperiodic**
  - **Energy and Power**
  - **Summability: bounded, absolutely summable, square-summable**

## ★卷积和（ convolution sum ）的计算

$$y[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]=\sum_{k=-\infty}^{\infty}x[n-k]h[k]$$

- 翻转平移法
- 列表法（不进位乘法）
- 利用性质

# ★翻转平移法

序列x(n)={1,2,3}, h(n)={1,2}

求两个序列的线性卷积y(n)=x(n)*h(n)

| | | | | | | |
|---|---|---|---|---|---|---|
| x(n) | | 1 | 2 | 3 | | |
| h(n) | | 1 | 2 | | | |
| 反转 h(-n) | 2 | 1 | | | | y(0)=1 |
| 平移 h(1-n) | | 2 | 1 | | | y(1)=4 |
| 平移 h(2-n) | | | 2 | 1 | | y(2)=7 |
| 平移 h(3-n) | | | | 2 | 1 | y(3)=6 |

# 采样

- 什么是采样？
- 信号经采样后的特征变化
- 信号内容是否丢失由离散信号恢复连续信号的条件（如何不失真地还原信号）

# 以正弦信号为例，分析采样过程

- **Consider the continuous-time signal**

模拟角频率

$$x(t) = A\cos(2\pi f_o t + \phi) = A\cos(\Omega_o t + \phi)$$

**The corresponding discrete-time signal is**

$$x[n] = A\cos(\Omega_o nT + \phi) = A\cos(\frac{2\pi\Omega_o}{\Omega_T} n + \phi)$$

$$= A\cos(\omega_o n + \phi)$$

**where** $\boldsymbol{\omega_o = 2\pi\,\Omega_o / \Omega_T = \Omega_o T = \Omega_o / F_T}$

数字角频率是模拟角频率对采样频率的归一化

- 采样所得序列的频谱是模拟信号频谱在频率轴上以采样频率为周期进行周期延拓的结果

$$G_p(j\Omega)=\frac{1}{T}\sum_{k=-\infty}^{\infty}G_a\big(j(\Omega-k\Omega_T)\big)$$

# Sampling theorem（采样定理）

**Let $g_a(t)$ be a band-limited signal with CTFT $G_a(j\Omega)=0$ for $|\Omega| > \Omega_m$**

**Then $g_a(t)$ is uniquely determined by its samples $g_a(nT)$ , $-\infty \le n \le \infty$, if**

$$\Omega_T \geq 2\,\Omega_m$$

**where $\Omega_T = 2\pi/T$**

# 第三章重点

- **DTFT**定义与性质

# DTFT and IDTFT

$$x[n] \leftrightarrow X(e^{j\omega})$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

# DTFT的性质

- 序列频谱具有周期性，周期为 $2\pi$
- **the absolute summability of x[n] is a sufficient condition（充分条件） for the existence of the DTFT**

$$\sum_{n=-\infty}^{\infty} \left| x[n] \right| < \infty$$

# Table 3.2: DTFT Properties: Symmetry Relations

| Sequence | Discrete-Time Fourier Transform |
| --- | --- |
| $x[n]$ | $X(e^{j\omega}) = X_{re}(e^{j\omega}) + jX_{im}(e^{j\omega})$ |
| $x_{ev}[n]$<br>$x_{od}[n]$ | $X_{re}(e^{j\omega})$<br>$jX_{im}(e^{j\omega})$ |
| Symmetry relations | $X(e^{j\omega}) = X^*(e^{-j\omega})$<br>$X_{re}(e^{j\omega}) = X_{re}(e^{-j\omega})$<br>$X_{im}(e^{j\omega}) = -X_{im}(e^{-j\omega})$<br>$\|X(e^{j\omega})\| = \|X(e^{-j\omega})\|$<br>$\arg\{X(e^{j\omega})\} = -\arg\{X(e^{-j\omega})\}$ |

Note: $x_{ev}[n]$ and $x_{od}[n]$ denote the even and odd parts of $x[n]$, respectively.

实偶——实偶

实奇——虚奇

$x[n]$: A real sequence

# 常用的 DTFT变换对

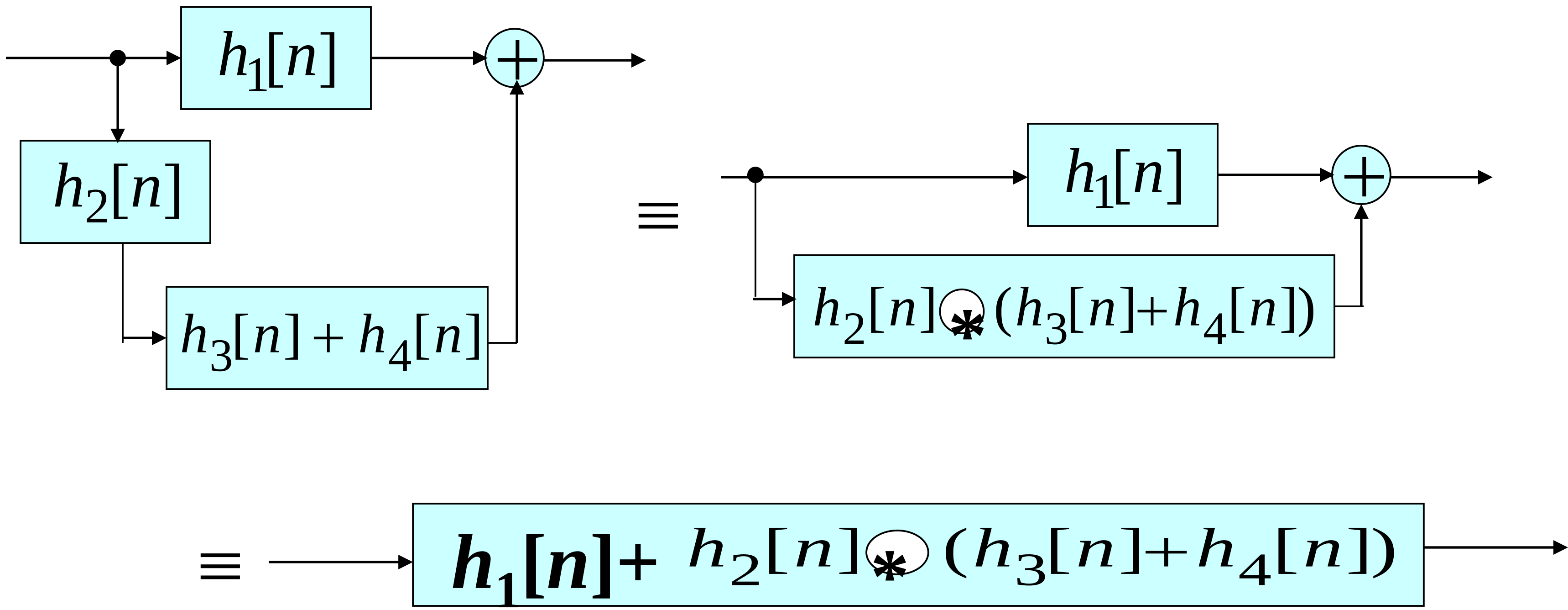
| Sequence | | DTFT |
|---|---|---|
| $\delta[n]$ | $\leftrightarrow$ | $1$ |
| $\alpha^n \mu[n], (\lvert\alpha\rvert < 1)$ | $\leftrightarrow$ | $\dfrac{1}{1-\alpha e^{-j\omega}}$ |

与时移、差分等性质结合

# 第四章 重点

- 系统的性质及其判定

# 系统之间的简单互连
# Simple Interconnection Schemes



例题：判断线性性和时不变性

- **$y(n)=2x(n)+5$** 非线性、时不变
- **$y(n)=x^2(n)$** 非线性、时不变
- **$y(n)=nx(n)$** 线性、时变
- **$y(n)=x(n-n_0)$** 线性、时不变
- $$y(n)=\sum_{m=-\infty}^{n} x(m)$$ 线性、时不变

# 判断下列系统的因果稳定性

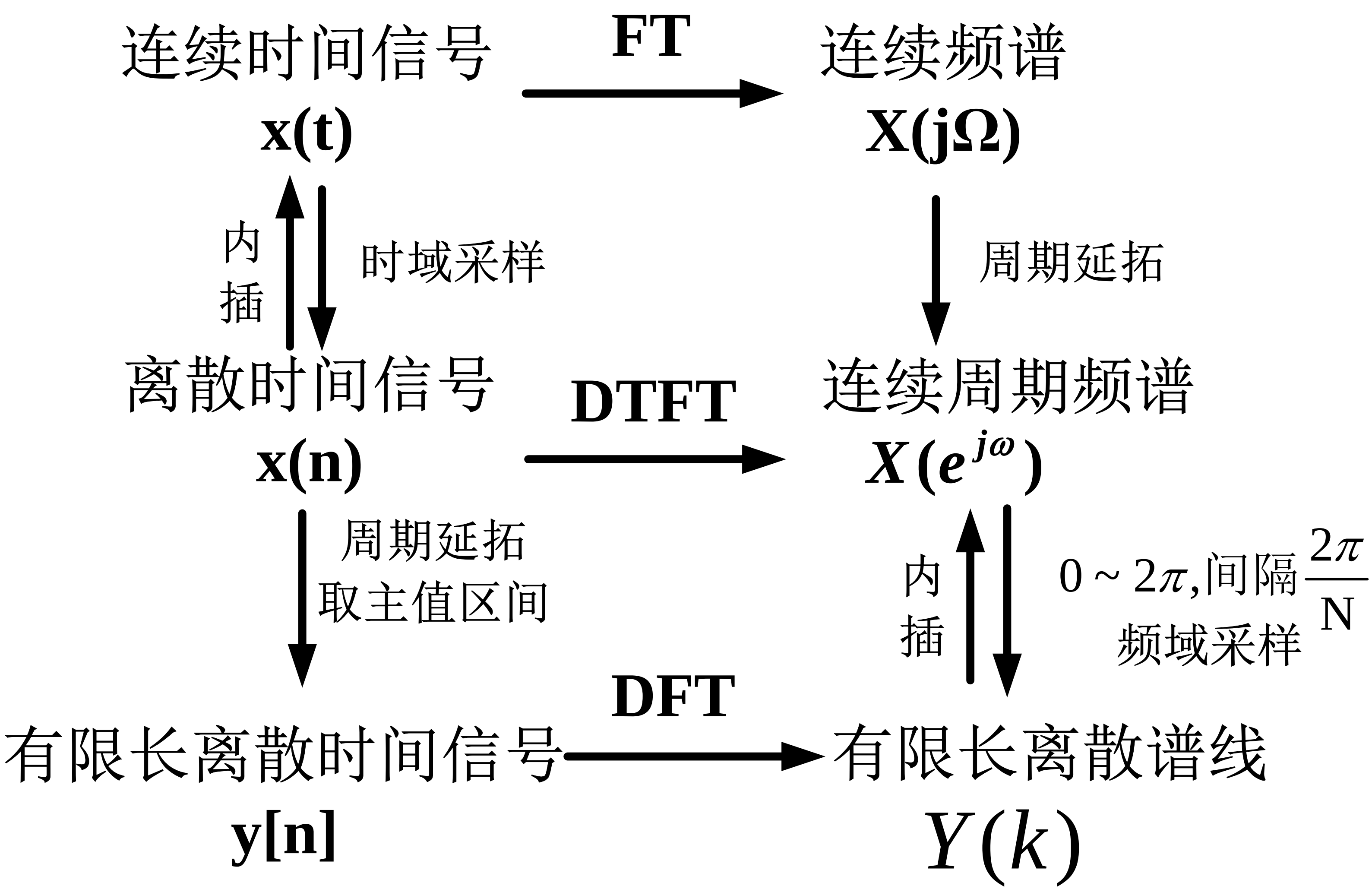
- $0.5^n u(n)$
- $2^n u(n)$
- $(-2)^n u(n)$
- $2^n u(-n)$
- $0.5^n u(-n-1)$
- $2^n R_{10}(n)$

# 第五、六章 变换

**DTFT、DFT、Z**

# 总结：几种变换之间的关系 ★

| | 变换式 | 变换域 |
|---|---|---|
| **Z变换** | $X(z)=\sum_{n=0}^{L-1}x(n)z^{-n}$ | **Z平面** |
| **DTFT** | $X(\omega)=\sum_{n=0}^{L-1}x(n)e^{-j\omega n}$ | **Z平面单位圆** |
| **DFT** | $X(k)=\sum_{n=0}^{L-1}x(n)e^{-j\omega_k n}$ <br> 其中 $\omega_k=\frac{2\pi}{N}k$ （***k***＝**0~*N*-1**） | **Z平面单位圆上等间隔的离散频点** |


连续时间信号
x(t)
FT
连续频谱
X(jΩ)
内插
时域采样
周期延拓
离散时间信号
x(n)
DTFT
连续周期频谱
$X(e^{j\omega})$
周期延拓
取主值区间
内插
$0 \sim 2\pi$,间隔$\frac{2\pi}{N}$
频域采样
有限长离散时间信号
y[n]
DFT
有限长离散谱线
$Y(k)$

# 第五章  重点

- DFT的定义、性质及其证明
- 循环卷积与线性卷积的关系
- 实序列的DFT
- 线性卷积的DFT实现
- 重叠相加法

# 1. DFT Definition

- Using the notation $W_N = e^{-j2\pi/N}$, the DFT is usually expressed as:

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}, \quad 0 \le k \le N-1$$

- The inverse discrete Fourier transform (IDFT) is given by

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn}, \quad 0 \le n \le N-1$$

- 长度为L的序列x[n]的N点DFT

$$X(k)=\sum_{n=0}^{L-1}x(n)e^{-j\frac{2\pi}{N}nk},0\le k\le N-1;$$

L反映序列长度

N反映DFT点数

- L和N可以相等，也可以不等；若L<N, 可以对输入序列补零，使补零后序列长度等于N

# 2. Matrix Relations

- where $\mathbf{X} = \begin{bmatrix} X(0) & X(1) & \cdots & X(N-1) \end{bmatrix}^T$

$$\mathbf{x} = \begin{bmatrix} x(0) & x(1) & \cdots & x(N-1) \end{bmatrix}^T$$

$$\mathbf{D}_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^1 & W_N^2 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}_{N\times N} \quad \bigstar$$

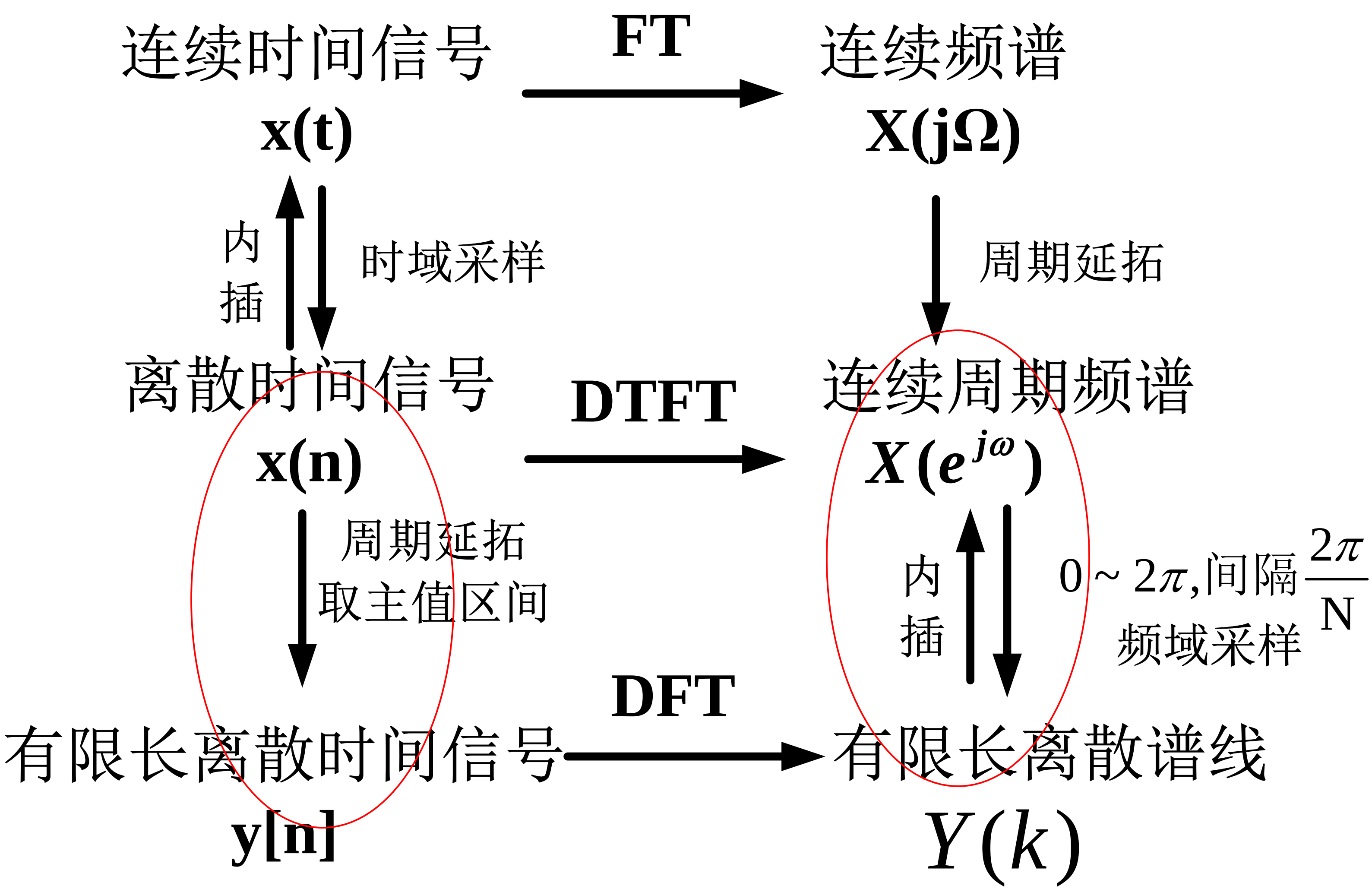
**twiddle factor (旋转因子):**

$$W_N = e^{-j2\pi/N}$$

$$W_N^0 = W_N^N = 1, \quad W_N^{N/2} = -1, \quad W_N^{N+k} = W_N^k,$$

N=L=2， $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

N=L=4， $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$



连续时间信号
x(t)
FT
连续频谱
X(jΩ)
内插
时域采样
周期延拓
离散时间信号
x(n)
DTFT
连续周期频谱
$X(e^{j\omega})$
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取主值区间
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$0 \sim 2\pi$,间隔$\frac{2\pi}{N}$
频域采样
DFT
有限长离散时间信号
y[n]
有限长离散谱线
$Y(k)$

# Sampling the DTFT

- To apply

$$y(n) = \sum_{m=-\infty}^{\infty} x(n + mN), \quad 0 \le n \le N - 1$$

to finite-length sequences, we assume that the samples outside the specified range are zeros

- Thus if $x(n)$ is a length-$M$ sequence with $M \leqslant N$, then $y(n) = x(n)$ for $0 \leqslant n \leqslant N-1$

# Sampling the DTFT

- If $M > N$, there is a time-domain aliasing of samples of $x(n)$ in generating $y(n)$ , and $x(n)$ cannot be recovered from $y(n)$
- This is called *Sampling Theory in Frequency-Domain ($N \geqslant M$)*

# 3、Circular Convolution

- **the relation between the circular convolution and the linear convolution**

时域周期延拓，周期为**N**

$$y_c(n) = \underline{y_L((n))_N} \cdot R_N(n)$$

if $N < L + K - 1$: aliasing $y_c(n) \neq y_L(n)$

if $N \geq L + K - 1$: not aliasing $y_c(n) = y_L(n)$

**the condition that the circular convolution to be equivalent to the linear convolution is** $N \geq L + K - 1$

序列x(n)={1,2,3}, h(n)={1,2}

求两个序列的线性卷积y(n)=x(n)*h(n)

| | | | | | | |
|---|---|---|---|---|---|---|
| x(n) | | 1 | 2 | 3 | | |
| h(n) | | 1 | 2 | | | |
| 反转 h(-n) | 2 | 1 | | | | y(0)=1 |
| 平移 h(1-n) | | 2 | 1 | | | y(1)=4 |
| 平移 h(2-n) | | | 2 | 1 | | y(2)=7 |
| 平移 h(3-n) | | | | 2 | 1 | y(3)=6 |

# 序列x(n)={1,2,3}, h(n)={1,2}　N＝3

求两个序列的循环卷积y1(n)

| | | |
|---|---|---|
| x(n) | 1 2 3 | |
| h(n) | 1 2 0 | |
| 反转 h(-n) | 1 0 2 | y1(0)=7 |
| 平移 h(1-n) | 2 1 0 | y1(1)=4 |
| 平移 h(2-n) | 0 2 1 | y1(2)=7 |

y1(n)是有限长序列，序列值为{7,4,7}

序列x(n)={1,2,3}, h(n)={1,2}  N＝4

求两个序列的循环卷积y2(n)

| | | | | | | |
|---|---|---|---|---|---|---|
| | x'(n) | 1 | 2 | 3 | 0 | |
| | h'(n) | 1 | 2 | 0 | 0 | |
| 反转 | h'(-n) | 1 | 0 | 0 | 2 | y2(0)=1 |
| 平移 | h'(1-n) | 2 | 1 | 0 | 0 | y2(1)=4 |
| 平移 | h'(2-n) | 0 | 2 | 1 | 0 | y2(2)=7 |
| 平移 | h'(3-n) | 0 | 0 | 2 | 1 | y2(3)=6 |

y2(n)是有限长序列，序列值为{1,4,7,6}

## Overlap-Add Method: ★

**Compute the convolution** $\mathbf{y} = \mathbf{h} * \mathbf{x}$ **of the filter and input,**

$$\mathbf{h} = [1 \quad 2 \quad 2 \quad 1], \quad \mathbf{x} = [2 \quad 3 \quad 3 \quad 5 \quad 3 \quad 1 \quad 1]$$

**using the overlap-add method of block convolution with length-5 blocks.**

**Solution:**

$$\mathbf{x} = [2 \quad 3 \quad 3 \quad 5 \quad 3 \mid 1 \quad 1]$$

$$\mathbf{y0} = \mathbf{h} * [2 \quad 3 \quad 3 \quad 5 \quad 3] = [2 \quad 7 \quad 13 \quad 19 \quad 22 \quad 19 \quad 11 \quad 3]$$

$$\mathbf{y}1 = \mathbf{h} * [1 \quad 1 \quad 0 \quad 0 \quad 0] = [1 \quad 3 \quad 4 \quad 3 \quad 1]$$

# How to Overlap Add?

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| y0 | 2 | 7 | 13 | 19 | 22 | 19 | 11 | 3 | | |
| y1 | | | | | | 1 | 3 | 4 | 3 | 1 |
| y | 2 | 7 | 13 | 19 | 22 | 20 | 14 | 7 | 3 | 1 |

# 第六章重点

- **Z变换、零极点求解、收敛域判定**
- **逆Z变换（部分分式法）**
- **传递函数**
- **几何作图法**
- **因果稳定性的Z域判决**

| Sequence | ZT | ROC |
|---|---|---|
| $\delta[n]$ | $1$ | All value of $z$ |
| $\mu[n]$ | $\frac{1}{1-z^{-1}}$ | $\lvert z\rvert > 1$ |
| $\alpha^n \mu[n]$ | $\frac{1}{1-\alpha z^{-1}}$ | $\lvert z\rvert > \lvert a\rvert$ |
| $-\alpha^n \mu[-n-1]$ | $\frac{1}{1-\alpha z^{-1}}$ | $\lvert z\rvert < \lvert a\rvert$ |
| $n\alpha^n \mu[n]$ | $\frac{\alpha z^{-1}}{\left(1-\alpha z^{-1}\right)^2}$ | $\lvert z\rvert > \lvert a\rvert$ |
| $(n+1)\alpha^n \mu[n]$ | $\frac{1}{\left(1-\alpha z^{-1}\right)^2}$ | $\lvert z\rvert > \lvert a\rvert$ |

## DTFT与Z变换的关系

★ $$X(e^{j\omega}) = X(z)\Big|_{z=e^{j\omega}} = \sum_{n=-\infty}^{\infty} x(n)e^{-jn\omega}$$

★采样序列在单位圆上的Z变换等于该序列的DTFT

# Rational z-Transform

- 零极点共轭成对出现、收敛域内无极点
- 需注意的是：求解零、极点时，为避免遗漏，需先将Z变换有理分式的分子和分母都转换成Z的正数次幂，再进行求解。

$$X(Z)=\frac{1}{1-az^{-1}}$$

$$=\frac{Z}{Z-a}$$

# 有限长序列的Z变换

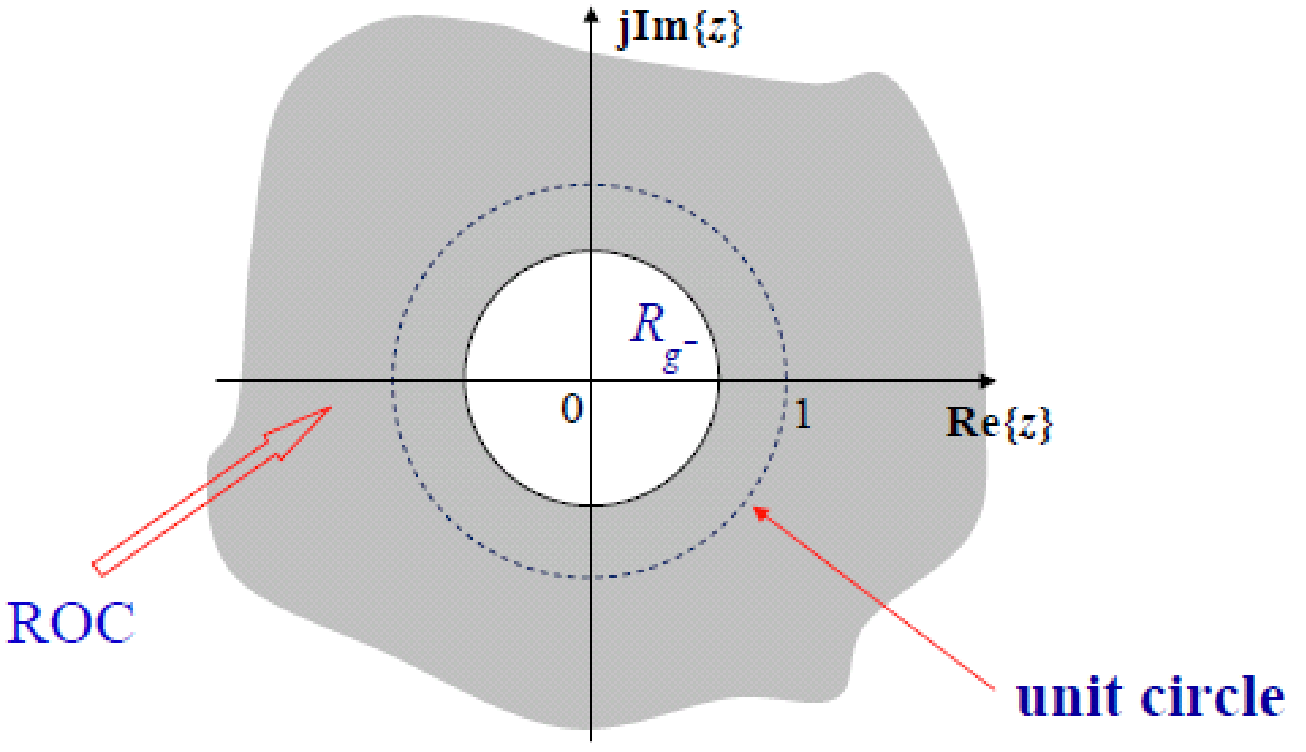
（1） 有限长序列的收敛域一定包含 $0<|z|<\infty$

（2） 如果对 $n_1, n_2$ 加以一定的限制，则：

$$\begin{cases} 0<|z|\le\infty & n_1\ge 0 \\ 0\le|z|<\infty & n_2\le 0 \end{cases}$$

## – Right-sided Sequence

A **right-sided sequence** $u(n)$ with nonzero sample values only for $n \geqslant M$



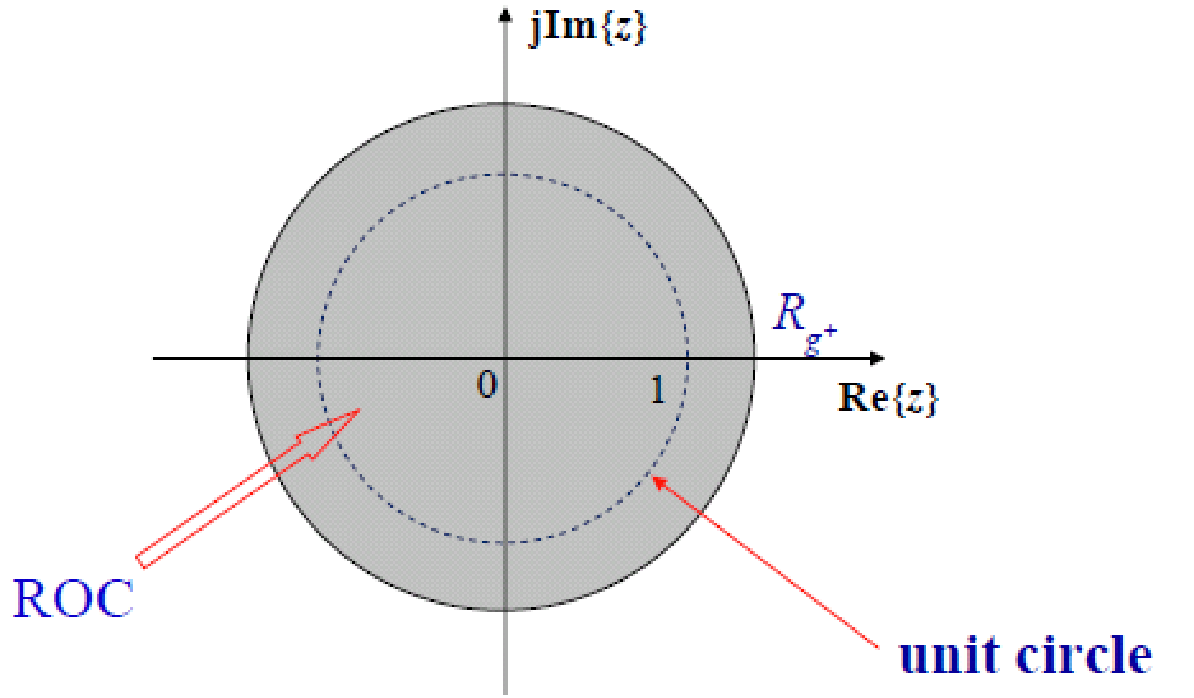

If $M \geqslant 0$, $R_{g-} < |z| \leq \infty \quad R_{g+} = \infty$

If $M < 0$, $R_{g-} < |z| < \infty \quad R_{g+} < \infty$

✓Z变换的收敛域包括∞点是因果序列的特征。

## – Left-sided Sequence

A **left-sided sequence** $v$(n) with nonzero sample values only for $n \leqslant N$




If $N > 0$, $0 < |z| < R_{g+}$ $R_{g-} > 0$

If $N \leqslant 0$, $0 \leq |z| < R_{g+}$ $R_{g-} = 0$

If $N$=0, $v(n)$ is called a **anticausal sequence**

## – Two-sided Sequence

The z-Transform of a **two-sided sequence** $w(n)$ can be expressed as

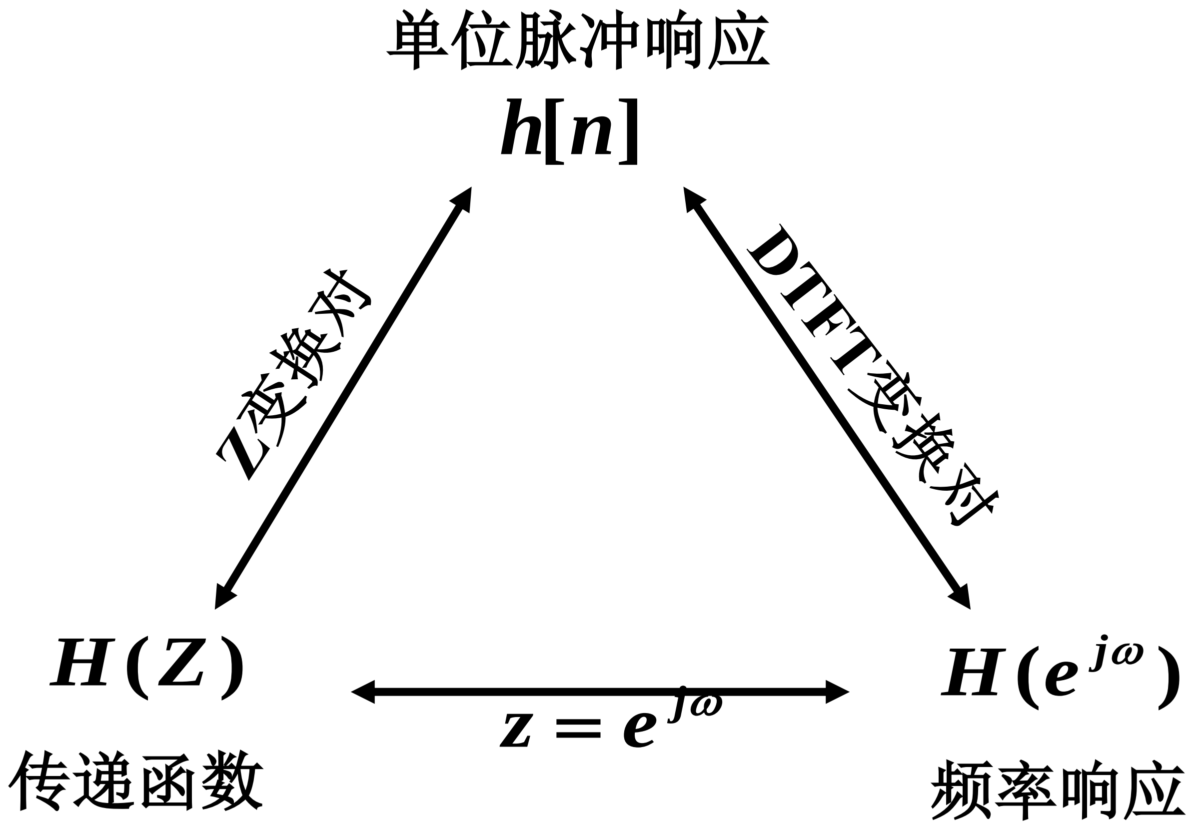
$$W(z)=\sum_{n=-\infty}^{\infty} w(n)z^{-n}=\sum_{n=0}^{\infty} w(n)z^{-n}+\sum_{n=-\infty}^{-1} w(n)z^{-n}$$

A right-sided sequence + A left-sided sequence

$|z|>R_{g-}$ $\qquad$ $|z|<R_{g+}$

| Property | Sequence | $z$ -Transform | ROC |
|---|---|---|---|
| | $g[n]$<br>$h[n]$ | $G(z)$<br>$H(z)$ | $\mathcal{R}_g$<br>$\mathcal{R}_h$ |
| Conjugation | $g^*[n]$ | $G^*(z^*)$ | $\mathcal{R}_g$ |
| Time-reversal | $g[-n]$ | $G(1/z)$ | $1/\mathcal{R}_g$ |
| Linearity | $\alpha g[n] + \beta h[n]$ | $\alpha G(z) + \beta H(z)$ | Includes $\mathcal{R}_g \cap \mathcal{R}_h$ |
| Time-shifting | $g[n - n_o]$ | $z^{-n_o} G(z)$ | $\mathcal{R}_g$, except possibly the point $z = 0$ or $\infty$ |
| Multiplication by an exponential sequence | $\alpha^n g[n]$ | $G(z/\alpha)$ | $\|\alpha\|\mathcal{R}_g$ |
| Differentiation of $G(z)$ | $ng[n]$ | $-z\dfrac{dG(z)}{dz}$ | $\mathcal{R}_g$, except possibly the point $z = 0$ or $\infty$ |
| Convolution | $g[n] \circledast h[n]$ | $G(z)H(z)$ | Includes $\mathcal{R}_g \cap \mathcal{R}_h$ |
| Modulation | $g[n]h[n]$ | $\frac{1}{2\pi j} \oint_C G(v)H(z/v)v^{-1}\, dv$ | Includes $\mathcal{R}_g \mathcal{R}_h$ |
| Parseval's relation | $\sum_{n=-\infty}^{\infty} g[n]h^*[n] = \frac{1}{2\pi j} \oint_C G(v)H^*(1/v^*)v^{-1}\, dv$ | | |

Note: If $\mathcal{R}_g$ denotes the region $R_{g-} < |z| < R_{g+}$ and $\mathcal{R}_h$ denotes the region $R_{h-} < |z| < R_{h+}$, then $1/\mathcal{R}_g$ denotes the region $1/R_{g+} < |z| < 1/R_{g-}$ and $\mathcal{R}_g\mathcal{R}_h$ denotes the region



单位脉冲响应
h[n]
Z变换对
DTFT变换对
H(Z)
传递函数
z = e^{jω}
H(e^{jω})
频率响应

- 课本例题6.35
- y[n]=x[n-1]-1.2x[n-2]+x[n-3]+1.3y[n-1]

  -1.04y[n-2]+0.222y[n-3]
- Its transfer function is therefore given by

$$H(z)=\frac{Y(z)}{X(z)}=\frac{z^{-1}-1.2z^{-2}+z^{-3}}{1-1.3z^{-1}+1.04z^{-2}-0.222z^{-3}}$$

**Consider a LTI *causal* system whose I/O difference equation is** $y(n) = \frac{5}{2} y(n-1) - y(n-2) + x(n-1)$

**1) Compute the transform function.**

**2) Determine the corresponding pole/zero pattern and the ROC.**

**3) Compute the impulse response.**

**4) It is easy to know this system is not stable. Determine another stable (but anticausal) system satisfying the same I/O difference equation.**

# Solution:

a) $$H(z)=\frac{z^{-1}}{1-\frac{5}{2}z^{-1}+z^{-2}}=\frac{z^{-1}}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}$$

b) zeroes： z＝0；

poles： z=2, z=1/2;

ROC: |z|>2

c) $$H(z)=\frac{2/3}{1-2z^{-1}}+\frac{-2/3}{1-\frac{1}{2}z^{-1}}$$

$$h[n]=\frac{2}{3}2^{n}\mu[n]-\frac{2}{3}(\frac{1}{2})^{n}\mu[n]$$

d) ROC: 1/2<|z|<2,

$$h[n] = -\frac{2}{3} 2^n \mu[-n-1] - \frac{2}{3} (\frac{1}{2})^n \mu[n]$$

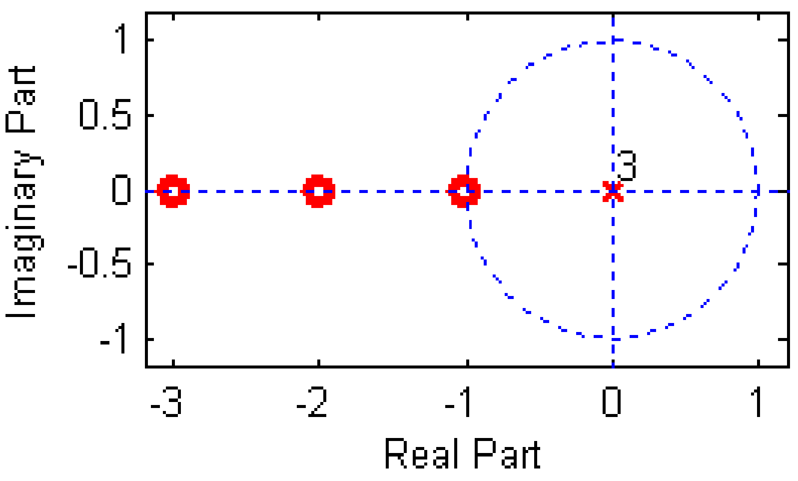
# LTI系统分类——根据h(n)的长度

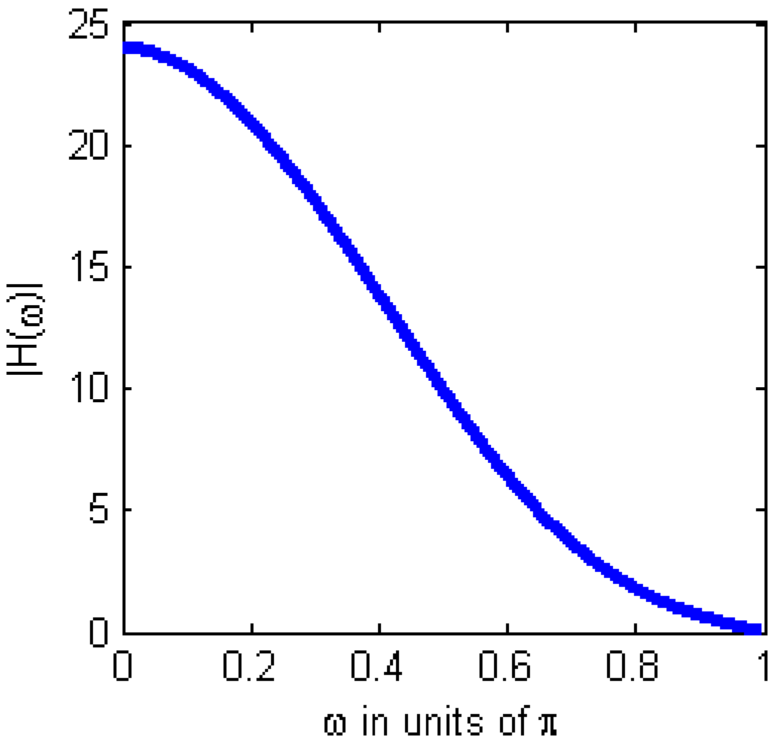
| IIR滤波器 | FIR滤波器 |
|---|---|
| $H(z)=\dfrac{\sum_{i=0}^{M}a_i z^{-i}}{1-\sum_{i=1}^{N}b_i z^{-i}}$，$b_i$不全为0 | $H(z)=\sum_{i=0}^{M}a_i z^{-i}$，$b_i$全为0 |
| $y(n)=\sum_{i=0}^{M}a_i x(n-i)+\sum_{i=1}^{N}b_i y(n-i)$ | $y(n)=\sum_{i=0}^{M}a_i x(n-i)$ |
| h(n)无限长 | $h(i)=a_i$，$i=0,\cdots,M$ |

**Example:** **h=[1, 6, 11, 6],** 求 ***H(z)*** 等。

$$y(n) = x(n) + 6x(n-1) + 11x(n-2) + 6x(n-3)$$

$$H(z) = 1 + 6z^{-1} + 11z^{-2} + 6z^{-3}$$



| 系统 | 时域条件 | Z域条件 |
| --- | --- | --- |
| 因果 | $h(n) \equiv 0 \ (n<0)$ | ROC: $R1 < \lvert Z \rvert \leq \infty$ |
| 稳定 | $\sum_{n=-\infty}^{\infty} \lvert h(n) \rvert < \infty$ | ROC: 包含单位圆 |
| 因果稳定 | 所有极点全在单位圆内部 | |

# 第7章

1、线性相位性：系统的相频特性是频率的线性函数

$$H(e^{j\omega}) = e^{-j\omega D}$$

2、线性相位的条件？

h[n]具有对称性——

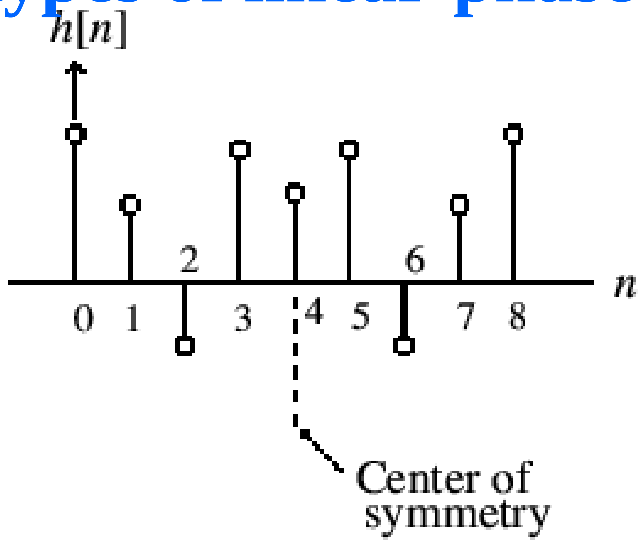
奇对称（h[n]= －h[N-n]）

偶对称 (h[n]= h[N-n])

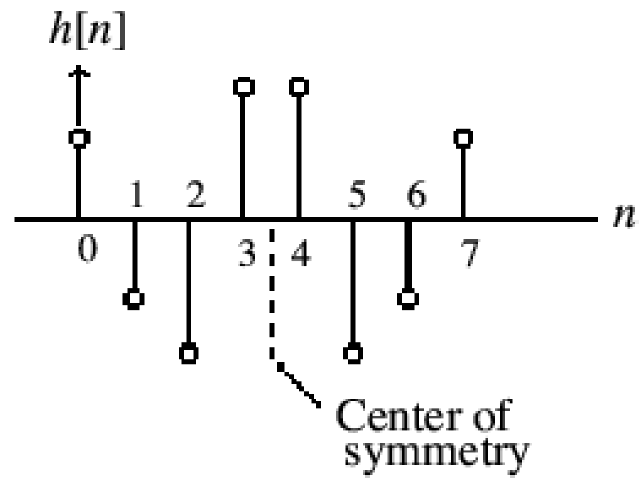
其中，群时延c＝－N/2

# Four types of linear-phase FIR transfer functions:

偶对称

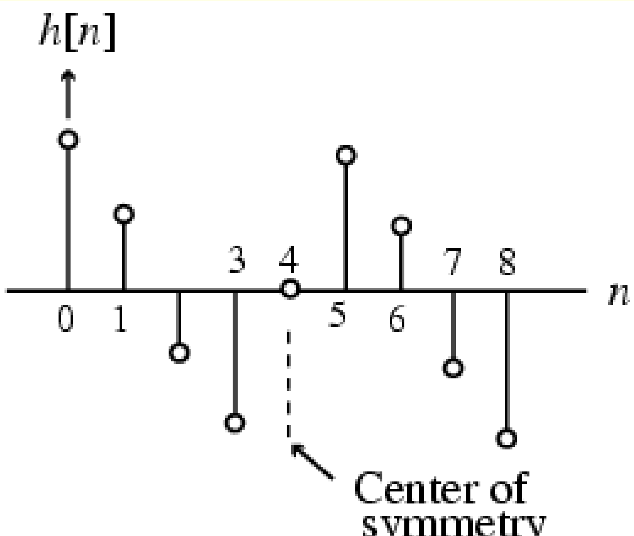



Type 1: $N = 8$

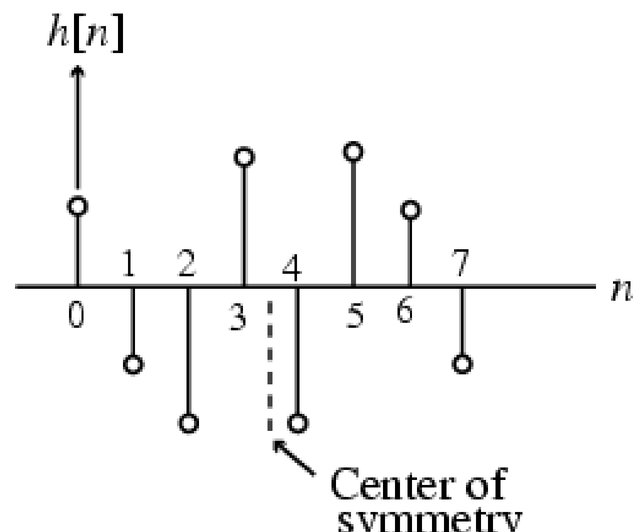



Type 2: $N = 7$

奇对称




Type 3: $N = 8$




Type 4: $N = 7$

# 线性相位FIR滤波器的零点特性

$$h(n) = \pm h(N-n)$$

$$\downarrow$$

$$H(z) = \pm z^{-N} H\left(z^{-1}\right)$$

若z= $z_{0i}$是H(z)的零点，则z= ${z_{0i}}^{-1}$也一定是H(z)的零点；由于h(n)是实数，H(z)的零点还必须共轭成对。

**结论：零点必须是互为倒数的共轭对**

# Zero Locations of Linear-Phase FIR Transfer Functions

| Type 1 | Type 2 | Type 3 | Type 4 |
|---|---|---|---|
| No restriction<br>Can design any type | Cannot design highpass and bandstop<br>Zero at $\omega = \pi$ | Cannot design lowpass, highpass, and bandstop<br>Zero at $\omega = 0$ and $\omega = \pi$ | Cannot design lowpass, and bandstop<br>Zero at $\omega = 0$ |

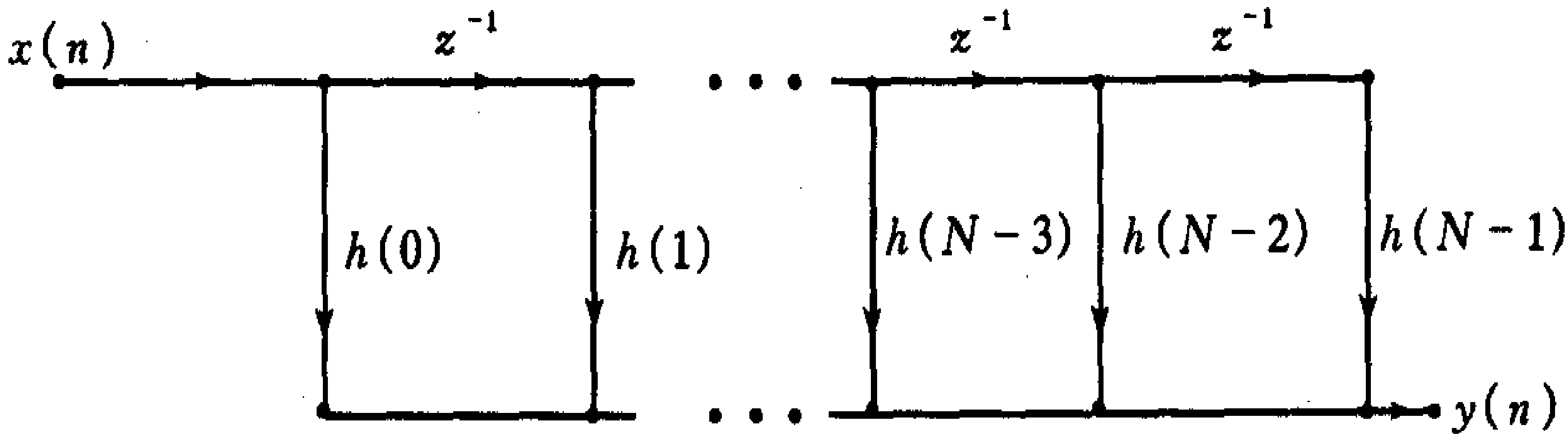
每种类型滤波器单位脉冲响应的长度、对称性、零点分布

# 第八章 滤波器结构

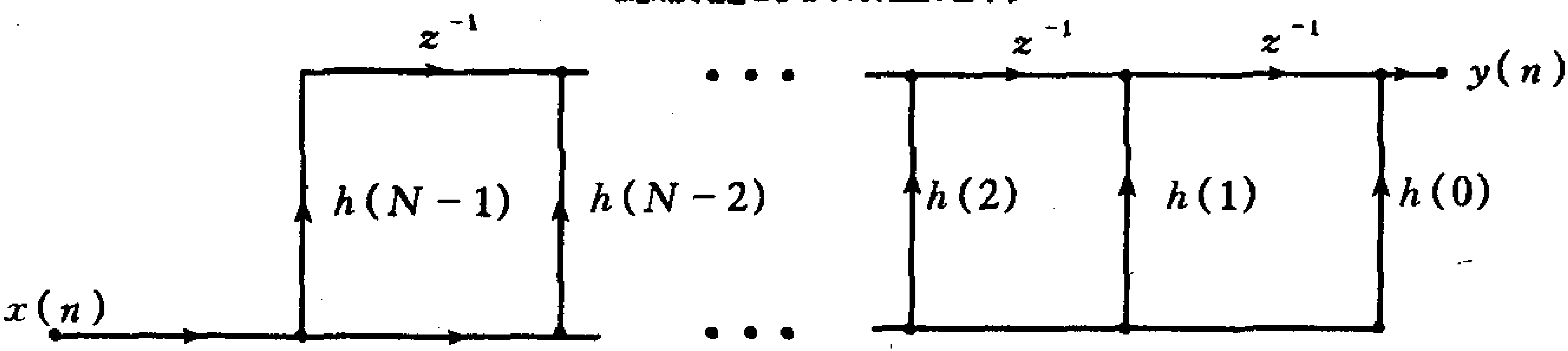
**Basic FIR Digital Filter Structures**

- **Direct Form**
- **Cascade Form**
- **Linear-phase Structure**

直接由差分方程可画出对应的网络结构：

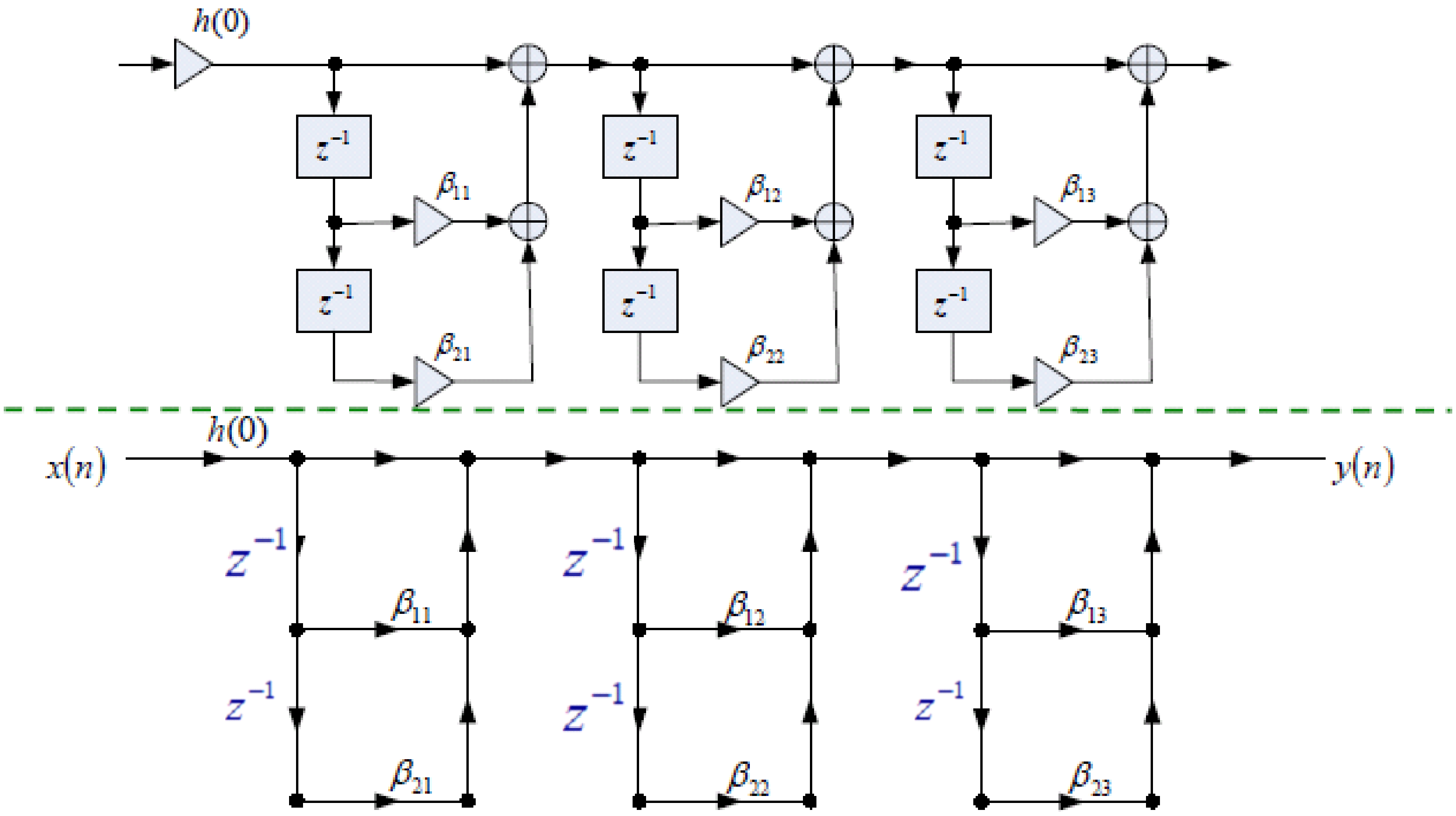



**FIR 滤波器的横截型结构**



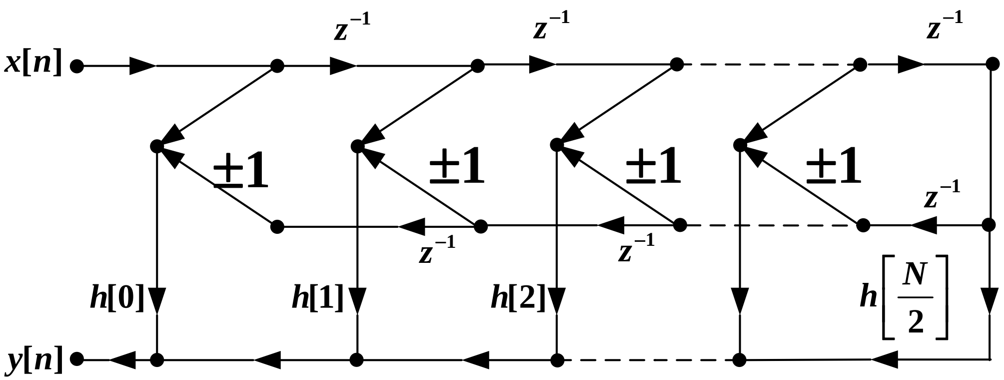

**横截型的转置结构**

- A cascade realization for $N = 6$ is shown below



# 3、线性相位型

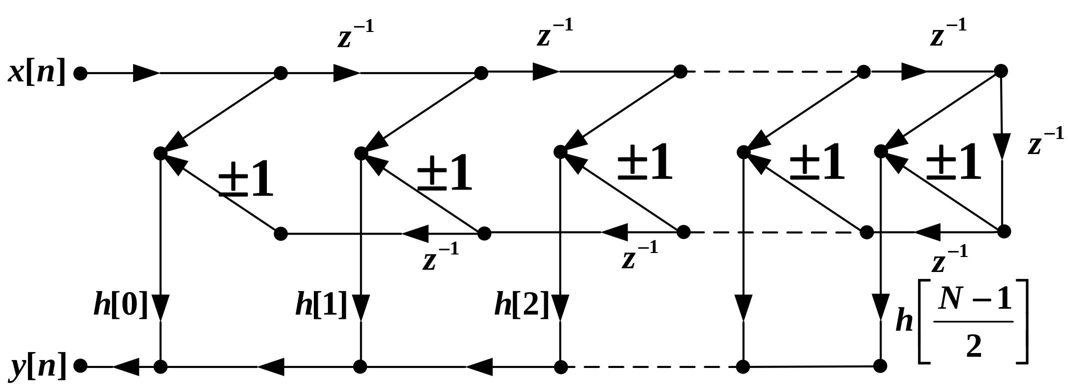
Type 1 and 3




（N/2+1）乘法器

直接型（N+1）个乘法器

Type 2 and 4




(N+1)/2乘法器

# Basic IIR Digital Filter Structures

- **Direct Form**
- **Cascade Form**
- **Parallel Form**

各种结构的优缺点比较：

■正准型比直接型节省一半的存储单元

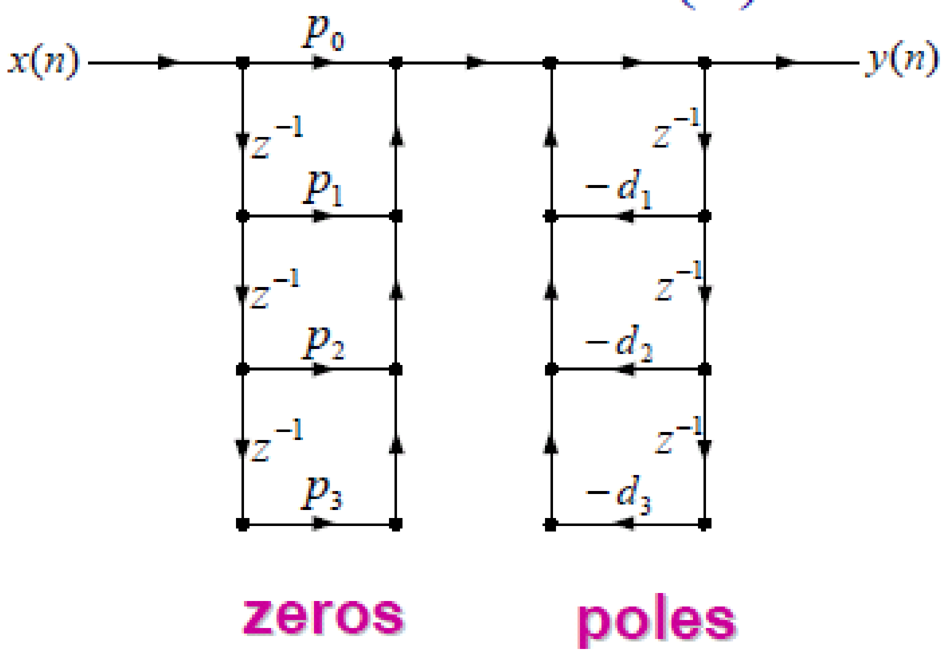
■级联型最易于控制零点和极点；

■并联型易于控制极点；

■并联型运算速度最快；

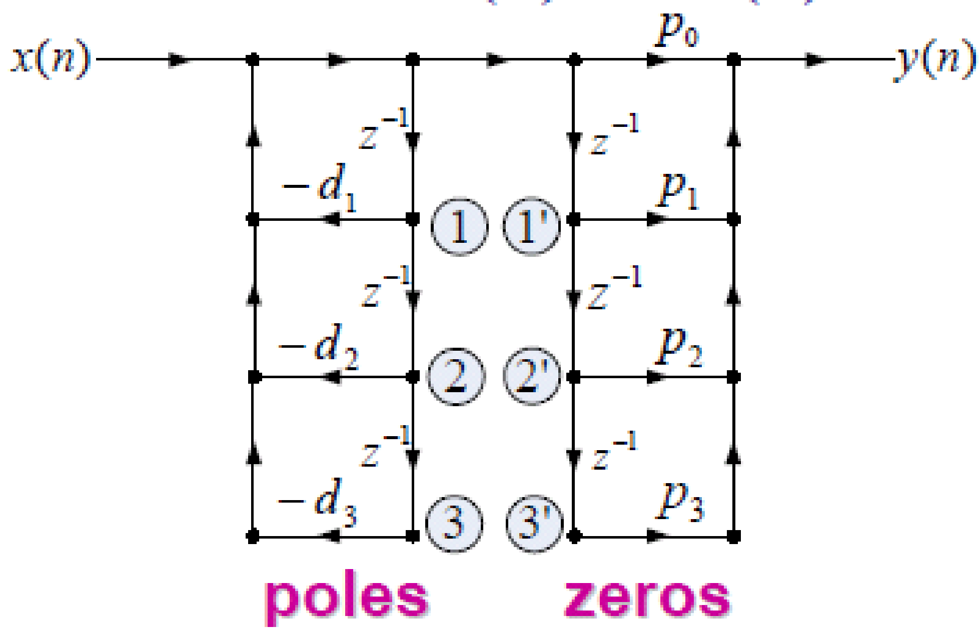
- Considering the basic cascade realization results in *Direct form* I :

$$H(z) = P(z) \cdot \frac{1}{D(z)}$$

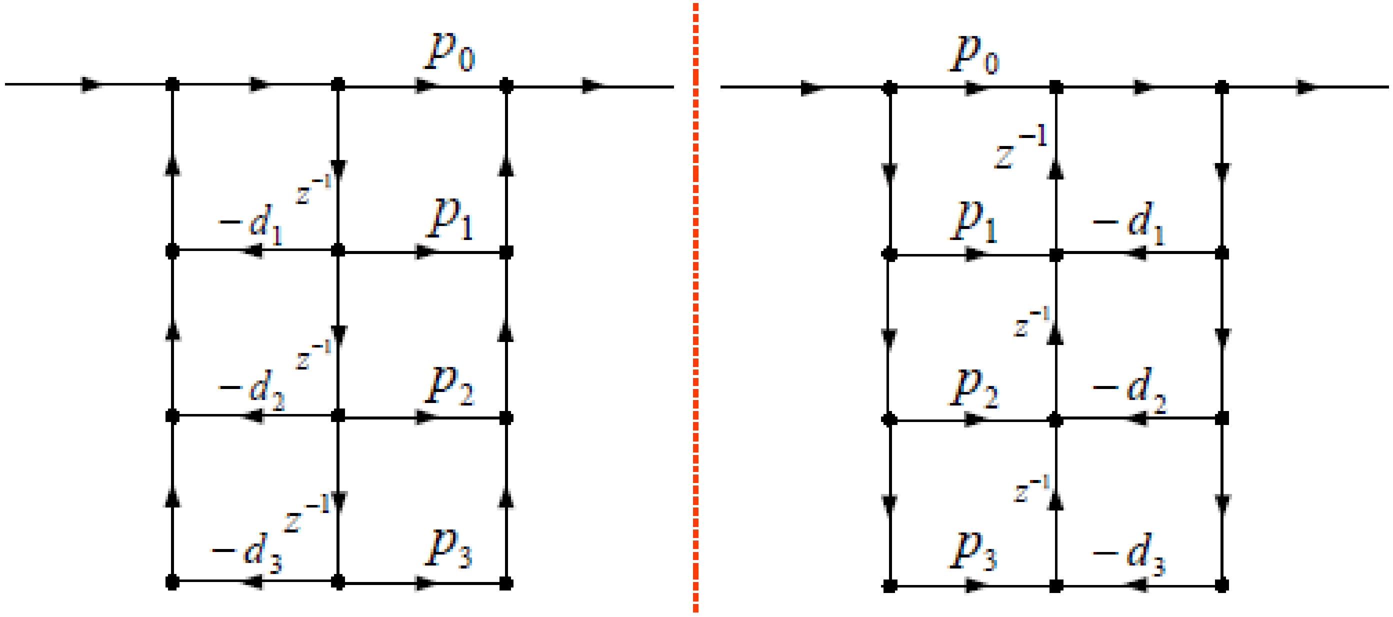


- Changing the order of blocks in cascade results in *Direct form* II :

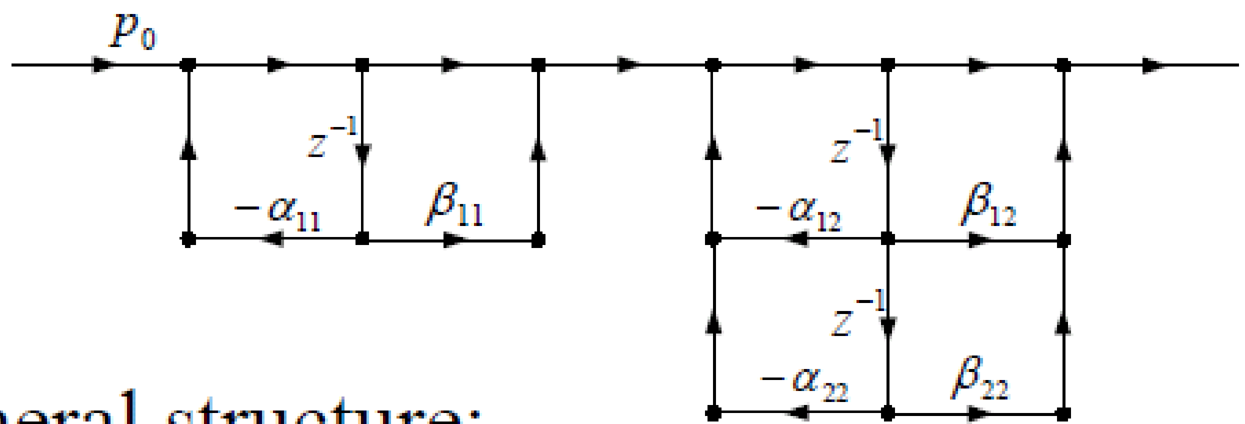
$$H(z) = P(z) \cdot \frac{1}{D(z)} = \frac{1}{D(z)} \cdot P(z)$$



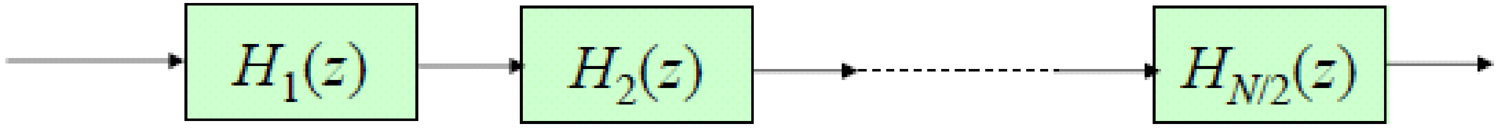
- Sharing of all delays reduces the total number of delays to 3 resulting in a canonic realization shown below along with its transpose structure.



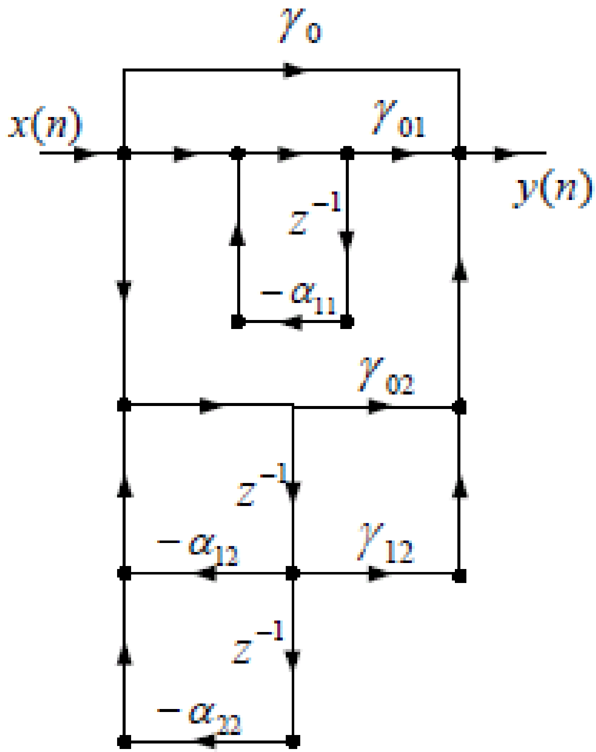
- One possible realization is shown below



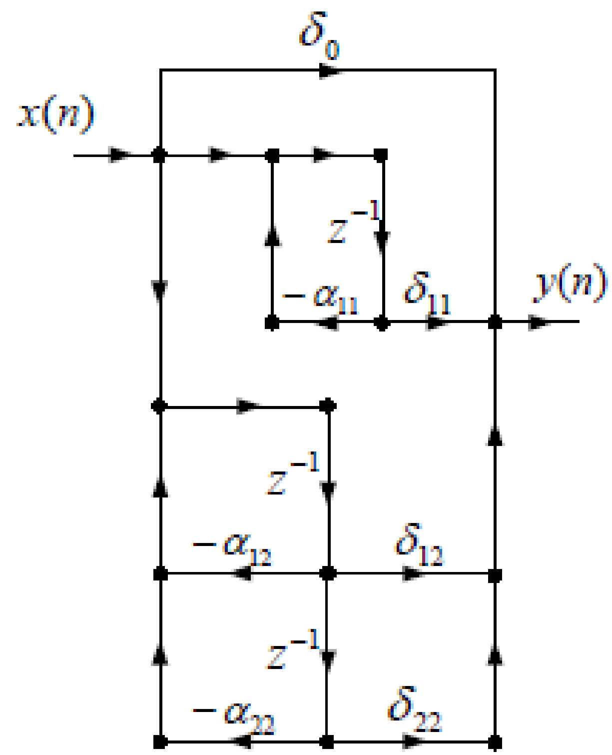

- General structure:



- The two basic parallel realizations of a 3rd order IIR transfer function are shown below




**Parallel Form I**




**Parallel Form II**

# 第九章 IIR滤波器设计

概念：

1、双线性变换法的映射规则

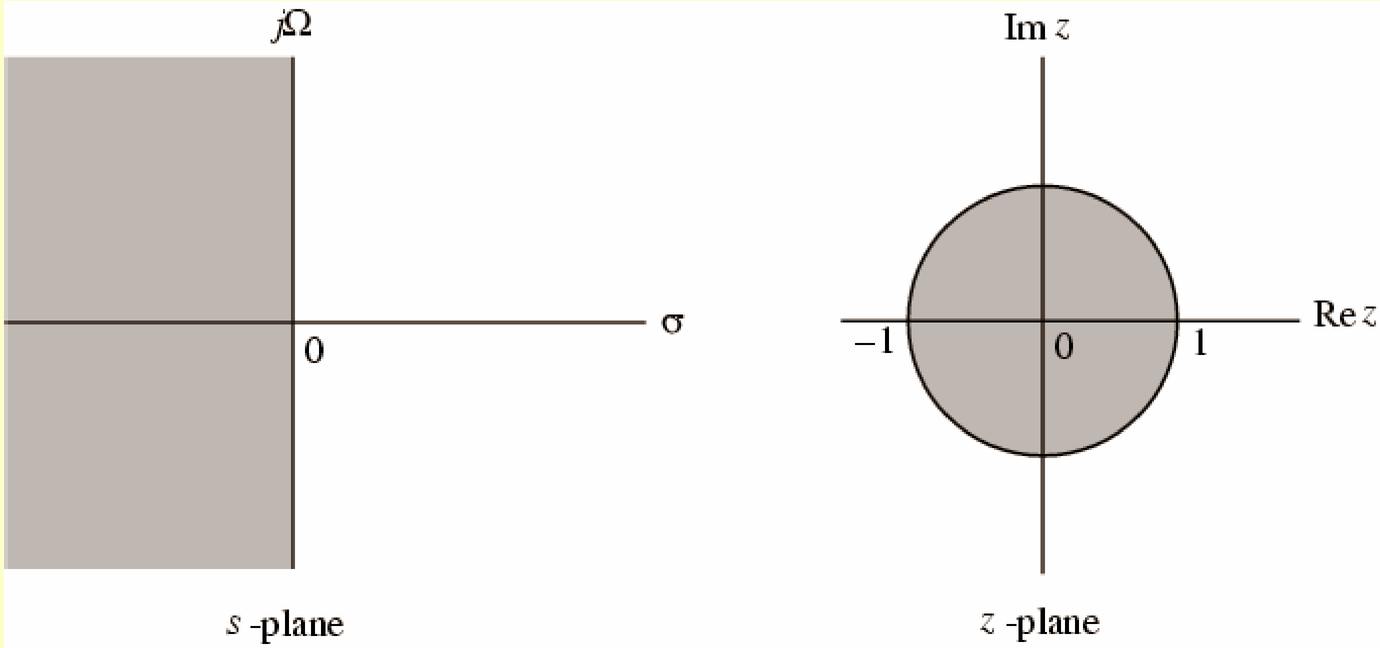
$$s = \frac{2(1-z^{-1})}{T(1+z^{-1})}$$

2、双线性变换法会导致峰点、谷点频率等临界频率点发生非线性变化，即畸变。这种频率点的畸变可以通过预畸来加以校正。预畸不能在整个频率段消除非线性畸变，只能消除模拟和数字滤波器在特征频率点的畸变。

$$\Omega = \frac{2}{T} tg\left(\frac{\omega}{2}\right)$$

# Bilinear Transformation

- Mapping of *s*-plane into the *z*-plane



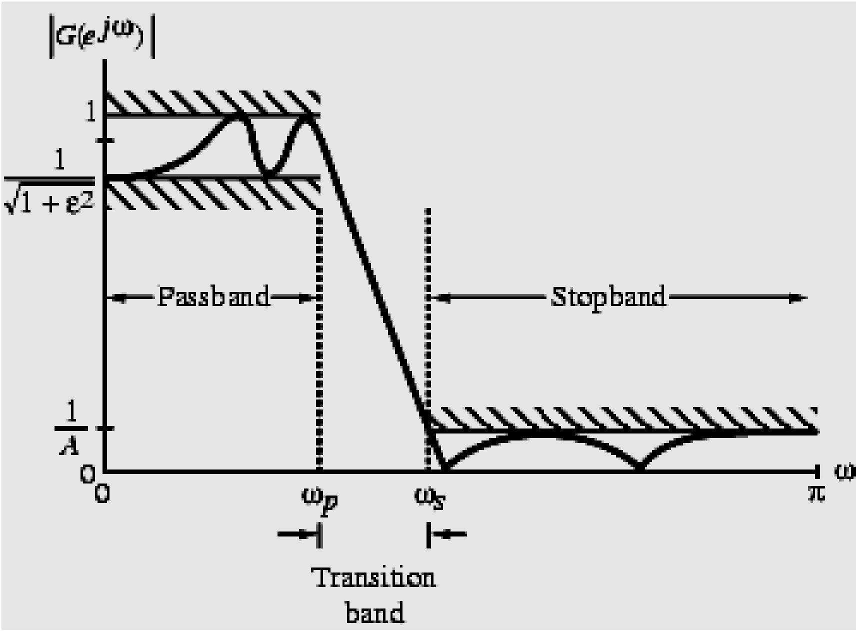
**Example** - Design a lowpass Butterworth digital filter with $\omega_p = 0.25\pi$, $\omega_s = 0.55\pi$, $\alpha_p = 0.5$ dB, and $\alpha_s = 15$ dB

**Analysis:**

If $|G(e^{j0})| = 1$ this implies

$$20\log_{10}|G(e^{j0.25\pi})| \geq -0.5$$

$$20\log_{10}|G(e^{j0.55\pi})| \leq -15$$



第一步：参数的预畸

- **Solution:**

**（1）Prewarping (T=2)**

$\Omega_p=\tan(\omega_p/2)=\tan(0.25\pi/2)=0.4142136$

$\Omega_s=\tan(\omega_s/2)=\tan(0.55\pi/2)=1.1708496$

第二步：模拟滤波器的设计

用以下公式计算N

$$N_{exact} = \frac{\log_{10}\sqrt{\dfrac{10^{\alpha_s/10}-1}{10^{\alpha_p/10}-1}}}{\log_{10}\left(\Omega_s/\Omega_p\right)}$$

Thus $$N = \frac{\log_{10}(1/k_1)}{\log_{10}(1/k)} = 2.6586997$$

**Choose $N = 3$**

To determine $\Omega_c$ we use

$$\left|H_a(j\Omega_p)\right|^2 = \frac{1}{1+(\Omega_p/\Omega_c)^{2N}} = \frac{1}{1+\varepsilon^2}$$

**We then get**

$\Omega_c = 0.588148$

# Butterworth Approximation

| N | $H_{an}(s)$ |
|---|---|
| 1 | $\dfrac{1}{1+s}$ |
| 2 | $\dfrac{1}{1+1.4142s+s^2}$ |
| 3 | $\dfrac{1}{1+2s+2s^2+s^3}$ |
| 4 | $\dfrac{1}{1+2.6131s+3.4142s^2+2.6131s^3+s^4}$ |
| 5 | $\dfrac{1}{1+3.2361s+5.2361s^2+5.2361s^3+3.2361s^4+s^5}$ |
| 6 | $\dfrac{1}{1+3.8637s+7.4641s^2+9.1416s^3+7.4641s^4+3.8637s^5+s^6}$ |
| 7 | $\dfrac{1}{1+4.4940s+10.0978s^2+14.5918s^3+14.5918s^4+10.0978s^5+4.4940s^6+s^7}$ |

- **3rd-order lowpass Butterworth transfer function for $\Omega_c$=1 is**

  **$H_{an}(s)=1/(s^3+2s^2+2s+1)=1/[(s+1)(s^2+s+1)]$**

- **Denormalizing to get**

  **we arrive at**

解归一化

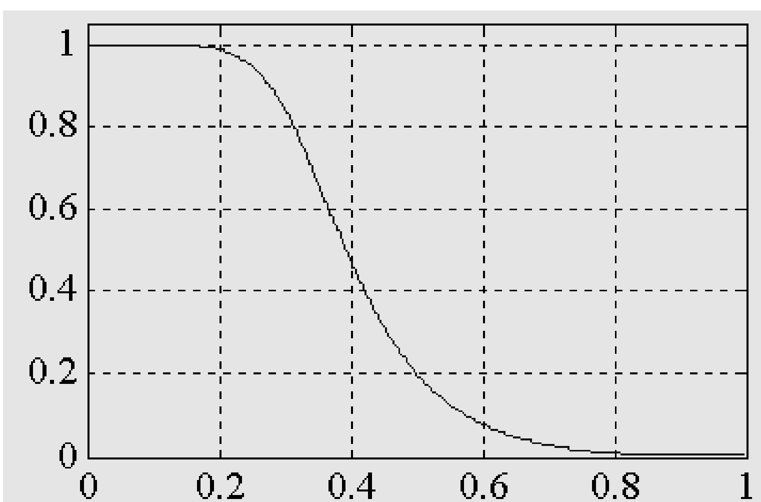
$$H_a(s) = H_{an}\left(\frac{s}{\Omega_c}\right) = H_{an}\left(\frac{s}{0.588148}\right)$$

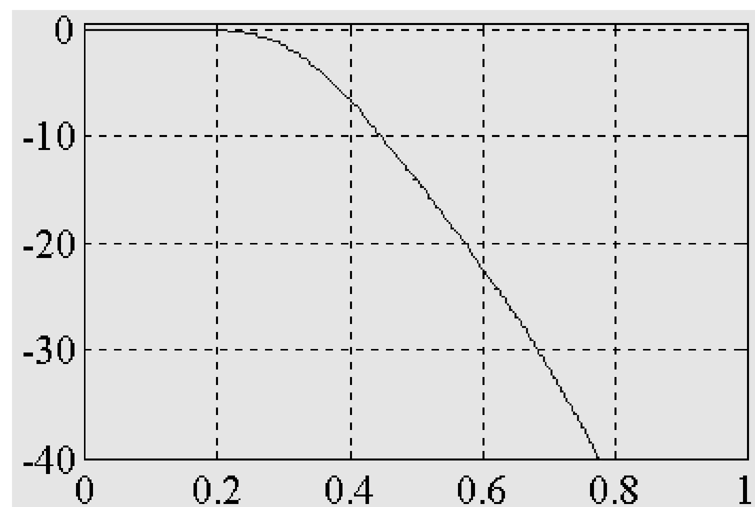
**第三步：映射（变量代换）**

- **Applying bilinear tra[illegible]rmation to $H_a(s)$ we get the desired digital tra[illegible]sfer function**

$$G(z) = H_a(s)\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}}$$

**Magnitude and gain responses of *G*(*z*) shown below:**



# 第十章 窗口法设计FIR滤波器

$$H_d(e^{j\omega}) \Rightarrow h_d(n) \Rightarrow h_d(n)w(n)$$

$$H(e^{j\omega}) \Leftarrow \quad h(n)$$

频域卷积 $$H(e^{j\omega}) = H_d(e^{j\omega}) * W(e^{j\omega})$$

**Steps:**

**1. Pick an odd length *N*=2*M*+1, and let *M*=(*N*-1)/2.**

**2. Calculate the *N* coefficients**

$$d(k) = \int_{-\pi}^{\pi} D(\omega) \cdot e^{j\omega k} \frac{d\omega}{2\pi}, \quad -M \le k \le M \ ,$$

**3. Make them causal by the delay**

$$h(n) = d(n - M) \qquad (n = 0 \sim N - 1)$$

# 窗口函数对理想特性的影响：

①改变了理想频响的边沿特性，形成过渡带，宽为

$4\pi/N$，等于$W_R(\omega)$的主瓣宽度。（决定于窗长）

②过渡带两旁产生肩峰和余振（带内、带外起伏），取决于 $W_R(\omega)$的旁瓣，旁瓣多，余振多；旁瓣相对值大，肩峰强，与 N无关。（决定于窗口形状）

③N增加, 过渡带宽减小, 肩峰值不变。最大肩峰永远为8.95%，这种现象称为吉布斯（Gibbs）效应。

肩峰值的大小决定了滤波器通带内的平稳程度和阻带内的衰减，所以对滤波器的性能有很大的影响。

改变窗函数的形状，可改善滤波器的特性，窗函数有许多种，但要满足以下两点要求：

①窗谱主瓣宽度要窄，以获得较陡的过渡带；

②相对于主瓣幅度，旁瓣要尽可能小，使能量尽量集中在主瓣中，这样就 可以减小肩峰和余振，以提高阻带衰减和通带平稳性。

但实际上这两点不能兼得，一般总是通过增加主瓣宽度来换取对旁瓣的抑制。

# 第11章 重点

- **Cooley-Tukey FFT算法的实现，包括DIT和DIF两种方法，掌握算法原理和蝶形图的画法**
- 算法计算量的分析
- 码位倒置

1、 FFT 的算法的思路：利用 W 因子的对称性和周期性将长序列 DFT 分解为短序列 DFT

2、 FFT 的算法分类——

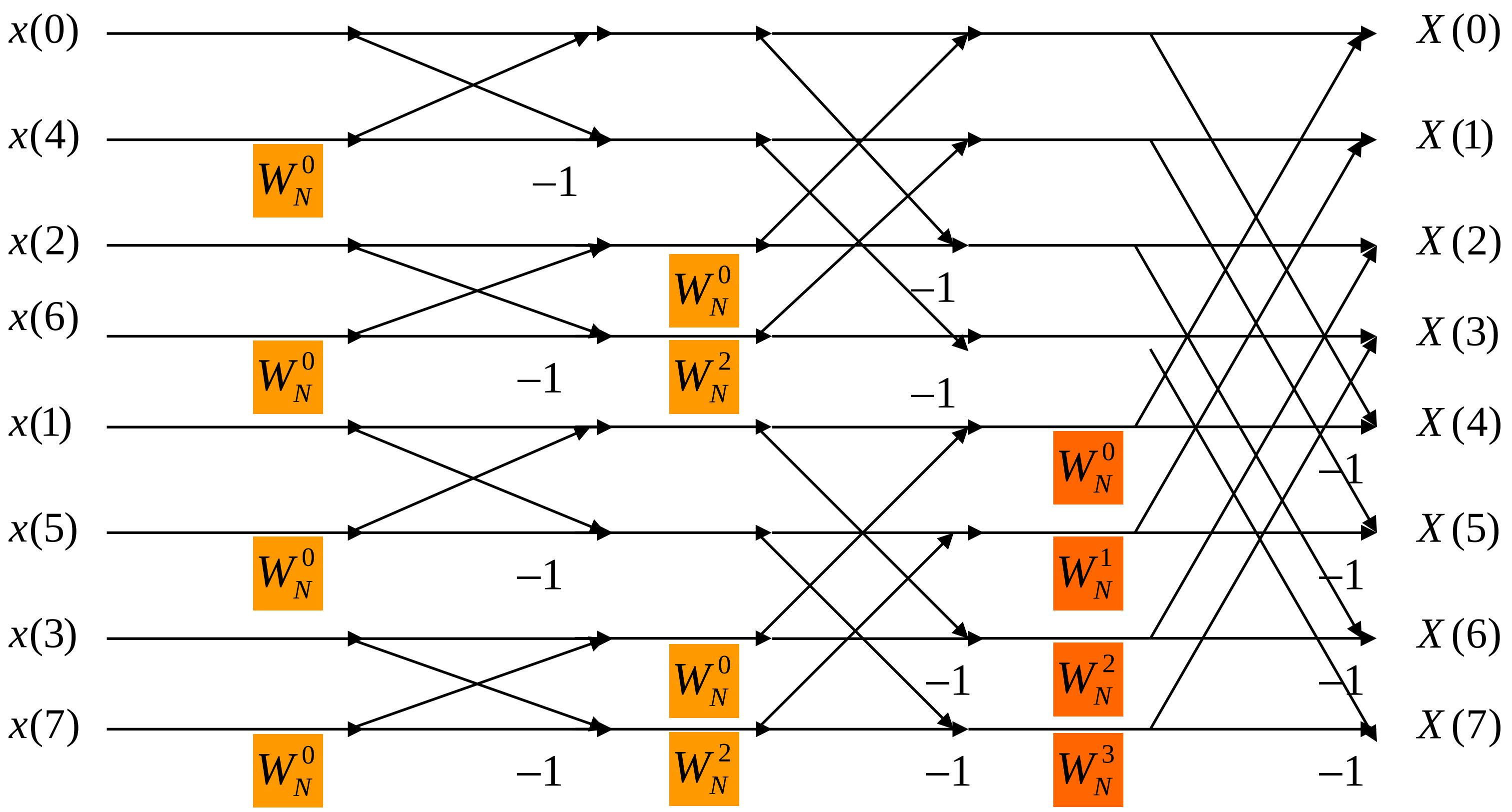
按时间抽取：将 x(n)按 n 的奇偶分成两半

按频率抽取：直接将 x(n)分成前后两半

3、三个步骤：Shuffling—Performing—Merging

| | FFT | 直接 DFT |
| --- | --- | --- |
| 乘法 | $\frac{N}{2}\log_2 N$ | $N^2$ |
| 加法 | $N\log_2 N$ | $N(N-1)$ |

# 按时间抽取 FFT Algorithms



# 按频率抽取 FFT Algorithms