Knowledge Representation and Processing

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⋆ **This examination contributes to 50% of the total mark of the course.**

Question 1. Basics of KR and Ontologies

- Describe the relationship between ontology, web ontology language (OWL), and description logic? (4 marks)

Model Solution. An ontology is a set of statements describing the knowledge of a subject domain using a fixed vocabulary. Model ontologies are specified in the Web Ontology Language, which is based on description logics, a family of logical formalisms for knowledge representation.

Question 2. KR formalism

- State the trade-off between the expressive power of the language available for making statements and the computational complexity of various reasoning tasks for this language. (4 marks)

Model Solution. A logic-based KR language should be designed as expressive as is able to satisfy the modelling requirements of an application. More expressivity brings more power and flexibility for making statements about domain knowledge. However, on the other hand, such power and flexibility come with a computational cost. The expressive power of the language is invariably constrained so as to at least ensure that reasoning is decidable, i.e., reasoning can always be correctly completed within a finite amount of time.

Marking Scheme. Award 3 mark when students have, somehow, pointed out that there is a trade-off between the expressive power of a language available for making statements and the computational complexity of various reasoning tasks for the language, and unlock the remaining 1 mark when students have mentioned that the expressiveness of a language is due to the fulfillment of the modelling requirements.

Question 3. DL Syntax and Semantics

Consider the interpretation $\mathcal{I}$ defined by

$$\begin{aligned}
\Delta^{\mathcal{I}} &= \{1, 2, 3, 4\} \\
A^{\mathcal{I}} &= \{1, 2, 3\} \\
B^{\mathcal{I}} &= \{3\} \\
r^{\mathcal{I}} &= \{(1, 3), (2, 3), (2, 4)\}
\end{aligned}$$

Determine the following sets (6 marks):

- $(A \sqcap \exists r.B)^{\mathcal{I}}$

- $(\forall r.B)^{\mathcal{I}}$
- $(\exists r.B \sqcap \forall r.\neg B)^{\mathcal{I}}$
- $(\neg A \sqcup \neg\exists r.B)^{\mathcal{I}}$
- $(\exists r.\forall r.\bot)^{\mathcal{I}}$
- $(\forall r.(\forall r.\top \sqcap (A \sqcup \neg A)))^{\mathcal{I}}$

Model Solution.

Determine the following sets (3 marks):

- $(A \sqcap \exists r.B)^{\mathcal{I}} = \{a, b\}$
- $(\forall r.B)^{\mathcal{I}} = \{a, c, d\}$
- $(\exists r.B \sqcap \forall r.\neg B)^{\mathcal{I}} = \emptyset$
- $(\neg A \sqcup \neg\exists r.B)^{\mathcal{I}} = \{c, d\}$
- $(\exists r.\forall r.\bot)^{\mathcal{I}} = \{a, b\}$
- $(\forall r.(\forall r.\top \sqcap (A \sqcup \neg A)))^{\mathcal{I}} = \{a, b, c, d\}$

Question 4. Tableau for $\mathcal{ALC}$-Concept

Consider the $\mathcal{ALC}$-concept:

$$C = P \sqcap \exists r.(\neg P \sqcup Q) \sqcap \forall r.\neg Q$$

- Apply the $\mathcal{ALC}$-tableau algorithm to C to determine whether C is satisfiable or not. In your answer, show how the completion rules are applied step by step to the constraint system $x : C$. If C is satisfiable, construct an interpretation $\mathcal{I}$ satisfying C. (6 marks)

Model Solution.

- As C is already in NNF, we immediately apply the ALC-Tableau algorithm to it.

  The ALC-Tableau algorithm starts with:
  $S_0 = \{x : P \sqcap \exists r.(\neg P \sqcup Q) \sqcap \forall r.\neg Q\}$

  An application of the $\sqcap$-rule gives:
  $S_1 = S_0 \cup \{x : P, x : \exists r.(\neg P \sqcup Q), x : \forall r.\neg Q\}$

  An application of the $\exists$-rule gives:
  $S_2 = S_1 \cup \{(x, y) : r, y : \neg P \sqcup Q\}$

  An application of the $\sqcup$-rule gives:
  $S_3 = S_2 \cup \{y : \neg P\}$ or $S_3^* = S_2 \cup \{y : Q\}$

  An application of the $\forall$-rule gives:
  $S_4^* = S_3^* \cup \{y : \neg Q\}$

  Clash obtained, thus we proceed with the other branch:

An application of the $\forall$-rule gives:
$S_4 = S_3 \cup \{y : \neg Q\}$

No rule is applicable to S_4 and S_4 contains no clash. Thus, C is *satisfiable*. A model $\mathcal{I}$ of C is given by:

$\Delta^{\mathcal{I}} = \{x, y\}$

$P^{\mathcal{I}} = \{x\}$

$Q^{\mathcal{I}} = \{x\}$

$r^{\mathcal{I}} = \{(x, y)\}$

Question 5. Tableau for $\mathcal{ALC}$ without TBox
Use the $\mathcal{ALC}$-Tableaux algorithm to determine whether

$$\emptyset \models \exists r.A \sqsubseteq \forall r.A$$

In words: determine whether the GCI $\exists r.A \sqsubseteq \forall r.A$ follows from the empty TBox (without TBox). (5 marks)

Model Solution.

- To show $\emptyset \models \exists r.A \sqsubseteq \forall r.A$ is to show that there is no an interpretation $\mathcal{I}$ such that $\mathcal{I} \not\models \exists r.A \sqsubseteq \forall r.A$. We prove by contradiction. We assume that there exists such an interpretation $\mathcal{I}$, that is, there exists an element $d \in \Delta^{\mathcal{I}}$ such that $d \in (\exists r.A)^{\mathcal{I}}$ but $d \notin (\forall r.A)^{\mathcal{I}}$, that is, $d \in (\exists r.A \sqcap \neg\forall r.A)^{\mathcal{I}}$. Therefore, our task is to check whether $\exists r.A \sqcap \neg\forall r.A$ is satisfiable.

  As $\exists r.A \sqcap \neg\forall r.A$ is not in NNF yet, the first step is to transform it into NNF using the transformation rules. This gives $\exists r.A \sqcap \exists r.\neg A$.

  The ALC-Tableau algorithm starts with:
  $S_0 = \{x : \exists r.A \sqcap \exists r.\neg A\}$

  An application of the $\sqcap$-rule gives:
  $S_1 = S_0 \cup \{x : \exists r.A, x : \exists r.\neg A\}$

  An application of the $\exists$-rule gives:
  $S_2 = S_1 \cup \{(x, y) : r, y : A\}$

  An application of the $\exists$-rule gives:
  $S_3 = S_2 \cup \{(x, z) : r, z : \neg A\}$

  No rule is applicable to S_3 and S_3 contains no clash. Thus, $\exists r.A \sqcap \neg\forall r.A$ is *satisfiable*, which means $\emptyset \not\models \exists r.A \sqsubseteq \forall r.A$.

Question 6. Tableau for $\mathcal{ALC}$ with TBox
Consider the following $\mathcal{ALC}$-TBox $\mathcal{T}$:

$$\text{Herbivore} \equiv \text{Animal} \sqcap \forall \text{eats.Plant}$$
$$\text{Omnivore} \equiv \text{Animal} \sqcap \forall \text{eats.(Plant} \sqcup \text{Meat)}$$

We want to know if $\mathcal{T} \models \text{Herbivore} \sqsubseteq \text{Omnivore}$. (5 marks)

Model Solution.

This amounts to showing $\mathcal{T} \models \mathsf{Animal} \sqcap \forall\mathsf{eats.Plant} \sqsubseteq \mathsf{Animal} \sqcap \forall\mathsf{eats.}(\mathsf{Plant} \sqcup \mathsf{Meat})$ (**unfolding**). As in Question 5, our task is to check whether $\forall\mathsf{eats.Plant} \sqcap \neg\forall\mathsf{eats.}(\mathsf{Plant} \sqcup \mathsf{Meat})$ is satisfiable. Since $\forall\mathsf{eats.Plant} \sqcap \neg\forall\mathsf{eats.}(\mathsf{Plant} \sqcup \mathsf{Meat})$ is not in NNF yet, the first step is to transform it into NNF using the transformation rules. This gives $\forall\mathsf{eats.Plant} \sqcap \exists\mathsf{eats.}(\neg\mathsf{Plant} \sqcap \neg\mathsf{Meat})$.

The ALC-Tableau algorithm starts with:
$S_0 = \{x : \forall\mathsf{eats.Plant} \sqcap \exists\mathsf{eats.}(\neg\mathsf{Plant} \sqcap \neg\mathsf{Meat})\}$

An application of the $\sqcap$-rule gives:
$S_1 = S_0 \cup \{x : \forall\mathsf{eats.Plant}, x : \exists\mathsf{eats.}(\neg\mathsf{Plant} \sqcap \neg\mathsf{Meat})\}$

An application of the $\exists$-rule gives:
$S_2 = S_1 \cup \{(x, y) : \mathsf{eats}, y : \neg\mathsf{Plant} \sqcap \neg\mathsf{Meat}\}$

An application of the $\sqcap$-rule gives:
$S_3 = S_2 \cup \{y : \neg\mathsf{Plant}, y : \neg\mathsf{Meat}\}$

An application of the $\forall$-rule gives:
$S_4 = S_3 \cup \{y : \mathsf{Plant}\}$

Clash, thus $\forall\mathsf{eats.Plant} \sqcap \neg\forall\mathsf{eats.}(\mathsf{Plant} \sqcup \mathsf{Meat})$ is unsatisfiable, and $\mathcal{T} \models \mathsf{Herbivore} \sqsubseteq \mathsf{Omnivore}$.

Question 7. Reasoning in $\mathcal{ALC}$

Consider the following $\mathcal{ALC}$-TBox $\mathcal{T}$:

$$\begin{aligned} \mathsf{PI} &\sqsubseteq \exists\mathsf{work\_for.School} \\ \mathsf{PI} &\sqsubseteq \exists\mathsf{supervises.Research\_Group} \\ \mathsf{PI} &\sqsubseteq \forall\mathsf{supervises.Research\_Student} \\ \mathsf{School} &\sqsubseteq \exists\mathsf{affiliated\_with.University} \end{aligned}$$

- Define a model $\mathcal{I}$ of $\mathcal{T}$ in which PI is satisfiable.
- Add $\mathsf{ResearchGroup} \sqcap \mathsf{ResearchStudent} \sqsubseteq \bot$ to $\mathcal{T}$. Is the resulting $\mathcal{T}$ consistent? Why? (5 marks)

Model Solution.

The interpretation $\mathcal{I}$ that is a model of $\mathcal{T}$ with $\mathsf{PI} \neq \emptyset$ is given by setting

$\Delta^{\mathcal{I}} = \{a, b, c, d\}$,

$\mathsf{PI}^{\mathcal{I}} = \{a\}$,

$\mathsf{School}^{\mathcal{I}} = \{b\}$,

$\mathsf{Research\_Group}^{\mathcal{I}} = \{c\}$,

$\mathsf{Research\_Student}^{\mathcal{I}} = \{c\}$,

$\mathsf{University}^{\mathcal{I}} = \{d\}$,

$\mathsf{works\_for}^{\mathcal{I}} = \{(a, b)\}$,

$\mathsf{supervises}^{\mathcal{I}} = \{(a, c)\}$,

$\mathsf{affiliated\_with}^{\mathcal{I}} = \{(b, d)\}$.

The resulting TBox is satisfiable. For example, every $\mathcal{I}$ in which $\mathsf{PI}^{\mathcal{I}} = \emptyset$ and $\mathsf{School}^{\mathcal{I}} = \emptyset$ is a model of the extended TBox. Note, however, that PI is not satisfiable anymore because Research_Group and Research_Student are disjoint (according to the new inclusion) but every PI supervises a Research_Group and any such Research_Group is, by the value restriction in the third inclusion, a Research_Student.

Question 8. Ontology-Based Data Access

Consider the following $\mathcal{ALC}$-TBox $\mathcal{T}$:

$$\begin{aligned} \mathsf{Cat} \sqcap \mathsf{Dog} \sqcap \mathsf{Bird} &\sqsubseteq \bot \\ \mathsf{Cat} &\sqsubseteq \mathsf{Mammal} \\ \mathsf{Dog} &\sqsubseteq \mathsf{Mammal} \\ \exists \mathsf{breatheWith}.\mathsf{Lung} &\sqsubseteq \mathsf{Mammal} \\ \mathsf{Mammal} &\sqsubseteq \forall \mathsf{breatheWith}.\mathsf{Lung} \\ \top &\sqsubseteq \exists \mathsf{breatheWith}.\mathsf{Lung} \end{aligned}$$

Consider the following $\mathcal{ALC}$-ABox $\mathcal{A}$:

$$\begin{aligned} \mathsf{Cat}(\mathsf{paper}) \\ \neg\mathsf{Dog}(\mathsf{cotton\_candy}) \\ \mathsf{Bird}(\mathsf{polly}) \\ \mathsf{breatheWith}(\mathsf{dudu}, \mathsf{krp}) \end{aligned}$$

Recall that the answers to Boolean queries given by knowledge bases are "yes", "no", or "do not know" (OWA). Find the answers given by the knowledge base $(\mathcal{T}, \mathcal{A})$ to the following Boolean queries: (5 marks)

- Cat(cotton_candy)   Don't know
- Dog(paper)   Don't know
- Mammal(cotton_candy)   Yes
- Mammal(dudu)   Yes
- Lung(krp)   Yes
- $\forall$breatheWith.Lung(dudu)   Yes
- $\forall$breatheWith.Lung(polly)   Yes
- $\neg$Dog(polly)   Don't know

Question 9. Reasoning in $\mathcal{EL}$

Let $\mathcal{T}$ be an $\mathcal{EL}$-TBox containing:

$$\mathsf{X} \sqsubseteq \exists \mathsf{r}.\mathsf{Y},\ \mathsf{Y} \sqsubseteq \exists \mathsf{r}.\mathsf{Y},$$

and let $\mathcal{T}$ be an $\mathcal{EL}$-ABox containing:

$$\mathsf{r(a,b)}, \mathsf{X(b)},$$

Compute the interpretation $\mathcal{I}_{\mathcal{T},\mathcal{A}}$ so that all $\mathcal{EL}$-concepts C and d $\in \{a, b\}$:

$$\mathcal{T}, \mathcal{A} \models \mathsf{C(d)} \iff \mathcal{I}_{\mathcal{T},\mathcal{A}} \models \mathsf{C(d)}$$

(5 marks)

Question 10. Ontology-Mediated Query Answering
Consider the following $\mathcal{ALC}$-TBox $\mathcal{T}$:

$$\top \sqsubseteq \mathsf{square} \sqcup \mathsf{diamond}, \mathsf{square} \sqcap \exists \mathsf{r}.\mathsf{diamond} \sqsubseteq \mathsf{clash}, \mathsf{diamond} \sqcap \exists \mathsf{r}.\mathsf{square} \sqsubseteq \mathsf{clash}$$

and the following $\mathcal{ALC}$-ABox $\mathcal{A}$:

$$\mathsf{r}(0, 1), \mathsf{r}(1, 2), \mathsf{r}(2, 3), \mathsf{r}(3, 0).$$

What is the answer of $(\mathcal{T}, \mathcal{A})$ to the Boolean query $\exists x\ \mathsf{clash}(x)$? Explain your answer. (5 marks)

Question 11 (BONUS question). Non-standard Reasoning
Consider the following $\mathcal{EL}$-TBox $\mathcal{T}$:

$$\mathsf{A} \sqcap \mathsf{C} \sqsubseteq \mathsf{D}, \mathsf{B} \sqsubseteq \exists \mathsf{r}.\mathsf{A}.$$

For any $\mathcal{EL}$-GCI α with (i) $\mathsf{sig}(\alpha) \subseteq \mathsf{sig}(\mathcal{T})$ and (ii) $\mathsf{A} \notin \mathsf{sig}(\alpha)$, show (on the semantics level) that $\alpha = \mathsf{B} \sqsubseteq \exists \mathsf{r}.\top$ is the strongest logical consequence of $\mathcal{T}$.
(Hint: α is the logical consequence of $\mathcal{T}$ means: $\mathcal{T} \models \mathsf{B} \sqsubseteq \exists \mathsf{r}.\top$, while α is the STRONGEST logical consequence of $\mathcal{T}$ means: there is NO another β with (i) $\mathsf{sig}(\beta) \subseteq \mathsf{sig}(\mathcal{T})$ and (ii) $\mathsf{A} \notin \mathsf{sig}(\beta)$ such that $\mathcal{T} \models \beta \models \alpha$).
(5 marks)