Date. / /

Problem Set 5.

1. a) 假. 举反例: $A=\{1,2,3\}$ $B=\{1\}$ $C=\{1,2\}$

b) 假. 举反例: $A=\{1,2\}$ $B=\{1,2,3\}$ $C=\{3\}$

c) 真. 证: $2^{A\cup B} = \{x \mid \exists y(y\in A\cup B \wedge y\in x)\}$

$= \{x \mid \exists y[(y\in A \cup y\in B) \wedge y\in x]\}$

$= \{x \mid \exists y[(y\in A \wedge y\in x) \cup (y\in B \wedge y\in x)]\}$

$= \{x \mid \exists y(y\in A \wedge y\in x) \cup \exists y(y\in B \wedge y\in x)\}$

$= 2^A \cup 2^B$

d) 真. 证: $2^{A\cap B} = \{x \mid \exists y(y\in A\cap B \wedge y\in x)\}$

$= \{x \mid \exists y(y\in A \wedge y\in B \wedge y\in x)\}$

$= \{x \mid \exists y(y\in A \wedge y\in x) \wedge \exists y(y\in B \wedge y\in x)\}$

$= 2^A \cap 2^B$

2. 满足 $(A\oplus B)\oplus C = A\oplus(B\oplus C)$

$(A\oplus B)\oplus C = ((A\oplus B)-C)\cup(C-(A\oplus B))$

$= ((A\oplus B)\cap\bar{C})\cup(C\cap\overline{(A\oplus B)})$

$= (((A\cap\bar{B})\cup(B\cap\bar{A}))\cap\bar{C})\cup((A\cap\bar{B})\cap(B\cap\bar{A})\cap C)$

$= (A\cap\bar{B}\cap\bar{C})\cup(\bar{A}\cap B\cap\bar{C})\cup(\bar{A}\cap\bar{B}\cap C)\cup(A\cap B\cap C)$

$A\oplus(B\oplus C) = (A-(B\oplus C))\cup((B\oplus C)-A)$

$= (A\cap\overline{(B\oplus C)})\cup((B\oplus C)\cap\bar{A})$

$= (A\cap(\bar{B}\cup C)\cap(B\cap\bar{C}))\cup(((B\cap\bar{C})\cup(C\cap\bar{B}))\cap\bar{A})$

$= (A\cap\bar{B}\cap\bar{C})\cup(\bar{A}\cap B\cap\bar{C})\cup(\bar{A}\cap\bar{B}\cap C)\cup(A\cap B\cap C)$

$\therefore (A\oplus B)\oplus C = A\oplus(B\oplus C)$

Date.　　/　　/

3. a) $\{-1, 0, 1\}$　　b) ϕ

4. a) $A-B = A\cap\overline{B}$

证：$A-B = \{x \mid x\in A \wedge x\notin B\}$

$= \{x \mid x\in A \wedge \neg(x\in B)\}$

$= A\cap\overline{B}$

b) $(A\cap B)\cup(A\cap\overline{B}) = A$

证：$(A\cap B)\cup(A\cap\overline{B})$

$= (A\cup A)\cap(A\cup\overline{B})\cap(B\cup A)\cap(B\cup\overline{B})$

$= A\cap(A\cup\overline{B})\cap(B\cup A)\cap E$

$= A\cap(A\cup(B\cap\overline{B}))$

$= A\cap(A\cup\phi)$

$= A\cap A = A$.

5. a) 不能．举反例：$A=\{1,2,3\}$　$B=\{3\}$　$C=\{1,2\}$

b) 不能．举反例：$A=\{1,2\}$　$B=\{1,2,3,4\}$　$C=\{1,2,5\}$

c) 能．对于 $\forall x\in A$，$x\in A\cup C$，$x\in B\cup C$

∴ x 必属于 B 或 C

若 $x\in \cancel{B}C$，又 $x\in A$　∴ $x\in A\cap C$，$x\in B\cap C$

∴ $x\in B$　∴ $A\subseteq B$

对于 $\forall x\in B$，$x\in B\cup C$　$x\in A\cup C$

∴ x 必属于 A 或 C

若 $x\in C$，又 $x\in B$，则 $x\in B\cap C$，$x\in A\cap C$

∴ $x\in A$　∴ $B\subseteq A$

∴ $A=B$

Date.

6. 证：$A \subseteq B \Leftrightarrow \overline{B} \subseteq \overline{A}$　$A \subseteq B$ 表示 $\forall x(x \in A \to x \in B)$

① 先证必要性

则有 $\forall x(x \notin B \to x \notin A)$

即 $\overline{B} \subseteq A$

② 充分性：$\overline{B} \subseteq \overline{A}$ 表示 $\forall x(x \notin B \to x \in A)$

则有 $\forall x(x \in A \to x \in B)$

即 $A \subseteq B$

7. a) $A \oplus A = \phi$

证：$A \oplus A = (A-A) \cup (A-A)$
$= \phi \cup \phi = \phi$

b) $A \oplus U = \overline{A}$

证：$A \oplus U = (A-U) \cup (U-A)$
$= \phi \cup \overline{A}$
$= \overline{A}$

8. a) $\bigcup_{i=1}^{n} A_i = \{1, 2, 3, \cdots, n\}$

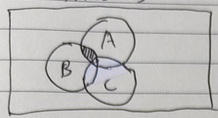
b) $\bigcap_{i=1}^{n} A_i = \{1\}$

9. 证：$(A-B) \cup B = A \cup B$

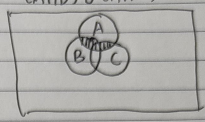
Pf. $(A-B) \cup B = (A \cap \overline{B}) \cup B$
$= (A \cup B) \cap (B \cup \overline{B})$
$= A \cup B$

Date. / /

10. a) $A \cap (B - C)$



b) $(A \cap B) \cup (A \cap C)$



c) $(A \cap \bar{B}) \cup (A \cap \bar{C})$