DATE 第三次作业. 14 (1) 对于任意一个模型 (M.6) AF YAEM, (X=X)) MIS] = Ya (X=X) MIS [X:=a]] TA=XIMA= (Va(q=a)XI) = (((ATXA=(AM))) = = T (ACKE ASSE) A = ACKE + ANTO : XX(X=X)为京人直式 (2) 对于任意一个模型 (M, 6) (∀x ∀y (x=y → y=x)) NEGT = Ha (∀y (x=y → y=x) MIGEX = a]]  $= \forall \alpha (\forall y (\alpha = y \rightarrow y = \alpha))$ = Va Vb (a=y - y=D) MECEX=677 =  $\forall a \forall b (a = b \rightarrow b = a)$ 其中 a.b EM の 若 a=b,则 a=b 为T, b=a为T, Va Vb(a=b→b=a)为丁 回 若 a ≠ b, 凤 a = b 为 F, b = a 为 (3) 对于任意一个模型 (M, 6) YX Yy YZ((X=y) N(y=z) → X=Z) MIG] = YaYbYc((x=y) ∧ (y=Z) → X=Z) MEG[x:=a][y:=b][z:=c]] = Ya+bVc (B+ (B, (a=b, b=c), a=c) 若 Bn (a=b, b=c) 为丁, 为等号传递性知 Q=b=C : 上式成立 ANETHER FIRE CONTRACT TO BELLEAR 若 B<sub>n</sub>(a=b, b=c) 为 F, B<sub>n</sub>(F, a=c) 为 T 1. 上式也成多 ·阿永真

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16. 证: ⊨(¬∀xA) ↔ (Ax¬A)

今对任何模型 M ⊨ (¬∀xA) ↔ (∃X¬A)

   ↔ 对任何模型 M = 6 ((¬YXA) → (∃X¬A)) ∧ ((∃X¬A) → (¬YXA))
   B, (TYXA) = ATA + (AXY) = T
    (ATXE, AXYT) = B+ (AXXE + AXYTO
       当 TKA为T BJ. YXA 为F
       · 存在:Q∈M, 使得 AMEGEX:=a]]=F
       T=(ATXE, AXYT) & TKATXE SA
    (AXYT. ATXE) = BXYT CATYE (S)
      当日X74为「时 那脏'a EM, 使得 TAME6:[X:=a]]=T
      · AMIGENERAL = F
      · WAAF, THXAAT
      = B>(JXTA, TVXA)=T
 i BA(B+CTXA, ATA), B+(AXTA, TXXA))=T
    (ArXY) \leftrightarrow (AxEr) \neq
  (AXET ← ATXY) A (ATXY ← AXET) = (
 (AXEr ← ArXY) Λ (ArXY ← AXEr) 3+M , (3,M) 里朝即到於 (A)
 (=> BA(B+(TEXA, HXTA), B+(HXTA, TEXA)) MEGEX = T for all & a & M.
    THAXE, THAXE D
      即任意a.使得 AMEGEX:=a]]=F
     I=(Arxy, Axer) & T = Arxy is
   D YXTA ST, AP 14 & a & M, TAMESEX = ATT = T, AMESEX = ATT = F
     TRAXET , 78 AXE.
     : B>(WYA, TEXA)=T
```

: BA(B+ (T=XA, VXTA), B+(KTA, T=XA))=T for all aEM.

```
18. (1) F ∀XA ↔ 60 A [Y/X]

18. (1) 即对任何模型. 有 M ⊨ 6 ∀XA ↔ ∀9 A [Y/X]
DATE
      (XA)MIB] = {T. 对所有aeM, AMIBEX=ajj=T
F, 否则
      (以A[y/x]) MICO = ST. 对所有图EM, A[y/x]AIGT yi= a]] = T
Y 从是新度元
      A [YIX] MEE y = a ] = AMEE [X = YMEE], Y = a]]
                      = AMIGIX:= AJJ
      i B> ( VXA, Vy AIy/x7) = T
       in | FXA A BY A [Y/x]
    即对任何模型,有MF6∃XA↔∃YAIY/x]
       (∃XA/MIG) = { T, 存在a∈M, AMBIXI=a]] = T
F, 在例
       1 3 y A [ y / x ] ) M [ 6] = } T, 存在 a ∈ M. A [ y / x ] M [ 6 [ y = a]] = T
                    AFE ATTA THE
        以提新变元
        t. 由精换引理 A [ y/X] M [ 6 E y: = 0.7]
                    = AMEGEX:= YMES] . Y: = a]]
```

= AMI 6 [X:= A]]

· AXA← JYA[Y/X] 对任意模型

```
19. (1) YXA(X)↔ Yt. A(X)庆7净~真
         那对任何模型,MF6 VXA⇔YA [t/x]
         AP Bes ( YXA. A[t/x]) MIGT = T
         EP B. (YXA, YHIE+/X]) MEGJ = T A
             B = (A[t/x], VXA) MIGJ = T
          B VXA=T 、 例 FF有 a ∈ M· AMISEX:=a]]= T
         由引理3.11、 tMCG EM
        · AMEGEX = tures ]] = T
        由替换引理,A[专]MEG] = AMEGEX:=tMEG]]=T
         · B + ( WXA, ALTK) MEGJ = T
         岩thA[t/x] = T
         即 A[+] MES] = AMESEX= tMES] = I , 对所有项七
         · tacos ∈ M, t为任意一个顶 / 例全ti= Xi, ti= Xi, · ti= Xi,
                                  FOR ALTYXIALD = AMEGEX = TIMED]
         : Va EM. AMEGEX:= a]]=T
         i. B + ( VXA, Alt/x)) MIGO = T
                                       = AMIGEX:= XI] = 7
         · YXA↔ A[t/x]永真
(2). H.AIt/x] → JXA 永真
                                     A [tr/x] MEGJ = AMEGEX = trAKEG]
    即对任何模型, M F 6 ATHXJ→3×A
                                                = AMTGTX = KaJJ=T
    T = (AXE, [x/+]A) = T
    当 /A[t/x]mtg = T 时,
    AP A[t/x] MIGJ = A MIGIX = tMIG] = T
    由引理3.11. tMICJ EM, 该 tMICJ = a.
    FIFUL TOTE a EM , AMIGIX:= a]] = T
    T = AXE 93
    : B > (AT+/X), 3XA)=T
    · A[t/x]→∃XA 永真
```

20. (1) # AM[8[x=0]] = T , AM[6[x=b]] = F (a+b) DATE 別(JXA)MEGT = T 因为 Jack,使得 AMESTAGET=T (YXA) MICO = F 因为 I b E M. 使得 AMIGNARIE F AMESON=10 = T, BMESON=1) F (a+b) 对其他 c∈M 且 c+a, c=b AMEGEX:=CJ]=T, BMEGEX:=CJ]=T WYX (AVB)MED = T Man Talman saper la (大A)MICT = F 因为 JacM, AMIGEX:=OU=F (YXB) MICT = F 因为 3 B E M. BMIGEX:=bJJ=F · ∀X(AVB)→((∀XA)V(∀XB))不是永真 21. (1) 设A的-个模型 (M, 6) M = N,  $6(X_n) = n$ (Xy) E R(Xy) 当区仅当 X<Y 刚对所有 a∈M, (a, a) ≠ R(x,y) : VX7R(xx)=T ∀a,b,c∈M, 芨 (a,b)∈R(x,y), (b,c)∈R(x,y) 取p axb 風b(c, か) a(c) = (a,c) = R(xy) 2 VXYYVZ ((R(X)) AR(Y,Z) -> R(X,Z)) = T Yaem, fatlem 使得 ● (a, a+1) ∈ R(Xy) : YX3yR(Xy) = T :、(M.6)为A的一个无穷模型

## (2) 假设A有一个无穷模型 (M, 6)

群 6(Xn) = a7 (i=0,1,·-, 人)

M= fail 0 < i < k }

楚(ai, aj) ∈ R(Xy).

记为 ai<aj (i<j)

· I VX By R(X, y) = T

: 30 Rt 取 XME6] = ak

刚 JajR(ak, aj)=T, ajeM

かりたド

う D≤j≤k矛盾

· A没有有写模型

```
(1) 证: case 1: t为某个变元 Xi (1≤n)
        1 4151, SI(XI) = 52 (XI)
        . toursis = Xintsis = Si(Xi) = Se(Xi) = toursis
       Case Z: t为某个常元 C
         A) this, ] = CMIS, ] = C = CMIS, = this, ]
       case 3: + 为 f(t1, t2, ..., tn) 英中 titz, ..., tn 均为项
         Ry thisis = f(ti, t2, ..., tn) MESIS = fn(timesis, ..., thinksis)
                             = Im(timeszo, --, tomesso)
                          = f(t1, t2, -- tn) MIS27
                                       = tmss]
         Q.E.D. (FERY) SAYE WE SAY (FEA OZE)
  12). Cast 1: 若A呈开 S=t (S称 t为政)
              My Amesij = (S = t)_{MEsij} = \{T, tf \# Smesij = tmesij\}
                 A_{MES2J} = (S = t)_{MES2J} = \begin{cases} T, & \text{if } S_{MES2J} = t_{MES2J} \\ F, & \text{else} \end{cases}
              由川知, 对任意场t, tMESI) = TMESI]
               : AMISO - AMISOT
       Case 2: A呈形 R(ti, tz,...,tm), 内n无谓钢铁, ti, ~ t为顶.
                別 Amis,j = R(ti, ti, ··, th) missj = 「T, 差<timisp, ··, thmisp > EPM;
F, 差<timisp, ··, thmisp> 手M.
               AMIGO = RCti, to, ..., tn) MIGO = { T, $ < timeso, ..., tn/miso) > & PM

F, $ < timeso, ..., tn/miso) > & PM
                由u), 大任產项士, thisi = thisis
```

1. AMESIJ = AMESIJ

DATE

case 3: A呈形 7B, 其中 B 为公式

AMIST = (B)MIST = B7 (BATED)

AMISJ = (B)MISJ = BT(BMISJ)

由 case 1 An case 2 Ao BNOST = BMISST

: AMTSIJ = AMISIJ

case 4: A27 B\*C, \* ∈ {1. V, >}

W AMISOJ = B\* ( BMBD, CMISO)

AMISSJ = B\* (BMISS) (MISS)

V BALEGJ = BALEGJ , CALEGJ = CALEGJ

AMISIJ = AMISIJ

cave 5: A呈形 以.B (X为变元)

AMISIJ = (长x.B)MISIJ = S.T. 差对所有 Q.G.M. BMISIX=aJJ=T

AMEGO = (以,B)MIGO = 「T. 共对所有QEM, BMISOIX=GJ]=T

! BMESJ = BMESJ

: Amiso = Amiso

Case 6: A \$# 3x.B

AMISJ=(3XB)MISJ = {T, 基本某个 a < M, BMISITX=aJJ=T

AMISO = (JX.B) MISO = (T. 炭杖菜 aem. BMISOIX:=a)=T F. 否则

" BMDS = BMDSJ

: AMIST - AMIST

13. 证: M =6 (A N B) (A N B) MIST ( BA ( AMEG) BMEG) = of T. Amiso = T and BMISO = T MESA and MESB = { T, AMEGS = T and BMEGS = T F, GAI) · MEG(AAB) ( IMEGA and MEGB) 24. 班: M = 6 YZ. A[表]. ⇔ (YZ. A[表]) M[6] = T 母 对任意 QEM, (A[表])MI6EZ=a]] Z] AMEGEX:= XJEZ:= aJJ = AMEGEX:=aJJ = T (卷换引理) ⇔ M = 6 tx. A