第六次作业.

1. $(\forall x \exists y \forall z \exists u P(x, y, z, u))^S$

$= \forall x(\exists y \forall z \exists u P(x, y, z, u))^S$

$= \forall x \forall z \exists u P(x, f_1(x), z, u)$

$= \forall x \forall z (\exists u P(x, f_1(x), z, u))^S$

$= \forall x \forall z P(x, f_1(x), z, f_2(x, z))$

其中 f_1、f_2 为新函数

2. $(\forall x P(x) \wedge \forall y Q(y)) \rightarrow \exists z P(z)$

$\Leftrightarrow \forall x(P(x) \wedge \forall y Q(y)) \rightarrow \exists z P(z)$

$\Leftrightarrow \forall x \forall y(P(x) \wedge Q(y)) \rightarrow \exists z P(z)$

$\Leftrightarrow \exists z(\forall x \forall y(P(x) \wedge Q(y)) \rightarrow P(z))$

$\Leftrightarrow \exists z(\exists x(\forall y(P(x) \wedge Q(y)) \rightarrow P(z))$

$\Leftrightarrow \exists z(\exists x(\exists y(P(x) \wedge Q(y) \rightarrow P(z))))$

证: $\vDash \exists x \forall y P(x,y) \to \forall y \exists x P(x,y)$

$\Leftrightarrow$ 对任何Model, 有 $M \vDash_{\sigma} \exists x \forall y P(x,y) \to \forall y \exists x P(x,y)$

$\Leftrightarrow$ 当 $(\exists x \forall y P(x,y))_{M[\sigma]} = T$ 时,

$(\forall y \exists x P(x,y))_{M[\sigma]} = T$.

而 $(\exists x \forall y P(x,y))_{M[\sigma]} = T \Leftrightarrow \exists a \in M$, 使得 $(\exists x \forall y P(x,y))_{M[\sigma[x:=a]]}$

$= (\forall y P(a,y))_{M[\sigma]} = T$

$\Leftrightarrow$ 对所有的 $b \in M$, 有 $(\forall y P(a,y))_{M[\sigma[y:=b]]} = P(a,b) = T$

$(\forall y \exists x P(x,y))_{M[\sigma]} = T \Leftrightarrow$ 对所有的 $b \in M$, 有 $(\forall y \exists x P(x,y))_{M[\sigma[y:=b]]}$

$= (\exists x P(x,b))_{M[\sigma]} = T$

$\Leftrightarrow$ 存在 $a \in M$, 使得 $(\exists x P(x,b))_{M[\sigma[x:=a]]} = P(a,b) = T$

$\therefore$ 当 $(\exists x \forall y P(x,y))_{M[\sigma]} = T$ 时, $(\forall y \exists x P(x,y))_{M[\sigma]} = T$

QED.

ATE

5. 反例： $M = \{a, b, c\}$

$I(P) = \{(a, b), (b, c), (c, a)\}$

则 $(\forall x \exists y P(x, y))_{M[I]} = T$

$(\exists y \forall x P(x, y))_{M[I]} = F$

$\therefore \nvDash \forall x \exists y P(x, y) \to \exists y \forall x P(x, y)$