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第七次作业.

6.

$$\text{Axiom}$$

$$\dfrac{P(z, f(z)),\ \forall x P(x, f(x)) \vdash P(z, f(z)),\ \exists y P(z, y)}{P(z, f(z)),\ \forall x P(x, f(x)) \vdash \exists y P(z, y)}\ \exists R$$

$$\dfrac{P(z, f(z)),\ \forall x P(x, f(x)) \vdash \exists y P(z, y)}{\forall x P(x, f(x)) \vdash \forall x \exists y P(x, y)}\ \forall L,\ \forall R$$

$$\dfrac{\forall x P(x, f(x)) \vdash \forall x \exists y P(x, y)}{\vdash \forall x P(x, f(x)) \to \forall x \exists y P(x, y)}\ \to R$$

7、证：设 $(M, I) \models \forall x \exists y P(x, y)$

从而对 $a \in M$ 存在 $b \in M$，使对任何 σ 有

$(M, I) \models_{\sigma[x:=a,\, y:=b]} P(x, y)$ (*)

令 $S_a = \{ b \mid (*) \text{成立} \}$

$\because S_a \neq \phi$ 且 $S_a \in P(M)$

$\therefore$ 由AC知，有 $\rho: P(M) \to M$ 使 $\rho(S_a) \in S_a$.

因此 $(M, I) \models_{\sigma[x:=a,\, y:=\rho(S_a)]} P(x, y)$

令 $F: M \to M$ 如下：$F(a) = \rho(S_a)$ $(a \in M)$

令 I' 为 I 的扩展使 $I'(f) = F$

$\therefore (M, I') \models_{\sigma[x:=a,\, y:=F(a)]} P(x, y)$

$\therefore (M, I') \models_{\sigma[x:=a]} P(x, \frac{F(x)}{y})$

从而 $(M, I') \models \forall x P(x, \frac{F(x)}{y})$.

$\therefore \forall x P(x, f(x))$ 可满足.

8. $H_0 = \{c\}$

$H_1 = \{c\} \cup \{f(c)\}$

$H_2 = \{c, f(c)\} \cup \{f(c), f(f(c))\}$

$\cdots$

$H_A = \cup\{H_n \mid n \in N\} = \{c, f(c), f^2(c), \cdots, f^n(c), \cdots\}$

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9. 证: $n=0$时, $H_0=\{c_0\}$ or $H_0=\{c \mid c$为常元且出现在A中$\}$

$\therefore$ $|H_0|$ 为一常数, $|H_0|<\aleph_0$

设 $n<k$时, $|H_n|<\aleph_0$

$n=k+1$时, $\cancel{|H_n|}=$ $H_n=H_k \cup \{f(t_1,\cdots,t_m) \mid f$为A中函数且 $t_1,\cdots,t_m \in H_k\}$

$\therefore$ $|H_n| < |H_k| + |\{f(t_1,\cdots,t_m) \mid f$为A中函数且 $t_1,\cdots,t_m \in H_k\}|$

$< |H_k| + |H_k|^m < \aleph_0$

$\therefore$ $\forall n \in N$, $|H_n|<\aleph_0$

$\because$ $H_A = \cup\{H_n \mid n \in N\} = H_1 \cup H_2 \cdots \cup H_n \cup H_{n+1} \cup \cdots$

显然 H_A 是可数无穷的

$\therefore$ $|H_A|=\aleph_0$