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第五次作业

1、证：(1) 对于任意的集合 Δ，若 Δ 为 $\Phi \cap \Psi$ 的子集

则 $\Delta \subseteq \Phi$ 且 $\Delta \subseteq \Psi$，

$\because$ $Con(\Phi)$、$Con\,\Psi$.

$\therefore$ $\Delta \vdash$ 不可证.

$\therefore$ 由定义知 $Con(\Phi \cap \Psi)$

(2) 令 $\Phi = \{A, B\}$

$\Psi = \{\neg A, C\}$

则 $\Phi \cup \Psi = \{A, \neg A, B, C\}$

对于公式 A，存在 $\Phi \cup \Psi$ 的有穷子集 $\Delta = \{A, \neg A\}$

使得 $\Delta \vdash A$ 和 $\Delta \vdash \neg A$ 可证

由命题 6.2 知 $Incon(\Phi \cup \Psi)$

3. 证: 对L的任何协调公式集Γ.

设L的全体公式为$\varphi_0, \varphi_1, \cdots, \varphi_n, \cdots \ (n \in N)$.

令
$$\begin{cases} \Phi_0 = \Gamma \\ \Phi_{n+1} = \begin{cases} \Phi_n, & \text{若 } Incon(\Phi_n \cup \{\varphi_n\} \\ \Phi_n \cup \{\varphi_n\}, & \text{若 } Con(\Phi_n \cup \{\varphi_n\}) \end{cases} \end{cases}$$

令$\Phi = \cup\{\Phi_n \mid n \in N\}$

现证Φ是极大协调的.

反设$Incon(\Phi)$, 从而存在Φ的有穷子集Δ使$\Delta \vdash$可证.

$\because$ Δ有穷, 不妨设$\Delta = \{A_1, \cdots, A_k\}$

$\therefore$ $A_i (i=1,2,\cdots,k) \in \Phi = \cup\{\Phi_n \mid n \in N\}$

故对每个$i \le k$, 有n_i使 $A_i \in \Phi_{n_i}$

故有l对每个$i \le k$, $A_i \in \Phi_l$

$\therefore$ $\Delta \subseteq \Phi_l$, 这与$Con(\Phi_l)$矛盾.

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证明极大性.

只需证若 $Con(\Phi_n \cup \{\varphi_n\})$，则 $\varphi_n \in \Phi$

设 $Con(\Phi \cup \{\varphi_n\})$，从而 $Con(\Phi_n \cup \{\varphi_n\})$

$\therefore \varphi_n \in \Phi_{n+1}$ $\therefore \varphi_n \in \Phi$.

4、(2) 证：

$$\frac{s\doteq t,\ P(s)\vdash P(t)}{\vdash (s\doteq t)\to(P(s)\to P(t))}\to R$$

定义 P，$m\in P \Leftrightarrow m\doteq s$

则有 $s\doteq t,\ s\doteq s\vdash t\doteq s$

$$\therefore\quad \cfrac{\cfrac{s\doteq t,\ s\doteq s\vdash t\doteq s \qquad \vdash s\doteq s}{s\doteq t\vdash t\doteq s}\,Cut}{\vdash s\doteq t\to t\doteq s}\to R$$

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(3)

$$\dfrac{\dfrac{\dfrac{t \doteq s\ ,\ t \doteq u \vdash s \doteq u \qquad s \doteq t \vdash t \doteq s}{s \doteq t,\ t \doteq u \vdash s \doteq u}\ \text{Cut}}{s \doteq t \vdash (t \doteq u \to s \doteq u)}\ {\to}R}{\vdash s \doteq t \to (t \doteq u \to s \doteq u)}\ {\to}R$$

的模型 $\langle M, \sigma \rangle$.

7. (1) $M = \{a, b, c\}$

σ: $\sigma(P) = \{a\}$

易知 $P(a) \to \forall y P(y)$ 为假 ∴ 有反例模型 $\langle M, \sigma \rangle$.

(2)

$$
\dfrac{
  \dfrac{
    \dfrac{P(a) \vdash P(a), \exists x P(x), Q(a)}{P(a) \vdash \exists x P(x), Q(a)}\ \exists R
    \qquad
    \dfrac{P(a), Q(y), \forall y Q(y) \vdash Q(a)}{P(a), \forall y Q(y) \vdash Q(a)}\ \forall L
  }{
    \exists x P(x) \to \forall y Q(y), P(a) \vdash Q(a)
  }\ \to L
}{
  \dfrac{
    \exists x P(x) \to \forall y Q(y) \vdash P(a) \to Q(a)
  }{
    \dfrac{\exists x P(x) \to \forall y Q(y) \vdash \forall z (P(z) \to Q(z))}{\vdash (\exists x P(x) \to \forall y Q(y)) \to \forall z (P(z) \to Q(z))}\ \to R
  }\ \forall R
}\ \to R
$$

(3) $M: \{a, b\}$ 则 $\forall z (P(z) \to Q(z)) = T$

$\sigma(P) = \{a\}$ $\exists x P(x) \to \forall y Q(y) = F$

$\sigma(Q) = \{a\}$

∴ $\forall z (P(z) \to Q(z)) \to (\exists x P(x) \to \forall y Q(y)) = F$.

(4)

Axiom

$R(z,z), \exists y \forall x(\cdots) \vdash R(z,z)$ $\neg R$

$\exists y \forall x(\cdots) \vdash \neg R(z,z), R(z,z)$

Axiom

$\exists y \forall x(\cdots), \neg R(z,z) \vdash \neg R(z,z)$ $\to L$

$R(z,z) \to \neg R(z,z), \exists y \forall x(R(x,y) \leftrightarrow \neg R(x,x)) \vdash \neg R(z,z)$

Axiom

$\exists y \forall x(\cdots), R(z,z) \vdash R(z,z)$

Axiom.

$R(z,z), \exists y \forall x(\cdots) \vdash R(z,z)$ $\neg L$

$\exists y \forall x(\cdots), R(z,z), \neg R(z,z) \vdash$ $\to L$

$R(z,z) \to \neg R(z,z), \exists y \forall x(\cdots), R(z,z) \vdash$ $\to L$

$R(z,z) \to \neg R(z,z), \neg R(z,z) \to R(z,z), \exists y \forall x\, R(x,y) \leftrightarrow \neg R(x,x)) \vdash$ $\wedge L$

$R(z,z) \to \neg R(z,z) \wedge \neg R(z,z) \to R(z,z), \exists y \forall x(R(x,y) \leftrightarrow \neg R(x,x)) \vdash$

$R(z,z) \leftrightarrow \neg R(z,z), \exists y \forall x(R(x,y) \leftrightarrow \neg R(x,x)) \vdash$ $\forall L$

$\forall x(R(x,z) \leftrightarrow \neg R(x,x)), \exists y \forall x(R(x,y) \leftrightarrow \neg R(x,x)) \vdash$ $\exists L$

$\exists y \forall x(R(x,y) \leftrightarrow \neg R(x,x)) \vdash$ $\neg R$

$\vdash \neg \exists y \forall x(R(x,y) \leftrightarrow \neg R(x,x))$