DATE

第四次作业.

1. (9).

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overset{\text{Axiom}}{A(y) \vdash A(y), \exists x \neg A(x)}}{A(y) \vdash \forall x A(x), \exists x \neg A(x)}\forall R}{\vdash \forall x A(x), \neg A(y), \exists x \neg A(x)}\neg L}{\vdash \forall x A(x), \exists x \neg A(x)}\exists R}{\neg \forall x A(x) \vdash \exists x \neg A(x)}\neg R}{\vdash \neg \forall x A(x) \to \exists x \neg A(x)}\to R$$

其中y为新变元.

(10) y为新变元

$$\dfrac{\dfrac{\dfrac{\dfrac{\overset{\text{Axiom}}{A(y) \vdash A(y), \exists x A(x)}}{\vdash A(y), \exists x A(x), \neg A(y)}\neg L}{\vdash \exists x A(x), \forall x \neg A(x)}\exists R, \forall R}{\neg \exists x A(x) \vdash \forall x \neg A(x)}\neg R}{\vdash \neg \exists x A(x) \to \forall x \neg A(x)}\Rightarrow R$$

3. 证: $\Leftarrow$:

∵ $\dfrac{A \vdash B}{\vdash A \to B} \to R$

∴ 若 $A \vdash B$ 可证, $\vdash A \to B$ 可证

$\Rightarrow$:

∵ $$\dfrac{\vdash A \to B \qquad \dfrac{\overset{\text{Axiom}}{A \vdash A, B \qquad A, B \vdash B}}{A, A \to B \vdash B}\to L}{A \vdash B}\text{Cut}$$

∴ 若 $\vdash A \to B$ 可证, $A \vdash B$ 可证.

DATE 模型 (M, σ)

6. 论域 $M = \{a, b, c\}$

$\sigma(P) = \{(a,a), (b,b), (c,c), (a,b), (b,a), (b,c), (c,b)\}$

则 $(\forall x P(x,x))_{M[\sigma]} = T$

$\forall x \forall y (P(x,y) \to P(y,x))_{M[\sigma]} = T$

$\forall x \forall y \forall z ((P(x,y) \land P(y,z)) \to P(x,z))_{M[\sigma]} = F$

因为若 $\sigma(x) = a$，$\sigma(y) = b$，$\sigma(z) = c$

$(a,b) \in P$，$(b,c) \in P$，但 $(a,c) \notin P$

$\therefore B_{\to}(B_{\land}(P(a,b), P(b,c)), P(a,c)) = F$

$\therefore$ 原矢列存在反例，不可证.