

习题一补充讲解:

3 (6) 解

$$\begin{vmatrix} a & 0 & b & 0 \\ 0 & c & 0 & d \\ x & 0 & y & 0 \\ 0 & u & 0 & v \end{vmatrix} \xrightarrow[r_2 \leftrightarrow r_3]{c_2 \leftrightarrow c_3} \begin{vmatrix} a & b & 0 & 0 \\ x & y & 0 & 0 \\ 0 & 0 & c & d \\ 0 & 0 & u & v \end{vmatrix} = \begin{vmatrix} a & b \\ x & y \end{vmatrix} \begin{vmatrix} c & d \\ u & v \end{vmatrix} = (ay - bx)(cv - du)$$

4 (2) 证明

$$\text{左式} \xrightarrow{2,3 \text{列加到第1列}} \begin{vmatrix} 1+a+b+c & b & c \\ 1+a+b+c & 1+b & c \\ 1+a+b+c & b & 1+c \end{vmatrix} \xrightarrow{c_1 \div (1+a+b+c)} \begin{vmatrix} 1 & b & c \\ 1 & 1+b & c \\ 1 & b & 1+c \end{vmatrix}$$

$$\xrightarrow[c_3 - c_1]{c_2 - b \cdot c_1} \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \text{右式}$$

4 (5) 证明一:

$$(a-b)D_n \xrightarrow{(a-b) \times c_1} \begin{vmatrix} a^2-b^2 & ab & 0 & \cdots & 0 \\ a-b & a+b & ab & \cdots & 0 \\ 0 & 1 & a+b & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & a+b \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & ab & 0 & \cdots & 0 \\ a & a+b & ab & \cdots & 0 \\ 0 & 1 & a+b & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & a+b \end{vmatrix} - \begin{vmatrix} b^2 & ab & 0 & \cdots & 0 \\ b & a+b & ab & \cdots & 0 \\ 0 & 1 & a+b & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & a+b \end{vmatrix}$$

$$= a \begin{vmatrix} a & ab & 0 & \cdots & 0 \\ 1 & a+b & ab & \cdots & 0 \\ 0 & 1 & a+b & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & a+b \end{vmatrix} - b \begin{vmatrix} b & ab & 0 & \cdots & 0 \\ 1 & a+b & ab & \cdots & 0 \\ 0 & 1 & a+b & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & a+b \end{vmatrix}$$

$$\begin{matrix} \text{前行列式: } c_2 - bc_1, c_3 - bc_2, \dots, c_n - bc_{n-1} \\ \text{后行列式: } c_2 - ac_1, c_3 - ac_2, \dots, c_n - ac_{n-1} \end{matrix} \begin{vmatrix} a & 0 & \cdots & 0 \\ 1 & a & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & a \end{vmatrix} - b \begin{vmatrix} b & 0 & \cdots & 0 \\ 1 & b & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & b \end{vmatrix} = a^{n+1} - b^{n+1}$$

故 $D_n = \frac{a^{n+1} - b^{n+1}}{a - b}$

证明二: $D_n = (a+b)D_{n-1} - abD_{n-2}$, 故

$$D_n - aD_{n-1} = b(D_{n-1} - aD_{n-2}) = \cdots = b^{n-2}(D_2 - aD_1) = b^n, \text{ 从而有}$$

$$D_n = aD_{n-1} + b^n = a^2D_{n-2} + ab^{n-1} + b^n = \cdots = a^n + a^{n-1}b + \cdots + b^n = \frac{a^{n+1} - b^{n+1}}{a - b}$$

证明三: 用数学归纳法

5(2) 解

$$\begin{vmatrix} x & a & b & c \\ a & x & c & b \\ b & c & x & a \\ c & b & a & x \end{vmatrix} \stackrel{\substack{r_1+r_3 \\ r_2+r_4}}{=} \begin{vmatrix} x+b & a+c & x+b & a+c \\ a+c & x+b & a+c & x+b \\ b & c & x & a \\ c & b & a & x \end{vmatrix} \stackrel{\substack{c_3-c_1 \\ c_4-c_2}}{=} \begin{vmatrix} x+b & a+c & 0 & 0 \\ a+c & x+b & 0 & 0 \\ b & c & x-b & a-c \\ c & b & a-c & x-b \end{vmatrix}$$

$$= \begin{vmatrix} x+b & a+c \\ a+c & x+b \end{vmatrix} \begin{vmatrix} x-b & a-c \\ a-c & x-b \end{vmatrix} = ((x+b)^2 - (a+c)^2)((x-b)^2 - (a-c)^2) = 0,$$

故解为 $x+b=\pm(a+c), x-b=\pm(a-c)$ 即

$$x=-b+a+c, x=-b-a-c, x=b+a-c, x=b-a+c$$

6(2) 解

$$D = \begin{vmatrix} 1-x^2-y^2-z^2 & x & y & z \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \stackrel{\substack{c_1-xc_2 \\ c_1-yc_3 \\ c_1-zc_3}}{=} 1-x^2-y^2-z^2=1$$

故有 $x^2+y^2+z^2=0$, 于是有 $x=y=z=0$

7(5) 解

4(5)特例

$$D_n = \begin{vmatrix} 1+1 & 1 & 0 & \cdots & 0 \\ 1+0 & 2 & 1 & \cdots & 0 \\ 0 & 1 & 2 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 1 \\ 0 & 0 & \cdots & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & 2 & 1 & \cdots & 0 \\ 0 & 1 & 2 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 1 \\ 0 & 0 & \cdots & 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 0 & \cdots & 0 \\ 0 & 2 & 1 & \cdots & 0 \\ 0 & 1 & 2 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 1 \\ 0 & 0 & \cdots & 1 & 2 \end{vmatrix},$$

$$\begin{vmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & 2 & 1 & \cdots & 0 \\ 0 & 1 & 2 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 1 \\ 0 & 0 & \cdots & 1 & 2 \end{vmatrix} \stackrel{\substack{r_2-r_1 \\ r_3-r_2 \\ \cdots \\ r_n-r_{n-1}}}{=} \begin{vmatrix} 1 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 1 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & 1 \\ 0 & 0 & \cdots & 0 & 1 \end{vmatrix} = 1, \text{ 故}$$

$$D_n = 1 + D_{n-1} = \cdots = n-1 + D_1 = n+1$$

法二：求出递推式： $D_n - D_{n-1} = D_{n-1} - D_{n-2}$, D_n 为等差数列，公差为 1

12 证明 显然 $f(x)$ 是 x 的多项式，故 $f(x)$ 连续可微。

$$\text{又 } f(0) = \begin{vmatrix} 1 & -1 & -1 \\ 1 & -2 & -2 \\ 1 & -3 & -3 \end{vmatrix} = 0, f(1) = \begin{vmatrix} 1 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 0, \text{ 由罗尔定理, 存在}$$

$\xi \in (0,1)$ 使得 $f'(\xi) = 0$