Estimation of Trend by Curve Fitting

SESSION 15

Structure

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15.1 INTRODUCTION

So far, you have learnt about different control charts for variables as well as attributes in the lab sessions of Part A and Part B, respectively. You have also learnt how to fit and analyse regression models and apply different variable selection methods in the lab sessions of Part C. In Unit 13 of MSTE-002 (Industrial Statistics-II), we have explained different components of the time series along with the estimation of trend using the methods of curve fitting. We have explained three methods of curve fitting: (i) linear, (ii) quadratic and (iii) exponential to fit an equation of the trend in Unit 13 of MSTE-002.

In this lab session, you will learn how to measure trend using all three methods of curve fitting. In the next lab session, we shall explain the methods of smoothing and filtering the time series data.

Prerequisite

- Lab Session 12 of MSTL-002 (Industrial Statistics Lab).
- Lab Session 5 of MSTL-001 (Basic Statistics Lab).
- Unit 13 of MSTE-002 (Industrial Statistics-II).

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- apply the method of curve fitting to fit linear, quadratic and exponential trends;
- identify the appropriate trend curve; and
- project the trend for a given period of time.

15.2 PROBLEM DESCRIPTION

We state three problems to illustrate the three trends:

1) Suppose the number of cars (in thousands) on a particular road were recorded for 48 months from 2007 to 2010 as a part of a hypothetical study of a car manufacturing company in a city. The data are given in Table 1.

Table 1: Number of cars on road

Month	Number of Cars (in thousands)	Month	Number of Cars (in thousands)	Month	Number of Cars (in thousands)
1	321	17	563	33	757
2	296	18	498	34	834
3	375	19	556	35	806
4	402	20	621	36	819
5	331	21	557	37	758
6	387	22	564	38	836
7	442	23	564	39	886
8	378	24	668	40	874
9	435	25	602	41	837
10	498	26	612	42	847
11	425	27	705	43	834
12	447	28	738	44	924
13	521	29	725	45	967
14	435	30	654	46	934
15	465	31	775	47	875
16	568	32	684	48	956

2) Suppose the owner of a store in Delhi wants to study the pattern of sales in his store and predict future sales. He records the monthly sales (in thousands) of the store for 48 months from 2010 to 2013. The data are given in Table 2.

Table 2: Monthly sales of a store

Month	Monthly Sales (in thousands)	Month	Monthly Sales (in thousands)	Month	Monthly Sales (in thousands)
1	589	17	495	33	473
2	754	18	369	34	750
3	517	19	521	35	575
4	487	20	354	36	734
5	690	21	345	37	795
6	606	22	478	38	669
7	478	23	387	39	845
8	425	24	494	40	645
9	504	25	523	41	715
10	458	26	398	42	918
11	523	27	578	43	804
12	584	28	587	44	994
13	423	29	467	45	989
14	548	30	624	46	897
15	365	31	645	47	1121
16	434	32	624	48	1087

3) The growth of a company in terms of the quarterly revenue generated (in crores) from 2002 to 2013 is given in Table 3.

Table 3 : Quarterly revenue of a production company

Year	Quarter	Revenue (in crores)	Year	Quarter	Revenue (in crores)
2002	1	25	2008	1	895
	2	180		2	2527
	3	864		3	1763
	4	683		4	1847
2003	1	244	2009	1	2630
	2	400		2	3224
	3	240		3	3076
	4	29		4	2699
2004	1	73	2010	1	4225
	2	135		2	4566
	3	884		3	4128
	4	384		4	4122
2005	1	827	2011	1	4900
	2	833		2	6526
	3	818		3	5949
	4	301		4	7022
2006	1	136	2012	1	8371
	2	1506		2	8906
	3	125		3	9325
	4	308		4	10450
2007	1	1531	2013	1	11891
	2	1709		2	13688
	3	1269		3	14068
	4	1294		4	16263

- Represent the data given in Tables 1, 2 and 3 graphically and fit the appropriate trend.
- Fit the appropriate trend using the method of curve fitting by matrix approach.
- Project the number of cars on the road in February 2012 for the data given in Table 1.
- Project the sales of May 2014 for the data given in Table 2.
- Project the revenue for the last quarter of the year 2014 from the data given in Table 3.

15.3 FITTING OF THE LINEAR TREND

You have studied about estimation of the linear trend by the method of curve fitting in Unit 13 of MSTE-002. We formulate the steps involved in this method as follows:

Step 1: Let x_t be the time period and y_t, the corresponding variable or trend values being studied. When the trend shows linear pattern, its equation can be obtained as

$$y_t = b_0 + b_1 x_t \quad ...(1)$$

where b_0 and b_1 are unknown constants.

Step 2: The normal equations for estimating b_0 and b_1 using the method of least squares are given by

$$\sum y_t = nb_0 + b_1 \sum x_t \quad ...(2)$$

$$\sum x_t y_t = b_0 \sum x_t + b_1 \sum x_t^2 \quad ...(3)$$

The values of $\sum y_t, \sum x_t$ $\sum x_t y_t$ and $\sum x_t^2$ are obtained from the given data.

Step 3: To solve the normal equations in Excel 2007, we represent equations (2) and (3) in matrix notation as

$$\mathbf{C} = \mathbf{AB} \quad ...(4)$$

where

$$\mathbf{C} = \begin{bmatrix} \sum y_t \\ \sum x_t y_t \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} n & \sum x_t \\ \sum x_t & \sum x_t^2 \end{bmatrix}, \text{ and}$$

$$\mathbf{B} = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}$$

Step 4: The optimum solution of b_0 and b_1, i.e., $\hat{b}_0$ and $\hat{b}_1$ can be obtained by solving the following equation:

$$\mathbf{B} = \mathbf{A}^{-1}\mathbf{C} \quad ...(5)$$

Step 5: The desired trend line using the optimum values $\hat{b}_0$ and $\hat{b}_1$ is given by

$$y_t = \hat{b}_0 + \hat{b}_1 x_t \quad ...(6)$$

Steps in Excel

We now describe the procedure for estimating the trend with the help of Problem 1. We first examine the data by plotting the line chart and appropriate trendline. For this, we follow the steps given below and fit the trend line using Excel 2007:

Step 1: We enter the data given in Table 1 in the Excel spreadsheet as shown in Fig. 15.1.

	A	B	C
1	Year	Month (x_t)	Number of Cars (y_t)
2	2007	1	321
3		2	296
4		3	375
5		4	402
6		5	331
7		6	387
8		7	442
9		8	378
10		9	435
11		10	498
12		11	425
13		12	447
14	2008	13	521
15		14	435
16		15	465
17		16	568
18		17	563
19		18	498
20		19	556

Linear

Fig. 15.1

Step 2: To plot the line chart for the given time series data, we

- ✓ select Cells C2:C49,
- ✓ select the ***Insert*** tab from the ***Charts*** group, and
- ✓ click on the ***Line*** chart as explained in Lab Session 5 of MSTL-001.

Step 3: We format the chart by

- ✓ eliminating the grid lines and border around the chart,
- ✓ changing the data series to bold solid lines with visible markers, and
- ✓ changing the marker option, size and colour.

We can also change the fonts, axes, titles, background, etc., as desired.

The resulting chart is shown in Fig. 15.2.

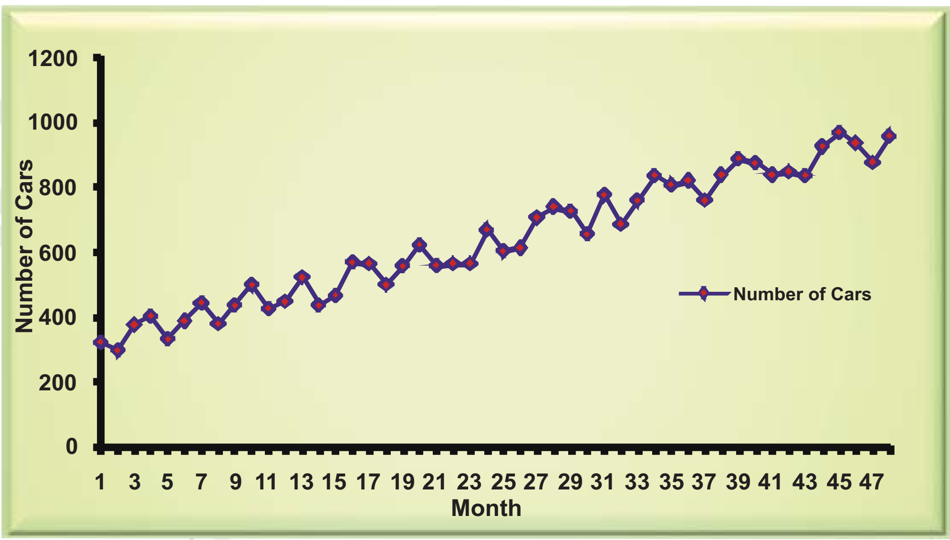


Fig. 15.2

Insert Tab→Charts Group→Line Chart

Step 4: To guess the appropriate trendline which will fit the given car data, we first click on the chart and select the ***Layout*** menu from ***Chart Tools*** as shown in Fig. 15.3.

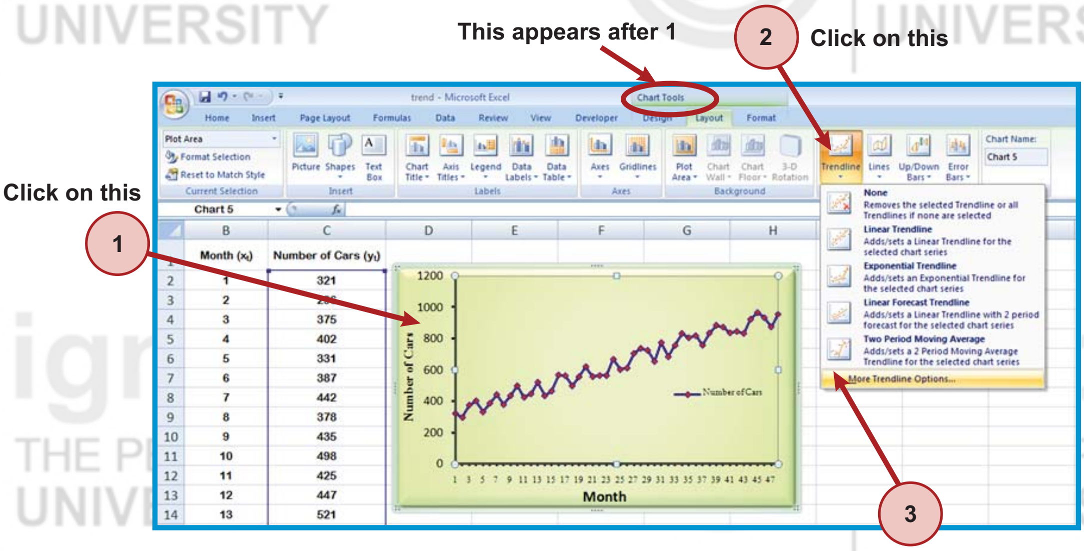


Fig. 15.3

Step 5: Then we select ***More Trendline Options***. A new dialog box gets opened as shown in Fig. 15.4.

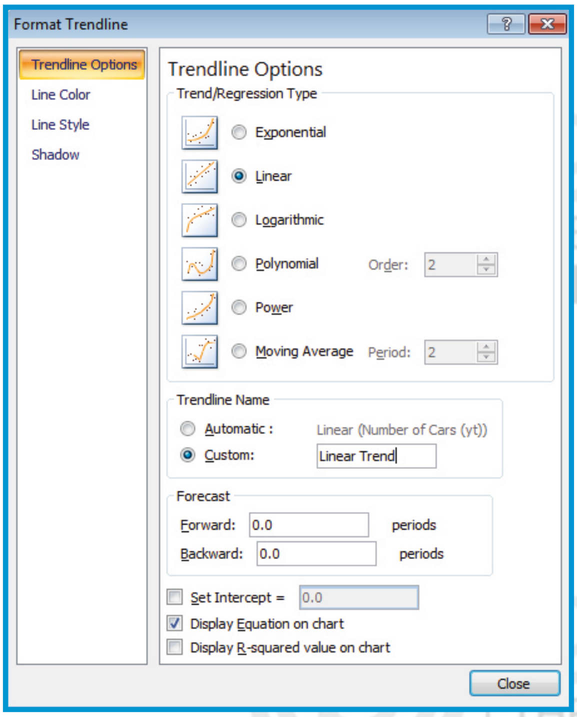


Fig. 15.4

Step 6: In the dialog box shown in Fig. 15.4, we can see different options such as **Exponential, Linear, Logarithmic, Polynomial, Power** and **Moving Averages**.

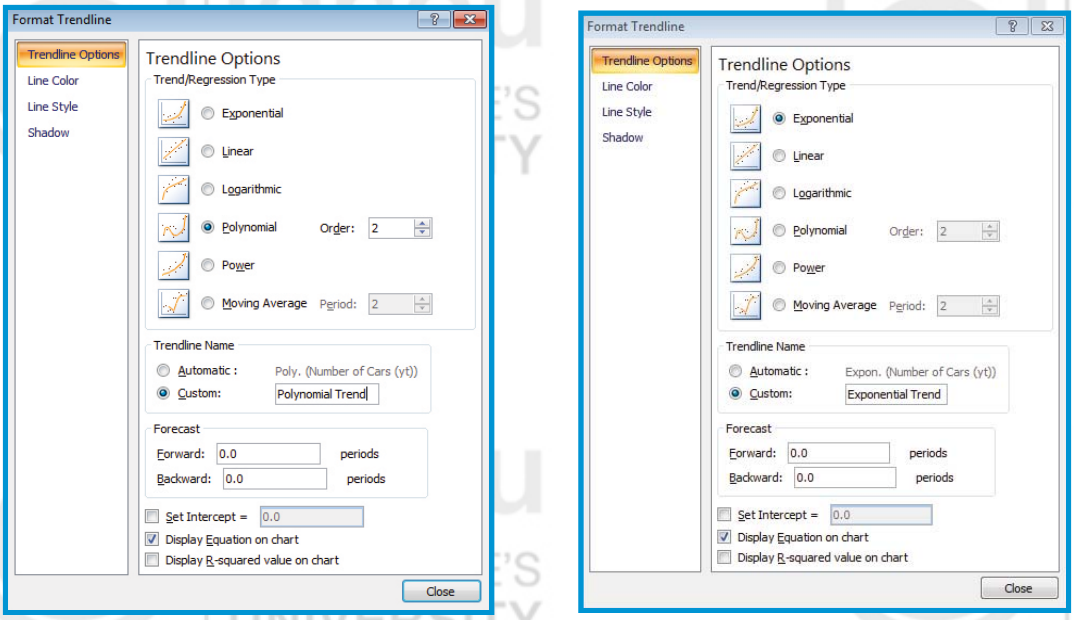
In Unit 13 of MSTE-002, we have discussed trends that are linear, polynomial of order 2 and exponential. So here we shall focus only on these three types. We fit these three trends together on the chart shown in Fig. 15.2 and then choose the best fitted trend.

We now fit the three trends one at a time.

✓ To fit linear trend, we tick on ***Linear***, ***Display Equation on chart*** and ***Custom***. Here we type "**Linear Trend**" as ***Trendline Name*** as shown in Fig. 15.4 and ***Close*** the box.

✓ To fit quadratic trend, we repeat Steps 4 and 5. We tick on ***Polynomial*** (select ***order 2***), ***Display Equation on chart*** and ***Custom***. Here we type "**Polynomial Trend**" as ***Trendline Name*** as shown in Fig. 15.5a and ***Close*** the box.

✓ To fit exponential trend, we repeat Steps 4 and 5. We now tick on ***Exponential***, ***Display Equation on chart*** and ***Custom***. Here we type "**Exponential Trend**" as ***Trendline Name*** as shown in Fig. 15.5b and ***Close*** the box.



(a) **Fig. 15.5** (b)

Step 7: From the ***Line Colour*** and ***Line Style*** specified on the left in the dialog boxes shown in Figs. 15.4, 15.5a and 15.5b, we can set different line formats. We have changed the trendlines to dashed lines of different colours. When we close the dialog box, we obtain the resulting chart with the three fits of the trend as shown in Fig. 15.6.

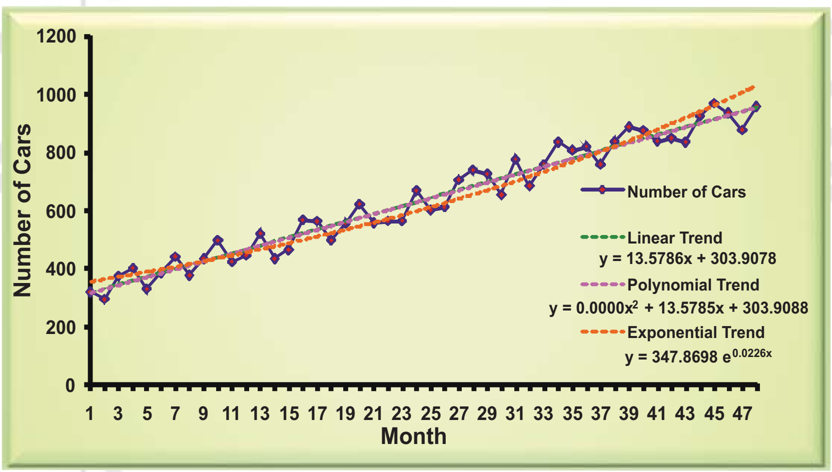


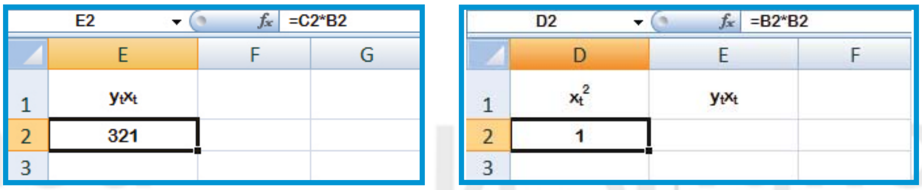
Fig. 15.6

Notice from Fig. 15.6 that the exponential curve is not fitting well and the fits of the linear and quadratic trends are approximately the same. The equation of the quadratic trend shown in Fig.15.6 is $y_t = 0.000\,x_t^2 + 13.578\,x_t + 303.909$, which is approximately the same as the equation of linear trend, i.e., $y_t = 13.579x_t + 303.908$. Note that the quadratic term (x_t^2) contributes approximately zero (0) to the equation. So we need not consider the quadratic trend. We choose the linear trend as it fits the given data well.

So far, we have obtained a linear trend for the data given in Table 1. We now explain how to fit the trend line using the method of least squares by the matrix approach in Excel 2007. You have already learnt this approach in Lab Session 12 of MSTL-002.

Step 8: As discussed in Sec. 15.3, we formulate the normal equations to find the equation of linear trend for which we need the values of $\sum x_t, \sum y_t, \sum y_t x_t$ and $\sum x_t^2$. The values of y_t and x_t are already given in Columns B and C, respectively (see Fig. 15.1). In Columns D and E, we compute the values of x_t^2 and $y_t x_t$, respectively.

In Cells D2 and E2, we type "=B2*B2" and "=C2*B2", respectively, as shown in Fig. 15.7.


E2 =C2*B2

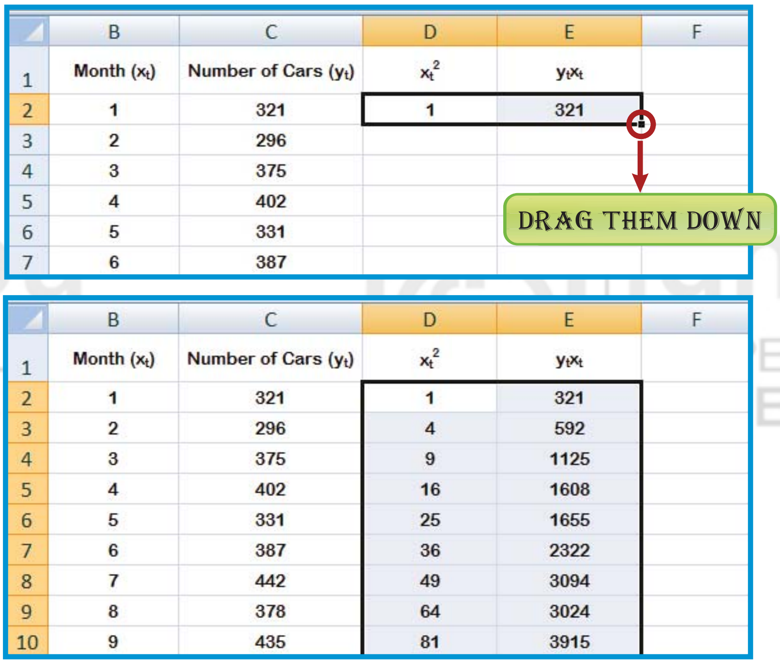
	E	F	G
1	$y_t x_t$		
2	321		
3			

D2 =B2*B2

	D	E	F
1	x_t^2	$y_t x_t$	
2	1		
3			

Fig. 15.7

Step 9: We select Cells D2:E2 and drag them down up to Row 49 to compute the values of x_t^2 and $y_t x_t$ for the remaining months as shown in Fig. 15.8.



	B	C	D	E	F
1	Month (x_t)	Number of Cars (y_t)	x_t^2	$y_t x_t$	
2	1	321	1	321	
3	2	296			
4	3	375			
5	4	402			
6	5	331			
7	6	387			

	B	C	D	E	F
1	Month (x_t)	Number of Cars (y_t)	x_t^2	$y_t x_t$	
2	1	321	1	321	
3	2	296	4	592	
4	3	375	9	1125	
5	4	402	16	1608	
6	5	331	25	1655	
7	6	387	36	2322	
8	7	442	49	3094	
9	8	378	64	3024	
10	9	435	81	3915	

Fig. 15.8

Step 10: We type "=Sum(B2:B49)" in Cell B50 to get the value of $\sum y_t$ as shown in Fig.15.9.

B50 f_x =SUM(B2:B49)

	A	B	C
48		47	875
49		48	956
50	Sum	1176	
51			
52			

Fig. 15.9

Step 11: By dragging Cell B50 right up to Cell E50, we determine the values of $\sum x_t$, $\sum x_t^2$ and $\sum y_t x_t$, in Cells C50, D50 and E50, respectively, as shown in Fig. 15.10.

	A	B	C	D	E	F
48		47	875	2209	41125	
49		48	956	2304	45888	
50	Sum	1176	30556	38024	873708	
51						
52						

Fig. 15.10

Step 12: We substitute the values of summations in equation (5) given in Sec. 15.3 to determine the optimum values $\hat{b}_0$ and $\hat{b}_1$ as follows:

$$\mathbf{B} = \mathbf{A}^{-1}\mathbf{C}$$

$$\text{i.e.,}\quad \begin{pmatrix} \hat{b}_0 \\ \hat{b}_1 \end{pmatrix} = \begin{pmatrix} 48 & 1176 \\ 1176 & 38024 \end{pmatrix}^{-1} \begin{pmatrix} 30556 \\ 873708 \end{pmatrix}$$

To find the solution of $\hat{b}_0$ and $\hat{b}_1$ in Excel, we type the matrix **C** in Cells A52: A53 and matrix **A** in Cells C52:D53 as shown in Fig. 15.11.

	A	B	C	D
51				
52	30556		48	1176
53	873708		1176	38024
54				

Fig. 15.11

Step 13: To find $\mathbf{A}^{-1}$, we first select the cells of the same size as the original matrix **A**, i.e., (2×2) where we want to put this A^{-1} matrix. Here we have selected Cells C56:D57. We then type "=Minverse(C52:D53)" and press ***Ctrl+Shift+Enter*** instead of simply ***Enter*** as shown in Fig. 15.12.

(a)

(b)

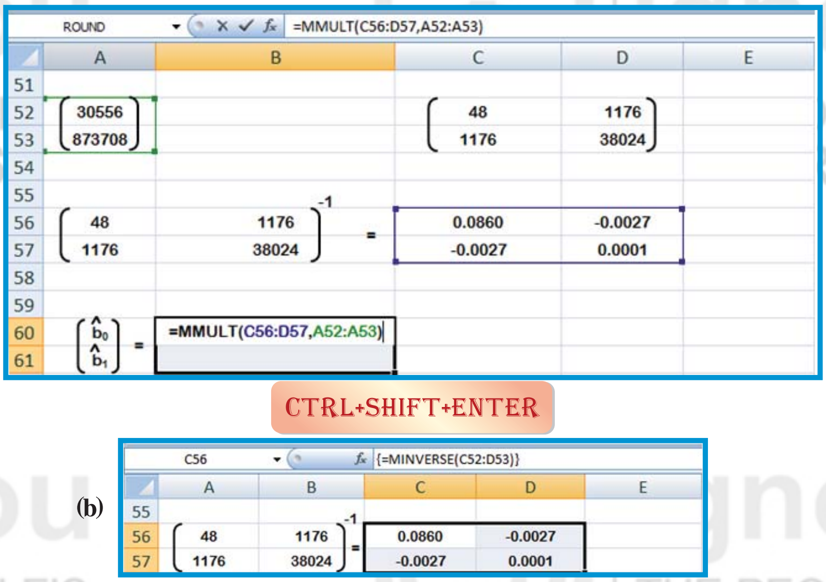


Fig. 15.12

Step 14: To find **B**, we select the cells of the same size as the resulting matrix, i.e., (2×1) where we want to put this **B** matrix. Here we have selected the Cells B60:B61. We then type "=Mmult(C56:D57, A52:A53)" and press ***Ctrl*+*Shift*+*Enter*** instead of simply ***Enter*** as shown in Fig. 15.13.

(a)

(b)

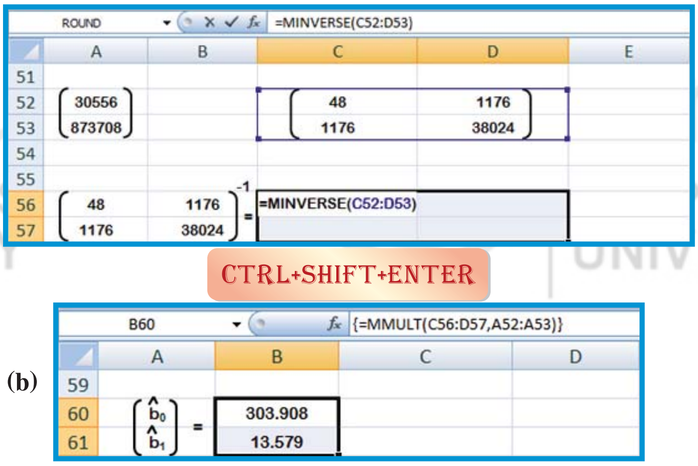


Fig. 15.13

Notice from Fig. 15.13 that the values of $\hat{b}_0 = 13.579$ and $\hat{b}_1 = 303.908$ are given in Cells B60 and B61, respectively. Hence, the equation of linear trend fitted to the given data is $y_t = 13.579 + 303.908x_t$ which is the same as obtained in Step 6.

Step 15: We type "=$B$60+$B$61*B2" in Cell F2 to obtain the trend value of the first month as shown in Fig. 15.14.

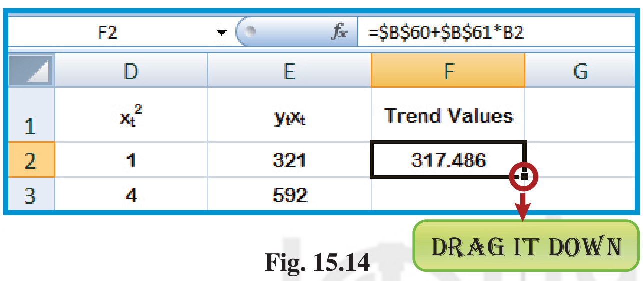


Fig. 15.14

Step 16: To determine the trend values for the remaining months, we drag down Cell F2 up to Row 49 as shown in Fig. 15.15.

	D	E	F	G
1	x_t^2	$y_t x_t$	Trend Values	
2	1	321	317.486	
3	4	592	331.065	
4	9	1125	344.644	
5	16	1608	358.222	
6	25	1655	371.801	
7	36	2322	385.379	
8	49	3094	398.958	
9	64	3024	412.537	
10	81	3915	426.115	
11	100	4980	439.694	
12	121	4675	453.272	
13	144	5364	466.851	
14	169	6773	480.430	
15	196	6090	494.008	
16	225	6975	507.587	
17	256	9088	521.165	
18	289	9571	534.744	
19	324	8964	548.322	
20	361	10564	561.901	

Linear

Fig. 15.15

Step 17: To compute the trend projection for February 2012, we first locate the number at which the month of February 2012 is placed in the sequence of data given in Table 1. Note that January 2007 is placed at number 1 and December 2010 is placed at number 48. In this sequence, February 2012 will be placed at number 62 in the table. Now, the trend projection for February 2012 can be computed by typing "=$B$60+$B$61*62" in any Cell. Here we use Cell A64 as shown in Fig. 15.16.

A64 f_x =$B$60+$B$61*62

	A	B	C
63	Trend projection for February 2012 is		
64	1145.781		
65			

Fig. 15.16

Fig. 15.16 reveals that the number of cars on road in February 2012 is approximately 1146 (Cell A64).

You may now like to do the following activity.

Activity 1

Consider $x'_t = x_t - \bar{x}_t$ to simplify the calculations, where $\bar{x}_t$ is the mean of all given time periods x_t as discussed in Sec. 13.6 of MSTE-002. In Problem 1, x_t represents the month. Repeat Steps 8 to 17 taking x'_t instead of x_t. Then match your results with the result obtained in Step 17 and state the underlying reason.

15.4 FITTING OF THE QUADRATIC TREND

Sometimes trend is not linear and shows some curvature. We may check whether the data fits a second degree polynomial. The formulae and procedure involved in quadratic fitting are given below:

Step 1: The equation of a second degree polynomial may be written as:

$$y_t = b_0 + b_1 x_t + b_2 x_t^2 \qquad \text{...(7)}$$

where b_0, b_1 and b_2 are constants.

Step 2: The normal equations for estimating b_0, b_1 and b_2 using the method of least squares are given by

$$\sum y_t = nb_0 + b_1 \sum x_t + b_2 \sum x_t^2 \qquad \text{...(8)}$$

$$\sum x_t y_t = b_0 \sum x_t + b_1 \sum x_t^2 + b_2 \sum x_t^3 \qquad \text{...(9)}$$

$$\sum x_t^2 y_t = b_0 \sum x_t^2 + b_1 \sum x_t^3 + b_2 \sum x_t^4 \qquad \text{...(10)}$$

The values of $\sum y_t, \sum x_t \; \sum x_t y_t, \sum x_t^2 y_t, \sum x_t^2, \sum x_t^3$ and $\sum x_t^4$ are obtained from the given data.

Step 3: It is the same as explained in Sec. 15.3. We represent equations (8), (9) and (10) in matrix notation as

$$\mathbf{C} = \mathbf{A}\ \mathbf{B} \qquad \text{...(11)}$$

where

$$\mathbf{C} = \begin{pmatrix} \sum y_t \\ \sum x_t y_t \\ \sum x_t^2 y_t \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} \text{ and}$$

$$\mathbf{A} = \begin{pmatrix} n & \sum x_t & \sum x_t^2 \\ \sum x_t & \sum x_t^2 & \sum x_t^3 \\ \sum x_t^2 & \sum x_t^3 & \sum x_t^4 \end{pmatrix}$$

Step 4: The optimum values of b_0 and b_1 and b_2, i.e., $\hat{b}_0$, $\hat{b}_1$ and $\hat{b}_2$ can be obtained by solving the following equation

$$\mathbf{B} = \mathbf{A}^{-1}\ \mathbf{C} \qquad \text{...(12)}$$

Step 5: The desired quadratic trend using the optimum values $\hat{b}_0$, $\hat{b}_1$ and $\hat{b}_2$ is given by

$$y_t = \hat{b}_0 + \hat{b}_1 x_t + \hat{b}_2 x_t^2 \qquad \text{...(13)}$$

Steps in Excel

Step 1: We enter the data given in Problem 2 in Sec. 15.2 of this lab session in Excel Sheet as shown in Fig. 15.17.

	A	B	C
1	Year	Month (x_t)	Monthly Sales (y_t)
2	2010	1	589
3		2	754
4		3	517
5		4	487
6		5	690
7		6	606
8		7	478
9		8	425
10		9	504
11		10	458
12		11	523
13		12	584
14	2011	13	423
15		14	548
16		15	365
17		16	434
18		17	495
19		18	369
20		19	521

Quadratic

Fig. 15.17

Step 2: As explained in Steps 2-7 of Sec. 15.3 under the heading 'Steps in Excel', we have plotted the data shown in Fig. 15.17 and also the three trends, i.e., linear, quadratic and exponential. The resulting plot is shown in Fig. 15.18.

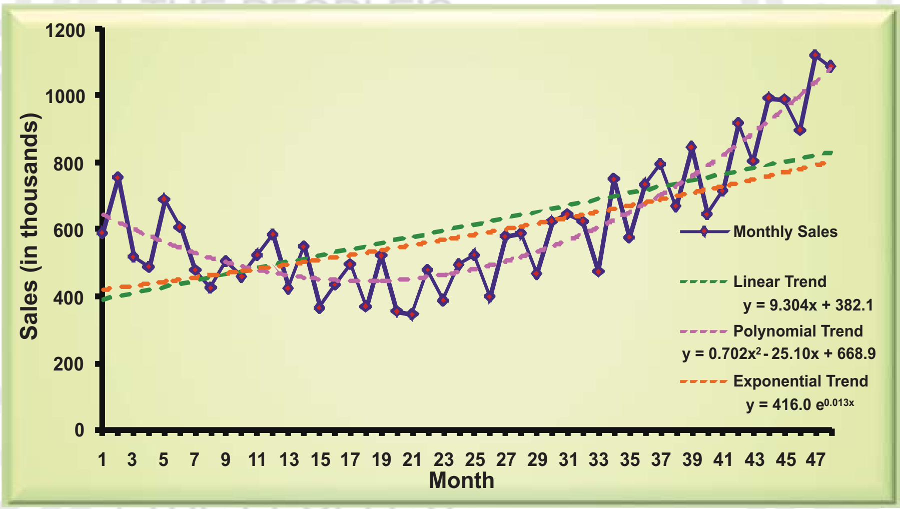


Fig. 15.18

From Fig. 15.18, we observe that the quadratic trend fits well in comparison with the linear and exponential trends. The equation of the quadratic trend is $y_t = 0.702x_t^2 - 25.108x_t + 668.908$.

We now explain how to fit the quadratic trend using the method of least squares in Excel 2007 in the steps given below:

Step 3: As explained in Sec. 15.3, we formulate the normal equations to find the optimum values $\hat{b}_0$, $\hat{b}_1$ and $\hat{b}_2$ using the method of least squares. For this, we need the values of $\sum x_t, \sum y_t, \sum y_t x_t$, $\sum y_t x_t^2$, $\sum x_t^2$, $\sum x_t^3$ and $\sum x_t^4$. The values of x_t and y_t are already given in Columns B and C, respectively (Fig. 15.17). We determine the values of x_t^2, x_t^3, x_t^4, $y_t x_t$ and $y_t x_t^2$ in Columns D, E, F, G and H, respectively. These are shown in Fig. 15.19.

	D	E	F	G	H
1	x_t^2	x_t^3	x_t^4	$y_t x_t$	$y_t x_t^2$
2	1	1	1	589	589
3	4	8	16	1508	3016
4	9	27	81	1551	4653
5	16	64	256	1948	7792
6	25	125	625	3450	17250
7	36	216	1296	3636	21816
8	49	343	2401	3346	23422
9	64	512	4096	3400	27200
10	81	729	6561	4536	40824
11	100	1000	10000	4580	45800
12	121	1331	14641	5753	63283
13	144	1728	20736	7008	84096
14	169	2197	28561	5499	71487
15	196	2744	38416	7672	107408
16	225	3375	50625	5475	82125
17	256	4096	65536	6944	111104
18	289	4913	83521	8415	143055
19	324	5832	104976	6642	119556
20	361	6859	130321	9899	188081

Quadratic

Fig. 15.19

Step 4: We obtain the values of $\sum x_t, \sum y_t, \sum x_t^2, \sum x_t^3, \sum x_t^4$, $\sum y_t x_t$ and $\sum y_t x_t^2$ in Cells B50:H50, respectively, as explained in Steps 10 and 11 of Sec. 15.3 (see Fig. 15.20).

	A	B	C	D	E	F	G	H
50	Sum	1176	29285	38024	1382976	53651864	803199	28390715

Fig. 15.20

Step 5: As explained in Step 12 given in Sec. 15.3, we can find the solutions of $\hat{b}_0, \hat{b}_1$ and $\hat{b}_2$ easily by substituting the values of the summations in equation (12) as

$$\mathbf{B} = \mathbf{A}^{-1}\,\mathbf{C}$$

$$\text{or} \quad \begin{pmatrix} \hat{b}_0 \\ \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} 48 & 1176 & 38024 \\ 1176 & 38024 & 1382976 \\ 38024 & 1382976 & 53651864 \end{pmatrix}^{-1} \begin{pmatrix} 29285 \\ 803199 \\ 28390715 \end{pmatrix}$$

We type the matrix **C** in Cells A52:A54 and matrix **A** in Cells D52:F54 as shown in Fig. 15.21.

	A	B	C	D	E	F
51						
52	29285			48	1176	38024
53	803199			1176	38024	1382976
54	28390715			38024	1382976	53651864
55						

Fig. 15.21

Step 6: We repeat Steps 13 and 14 of Sec. 15.3 under the heading 'Steps in Excel' to compute the optimum values of b_0, b_1 and b_2 shown in Cells B60:B62 of Fig. 15.22.

	A	B	C	D	E	F
56			-1			
57	48	1176	38024	0.20404	-0.01682	0.00029
58	1176	38024	1382976 =	-0.01682	0.00181	-0.00003
59	38024	1382976	53651864	0.00029	-0.00003	0.00000
60						
61	$\hat{b}_0$	668.9079				
62	$\hat{b}_1$ =	-25.1079				
63	$\hat{b}_2$	0.7023				
64						

Fig. 15.22

From Fig. 15.22, $\hat{b}_0 = 668.908$, $\hat{b}_1 = -25.108$ and $\hat{b}_2 = 0.7023$.

Hence, the equation of the quadratic trend fitted to the data given in Table 2 is $y_t = 0.702x_t^2 - 25.108x_t + 668.908$.

It is the same as obtained in Step 2.

Step 7: We now type "=$B$61+$B$62*B2+$B$63*B2*B2" in Cell I2 to get the trend value of the first month. Then we drag down the Cell I2 up to Row 49 to find the trend values for all the given months as shown in Fig. 15.23.

I2 | fx =$B$61+$B$62*B2+$B$63*B2*B2

	F	G	H	I	J
1	x_t^4	$y_t x_t$	$y_t x_t^2$	Trend Values	
2	1	589	589	644.502	
3	16	1508	3016		
4	81	1551	4653		
5	256	1948	7792		

DRAG IT DOWN

	F	G	H	I	J
1	x_t^4	$y_t x_t$	$y_t x_t^2$	Trend Values	
2	1	589	589	644.502	
3	16	1508	3016	621.501	
4	81	1551	4653	599.905	
5	256	1948	7792	579.713	
6	625	3450	17250	560.926	
7	1296	3636	21816	543.543	
8	2401	3346	23422	527.565	
9	4096	3400	27200	512.992	
10	6561	4536	40824	499.823	
11	10000	4580	45800	488.059	
12	14641	5753	63283	477.699	
13	20736	7008	84096	468.744	
14	28561	5499	71487	461.194	
15	38416	7672	107408	455.048	
16	50625	5475	82125	450.307	

Fig. 15.23

Step 8: To forecast the trend value (sales) for May 2014, we note that the month of May 2014 is placed at number 53 in the sequence of months given in Table 2. Now, the trend projection for May 2014 can be computed by typing "=$B$60+$B$61*53+$B$62*53*53" in any cell. Here we use Cell A65 as shown in Fig. 15.24.

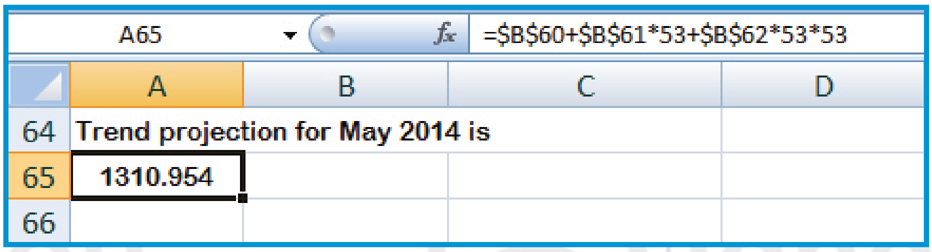


Fig. 15.24

So far, you have learnt the linear and quadratic curve fitting methods for trend. We now explain the exponential trend.

15.5 FITTING OF THE EXPONENTIAL TREND

In many situations, linear or quadratic trends do not give a good fit for the data. In this section, we explain how to use Excel 2007 to fit an exponential trend to the given data. We use the exponential function when the rate of change of a quantity is proportional to the initial amount of the quantity. We explain the procedure for fitting the exponential trend in the steps given below:

Step 1: The exponential model can be represented as

$$y_t = b_0 e^{b_1 x_t} \qquad ...(14)$$

Step 2: We transform the exponential model to a linear trend model by taking natural logarithm of equation (14):

$$\log_e(y_t) = \log_e(b_0) + x_t\, b_1 \qquad ...(15)$$

Step 3: We can represent the transformed model as a linear trend model given by

$$y'_t = b'_0 + b_1 x_t \qquad ...(16)$$

where $y'_t = \log_e(y_t)$ and $b'_0 = \log_e(b_0)$

Step 4: We determine the optimum values of $\hat{b}_0$ and b_1, i.e., $\hat{b}'_0$ and $\hat{b}_1$ as explained in Steps 3 and 4 of Sec. 15.3 using the matrix approach.

Step 5: We compute the value of b_0 as:

$$\hat{b}_0 = e^{\hat{b}'_0} \qquad ...(17)$$

Step 6: The desired exponential trend using the estimated values of $\hat{b}_0$ and $\hat{b}_1$ is given by

$$y_t = \hat{b}_0 e^{\hat{b}_1 x_t} \qquad ...(18)$$

Steps in Excel

Step 1: We enter the data given in Table 3 in Sec. 15.2 of this lab session in Excel Sheet as shown in Fig. 15.25.

	A	B	C
1	Year	Quarter (x_t)	Revenue (y_t)
2	2002	1	25
3		2	180
4		3	864
5		4	683
6	2003	5	244
7		6	400
8		7	240
9		8	29
10	2004	9	73
11		10	135
12		11	884
13		12	384
14	2005	13	827
15		14	833
16		15	818
17		16	301
18	2006	17	136
19		18	1506
20		19	125

Exponential

Fig. 15.25

Step 2: As explained in Steps 2-7 of Sec. 15.3 under the heading 'Steps in Excel', we plot the three trends, i.e., linear, quadratic and exponential along with the data of quarterly revenue shown in Fig. 15.25. The resulting chart is shown in Fig. 15.26.

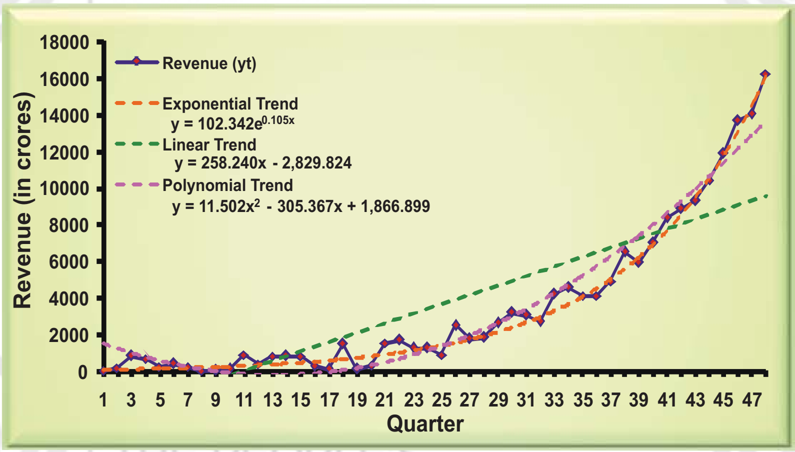


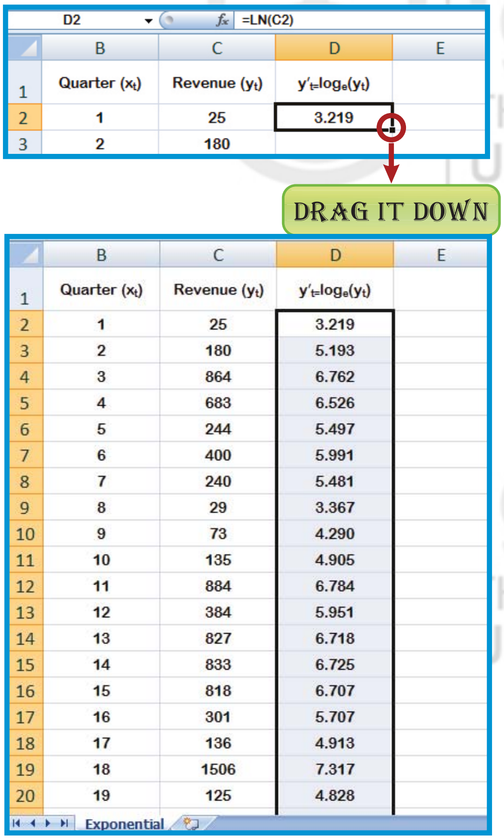
Fig. 15.26

From Fig. 15.26, it is observed that the exponential trend fits well in comparison with the linear and quadratic. The equation of exponential trend is given as

$$y_t = 102.342\, e^{0.105\, x_t}$$

We now explain how to fit exponential trend in Excel 2007 using the method of least squares in the following steps:

Step 3: We transform y_t, i.e., $Y_t = \log_e(y_t)$ by typing "=Ln(C2)" in Cell D2 and drag it down up to Row 49 as shown in Fig. 15.27.



	B	C	D	E
1	Quarter (x_t)	Revenue (y_t)	y'_t=log$_e$(y_t)	
2	1	25	3.219	
3	2	180		

	B	C	D	E
1	Quarter (x_t)	Revenue (y_t)	y'_t=log$_e$(y_t)	
2	1	25	3.219	
3	2	180	5.193	
4	3	864	6.762	
5	4	683	6.526	
6	5	244	5.497	
7	6	400	5.991	
8	7	240	5.481	
9	8	29	3.367	
10	9	73	4.290	
11	10	135	4.905	
12	11	884	6.784	
13	12	384	5.951	
14	13	827	6.718	
15	14	833	6.725	
16	15	818	6.707	
17	16	301	5.707	
18	17	136	4.913	
19	18	1506	7.317	
20	19	125	4.828	

Fig. 15.27

Step 4: We formulate the normal equations to determine the optimum values $\hat{b}'_0$ and $\hat{b}_1$. For this we need the values of $\sum y'_t, \sum x_t$ $\sum x_t y'_t$ and $\sum x_t^2$. The values of y'_t and x_t are already given in Columns D and B, respectively (Fig. 15.27). In Columns E and F, we compute the values of x_t^2 and $y'_t x_t$, respectively, as shown in Fig. 15.28.

	B	C	D	E	F
1	Quarter (x_t)	Revenue (y_t)	y'_t=log$_e$(y_t)	x_t^2	$y'_t x_t$
2	1	25	3.219	1	3.219
3	2	180	5.193	4	10.386
4	3	864	6.762	9	20.285
5	4	683	6.526	16	26.106
6	5	244	5.497	25	27.486
7	6	400	5.991	36	35.949
8	7	240	5.481	49	38.364
9	8	29	3.367	64	26.938
10	9	73	4.290	81	38.614
11	10	135	4.905	100	49.053
12	11	884	6.784	121	74.629
13	12	384	5.951	144	71.408
14	13	827	6.718	169	87.331
15	14	833	6.725	196	94.150
16	15	818	6.707	225	100.603
17	16	301	5.707	256	91.314
18	17	136	4.913	289	83.515
19	18	1506	7.317	324	131.710
20	19	125	4.828	361	91.738

Exponential

Fig. 15.28

Step 5: We compute the values of $\sum x_t$, $\sum y'_t$, $\sum y'_t x_t$ and $\sum x_t^2$ in Cells B50, D50, E50 and F50, respectively, as explained in Steps 10 and 11 of Sec. 15.3 (Fig. 15.29).

	A	B	C	D	E	F
50	Sum	1176	167859	346.145	38024	9451.779
51						

Fig. 15.29

Step 6: We now follow Steps 12-14 given in Sec. 15.3 under the heading 'Steps in Excel' to obtain the solutions of $\hat{b}'_0$ and $\hat{b}_1$.

The computation of $\hat{b}'_0$ and $\hat{b}_1$ is shown in Fig. 15.30 in terms of matrices.

	A	B	C	D
51				
52	346.145		48	1176.000
53	9451.779		1176	38024.000
54				
55		-1		
56	48	1176 =	0.0860	-0.003
57	1176	38024	-0.0027	0.000
58				
59				
60	$\hat{b}'_0$ =	4.628		
61	$\hat{b}_1$	0.105		
62				

Fig. 15.30

Step 7: We calculate the value of $\hat{b}_0$ by typing "=Exp(B60)" in Cell D60 as shown in Fig. 15.31.

D60 f_x =EXP(B60)

	A	B	C	D	E
59					
60	$\hat{b}'_0$ =	4.628	$\hat{b}_0$ =	102.342	
61	$\hat{b}_1$	0.105	$\hat{b}_1$ =	0.105	
62					

Fig. 15.31

From Fig. 15.31, $\hat{b}_0 = 102.342$ and $\hat{b}_1 = 0.105$. Hence, the equation of the exponential trend fitted to the given data is $y_t = 102.342\, e^{0.105\, x_t}$, which is the same as obtained in Step 2.

Step 8: We now type "=$D$60*Exp($D$61*B2)" in Cell G2 to get the trend value of the first month and then drag down the Cell G2 up to Row 49 to find the trend values for all given months as shown in Fig. 15.32.

G2 f_x =$D$60*EXP($D$61*B2)

	E	F	G	H
1	x_t^2	$y'_t x_t$	Trend Values	
2	1	3.219	113.721	
3	4	10.386		

DRAG IT DOWN

	E	F	G	H
1	x_t^2	$y'_t x_t$	Trend Values	
2	1	3.219	113.721	
3	4	10.386	126.365	
4	9	20.285	140.416	
5	16	26.106	156.028	
6	25	27.486	173.377	
7	36	35.949	192.654	
8	49	38.364	214.075	
9	64	26.938	237.878	
10	81	38.614	264.327	
11	100	49.053	293.718	
12	121	74.629	326.376	
13	144	71.408	362.665	
14	169	87.331	402.989	
15	196	94.150	447.797	
16	225	100.603	497.587	
17	256	91.314	552.913	
18	289	83.515	614.390	
19	324	131.710	682.703	
20	361	91.738	758.612	

Exponential

Fig. 15.32

Step 9: To forecast the trend value (revenue) for the last quarter of 2014, we note that the last quarter of 2014 is placed at number 52 in the sequence given in Table 3. Now, the trend projection for the last quarter of 2014 can be computed by typing "=$D$60*Exp($D$61*52)" in any cell. Here we use Cell A64 as shown in Fig. 15.33.

A64 f_x =$D$60*EXP($D$61*52)

	A	B	C	D
63	Trend projection for the last quarter of 2014 is			
64	24604.570			
65				

Fig. 15.33

You can now solve the following exercises to check whether you have learnt how to fit the appropriate trend to the given data and forecast the trend value for a specified period of time.

Activity 2

Fit an appropriate trend with the help of MS Excel 2007 and interpret the results for

A1) Examples 6, 7 and 8 given in Unit 13 of MSTE-002.

A2) Exercise E4 given in Unit 13 of MSTE-002.

Match the results with the calculation done in Unit 13 of MSTE-002.

Continuous Assessment 15

Consider the following situations:

1. Data of annual rainfall for the period of 1956–2011 were collected to study the rainfall patterns in a state. Data of rainfall for 56 years are given in Table 4.

Table 4: Annual rainfall data

Year	Annual Rainfall (in mm)	Year	Annual Rainfall (in mm)	Year	Annual Rainfall (in mm)
1	187.2	20	188.4	39	228.3
2	207.9	21	226.3	40	204.4
3	171.5	22	178.7	41	195.6
4	217.7	23	228.8	42	221.8
5	199.5	24	182.6	43	206.0
6	185.7	25	230.7	44	235.9
7	261.2	26	186.3	45	218.5
8	201.5	27	229.2	46	168.0
9	173.1	28	230.0	47	183.6
10	192.7	29	220.1	48	213.0
11	157.3	30	189.4	49	180.8
12	244.4	31	188.4	50	208.4
13	182.2	32	179.1	51	240.6
14	241.5	33	199.2	52	208.2
15	232.4	34	228.8	53	187.2
16	182.4	35	190.0	54	234.7
17	182.7	36	251.1	55	206.4
18	149.6	37	207.6	56	227.8
19	191.6	38	211.9		

2. Data of the annual yield of a particular crop for the period of 1966-2013 were collected to study the annual cropping pattern of a state and are given in Table 5.

Table 5: Yearly yield data

Year	Yield (in hectares)	Year	Yield (in hectares)	Year	Yield (in hectares)
1966	133	1982	95	1998	341
1967	72	1983	143	1999	237
1968	91	1984	113	2000	290
1969	59	1985	151	2001	323
1970	90	1986	144	2002	397
1971	84	1987	210	2003	334
1972	115	1988	132	2004	422
1973	112	1989	171	2005	322
1974	96	1990	169	2006	357
1975	61	1991	220	2007	459
1976	168	1992	157	2008	402
1977	92	1993	219	2009	497
1978	97	1994	200	2010	494
1979	74	1995	218	2011	448
1980	105	1996	287	2012	560
1981	136	1997	203	2013	543

3. Suppose a manufacturing company produces spare parts for two wheelers. Data for monthly production of the company for 44 months are given in Table 6.

Table 6: Monthly production of spare parts

Month	Production (in thousands)	Month	Production (in thousands)
1	81	23	587
2	133	24	615
3	80	25	876
4	9	26	1074
5	24	27	1025
6	45	28	899
7	294	29	1408
8	128	30	1522
9	275	31	1376
10	277	32	1374
11	272	33	1633
12	100	34	2175
13	45	35	1983
14	502	36	2340
15	41	37	2790
16	102	38	2968
17	510	39	3108
18	569	40	3483
19	423	41	3963
20	431	42	4875
21	298	43	5039
22	842	44	5967

- Represent the data given in Tables 4, 5 and 6 graphically and fit the appropriate trend.
- Fit the appropriate trend using the method of curve fitting by matrix approach.
- Project the annual rainfall in 2014 for the data given in Table 4.
- Project the yield in year 2015 for the data given in Table 5.
- Project the production of spare parts for the 50^{th} month from the data given in Table 6.

Home Work: Do It Yourself

1) Follow the steps explained in Sec. 15.3 to 15.5 to fit an appropriate trend for the data of Tables 1, 2 and 3. Use a different format for plotting the given data along with the fitted trends and take the screenshots.
2) Develop the spreadsheets for the exercises given in "Continuous Assessment 15" as explained in this lab session. Take screenshots of the final spreadsheets and the charts.
3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.