Control Chart for Fraction Defective with Variable Sample Size

SESSION 7

Structure

7.1 INTRODUCTION

Prerequisite

- Lab Sessions 1 and 3 of MSTL-001 (Basic Statistics Lab).
- Lab Session 6 of MSTL-002 (Industrial Statistics Lab).
- Unit 3 of MSTE-001 (Industrial Statistics-I).

In Lab Session 6, you have learnt how to construct the control chart for fraction defective (p-chart) when we have constant sample size, i.e., we have inspected the same number of units in each sample and counted the number of defective items. But sometimes, we cannot take the same number of units in each sample. Generally, such a situation arises when the p-chart is used for 100% inspection of output that varies from day to day. The variation in the output per day can be due to breakdown of machines, different production requirements, etc. When the sample size is not uniform, we use the p-chart for variable sample size using two approaches as discussed in Sec. 3.4 of Unit 3 in MSTE-001 (Industrial Statistics-I). Since the control limits for the p-chart are functions of sample size (n_i), these vary with the sample size. We calculate the control limits separately for each sample. The control limits in such a situation are known as variable control limits.

In this lab session, you will learn how to construct the p-chart for variable sample size in MS Excel 2007. The procedure for drawing the p-chart for variable sample size is similar to the p-chart with constant sample size except for the control limits.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine control limits for the control chart for fraction defective with variable sample size;
- construct the control chart for fraction defective with variable sample size; and
- interpret the control chart for fraction defective.

7.2 PROBLEM DESCRIPTION

To monitor the manufacturing of laptops, a quality controller randomly selected laptops from the production line each day over a period of 25 days. The laptops were inspected for defectives and the number of defective laptops found each day is recorded in Table 1.

Table 1: Number of defective laptops

Day	Number of Laptops Inspected	Number of Defective Laptops	Day	Number of Laptops Inspected	Number of Defective Laptops
1	50	2	14	55	5
2	52	4	15	52	1
3	57	4	16	48	4
4	50	11	17	50	3
5	50	4	18	56	6
6	48	2	19	52	2
7	51	4	20	53	4
8	54	6	21	50	3
9	52	5	22	55	1
10	50	1	23	50	5
11	55	6	24	50	3
12	60	3	25	47	4
13	55	6			

The quality control inspector of this company needs to construct the control chart with the help of both approaches to check whether the process is under statistical control or not. He/she also computes the revised control limits, if necessary.

Therefore, the problem for this session is to construct the control chart for fraction defective, i.e., the p-chart with variable sample size for the data given in Table 1.

7.3 PROCEDURE FOR THE CONSTRUCTION OF p-CHART

You have already learnt all formulae for calculating control limits for the p-chart and the method to draw the p-chart manually in Unit 3 of MSTE-001. The steps involved in the computation of p-chart for variable sample size are given below:

First Approach

According to the first approach, we calculate the control limits for each sample as follows:

Step 1: If $d_1, d_2, ..., d_k$ represent the number of defective, items in 1st, 2nd, …, kth samples of size $n_1, n_2, ..., n_k$, respectively, the unknown value of fractions defective (P) can be estimated by

$$\bar{p} = \frac{d_1 + d_2 + ... + d_k}{n_1 + n_2 + ... + n_k} = \frac{\sum_{i=1}^{k} d_i}{\sum_{i=1}^{k} n_i} \quad ...(1)$$

Step 2: The centre line remains constant for each sample:

$$\text{Centre line }(CL) = \bar{p} \qquad \ldots(2)$$

Step 3: The upper and lower control limits change due to variable sample size. The control limits for i^{th} sample are given by

✓ $$\text{Upper control limit }(UCL_i) = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}} \qquad \ldots(3)$$

✓ $$\text{Lower control limit }(LCL_i) = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}} \qquad \ldots(4)$$

Step 4: Interpretation of the p-chart.

Second Approach

According to the second approach, the control limits are calculated using the average sample size. This approach is used only when there is no large variation in the sample sizes and future sample sizes are not expected to differ significantly from the average sample size. We get constant control limits using this approach as we get with the constant sample size discussed in Lab Session 6.

Step 1: We calculate the average sample size as follows:

$$\bar{n} = \frac{n_1 + n_2 + \ldots + n_k}{k} = \frac{1}{k}\sum_{i=1}^{k} n_i \qquad \ldots(5)$$

Step 2: The unkown value of fraction defective (P) can be estimated by

$$\bar{p} = \frac{\sum_{j=1}^{k} d_i}{k\bar{n}} = \frac{\sum_{i=1}^{k} d_i}{\sum_{i=1}^{k} n_i} \qquad \ldots(6)$$

Step 3: The centre line, upper and lower control limits are given by

✓ $$\text{Centre line }(CL) = \bar{p} \qquad \ldots(7)$$

✓ $$\text{Upper control limit }(UCL) = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}} \qquad \ldots(8)$$

✓ $$\text{Lower control limit }(LCL) = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}} \qquad \ldots(9)$$

Step 4: Interpretation of the p-chart.

7.4 STEPS INVOLVED IN THE CONSTRUCTION OF p-CHART IN EXCEL 2007

We now describe the procedure for the construction of p-chart for the problem given in Sec. 7.2. For plotting a p-chart in Excel 2007, we follow the steps given ahead.

Step 1: We enter the given data in MS Excel spreadsheet as shown in Fig. 7.1.

	A	B	C
1			
2	Sample No.	Laptops Inspected	Defective Laptops
3	1	50	2
4	2	52	4
5	3	57	4
6	4	50	11

p Chart

Fig. 7.1: Partial screenshot of the spreadsheet for given data.

Step 2: Following Step 2 of Lab Session 6, we compute the fraction defective for each sample as shown in Fig. 7.2.

D3 =C3/B3

	A	B	C	D
1				
2	Sample No.	Laptops Inspected	Defective Laptops	Fraction Defective (p)
3	1	50	2	0.040
4	2	52	4	0.077
5	3	57	4	0.070
6	4	50	11	0.220
7	5	50	4	0.080
8	6	48	2	0.042

Fig. 7.2

Step 3: We determine the total number of laptops inspected and total number of defective laptops in Cells B28 and C28, respectively, as shown in Fig. 7.3.

	A	B	C
28	Total	1302	99

Used **"=Sum(B3:B27)"** function   Used **"=Sum(C3:C27)"** function

Fig. 7.3

Step 4: We now compute the average fraction defective, i.e., $\bar{p}$ by typing "=C28/B28" in Cell B29 and clicking on ***Enter*** as shown in Fig. 7.4.

ROUND =C28/B28

	A	B	C
28	Total	1302	99
29	$\bar{p}$	=C28/B28	

	A	B	C
28	Total	1302	99
29	$\bar{p}$	0.076	
30			

Fig. 7.4

The formula with dollar sign ($) is used for an absolute reference.

Step 5: We now determine the centre line, upper and lower control limits in Columns E, F and G, respectively, by typing

i) "=$B$29" in Cell E3 to find centre line (Fig. 7.5a).

ii) "=E3+3*Sqrt(($B$29*(1-$B$29))/(B3))" in Cell F3 (Fig. 7.5b).

iii) "=E3-3*Sqrt(($B$29*(1-$B$29))/(B3))" in Cell G3 (Fig. 7.5c).

(a)

E3 =$B$29

	E	F	G	H	I	J
1		Control Limits				
2	Centre Line	UCL	LCL			
3	0.076					
4						

(b)

F3 =E3+3*SQRT(($B$29*(1-$B$29))/(B3))

	F	G	H	I	J	K
1	ontrol Limits					
2	UCL	LCL				
3	0.188					
4						

(c)

G3 =E3-3*SQRT(($B$29*(1-$B$29))/(B3))

	G	H	I	J	K	L
1						
2	LCL					
3	-0.036					
4						

Fig. 7.5

Step 6: We select Cells E3:G3 and drag them down up to Row 27 to get the centre line, upper and lower control limits corresponding to each sample as shown in Fig. 7.6. Here we get different values for UCL and LCL because we have different sample sizes.

	E	F	G
1		Control Limits	
2	Centre Line	UCL	LCL
3	0.076	0.188	-0.036
4	0.076	0.186	-0.034
5	0.076	0.181	-0.029
6	0.076	0.188	-0.036
7	0.076	0.188	-0.036
8	0.076	0.191	-0.039
9	0.076	0.187	-0.035
10	0.076	0.184	-0.032
11	0.076	0.186	-0.034
12	0.076	0.188	-0.036
13	0.076	0.183	-0.031
14	0.076	0.179	-0.027
15	0.076	0.183	-0.031
16	0.076	0.183	-0.031
17	0.076	0.186	-0.034
18	0.076	0.191	-0.039
19	0.076	0.188	-0.036
20	0.076	0.182	-0.030
21	0.076	0.186	-0.034
22	0.076	0.185	-0.033

p Chart

Fig. 7.6

Step 7: We notice from Fig. 7.6 that all values of LCL are negative, which is not possible and not acceptable. Hence, we set these values as 0 (zero) and denote the limit by LCL* as shown in Fig. 7.7.

If the value of lower control limits are negative, we set the lower control limit as zero because a negative fraction defective is not possible.

	E	F	G	H
1	Control Limits			
2	Centre Line	UCL	LCL	LCL*
3	0.076	0.188	-0.036	0
4	0.076	0.186	-0.034	0
5	0.076	0.181	-0.029	0
6	0.076	0.188	-0.036	0
7	0.076	0.188	-0.036	0
8	0.076	0.191	-0.039	0
9	0.076	0.187	-0.035	0
10	0.076	0.184	-0.032	0
11	0.076	0.186	-0.034	0
12	0.076	0.188	-0.036	0
13	0.076	0.183	-0.031	0
14	0.076	0.179	-0.027	0
15	0.076	0.183	-0.031	0
16	0.076	0.183	-0.031	0
17	0.076	0.186	-0.034	0
18	0.076	0.191	-0.039	0
19	0.076	0.188	-0.036	0
20	0.076	0.182	-0.030	0
21	0.076	0.186	-0.034	0
22	0.076	0.185	-0.033	0

p Chart

Fig. 7.7

Step 8: To obtain the p-chart in Excel 2007, we follow the procedure explained in Step 10 of Sec.1.4, Lab Session 1. It means that we

1. select Cells D2:F27 and H2:H27 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1. Thus, we obtain the p-chart shown in Fig. 7.8.

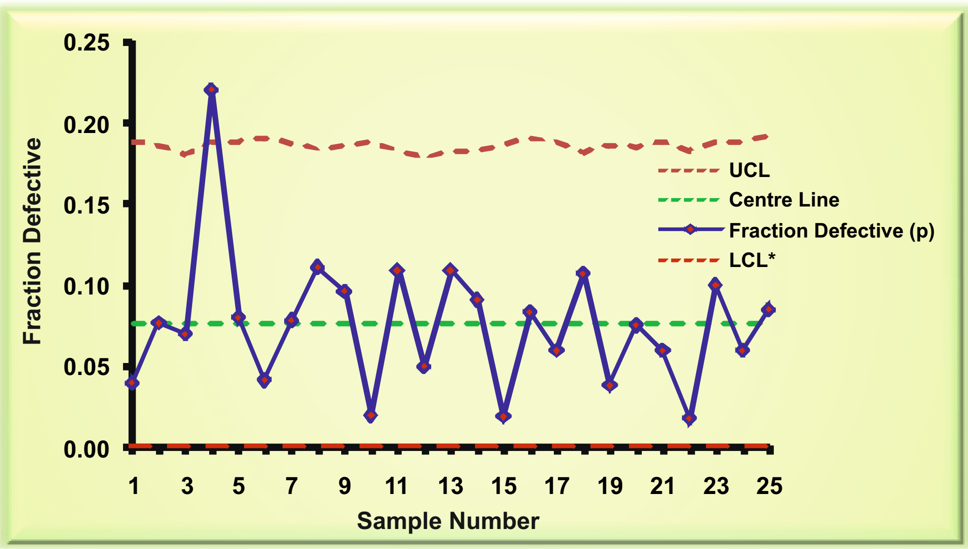

Fig. 7.8

Interpretation

The control chart for defective laptops (shown in Fig. 7.8) indicates that the process is not under statistical control. The point corresponding to Sample 4 is outside the upper control limit. Some assignable causes are present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them. Once this is done, we remove the out-of-control Sample 4 and calculate the revised centre line and control limits for the p-chart using the remaining samples. These limits are known as revised control limits and will be discussed in the next section.

We hope that you have grasped the method for plotting the p-chart for variable sample size. Before going to the next section, you should perform Activity 1.

Activity 1

- ✓ Compute the centre line and control limits in Excel 2007 using the second approach which you have learnt in Sec. 3.4 of MSTE-001.
- ✓ Plot the p-chart and compare the result with the output obtained above. If any point goes out of control, compute the revised control limits in the same manner as discussed in Sec. 7.5.

7.5 REVISED p-CHART

From the p-chart shown in Fig. 7.8, we observe that the point corresponding to Sample 4 lies outside the upper control limit. So we plot the p-chart for the revised control limits until all points come within UCL and LCL as discussed in the Lab Sessions 2 to 5. The main steps for computing the revised control limits are as follows:

<u>First Approach</u>

Step 1: To calculate the revised limits of p-chart, we first calculate new $\bar{p}$ as follows:

$$\bar{p}_{new} = \frac{\sum_{i=1}^{k} d_i - \sum_{j=1}^{d} d_j}{\sum_{i=1}^{k} n_i - \sum_{j=1}^{d} n_j} \qquad \ldots(10)$$

where d – number of discarded samples, and

$\sum_{j=1}^{d} d_j$ – sum of number of defectives in the discarded samples.

Step 2: After finding $\bar{p}_{new}$, we reconstruct the centre line and control limits of the chart by replacing $\bar{p}$ by $\bar{p}_{new}$ as follows:

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✓ Centre line = $\overline{p}_{new}$ ...(11)

✓ Upper control limit (UCL_i) $= \overline{p}_{new} + 3\sqrt{\dfrac{\overline{p}_{new}(1-\overline{p}_{new})}{n_i}}$ ...(12)

✓ Lower control limit (LCL_i) $= \overline{p}_{new} - 3\sqrt{\dfrac{\overline{p}_{new}(1-\overline{p}_{new})}{n_i}}$ ...(13)

Step 3: Interpretation of the p-chart.

Second Approach

Step 1: To calculate the revised limits of p-chart, the value of new $\overline{p}$ will be the same as equation (10).

Step 2: After finding $\overline{p}_{new}$, we reconstruct the centre line and control limits of the chart by replacing $\overline{p}$ by $\overline{p}_{new}$ as follows:

✓ Centre line = $\overline{p}_{new}$ ...(14)

✓ Upper control limit (UCL) $= \overline{p}_{new} + 3\sqrt{\dfrac{\overline{p}_{new}(1-\overline{p}_{new})}{\overline{n}}}$ ...(15)

✓ Lower control limit (LCL) $= \overline{p}_{new} - 3\sqrt{\dfrac{\overline{p}_{new}(1-\overline{p}_{new})}{\overline{n}}}$ ...(16)

Step 3: Interpretation of the p-chart.

Steps in Excel

The revised centre line and control limits for the p-chart using the remaining samples are described below:

Step 1: We highlight the samples outside the control limits, i.e., the 4^{th} sample, with light orange colour (Fig. 7.9).

	A	B	C	D	E	F	G	H
1						Control Limits		
2	Sample No.	Laptops Inspected	Defective Laptops	Fraction Defective (p)	Centre Line	UCL	LCL	LCL*
3	1	50	2	0.040	0.076	0.188	-0.036	0
4	2	52	4	0.077	0.076	0.186	-0.034	0
5	3	57	4	0.070	0.076	0.181	-0.029	0
6	4	50	11	0.220	0.076	0.188	-0.036	0
7	5	50	4	0.080	0.076	0.188	-0.036	0
8	6	48	2	0.042	0.076	0.191	-0.039	0
9	7	51	4	0.078	0.076	0.187	-0.035	0
10	8	54	6	0.111	0.076	0.184	-0.032	0
11	9	52	5	0.096	0.076	0.186	-0.034	0
12	10	50	1	0.020	0.076	0.188	-0.036	0

Fig. 7.9

Step 2: We calculate $\overline{p}_{new}$ by typing "=(C28-C6)/(B28-B6)" in Cell B30 as shown in Fig. 7.10.

B30 f_x =(C28-C6)/(B28-B6)

	A	B	C
28	Total	1302	99
29	$\overline{p}$	0.076	
30	$\overline{p}_{new}$	0.070	
31			

Fig. 7.10

Step 3: We determine the centre line and control limits by replacing $\overline{p}$ by $\overline{p}_{new}$ as shown in Fig. 7.11 and also replace the negative LCL by zero (0) and denote the limit by LCL* as discussed in Step 5 of Sec. 7.4.

	I	J	K	L
1	Revised Control Limits			
2	Centre Line	UCL	LCL	LCL*
3	0.070	0.179	-0.038	0
4	0.070	0.177	-0.036	0
5	0.070	0.172	-0.031	0
6	0.070	0.179	-0.038	0
7	0.070	0.179	-0.038	0
8	0.070	0.181	-0.040	0
9	0.070	0.178	-0.037	0
10	0.070	0.175	-0.034	0
11	0.070	0.177	-0.036	0
12	0.070	0.179	-0.038	0

p Chart

Fig. 7.11

Step 4: We now plot fraction defectives, revised centre line, UCL and LCL* as discussed in Step 8 of Sec. 7.4. We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting chart is shown in Fig. 7.12.

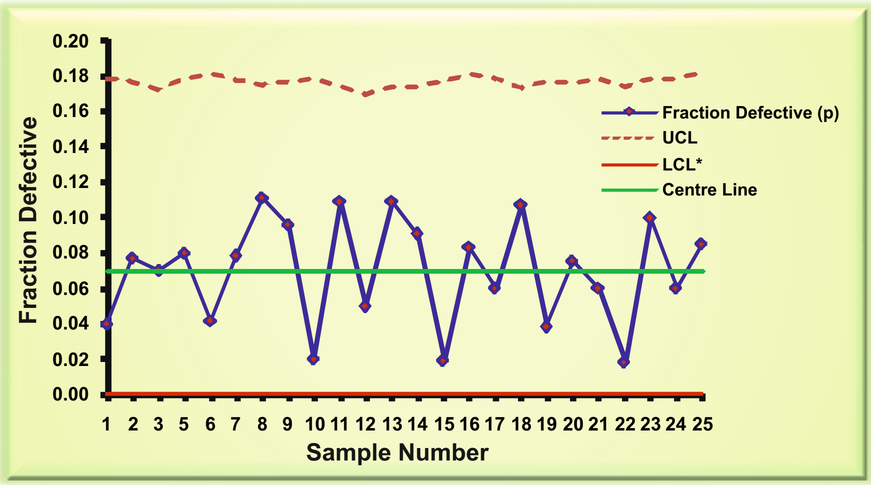

Fig. 7.12

Interpretation

The p-chart given in Fig. 7.12 reveals that no point lies outside the control limits. So we may conclude that the process is under statistical control with respect to fraction defectives.

You should now apply this method on other problems for practice.

Activity 2

Construct the control charts for fraction defective with the help of MS Excel 2007 and interpret the results for

A1) Example 4 given in Unit 3 of MSTE-001.

A2) Exercise E6 given in Unit 3 of MSTE-001.

Match the results with the manual calculation done in Unit 3 of MSTE-001.

Continuous Assessment 7

In the production of tyres, the output of given size was inspected every day prior to the tyres being given to finished goods stores. The number of defective tyres found everyday is summarised in the following table:

Table 2: Number of defective tyres

Sample No.	Number of Tyres Inspected	Number of Defective Tyres	Sample No.	Number of Tyres Inspected	Number of Defective Tyres
1	650	70	14	550	78
2	510	74	15	540	64
3	600	58	16	610	90
4	590	61	17	670	96
5	630	65	18	660	99
6	650	108	19	650	78
7	700	82	20	590	60
8	740	51	21	650	56
9	580	80	22	640	55
10	600	90	23	580	57
11	670	71	24	530	58
12	660	75	25	520	44
13	600	77			

Draw the p-chart using both approaches and comment about the state of the process. Also plot the revised control chart, if necessary.

Home Work: Do It Yourself

1) Follow the steps explained in Secs. 7.4 and 7.5 to construct the control chart for the data of Table 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 7" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.

3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.