Control Chart for Number of Defects

SESSION 9

Structure

9.1 INTRODUCTION

Prerequisite

- Lab Sessions 1 and 3 of MSTL-001 (Basic Statistics Lab).
- Lab Session 6 of MSTL-002 (Industrial Statistics Lab).
- Unit 4 of MSTE-001 (Industrial Statistics-I).

You have learnt in Lab Sessions 6 and 8 that the p-chart and np-chart are used to control the fraction of defectives and number of defectives, respectively, by considering only one criterion, i.e., whether the unit or item is defective or non-defective. But in many situations, it is economically advantageous to know the number of defects within the unit rather than knowing if a unit is defective or non-defective. For example, suppose a glass bottle has a small air bubble and another one has many small air bubbles in it. Even though both bottles are defective, the second bottle is of inferior quality as compared to the first because it has many more defects. Therefore, it is important to control the number of defects within the unit. In such situations, we use the c-chart and u-chart to monitor and control the number of defects.

You have studied in Sec. 4.2 of Unit 4 in MSTE-001 (Industrial Statistics-I) that we use the c-chart when the **sample size is 1 or constant**, i.e., we have inspected the same number of units in each sample, e.g., one TV, one aeroplane, twenty laptops, ten bottles, one bolt of cloth, and so on. The u-chart is used to **control the number of defects per unit**.

In this lab session, you will learn how to construct the c-chart to monitor and **control the number of defects within the unit** with the help of MS Excel 2007. In the next lab session, we shall discuss the application of u-chart when the sample size varies.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;

- determine the control limits for c-chart;
- construct the c-chart; and
- interpret the c-chart.

9.2 PROBLEM DESCRIPTION

Let us look at the problem faced by a textile company observing some defects in its woven fabric. As a part of overall quality improvement programme, the company decided to monitor the number of defects found in the woven fabric each day. The fabric produced each day is one metre wide and 60 metres in length. For this purpose, the total number of defects in the woven fabric each day were counted for 35 days. These are recorded in Table 1.

Table 1: Number of defects in fabric

Day	Number of Defects	Day	Number of Defects
1	7	19	8
2	3	20	4
3	1	21	2
4	3	22	5
5	6	23	12
6	2	24	1
7	4	25	5
8	5	26	3
9	7	27	6
10	3	28	5
11	12	29	7
12	2	30	2
13	4	31	4
14	8	32	5
15	3	33	4
16	7	34	3
17	6	35	6
18	3		

The quality control inspector of the company needs to construct a control chart to check whether the process is under statistical control or not and also computes the revised control limits, if necessary.

Therefore, the problem for this session is to construct the control chart for the number of defects, i.e., the c-chart for the data given in Table 1.

9.3 PROCEDURE FOR THE CONSTRUCTION OF C-CHART

You have already studied about the construction of c-chart in Unit 4 of MSTE-001 in depth. Here we briefly discuss how to compute control limits.

Generally, the average number of defects within a unit is not known. So we estimate it from the sample information as follows:

Step 1: We draw k samples of constant size and $c_1, c_2, ..., c_k$ are the numbers of defects the in the 1st, 2nd, …, kth sample, respectively. We estimate the average number of defects within a unit by the average number of defects in the sample which is calculated by

$$\bar{c} = \frac{\text{Total number of defects}}{\text{Total number of inspected samples}} = \frac{1}{k}\sum_{i=1}^{k} c_i \qquad ...(1)$$

Step 2: The control limits of c-chart are given by

- ✓ Centre Line $(CL) = \bar{c}$ …(2)
- ✓ Upper control limit $(UCL) = \bar{c} + 3\sqrt{\bar{c}}$ …(3)
- ✓ Lower control limit $(LCL) = \bar{c} - 3\sqrt{\bar{c}}$ …(4)

Step 3: We construct the c-chart by taking sample number on the X-axis and number of defects in the sample (c) on the Y-axis.

Step 4: Interpretation of the c-chart.

9.4 STEPS INVOLVED IN THE CONSTRUCTION OF c-CHART IN EXCEL 2007

We now describe the construction of c-chart in Excel 2007 with the help of the data given in Sec. 9.2. We follow the steps given below:

Step 1: We enter the given data in MS Excel spreadsheet as shown in Fig. 9.1.

	A	B
1		
2	Days	Number of Defects
3	1	7
4	2	3
5	3	1
6	4	3
7	5	6
8	6	2
9	7	4
10	8	5
11	9	7
12	10	3
13	11	12
14	12	2
15	13	4
16	14	8
17	15	3
18	16	7
19	17	6
20	18	3
21	19	8

c Chart

Fig. 9.1: Partial screenshot of the spreadsheet for the given data.

Step 2: We determine the total number of defects in the fabric by typing "=Sum(B3:B37)" in Cells B38 as represented in Fig. 9.2.

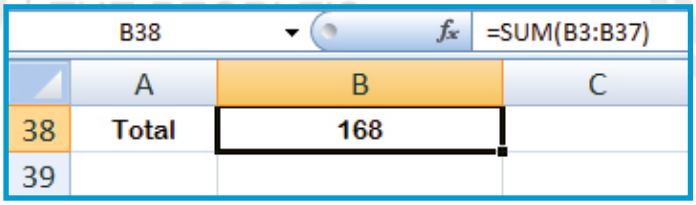


Fig. 9.2

Step 3: We type the values of n and k in Cells B41 and B42, respectively, as shown in Fig. 9.3.

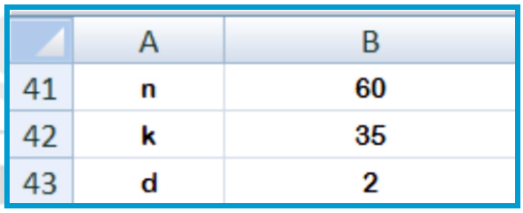


Fig. 9.3

Step 4: We use equation (1) to compute the average defects in fabric, i.e., $\bar{c}$ by typing "=B38/B42" in Cell B39 and then press ***Enter*** (Fig. 9.4).

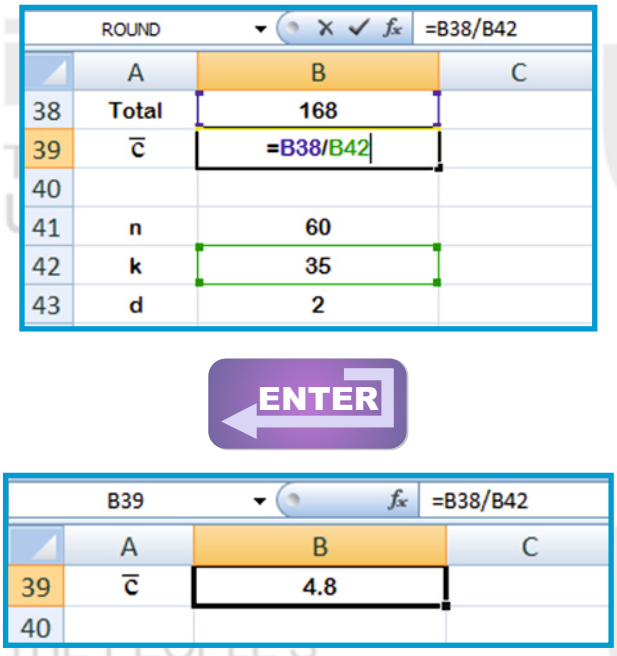


Fig. 9.4

Step 5: We compute the values of the centre line, upper and lower control limits using equations (2) to (4) in Columns C, D and E, respectively, by typing

i) "=$B$39" in Cell C3 to find the centre line as shown in Fig. 9.5a.

ii) "=C3+3*Sqrt(C3)" in Cell D3 as shown in Fig. 9.5b.

iii) "=C3-3*Sqrt(C3)" in Cell E3, as shown in Fig. 9.5c.

(a)

C3 | =$B$39

	C	D	E	F
1		Control Limits		
2	Centre Line	UCL	LCL	
3	4.8			
4				

(b)

D3 | =C3+3*SQRT(C3)

	D	E	F	G
1	Control Limits			
2	UCL	LCL		
3	11.373			
4				

(c)

E3 | =C3-3*SQRT(C3)

	E	F	G	H
1				
2	LCL			
3	-1.773			
4				

Fig. 9.5

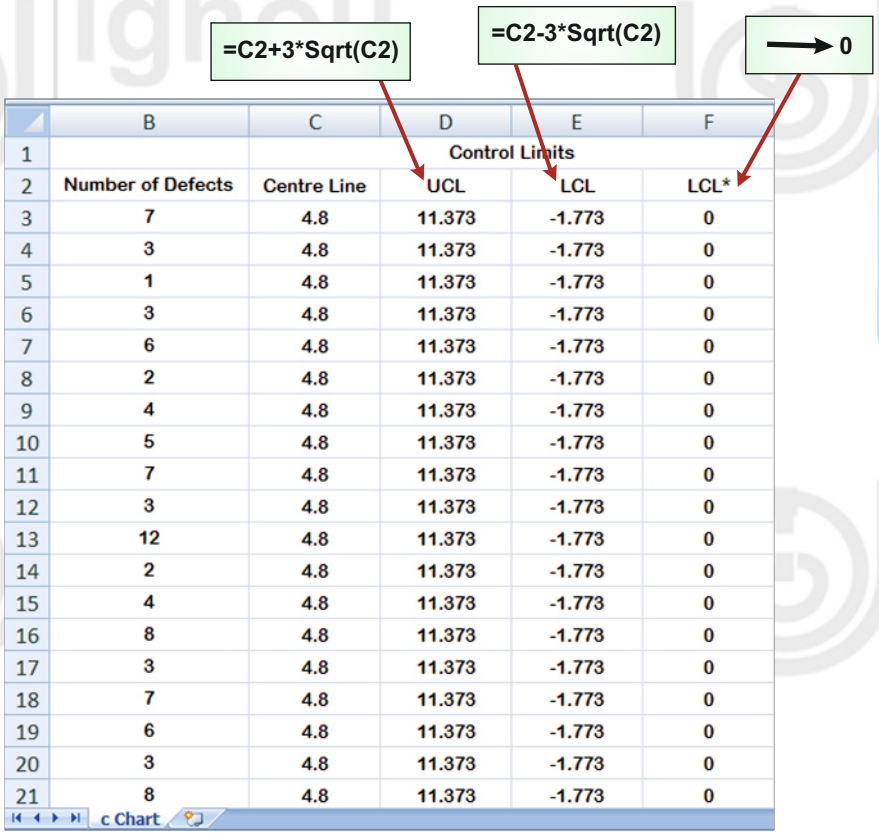
Step 6: We select Cells C3:E3 and drag them down up to Row 37 to get the control limits corresponding to each sample as shown in Fig. 9.6.

	B	C	D	E
1		Control Limits		
2	Number of Defects	Centre Line	UCL	LCL
3	7	4.8	11.373	-1.773
4	3	4.8	11.373	-1.773
5	1	4.8	11.373	-1.773
6	3	4.8	11.373	-1.773
7	6	4.8	11.373	-1.773
8	2	4.8	11.373	-1.773
9	4	4.8	11.373	-1.773
10	5	4.8	11.373	-1.773
11	7	4.8	11.373	-1.773
12	3	4.8	11.373	-1.773
13	12	4.8	11.373	-1.773
14	2	4.8	11.373	-1.773
15	4	4.8	11.373	-1.773
16	8	4.8	11.373	-1.773
17	3	4.8	11.373	-1.773
18	7	4.8	11.373	-1.773
19	6	4.8	11.373	-1.773
20	3	4.8	11.373	-1.773
21	8	4.8	11.373	-1.773

c Chart

Fig. 9.6

Step 7: Look at Fig. 9.6. The value of LCL is negative, which is not possible and, therefore, not acceptable. So we consider the value 0 (zero) instead of the negative value of LCL to plot the control chart as shown in Fig. 9.7.



	B	C	D	E	F
1			Control Limits		
2	Number of Defects	Centre Line	UCL	LCL	LCL*
3	7	4.8	11.373	-1.773	0
4	3	4.8	11.373	-1.773	0
5	1	4.8	11.373	-1.773	0
6	3	4.8	11.373	-1.773	0
7	6	4.8	11.373	-1.773	0
8	2	4.8	11.373	-1.773	0
9	4	4.8	11.373	-1.773	0
10	5	4.8	11.373	-1.773	0
11	7	4.8	11.373	-1.773	0
12	3	4.8	11.373	-1.773	0
13	12	4.8	11.373	-1.773	0
14	2	4.8	11.373	-1.773	0
15	4	4.8	11.373	-1.773	0
16	8	4.8	11.373	-1.773	0
17	3	4.8	11.373	-1.773	0
18	7	4.8	11.373	-1.773	0
19	6	4.8	11.373	-1.773	0
20	3	4.8	11.373	-1.773	0
21	8	4.8	11.373	-1.773	0

c Chart

If the value of lower control limit is negative, we change the lower control limit to zero because a negative number of defects is not possible.

Fig. 9.7

Step 8: To plot the c-chart in Excel 2007, we follow the procedure explained in Step 10 of Sec. 1.4, Lab Session 1. It means that we

1. select Cells B3:D37 and F3:F37 by holding *Ctrl* key,
2. click on the *Insert* tab,
3. select the *Line* option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting c-chart is shown in Fig. 9.8.

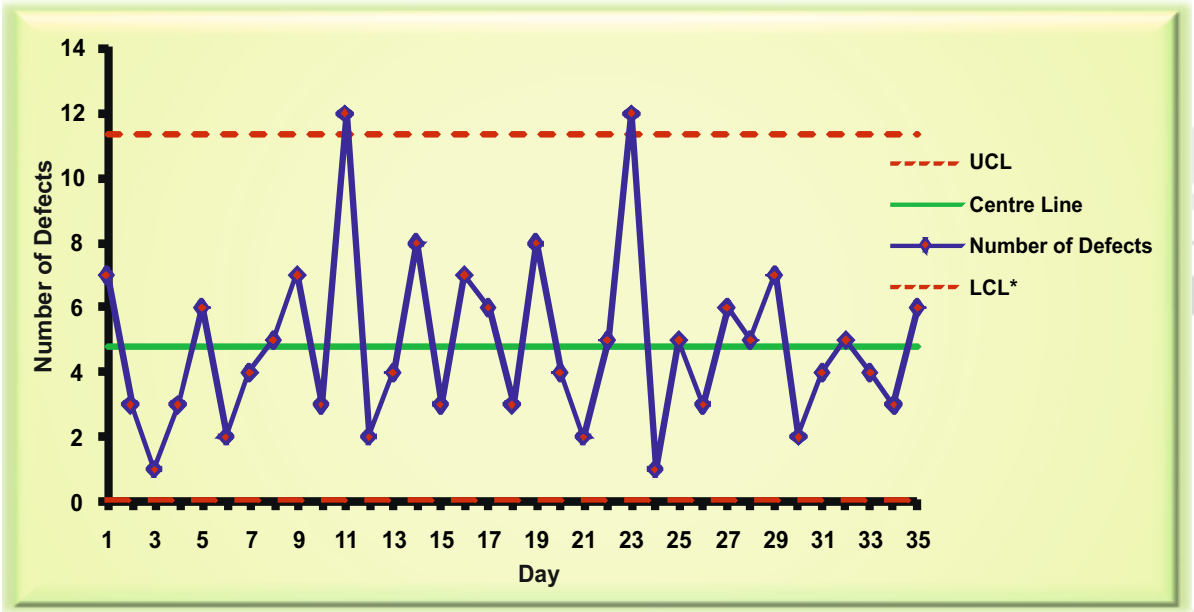


Fig. 9.8

Interpretation

The control chart for the number of defects in a fabric (shown in Fig. 9.8) indicates that the process is not under statistical control and some assignable causes are present in the process. The samples corresponding to the 11th and 23rd days are outside the upper control limit. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them. Then we eliminate out-of-control Samples 11 and 23 and calculate the revised centre line, upper and lower control limits for the c-chart using the remaining samples. These limits are known as revised control limits and are discussed in Sec. 9.5.

9.5 REVISED c-CHART

We plot the c-chart for the revised control limits using the remaining samples in the same way as explained in Lab Session 7. We repeat the procedure until all points come within UCL and LCL. The main steps for computing the revised control limits are as follows:

Step 1: For revised control limits for c-chart, we first calculate new $\bar{c}$ as follows:

$$\bar{c}_{new} = \frac{\sum_{i=1}^{k} c_i - \sum_{j=1}^{d} c_j}{k - d} \quad \ldots(5)$$

where d – number of discarded samples, and

$\sum_{j=1}^{d} c_j$ – Total number of defects within the discarded samples.

Step 2: After finding the new $\bar{c}$, we reconstruct the centre line and control limits for the c-chart by replacing $\bar{c}$ by $\bar{c}_{new}$ as given below:

- ✓ Centre line (CL) $= \bar{c}_{new}$ …(6)
- ✓ Upper control limit (UCL) $= \bar{c}_{new} + 3\sqrt{\bar{c}_{new}}$ …(7)
- ✓ Lower control limit (LCL) $= \bar{c}_{new} - 3\sqrt{\bar{c}_{new}}$ …(8)

Step 3: Interpretation of the c-chart.

Steps in Excel

The revised centre line and control limits for constructing the c-chart in Excel 2007 using the remaining samples are described below:

Step 1: From the c-chart given in Fig. 9.8, we observe that the samples corresponding to the 11th and 23rd days lie outside the upper control limit. We highlight these samples with light orange colour as shown in Fig. 9.9.

	A	B	C	D	E	F
1			Control Limits			
2	Days	Number of Defects	Centre Line	UCL	LCL	LCL*
3	1	7	4.8	11.373	-1.773	0
4	2	3	4.8	11.373	-1.773	0
5	3	1	4.8	11.373	-1.773	0
6	4	3	4.8	11.373	-1.773	0
7	5	6	4.8	11.373	-1.773	0
8	6	2	4.8	11.373	-1.773	0
9	7	4	4.8	11.373	-1.773	0
10	8	5	4.8	11.373	-1.773	0
11	9	7	4.8	11.373	-1.773	0
12	10	3	4.8	11.373	-1.773	0
13	11	12	4.8	11.373	-1.773	0
14	12	2	4.8	11.373	-1.773	0

	A	B	C	D	E	F
24	22	5	4.8	11.373	-1.773	0
25	23	12	4.8	11.373	-1.773	0
26	24	1	4.8	11.373	-1.773	0
27	25	5	4.8	11.373	-1.773	0

Fig. 9.9

Step 2: For revised limits of the c-chart, we calculate new $\bar{c}$ by typing "=(B38-B13-B25)/(B42-B43)" in Cell B40. We put the value d = 2 in Cell B43 as shown in Fig. 9.10.

B40 =(B38-B13-B25)/(B42-B43)

Name Box

	A	B	C	D
38	Total	168		
39	$\bar{c}$	4.8		
40	$\bar{c}_{new}$	4.364		
41	n	60		
42	k	35		
43	d	2		

Fig. 9.10

Step 3: After finding the new $\bar{c}$, we reconstruct the centre line and control limits by replacing $\bar{c}$ by $\bar{c}_{new}$ and also replace the negative LCL by zero (0) and denote the limit by LCL* as shown in Fig. 9.11.

	G	H	I	J
1	Revised Control Limits			
2	Centre Line	UCL	LCL	LCL*
3	4.4	10.630	-1.903	0
4	4.4	10.630	-1.903	0
5	4.4	10.630	-1.903	0
6	4.4	10.630	-1.903	0
7	4.4	10.630	-1.903	0
8	4.4	10.630	-1.903	0
9	4.4	10.630	-1.903	0
10	4.4	10.630	-1.903	0
11	4.4	10.630	-1.903	0
12	4.4	10.630	-1.903	0
13	4.4	10.630	-1.903	0
14	4.4	10.630	-1.903	0
15	4.4	10.630	-1.903	0
16	4.4	10.630	-1.903	0
17	4.4	10.630	-1.903	0

Fig. 9.11

Step 4: Finally, we plot the number of defects, centre line, UCL and LCL* as discussed in Step 8 of Sec. 9.4. We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting chart is shown in Fig. 9.12.

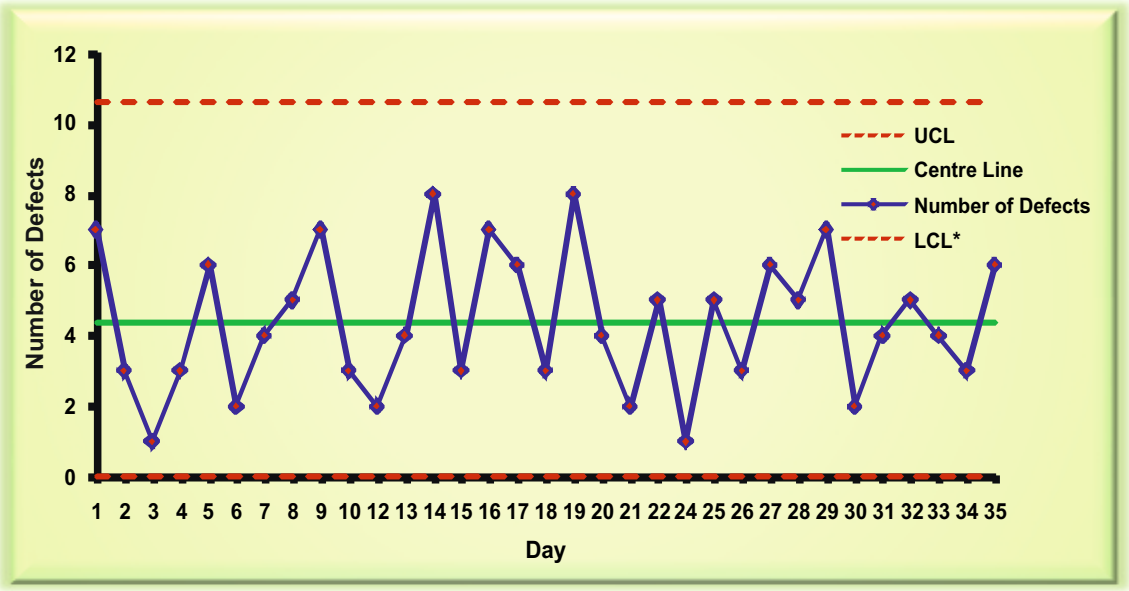


Fig. 9.12

Interpretation

From the revised c-chart given in Fig. 9.12, we observe that no point lies outside the control limits. This indicates that the process is under statistical control with respect to the number of defects.

The following activity will help you understand the c-chart more clearly.

Activity

Construct the control charts for the number of defects with the help of MS Excel 2007 and interpret the results for

A1) Examples 1, 2 and 3 given in Unit 4 of MSTE-001.

A2) Exercises E3 and E4 given in Unit 4 of MSTE-001.

Match the results with the manual calculation done in Unit 4 of MSTE-001.

Continuous Assessment 9

Suppose an almirah manufacturing company wants to judge the quality of the almirahs produced. The number of scratch marks on each almirah is recorded for this purpose. The data for 30 samples are given in Table 2.

Table 2: Number of scratch marks

Sample No.	1	2	3	4	5	6	7	8	9	10
Scratch Marks	7	3	1	3	6	2	4	5	7	3
Sample No.	11	12	13	14	15	16	17	18	19	20
Scratch Marks	6	3	14	7	2	5	9	4	7	3
Sample No.	21	22	23	24	25	26	27	28	29	30
Scratch Marks	2	7	6	8	4	10	5	4	6	7

Which chart will be used in this case? Construct the suitable control chart. Also plot the revised control chart, if necessary.

Home Work: Do It Yourself

1) Follow the steps explained in Secs. 9.4 and 9.5 to construct the control chart for the data of Table 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 9" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.

3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.