

Control Chart for Range

SESSION
4

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4.1 INTRODUCTION

Prerequisite

- Lab Sessions 1, 3 and 6 of MSTL-001 (Basic Statistics Lab).
- Lab Session 2 of MSTL-002 (Industrial Statistics Lab).
- Unit 2 of MSTE-001 (Industrial Statistics-I).

R-chart is more widely used in quality control than the S-chart because in quality control, we generally use samples of sizes less than 10. We know that for $n \leq 10$, range and standard deviation reflect almost the same information about the variability. Range is easier to calculate and interpret as compared to standard deviation.

In Lab Sessions 1 to 3, you have learnt about the \bar{X} -chart which is used to control the process mean. We have discussed in Sec. 2.5 of Unit 2 of MSTE-001(Industrial Statistics-I) that process variability also affects the product quality. When variation in the production process is high, that is, the produced items have a wider range of values of the quality characteristics, the quality of the product is assumed to be poor. So we must reduce the process variability to improve the quality of the product. We use control chart for range (R-chart) and control chart for standard deviation (S-chart) to **control the process variability**.

The underlying logic and basic form of R-chart are similar to the \bar{X} -chart. The primary difference between \bar{X} and R-chart is that instead of plotting the sample means and monitoring their variation, we plot the **sample range** and monitor the **variation of sample ranges**. In this lab session, you will learn how to **construct R-chart** in MS Excel 2007. We shall explain the construction of **S-chart** in the next lab session.

Objectives

After performing the activities of this session, you should be able to:

- prepare spreadsheet in MS Excel 2007;
- determine the control limits for control chart for range;
- construct the control chart for range;
- obtain the revised control limits for control charts for range and mean; and
- interpret the control charts.

4.2 PROBLEM DESCRIPTION

Control Chart for Range

Suppose a bulb manufacturing company wants to check whether the variation in the life of bulbs produced by a particular machine is due to chance causes or assignable causes. For this purpose, a quality control inspector at this company selects 35 samples, each of size 5, and measures the life of each bulb (in hours).

Table 1: Life of bulbs

| Sample Number | Life of Bulbs (in hours) | | | | |
|---------------|--------------------------|--------|--------|--------|--------|
| | Obs. 1 | Obs. 2 | Obs. 3 | Obs. 4 | Obs. 5 |
| 1 | 1152 | 650 | 1090 | 708 | 1065 |
| 2 | 688 | 540 | 1074 | 598 | 1049 |
| 3 | 876 | 1342 | 1187 | 1214 | 1162 |
| 4 | 725 | 1070 | 847 | 1128 | 822 |
| 5 | 900 | 846 | 1254 | 904 | 1229 |
| 6 | 1080 | 790 | 871 | 848 | 846 |
| 7 | 952 | 1087 | 1126 | 1145 | 1101 |
| 8 | 741 | 986 | 578 | 1044 | 837 |
| 9 | 1005 | 1244 | 1140 | 1302 | 1115 |
| 10 | 1494 | 1005 | 614 | 1063 | 575 |
| 11 | 1200 | 870 | 859 | 928 | 834 |
| 12 | 745 | 964 | 1050 | 1022 | 1025 |
| 13 | 926 | 1172 | 1200 | 1230 | 1175 |
| 14 | 781 | 876 | 981 | 934 | 956 |
| 15 | 1120 | 1350 | 750 | 1408 | 725 |
| 16 | 880 | 975 | 768 | 1033 | 743 |
| 17 | 1138 | 636 | 1076 | 694 | 1051 |
| 18 | 674 | 526 | 1060 | 584 | 1035 |
| 19 | 862 | 1328 | 1173 | 1200 | 1148 |
| 20 | 711 | 1056 | 833 | 1114 | 808 |
| 21 | 886 | 832 | 1240 | 890 | 1215 |
| 22 | 1066 | 776 | 857 | 834 | 832 |
| 23 | 938 | 1073 | 1112 | 1131 | 1087 |
| 24 | 727 | 972 | 564 | 1030 | 823 |
| 25 | 991 | 1230 | 1126 | 1288 | 1101 |
| 26 | 1227 | 991 | 600 | 1049 | 561 |
| 27 | 1186 | 856 | 845 | 914 | 820 |
| 28 | 731 | 950 | 1036 | 1008 | 1011 |
| 29 | 912 | 1158 | 1186 | 1216 | 1161 |
| 30 | 1124 | 862 | 967 | 920 | 942 |
| 31 | 1106 | 1336 | 578 | 1490 | 711 |
| 32 | 866 | 961 | 754 | 1019 | 729 |
| 33 | 1124 | 622 | 1062 | 680 | 1037 |
| 34 | 660 | 658 | 1046 | 784 | 1021 |
| 35 | 848 | 1314 | 1159 | 1186 | 1134 |

The quality control inspector of the company has to construct a control chart for range and mean to infer whether the production variability is under control or not. If it is out-of-control, he/she computes the revised control limits.

Therefore, the problem for this session is to construct the control charts for range and mean for the data given in Table 1.

4.3**PROCEDURE FOR THE CONSTRUCTION OF R-CHART**

You have already learnt all the formulae for calculating control limits for R-chart and the method to draw R-chart manually in Sec. 2.5 of Unit 2 of MSTE-001. So here we focus on the main steps involved in the construction of R-chart for the data given in Table 1:

When the value of σ is known

Step 1: The control limits for R-chart for known σ are given by

$$\checkmark \text{ Centre line (CL)} = d_2\sigma \quad \dots(1)$$

$$\checkmark \text{ Upper control limit (UCL)} = D_2\sigma \quad \dots(2)$$

$$\checkmark \text{ Lower control limit (LCL)} = D_1\sigma \quad \dots(3)$$

where d_2 , D_1 and D_2 are constants and depend on the size of the sample. These constants have been tabulated for various sample sizes in the Appendix given at the end of this lab course.

Step 2: To construct the R-chart, we take sample number on the X-axis and sample range R on the Y-axis and plot the value of sample range for each sample against the sample number. The consecutive sample points are joined by line segments.

Step 3: Interpretation of the R-chart.

When the value of σ is unknown

In practice, the value of σ is not known. Therefore, it is estimated from the samples which are taken when the process is assumed to be under control. In R-chart, σ is estimated by sample range (R).

Step 1-2: The first two steps are the same as Steps 4 and 5 of Sec. 2.3 of Lab Session 2.

Step 3: The control limits for R-chart when σ is estimated by \bar{R}/d_2 are given by

$$\checkmark \text{ Centre line (CL)} = \bar{R} \quad \dots(4)$$

$$\checkmark \text{ Upper control limit (UCL)} = D_4\bar{R} \quad \dots(5)$$

$$\checkmark \text{ Lower control limit (LCL)} = D_3\bar{R} \quad \dots(6)$$

where D_3 and D_4 are constants and depend on the size of the sample. These constants have been tabulated for various sample sizes in the Appendix given at the end of this lab course.

Step 4: Interpretation of the R-chart.

4.4

STEPS INVOLVED IN THE CONSTRUCTION OF R-CHART IN EXCEL 2007

In order to calculate the control limits of R-chart and to plot the control chart in Excel 2007 for the given data, we follow the steps given below:

- Step 1:** We enter the given data in Excel Sheet and calculate range for the first sample by typing “=Max(B3:F3)-Min(B3:F3)” in Cell G3. After that, we drag down Cell G3 up to Cell G37 as shown in Fig. 4.1.

Generally, in order to judge whether the process is in control, we use \bar{X} and R-charts together. We first control the process variability and so we analyse the R-chart prior to the \bar{X} -chart.

| | A | B | C | D | E | F | G |
|----|------------|--------------------------|---------|---------|---------|---------|---------|
| 1 | | | | | | | |
| 2 | Sample No. | Life of Bulbs (in hours) | | | | | Range |
| 3 | 1 | 1152.00 | 650.00 | 1090.00 | 708.00 | 1065.00 | 502.000 |
| 4 | 2 | 688.00 | 540.00 | 1074.00 | 598.00 | 1049.00 | |
| 5 | 3 | 876.00 | 1342.00 | 1187.00 | 1214.00 | 1162.00 | |
| 6 | 4 | 725.00 | 1070.00 | 847.00 | 1128.00 | 822.00 | |
| 7 | 5 | 900.00 | 846.00 | 1254.00 | 904.00 | 1229.00 | |
| 8 | 6 | 1080.00 | 790.00 | 871.00 | 848.00 | 846.00 | |
| 9 | 7 | 952.00 | 1087.00 | 1126.00 | 1145.00 | 1101.00 | |
| 10 | 8 | 741.00 | 986.00 | 578.00 | 1044.00 | 837.00 | |
| 11 | 9 | 1005.00 | 1244.00 | 1140.00 | 1302.00 | 1115.00 | |
| 12 | 10 | 1494.00 | 1005.00 | 614.00 | 1063.00 | 575.00 | |

| | A | B | C | D | E | F | G |
|----|------------|--------------------------|------|------|------|------|---------|
| 1 | | | | | | | |
| 2 | Sample No. | Life of Bulbs (in hours) | | | | | Range |
| 3 | 1 | 1152 | 650 | 1090 | 708 | 1065 | 502.000 |
| 4 | 2 | 688 | 540 | 1074 | 598 | 1049 | 534.000 |
| 5 | 3 | 876 | 1342 | 1187 | 1214 | 1162 | 466.000 |
| 6 | 4 | 725 | 1070 | 847 | 1128 | 822 | 403.000 |
| 7 | 5 | 900 | 846 | 1254 | 904 | 1229 | 408.000 |
| 8 | 6 | 1080 | 790 | 871 | 848 | 846 | 290.000 |
| 9 | 7 | 952 | 1087 | 1126 | 1145 | 1101 | 193.000 |
| 10 | 8 | 741 | 986 | 578 | 1044 | 837 | 466.000 |
| 11 | 9 | 1005 | 1244 | 1140 | 1302 | 1115 | 297.000 |
| 12 | 10 | 1494 | 1005 | 614 | 1063 | 575 | 919.000 |
| 13 | 11 | 1200 | 870 | 859 | 928 | 834 | 366.000 |

Fig. 4.1

- Step 2:** In Cell G38, we compute the average range (\bar{R}) by typing “=Average(G3:G37)” and click on **Enter** key as shown in Fig. 4.2.

| | F | G | H |
|----|---------|------------------|---|
| 38 | Average | =AVERAGE(G3:G37) | |
| 39 | | | |

| | F | G | H | I |
|----|---------|---------|---|---|
| 38 | Average | 418.457 | | |
| 39 | | | | |

Fig. 4.2

Step 3: Next we type the values of k and n in Cells G40 and G41, respectively. We also have to type the values of D_3 and D_4 for $n = 5$ in Cell G42 and G43, respectively, as shown in Fig. 4.3. We take them from the Appendix given at the end of this lab course.

| | D | E | F | G |
|----|---|-------------------------------------------|---|-------|
| 40 | | | k | 35 |
| 41 | | | n | 5 |
| 42 | | D ₃ Value from Table (for n=5) | | 0 |
| 43 | | D ₄ Value from Table (for n=5) | | 2.114 |

Fig. 4.3

The formula with dollar sign (\$) is used for an absolute reference.

Step 4: We recall the method for computing the centre line and both control limits described in Lab Sessions 1-3. Here we use Columns H, I and J for putting the values of centre line, upper and lower control limits, respectively. We calculate these values as follows:

- i) The centre line is given in equation (4) and \bar{R} is given in Cell G38 (see Fig. 4.2). So we type “=\$G\$38” in Cell H3 to find the centre line as shown in Fig. 4.4a.
- ii) We use equation (5) for the upper control limit. The values of D_4 and \bar{R} are given in Cells G43 and G38, respectively (see Figs. 4.2 and 4.3). We type “=\$G\$43*\$G\$38” in Cell I3 as shown in Fig. 4.4b.
- iii) Similarly, we calculate the lower control limit from equation (6) and type “=\$G\$43*\$G\$38” in Cell J3 as shown in Fig. 4.4c.

| Control Limits | | | |
|----------------|-------------|-----|-----|
| H | I | J | K |
| 1 | | | |
| 2 | Centre Line | UCL | LCL |
| 3 | 418.457 | | |
| 4 | | | |

| Control Limits | | | |
|----------------|---------|-----|---|
| I | J | K | L |
| 1 | | | |
| 2 | UCL | LCL | |
| 3 | 884.618 | | |
| 4 | | | |

| Control Limits | | | | |
|----------------|-------|---|---|---|
| J | K | L | M | N |
| 1 | s | | | |
| 2 | LCL | | | |
| 3 | 0.000 | | | |
| 4 | | | | |

Fig. 4.4

Step 5: For plotting control limits using Excel, we first select Cells H3:J3 and drag them down up to Row 37 as shown in Fig. 4.5.

| G | H | I | J | K |
|----|----------------|-------------|---------|-------|
| 1 | Control Limits | | | |
| 2 | Range | Centre Line | UCL | LCL |
| 3 | 502.000 | 418.457 | 884.618 | 0.000 |
| 4 | 534.000 | 418.457 | 884.618 | 0.000 |
| 5 | 466.000 | 418.457 | 884.618 | 0.000 |
| 6 | 403.000 | 418.457 | 884.618 | 0.000 |
| 7 | 408.000 | 418.457 | 884.618 | 0.000 |
| 8 | 290.000 | 418.457 | 884.618 | 0.000 |
| 9 | 193.000 | 418.457 | 884.618 | 0.000 |
| 10 | 466.000 | 418.457 | 884.618 | 0.000 |
| 11 | 297.000 | 418.457 | 884.618 | 0.000 |
| 12 | 919.000 | 418.457 | 884.618 | 0.000 |
| 13 | 366.000 | 418.457 | 884.618 | 0.000 |
| 14 | 305.000 | 418.457 | 884.618 | 0.000 |
| 15 | 304.000 | 418.457 | 884.618 | 0.000 |
| 16 | 200.000 | 418.457 | 884.618 | 0.000 |
| 17 | 683.000 | 418.457 | 884.618 | 0.000 |
| 18 | 290.000 | 418.457 | 884.618 | 0.000 |
| 19 | 502.000 | 418.457 | 884.618 | 0.000 |
| 20 | 534.000 | 418.457 | 884.618 | 0.000 |
| 21 | 466.000 | 418.457 | 884.618 | 0.000 |
| 22 | 403.000 | 418.457 | 884.618 | 0.000 |
| 23 | 408.000 | 418.457 | 884.618 | 0.000 |

Fig. 4.5

Step 6: For plotting the R-chart in Excel 2007, we refer to Fig. 4.6. It means that we

1. select Cells G2:J37,
2. click on the **Insert** tab,
3. select the **Line** option, and
4. choose the chart subtype.

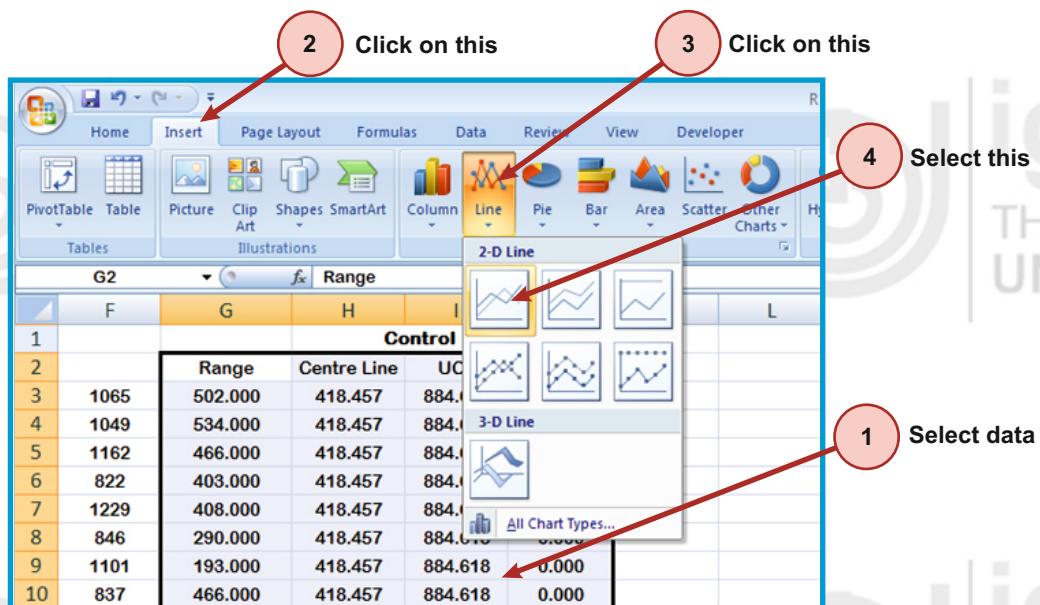


Fig. 4.6

Step 7: We find that Columns H, I and J will provide horizontal lines on the chart representing the centre line, UCL and LCL, respectively. Column G will provide the ranges for 35 samples on the chart. We format the chart as explained in Sec. 1.4 of Lab Session 1. Thus, we obtain the R-chart shown in Fig. 4.7.

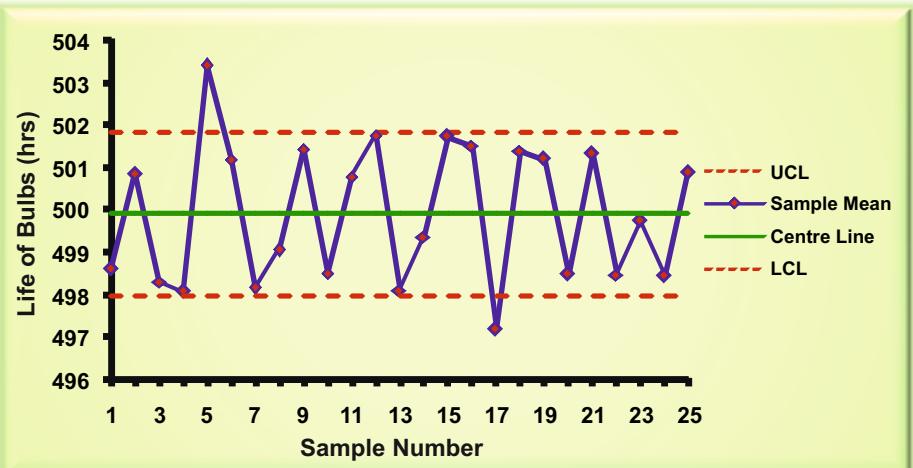


Fig. 4.7

Interpretation

Note from Fig. 4.7 that two points corresponding to Samples 10 and 31 lie outside the UCL. So the control chart indicates that the process is **not under statistical control**. Some **assignable causes** are present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them.

4.5 REVISED R-CHART

After eliminating the assignable causes from the process, we revise the control chart of Fig. 4.7 by deleting out-of-control samples. For this purpose, we obtain the revised control limits for the R-chart using only the remaining samples as explained below:

Step 1: For revised limits for R-chart, we first calculate new \bar{R} as follows:

$$\bar{R}_{\text{new}} = \frac{\sum_{i=1}^k R_i - \sum_{j=1}^d R_j}{k-d} \quad \dots(7)$$

where $\sum_{j=1}^d R_j$ – sum of the ranges of discarded samples, and

d – number of discarded samples.

Step 2: After finding the new \bar{R} , we reconstruct the centre line and control limits by replacing \bar{R} by \bar{R}_{new} as follows:

$$\checkmark \text{ Centre line (CL)} = \bar{R}_{\text{new}} \quad \dots(8)$$

$$\checkmark \text{ Upper control limit (UCL)} = D_4 \bar{R}_{\text{new}} \quad \dots(9)$$

$$\checkmark \text{ Lower control limit (LCL)} = D_3 \bar{R}_{\text{new}} \quad \dots(10)$$

Step 3: Interpretation of the R-chart.

Steps in Excel

The main steps for calculating the revised centre line and control limits for the R-chart in Excel 2007 for the remaining samples are as follows:

Step 1: We highlight the samples outside the control limits, i.e., the 10th and 31st samples, with light orange colour (Fig. 4.8).

| | A | B | C | D | E | F | G | H | I | J |
|-------------------------------------------------------------------|------------|--------------------------|---------|---------|---------|---------|---------|-------------|---------|-------|
| 1 | | | | | | | | | | |
| 2 | Sample No. | Life of Bulbs (in hours) | | | | | Range | Centre Line | UCL | LCL |
| 3 | 1 | 1152 | 650 | 1090 | 708 | 1065 | 502.000 | 418.457 | 884.618 | 0.000 |
| 4 | 2 | 688 | 540 | 1074 | 598 | 1049 | 534.000 | 418.457 | 884.618 | 0.000 |
| 5 | 3 | 876 | 1342 | 1187 | 1214 | 1162 | 466.000 | 418.457 | 884.618 | 0.000 |
| 6 | 4 | 725 | 1070 | 847 | 1128 | 822 | 403.000 | 418.457 | 884.618 | 0.000 |
| 7 | 5 | 900 | 846 | 1254 | 904 | 1229 | 408.000 | 418.457 | 884.618 | 0.000 |
| 8 | 6 | 1080 | 790 | 871 | 848 | 846 | 290.000 | 418.457 | 884.618 | 0.000 |
| 9 | 7 | 952 | 1087 | 1126 | 1145 | 1101 | 193.000 | 418.457 | 884.618 | 0.000 |
| 10 | 8 | 741 | 986 | 578 | 1044 | 837 | 466.000 | 418.457 | 884.618 | 0.000 |
| 11 | 9 | 1005 | 1244 | 1140 | 1302 | 1115 | 297.000 | 418.457 | 884.618 | 0.000 |
| 12 | 10 | 1494 | 1005 | 614 | 1063 | 575 | 919.000 | 418.457 | 884.618 | 0.000 |
| 13 | 11 | 1200 | 870 | 859 | 928 | 834 | 366.000 | 418.457 | 884.618 | 0.000 |
| 14 | 12 | 745 | 964 | 1050 | 1022 | 1025 | 305.000 | 418.457 | 884.618 | 0.000 |
| 15 | 13 | 926 | 1172 | 1200 | 1230 | 1175 | 304.000 | 418.457 | 884.618 | 0.000 |
| 16 | 14 | 781 | 876 | 981 | 934 | 956 | 200.000 | 418.457 | 884.618 | 0.000 |
| 17 | 15 | 1120 | 1350 | 750 | 1408 | 725 | 683.000 | 418.457 | 884.618 | 0.000 |
| 18 | 16 | 880 | 975 | 768 | 1033 | 743 | 290.000 | 418.457 | 884.618 | 0.000 |
| 19 | 17 | 1138 | 636 | 1076 | 694 | 1051 | 502.000 | 418.457 | 884.618 | 0.000 |
| 20 | 18 | 674 | 526 | 1060 | 584 | 1035 | 534.000 | 418.457 | 884.618 | 0.000 |
| 21 | 19 | 862 | 1328 | 1173 | 1200 | 1148 | 466.000 | 418.457 | 884.618 | 0.000 |
| ----- ----- ----- ----- ----- ----- ----- ----- ----- ----- ----- | | | | | | | | | | |
| | A | B | C | D | E | F | G | H | I | J |
| 31 | 29 | 912.00 | 1158.00 | 1186.00 | 1216.00 | 1161.00 | 304.000 | 418.457 | 884.618 | 0.000 |
| 32 | 30 | 1124.00 | 862.00 | 967.00 | 920.00 | 942.00 | 262.000 | 418.457 | 884.618 | 0.000 |
| 33 | 31 | 1106.00 | 1336.00 | 578.00 | 1490.00 | 711.00 | 912.000 | 418.457 | 884.618 | 0.000 |
| 34 | 32 | 866.00 | 961.00 | 754.00 | 1019.00 | 729.00 | 290.000 | 418.457 | 884.618 | 0.000 |
| 35 | 33 | 1124.00 | 622.00 | 1062.00 | 680.00 | 1037.00 | 502.000 | 418.457 | 884.618 | 0.000 |
| 36 | 34 | 660.00 | 658.00 | 1046.00 | 784.00 | 1021.00 | 388.000 | 418.457 | 884.618 | 0.000 |
| 37 | 35 | 848.00 | 1314.00 | 1159.00 | 1186.00 | 1134.00 | 466.000 | 418.457 | 884.618 | 0.000 |

Fig. 4.8

Step 2: We have two samples outside the UCL, so we put $d = 2$ in Cell G44 as shown in Fig. 4.9.

| | D | E | F | G |
|----|-------------------------------------------|---|---|-------|
| 40 | | | k | 35 |
| 41 | | | n | 5 |
| 42 | D ₃ Value from Table (for n=5) | | | 0 |
| 43 | D ₄ Value from Table (for n=5) | | | 2.114 |
| 44 | | | d | 2 |

Fig. 4.9

Step 3: For calculating the value of \bar{R}_{new} given in equation (7), we type “=(Sum(N3:N37)-N12-N33)/(G40-G44)” in Cell G39 and then press **Enter**. We get the value of revised \bar{R} i.e., \bar{R}_{new} as shown in Fig. 4.10.

| | E | F | G |
|----|----------------------------|---------------------------------|---|
| 39 | Revised Average | =SUM(G3:G37)-G12-G33)/(G40-G44) | |
| 40 | k | 35 | |
| 41 | n | 5 | |
| 42 | Value from Table (for n=5) | 0 | |
| 43 | Value from Table (for n=5) | 2.114 | |
| 44 | d | 2 | |

ENTER

| | E | F | G |
|----|-----------------|---------|---|
| 39 | Revised Average | 388.333 | |

Fig. 4.10

Step 4: We now calculate the revised centre line, upper and lower control limits as discussed in Step 4 of Sec. 4.3. Here we use Columns K, L and M for putting the values of the revised centre line, upper and lower control limits, respectively.

The formula with dollar sign (\$) is used for an absolute reference.

- The revised centre line is given by equation (8). The value of \bar{R}_{new} is given in Cell G39 (see Fig. 4.10). So we type “=\$G\$39” in Cell K3 to find the centre line as shown in Fig. 4.11a.
- The upper control limit is given by equation (9). The values of D_4 and \bar{R}_{new} are given in Cells G43 and G39, respectively (see Fig. 4.10). So we type “=\$G\$43*\$G\$39” in Cell L3 as shown in Fig. 4.11b.
- Similarly, we calculate the lower control limit given by equation (10) by typing “=\$G\$42*\$G\$39” in Cell M3 as shown in Fig. 4.11c.

| K3 | | | |
|----|-------------------------------|-----|-----|
| | K | L | M |
| 1 | Revised Control Limits | | |
| 2 | Centre Line | UCL | LCL |
| 3 | 388.333 | | |
| 4 | | | |

| L3 | | | |
|----|-----------------------|-----|---|
| | L | M | N |
| 1 | Control Limits | | |
| 2 | UCL | LCL | |
| 3 | 820.937 | | |
| 4 | | | |

| M3 | | | |
|----|--------|---|---|
| | M | N | O |
| 1 | limits | | |
| 2 | LCL | | |
| 3 | 0.000 | | |
| 4 | | | |

Fig. 4.11

| | H | I | J | K | L | M | N |
|----|----------------|---------|-------|------------------------|---------|-------|---|
| 1 | Control Limits | | | Revised Control Limits | | | |
| 2 | Centre Line | UCL | LCL | Centre Line | UCL | LCL | |
| 3 | 418.457 | 884.618 | 0.000 | 388.333 | 820.937 | 0.000 | |
| 4 | 418.457 | 884.618 | 0.000 | | | | |
| 5 | 418.457 | 884.618 | 0.000 | | | | |
| 6 | 418.457 | 884.618 | 0.000 | | | | |
| 7 | 418.457 | 884.618 | 0.000 | | | | |
| 8 | 418.457 | 884.618 | 0.000 | | | | |
| 9 | 418.457 | 884.618 | 0.000 | | | | |
| 10 | 418.457 | 884.618 | 0.000 | | | | |

| | H | I | J | K | L | M | N |
|----|----------------|---------|-------|------------------------|---------|-------|---|
| 1 | Control Limits | | | Revised Control Limits | | | |
| 2 | Centre Line | UCL | LCL | Centre Line | UCL | LCL | |
| 3 | 418.457 | 884.618 | 0.000 | 388.333 | 820.937 | 0.000 | |
| 4 | 418.457 | 884.618 | 0.000 | 388.333 | 820.937 | 0.000 | |
| 5 | 418.457 | 884.618 | 0.000 | 388.333 | 820.937 | 0.000 | |
| 6 | 418.457 | 884.618 | 0.000 | 388.333 | 820.937 | 0.000 | |
| 7 | 418.457 | 884.618 | 0.000 | 388.333 | 820.937 | 0.000 | |
| 8 | 418.457 | 884.618 | 0.000 | 388.333 | 820.937 | 0.000 | |
| 9 | 418.457 | 884.618 | 0.000 | 388.333 | 820.937 | 0.000 | |
| 10 | 418.457 | 884.618 | 0.000 | 388.333 | 820.937 | 0.000 | |
| 11 | 418.457 | 884.618 | 0.000 | 388.333 | 820.937 | 0.000 | |
| 12 | 418.457 | 884.618 | 0.000 | 388.333 | 820.937 | 0.000 | |
| 13 | 418.457 | 884.618 | 0.000 | 388.333 | 820.937 | 0.000 | |

Fig. 4.12

Step 5: For plotting the R-chart in Excel 2007, we refer to Fig. 4.13. It means that we

1. select Cells G2:G11, G13:G32, G34:G37, K2:M11, K13:M32 and K34:M37 by holding **Ctrl** key,
 2. click on the **Insert** tab,
 3. select the **Line** option, and
 4. choose the chart subtype.

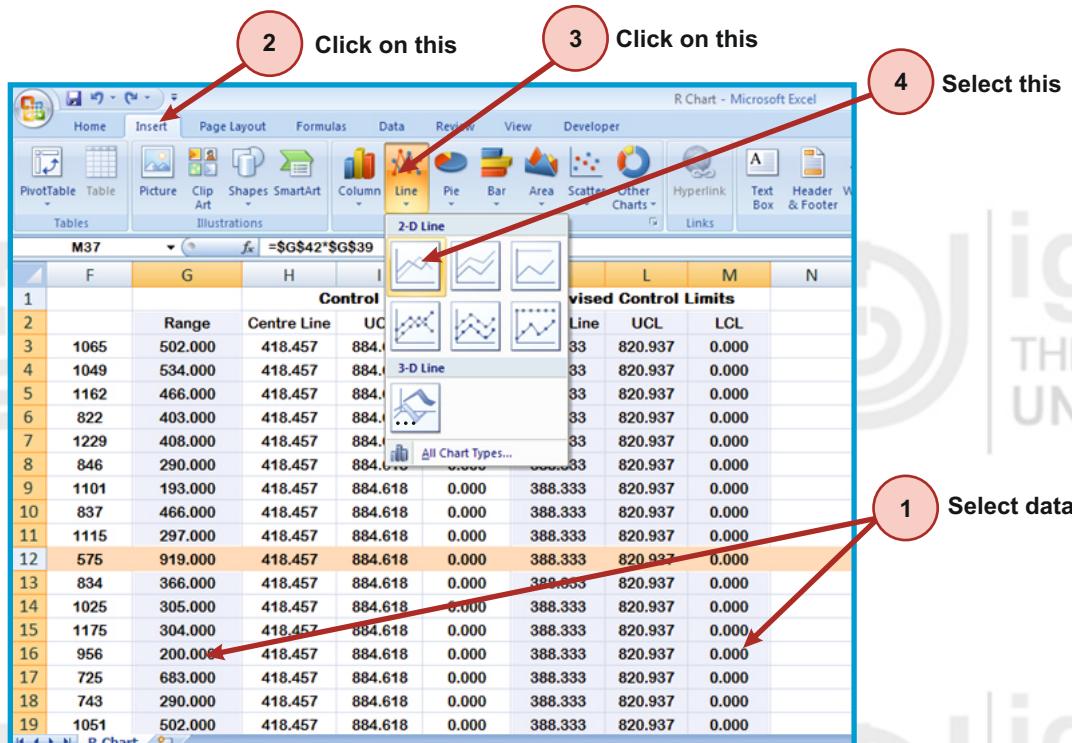


Fig. 4.13

Step 7: We format and change the horizontal axis as discussed in Steps 9 and 10 of Sec. 2.5 of Lab Session 2. The resulting R-chart is shown in Fig. 4.14.

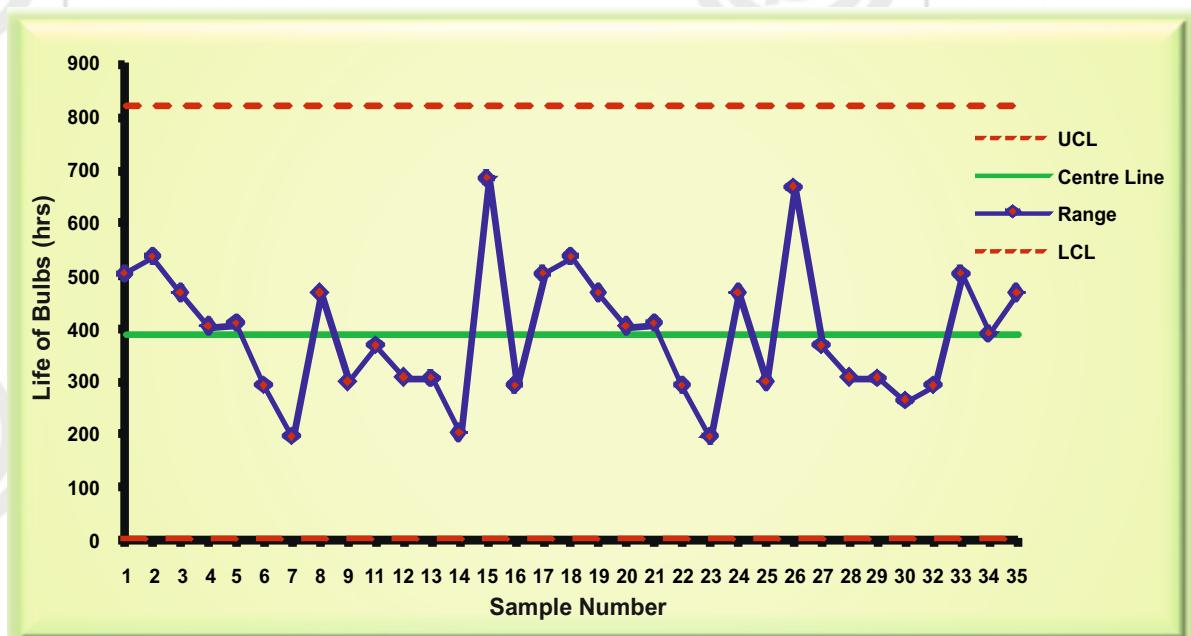


Fig. 4.14

If one or more points lie outside the revised control limits, we calculate the revised control limits of R-chart again. This continues until the process comes under control.

Interpretation

The revised R-chart shown in Fig. 4.14 indicates that all points lie within the control limits. So the process is under statistical control with respect to process variability.



Activity 1

You can also determine the revised control limits using another approach. For this purpose, follow the steps given below:

- ✓ Select Cells A2:F11, A13:F32 and A34:F37.
- ✓ Choose Cell A46 or any other cell and paste the values. You can also use a separate Excel sheet where you can paste these values.
- ✓ Repeat all the steps given in Sec. 4.4.

It will give you the same results as you have obtained in Sec. 4.5.

So far you have learnt how to control the process variability using the R-chart. We shall now study the process mean. For this, we calculate the control limits of the \bar{X} -chart using the remaining samples.

4.6 REVISED \bar{X} -CHART

Control Chart for Range

We have already controlled the process variability using R-chart in Sec. 4.5 after eliminating the assignable causes present in the process. We now have to control the process mean. Here we use \bar{R}_{new} to estimate the process variability. We calculate $\bar{\bar{X}}_{\text{new}}$ by deleting the out-of-control points. We compute the revised control limits for the \bar{X} -chart as explained in Sec. 2.5 of Lab Session 2.

Step 1: For the revised limits for \bar{X} -chart, we calculate new $\bar{\bar{X}}$ and new \bar{R} as follows:

$$\bar{\bar{X}}_{\text{new}} = \frac{\sum_{i=1}^k \bar{X}_i - \sum_{j=1}^d \bar{X}_j}{k-d} \quad \dots(11)$$

$$\text{and } \bar{R}_{\text{new}} = \frac{\sum_{i=1}^k R_i - \sum_{j=1}^d R_j}{k-d} \quad \dots(12)$$

where $\sum_{j=1}^d \bar{X}_j$ – sum of the averages of discarded samples,

d – number of discarded samples, and

$\sum_{j=1}^d R_j$ – sum of the ranges of discarded samples.

If R-chart indicates that the process variability is out-of-control, we first control the process variability by investigating the assignable causes and take corrective action to eliminate them. Then we analyse \bar{X} -chart because when R-chart is brought to control, many assignable causes for \bar{X} -chart are also eliminated.

If R-chart indicates that the process variability is under control but the \bar{X} -chart indicates that the average of the process is out-of-control, we continue to use the same R-chart and revise the centre line and control limits of the \bar{X} -chart by eliminating these points.

Step 2: Then we reconstruct the centre line and control limits of the chart by replacing \bar{X} by $\bar{\bar{X}}_{\text{new}}$ and \bar{R} by \bar{R}_{new} as follows:

✓ Centre line = $\bar{\bar{X}}_{\text{new}}$... (13)

✓ Upper control limit (UCL) = $\bar{\bar{X}}_{\text{new}} + A_2 \bar{R}_{\text{new}}$... (14)

✓ Lower control limit (LCL) = $\bar{\bar{X}}_{\text{new}} - A_2 \bar{R}_{\text{new}}$... (15)

Step 3: Interpretation of the \bar{X} -chart.

Steps in Excel

The main steps for calculating the revised centre line and control limits for the \bar{X} -chart in Excel 2007 using the remaining samples are as follows:

Step 1: We compute the sample means for all given samples in Cells N3:N37 as explained in Steps 1-5 of Sec. 1.4 of Lab Session 1. The output is shown in Fig. 4.15.

| | K | L | M | N | O |
|----|-------------------------------|---------|-------|-------------|---|
| 1 | Revised Control Limits | | | | |
| 2 | Centre Line | UCL | LCL | Sample Mean | |
| 3 | 388.333 | 820.937 | 0.000 | 933.000 | |
| 4 | 388.333 | 820.937 | 0.000 | 789.800 | |
| 5 | 388.333 | 820.937 | 0.000 | 1156.200 | |
| 6 | 388.333 | 820.937 | 0.000 | 918.400 | |
| 7 | 388.333 | 820.937 | 0.000 | 1026.600 | |
| 8 | 388.333 | 820.937 | 0.000 | 887.000 | |
| 9 | 388.333 | 820.937 | 0.000 | 1082.200 | |
| 10 | 388.333 | 820.937 | 0.000 | 837.200 | |
| 11 | 388.333 | 820.937 | 0.000 | 1161.200 | |
| 12 | 388.333 | 820.937 | 0.000 | 950.200 | |
| 13 | 388.333 | 820.937 | 0.000 | 938.200 | |
| 14 | 388.333 | 820.937 | 0.000 | 961.200 | |
| 15 | 388.333 | 820.937 | 0.000 | 1140.600 | |

Fig. 4.15

Step 2: We calculate \bar{X}_{new} by typing

“=(Sum(N3:N37)-N12-N33)/(G40-G44)” in Cell N38 (Fig. 4.16).

| | L | M | N | O | P | Q |
|----|-----------------|---|---------|---|---|---|
| 38 | Revised Average | | 967.655 | | | |
| 39 | | | | | | |

Fig. 4.16

Step 3: We already have the value of $d = 2$ in Cell G44 (see Fig. 4.9). We now type the value of A_2 for $n = 5$ in Cell G45 (Fig. 4.17).

| | D | E | F | G |
|----|-------------------------------|---|---|-------|
| 44 | | | d | 2 |
| 45 | A2 Value from Table (for n=5) | | | 0.577 |

Fig. 4.17

Step 4: We compute the centre line, UCL and LCL in Columns O, P and Q, respectively, as explained in Sec. 2.5 of Lab Session 2 (Fig. 4.18).

| | O | P | Q |
|----|-----------------------|----------|---------|
| 1 | Control Limits | | |
| 2 | Centre Line | UCL | LCL |
| 3 | 967.655 | 1191.723 | 743.586 |
| 4 | 967.655 | 1191.723 | 743.586 |
| 5 | 967.655 | 1191.723 | 743.586 |
| 6 | 967.655 | 1191.723 | 743.586 |
| 7 | 967.655 | 1191.723 | 743.586 |
| 8 | 967.655 | 1191.723 | 743.586 |
| 9 | 967.655 | 1191.723 | 743.586 |
| 10 | 967.655 | 1191.723 | 743.586 |
| 11 | 967.655 | 1191.723 | 743.586 |
| 12 | 967.655 | 1191.723 | 743.586 |
| 13 | 967.655 | 1191.723 | 743.586 |
| 14 | 967.655 | 1191.723 | 743.586 |
| 15 | 967.655 | 1191.723 | 743.586 |
| 16 | 967.655 | 1191.723 | 743.586 |
| 17 | 967.655 | 1191.723 | 743.586 |
| 18 | 967.655 | 1191.723 | 743.586 |
| 19 | 967.655 | 1191.723 | 743.586 |
| 20 | 967.655 | 1191.723 | 743.586 |
| 21 | 967.655 | 1191.723 | 743.586 |
| 22 | 967.655 | 1191.723 | 743.586 |
| 23 | 967.655 | 1191.723 | 743.586 |

Fig. 4.18

Step 5: To plot the \bar{X} -chart, we

1. select Cells N2:Q11, N13:Q32 and N34:Q37 by holding ***Ctrl*** key,
2. click on the **Insert** tab,
3. select the **Line** option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1.
The resulting control chart is shown in Fig. 4.19.

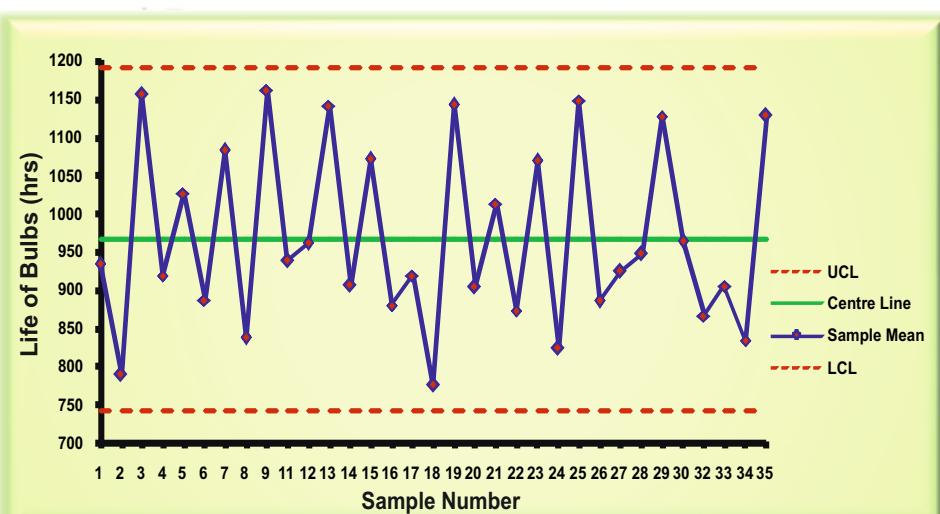


Fig. 4.19

Interpretation

Note from Fig. 4.19 that no point lies outside the control limits of the revised X-chart. This indicates that the process mean is under control. Hence, we may conclude that the process is under statistical control with respect to the process variability and process mean.

Now, you can try the following exercises to revise the R-chart and \bar{X} -chart which are used to control the process variability and mean, respectively.



Activity 2

Construct the control charts for range with the help of MS Excel 2007 and interpret the results for

- A1) Example 5 given in Unit 2 of MSTE-001.
- A2) Exercise E10 given in Unit 2 of MSTE-001.

Match the results with the manual calculation done in Unit 2 of MSTE-001



Continuous Assessment 4

Consider a problem related to a process which maintains the quality of bottling procedure. Suppose a fruit juice manufacturing company uses automatic machines to fill 500 ml juice bottles. A quality control inspector at the company collects 25 samples each of four observations at different times and measures the volume of each filled bottle. The data is given in Table 2.

Table 2: Volume of fruit juice in the filled bottles

| Sample | Volume of Juice per bottle (in ml) | | | | Number |
|--------|------------------------------------|--------|--------|--------|--------|
| | Obs. 1 | Obs. 2 | Obs. 3 | Obs. 4 | |
| 1 | 497.32 | 500.62 | 498.68 | 497.82 | |
| 2 | 504.76 | 500.00 | 498.32 | 500.32 | |
| 3 | 499.24 | 497.18 | 498.12 | 498.68 | |
| 4 | 499.26 | 496.32 | 498.88 | 497.82 | |
| 5 | 498.32 | 500.62 | 499.56 | 500.12 | |
| 6 | 499.12 | 500.32 | 499.38 | 500.94 | |
| 7 | 499.34 | 498.32 | 497.32 | 497.62 | |
| 8 | 499.38 | 498.12 | 500.62 | 498.12 | |
| 9 | 499.26 | 498.38 | 500.68 | 500.38 | |
| 10 | 498.60 | 497.62 | 499.25 | 498.56 | |
| 11 | 499.44 | 500.00 | 501.32 | 499.38 | |

| Sample | Volume of Juice per bottle (in ml) | | | Number |
|--------|------------------------------------|--------|--------|--------|
| | Obs. 1 | Obs. 2 | Obs. 3 | |
| 12 | 498.26 | 500.32 | 500.76 | 499.68 |
| 13 | 497.32 | 498.50 | 497.18 | 499.38 |
| 14 | 499.56 | 498.00 | 498.76 | 501.12 |
| 15 | 500.24 | 500.32 | 499.12 | 499.25 |
| 16 | 500.76 | 500.50 | 499.68 | 498.12 |
| 17 | 500.65 | 497.82 | 494.06 | 496.25 |
| 18 | 499.12 | 500.26 | 500.44 | 498.76 |
| 19 | 499.50 | 500.50 | 499.56 | 500.76 |
| 20 | 497.50 | 498.82 | 499.76 | 497.82 |
| 21 | 499.44 | 500.62 | 500.00 | 501.26 |
| 22 | 499.38 | 498.38 | 497.56 | 498.56 |
| 23 | 501.56 | 499.56 | 498.00 | 499.82 |
| 24 | 498.32 | 497.32 | 499.56 | 498.62 |
| 25 | 499.50 | 500.12 | 498.50 | 500.38 |

Develop the \bar{X} and R-charts to check whether the process of bottling is under control or out-of-control. Also plot the revised control chart, if necessary.



Home Work: Do It Yourself

- 1) Follow the steps explained in Secs. 4.4, 4.5 and 4.6 to construct the control charts for the data of Table 1. Use a different format for the control charts. Take their screenshots and keep them in your record book.
- 2) Develop the spreadsheet for the exercise “Continuous Assessment 4” as explained in this lab session. Take screenshots of the final spreadsheet and the charts.
- 3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.