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TECHNOLOGY, RESEARCH, SOCIAL INNOVATION & PARTNERSHIPS

CS231 Data Structure-II

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

CS231 Data Structure-II 3-4-2-2

Examination scheme: Marks-50 [Continuous Assessment]

Course Objectives:

1. To study algorithms related to trees and graphs.
2. To understand the concept of symbol table.
3. To realize appropriate data structures to solve problems in various domains.

Course Outcomes:

1. To select appropriate data structures in problem solving
2. To implement various algorithms related to trees and graphs
3. To demonstrate the use of trees for symbol table implementation.

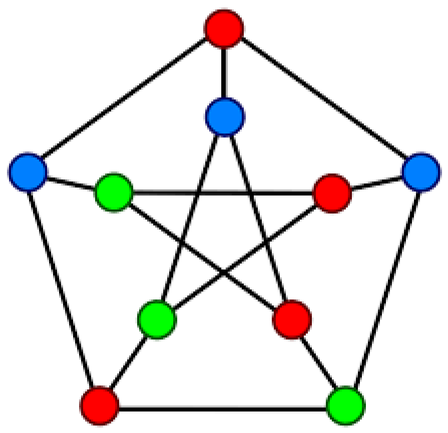
Graph

Graph- Basic Terminology, Storage representation**:** Adjacency matrix, Adjacency list, Creation of Graph and Traversals,

Minimum spanning Tree- Prim's and Kruskal's Algorithms, Dikjtra's Single source shortest path, Topological sorting

Graph

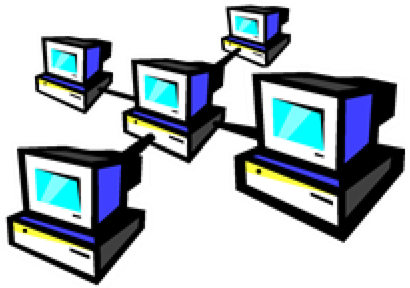
- Basic Terminology
- Storage representation
- Creation of graph and traversals
- Minimum spanning tree
- Topological sorting



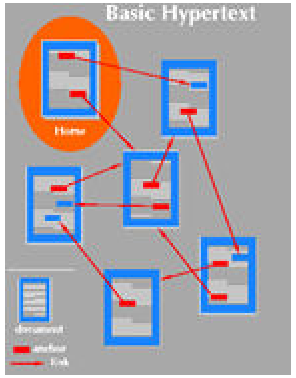
Graph Applications

Social Network

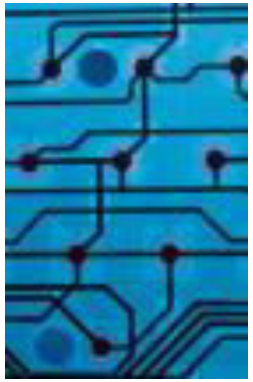
Maps



Computer Network



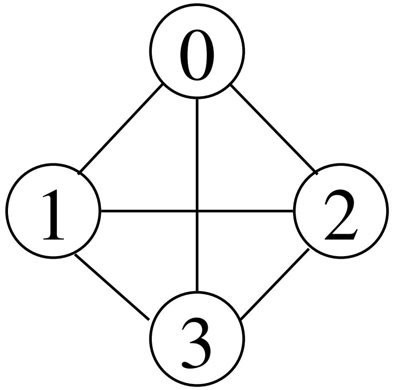
Hypertext



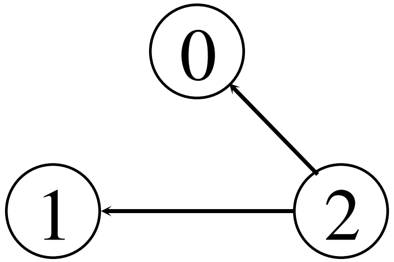
Circuits

Definition

- A graph G consists of two sets
  - a finite, nonempty set of vertices V(G)
  - a finite, possible empty set of edges E(G)
  - G(V,E) represents a graph



Graph G_1



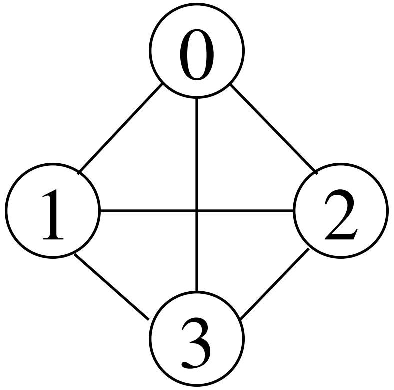
Graph G2

Vertex Set: $V(G_1)=\{0,1,2,3\}$
Edge Set:
$E(G_1)=\{(0,1),(0,2),(0,3),(1,2),(1,3),(2,3)\}$

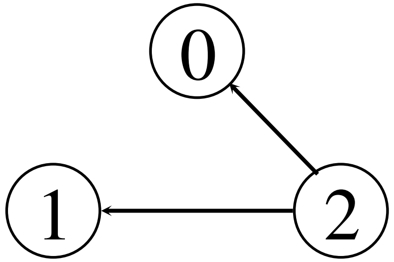
Vertex Set: $V(G2)=\{0,1,2\}$
Edge Set:
$E(G2)=\{(2,0),(2,1)\}$

Directed and Undirected Graph

- An undirected graph is one in which the pair of vertices in a edge is unordered, (v0, v1) = (v1,v0)
- A directed graph is one in which each edge is a directed pair of vertices, <v0, v1> != <v1,v0>



Undirected Graph



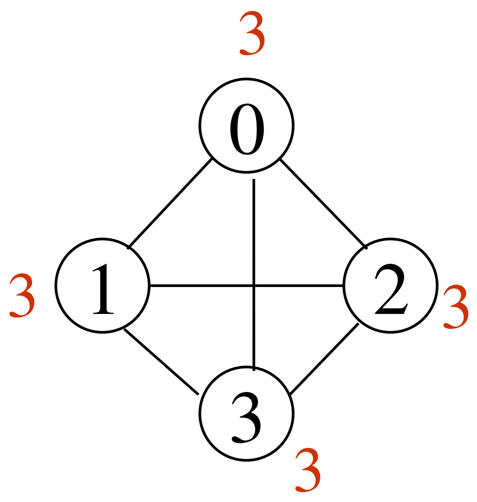
Directed Graph

Degree

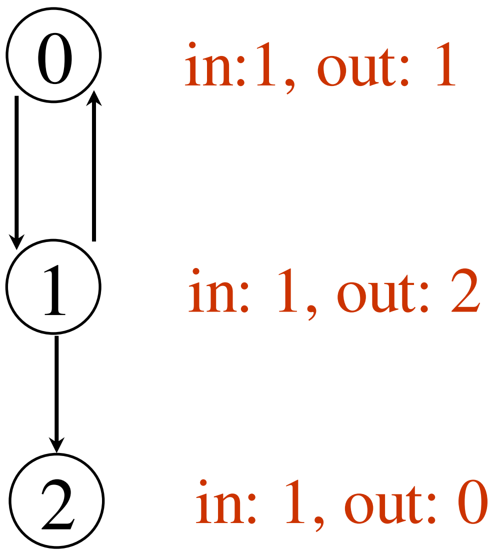
- The degree of a vertex is the number of edges incident to that vertex.
- For directed graph,
  - the in-degree of a vertex *v* is the number of edges that have *v* as the head
  - the out-degree of a vertex *v* is the number of edges that have *v* as the tail
  - if *di* is the degree of a vertex *i* in a graph *G* with *n* vertices and *e* edges, the number of edges is

$$e = \left(\sum_{0}^{n-1} d_i\right) / 2$$

Example for Degree



Undirected Graph : G_1



Directed Graph: G_3

Adjacent and Incident

- If (v0, v1) is an edge in an undirected graph,
  - for v0 and v1 are adjacent
  - The edge (v0, v1) is incident on vertices v0 and v1

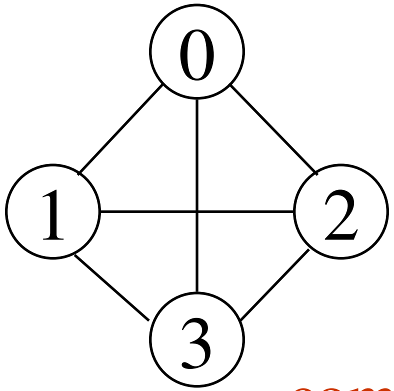
- If <v0, v1> is an edge in a directed graph
  - v0 is adjacent to v1, and v1 is adjacent from v0
  - The edge <v0, v1> is incident on v0 and v1

Complete graph

A complete graph is a graph that has the maximum number of edges

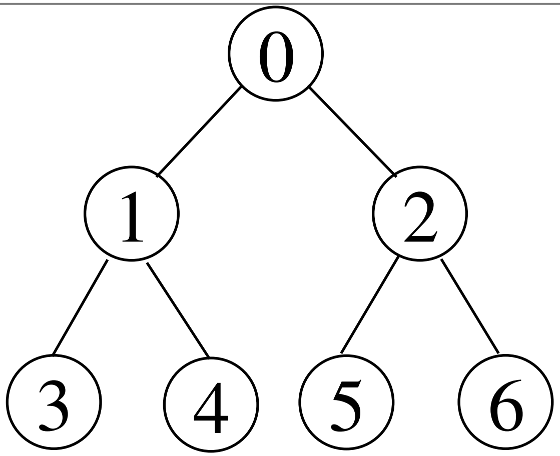
- for undirected graph with n vertices, the maximum number of edges is n(n-1)/2
- for directed graph with n vertices, the maximum number of edges is n(n-1)

Examples for Graph

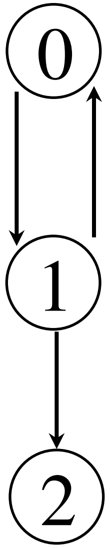


complete graph

G_1



G_2 incomplete graph



G_3

Directed graph

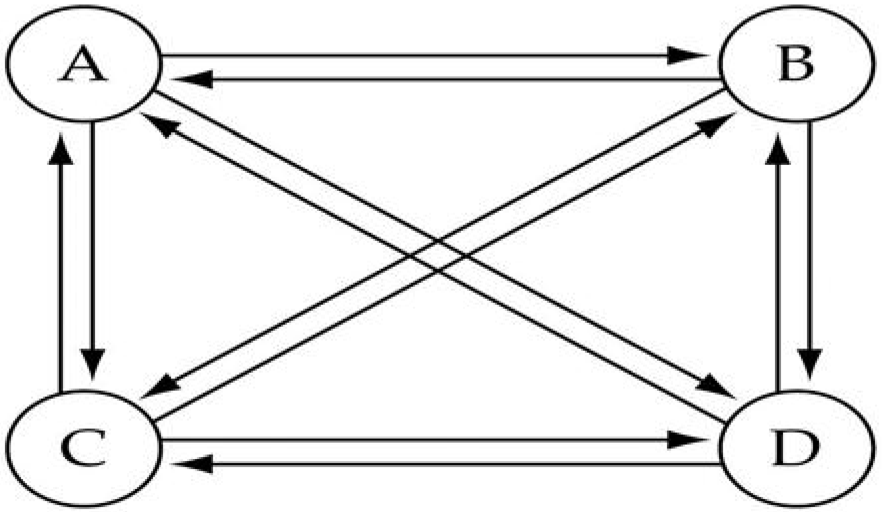
$V(G_1)=\{0,1,2,3\}$ $E(G_1)=\{(0,1),(0,2),(0,3),(1,2),(1,3),(2,3)\}$

$V(G_2)=\{0,1,2,3,4,5,6\}$ $E(G_2)=\{(0,1),(0,2),(1,3),(1,4),(2,5),(2,6)\}$

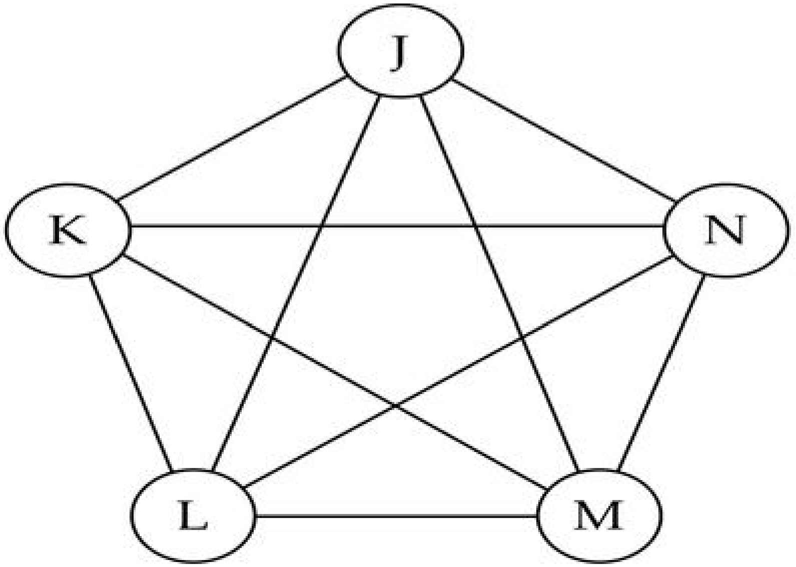
$V(G_3)=\{0,1,2\}$ $E(G_3)=\{\langle 0,1\rangle,\langle 1,0\rangle,\langle 1,2\rangle\}$

No. of edges (complete undirected graph) : n(n-1)/2
No. of edges (complete directed graph): n(n-1)

Complete Graph



(a) Complete directed graph.

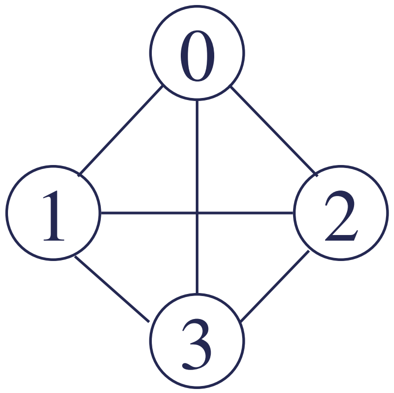


(b) Complete undirected graph.

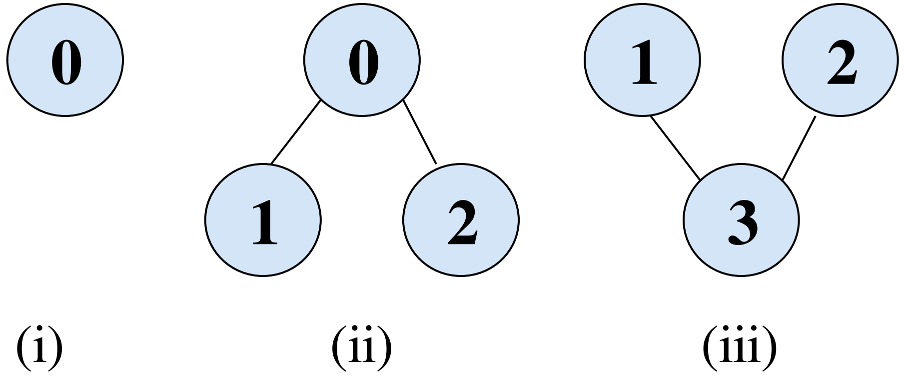
Subgraph and Path

- A subgraph of G is a graph G' such that V(G') is a subset of V(G) and E(G') is a subset of E(G)
- A path from vertex v_p to vertex v_q in a graph G, is a sequence of vertices, $v_p, v_{i1}, v_{i2}, \ldots, v_{in}, v_q$, such that $(v_p, v_{i1}), (v_{i1}, v_{i2}), \ldots, (v_{in}, v_q)$ are edges in an undirected graph
- The length of a path is the number of edges on it

Example for Subgraph

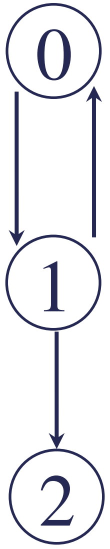


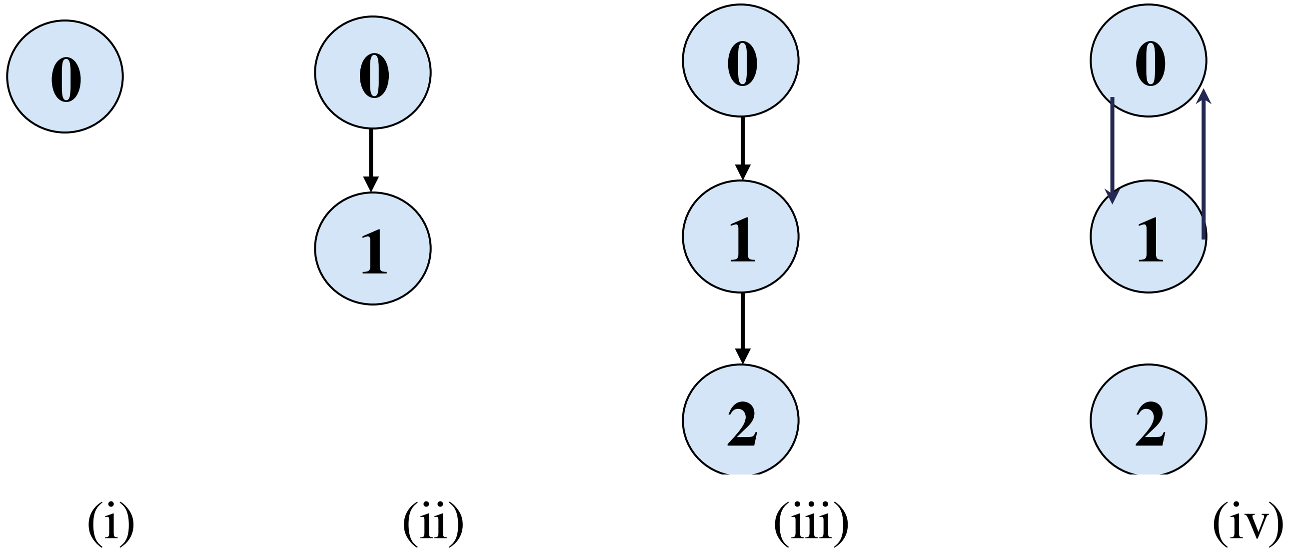
G_1



(a) Some of the subgraph of G_1

G_3



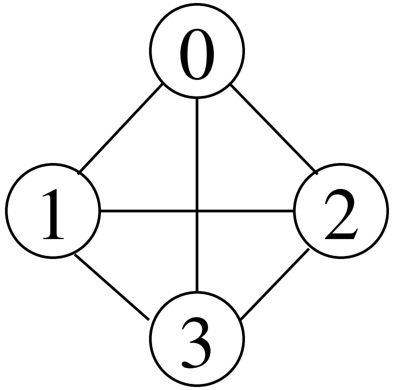


(b) Some of the subgraph of G_3

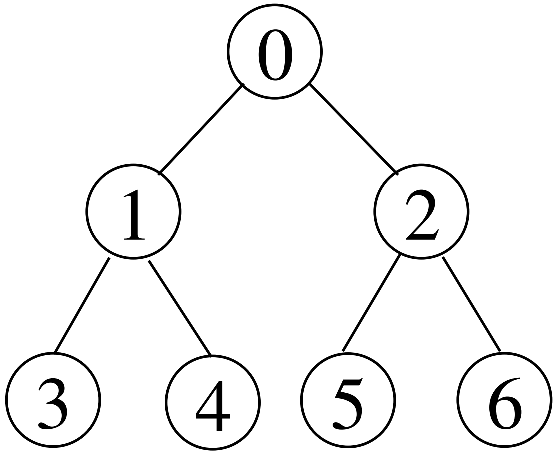
Simple path and cycle

- A simple path is a path in which all vertices, except possibly the first and the last, are distinct
- In an undirected graph G, two vertices, v0 and v1, are connected if there is a path in G from v0 to v1
- A cycle is a simple path in which the first and the last vertices are the same
- An undirected graph is connected if, for every pair of distinct vertices vi, vj, there is a path from vi to vj

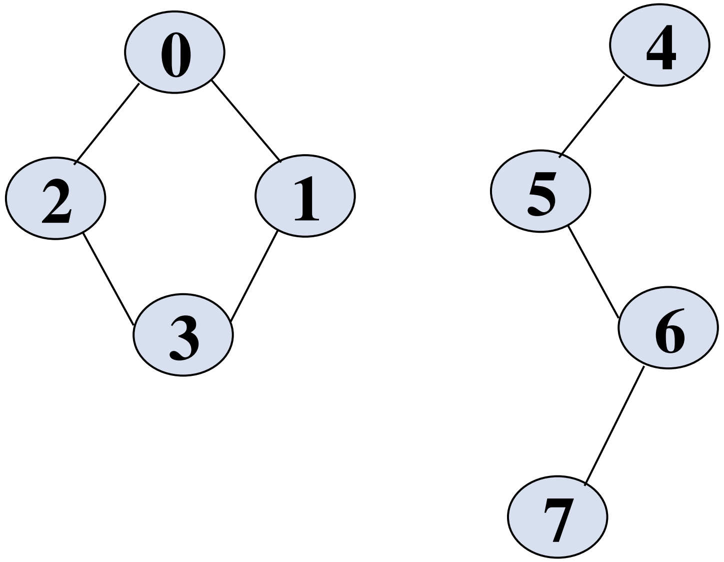
Examples for Graph



G1



G2



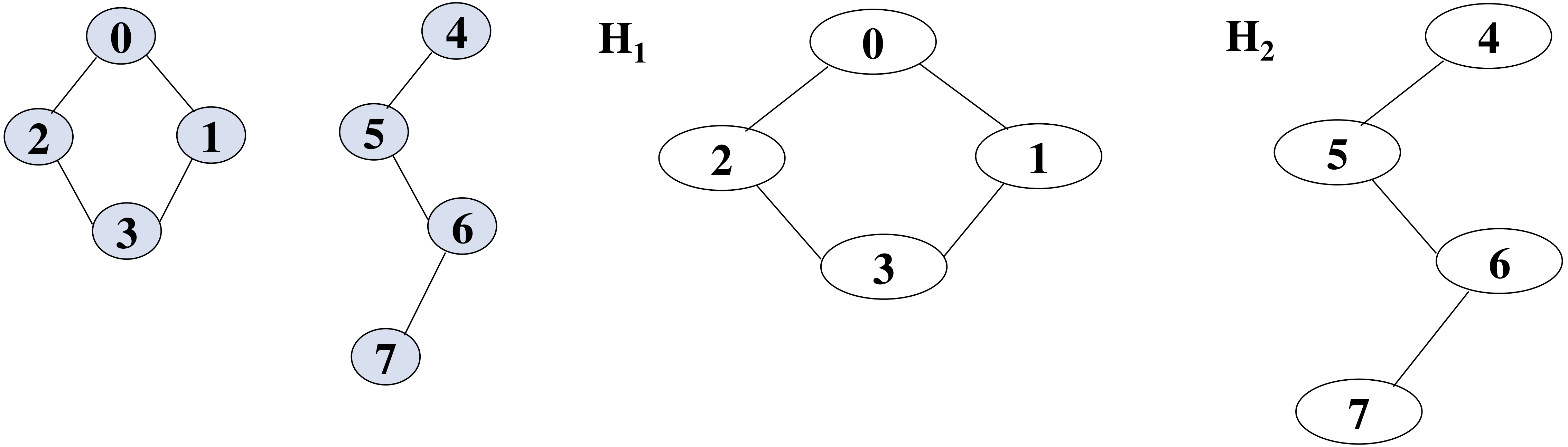
Connected Graphs: G_1 & G_2

Graph G_4 : (not connected)

Connected Component

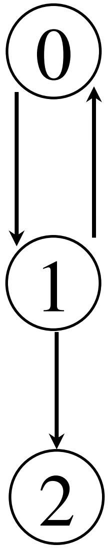
- A connected component of an undirected graph is a maximal connected subgraph.
- A tree is a graph that is connected and acyclic.
- A directed graph is strongly connected if there is a directed path from vi to vj and also from vj to vi.
- A strongly connected component is a maximal subgraph that is strongly connected.

Example for Connected Component

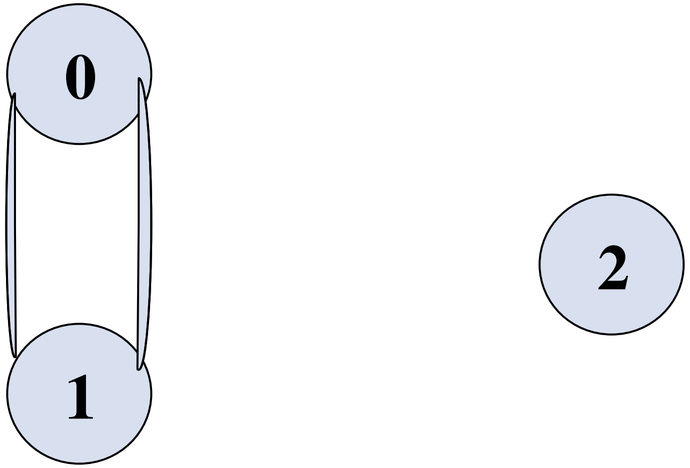


Two Connected Components for Graphs G_4: H_1 and H_2

Example for Strongly Connected Component



G3 (Not strongly connected)



Strongly connected components of G_3

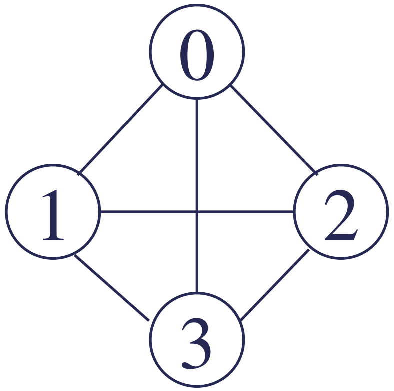
Graph Representation

- Adjacency Matrix
- Adjacency Lists

Adjacency Matrix

- Let G=(V,E) be a graph with n vertices.
- The adjacency matrix of G is a two-dimensional n by n array, say adj_mat
  - If the edge (vi, vj) is in E(G), adj_mat[i][j]=1
  - If there is no such edge in E(G), adj_mat[i][j]=0
- The adjacency matrix for an undirected graph is symmetric; the adjacency matrix for a digraph need not be symmetric

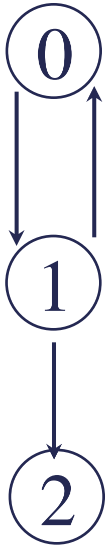
Adjacency Matrix



Graph G_1

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Adjacency Matrix for Graph G_1



Graph G_2

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Adjacency Matrix for Graph G_2

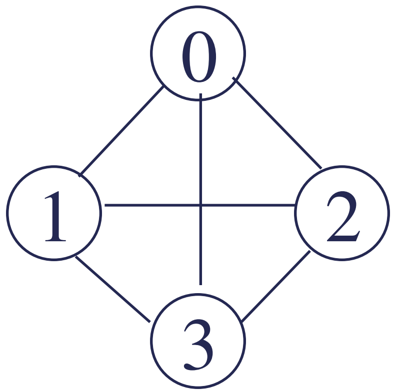
Merits: Adjacency Matrix

- From the adjacency matrix, to determine the connection of vertices is easy
- The degree of a vertex is $\sum_{j=0}^{n-1} adj\_mat[i][j]$
- For a digraph, the row sum is the out_degree, while the column sum is the in_degree

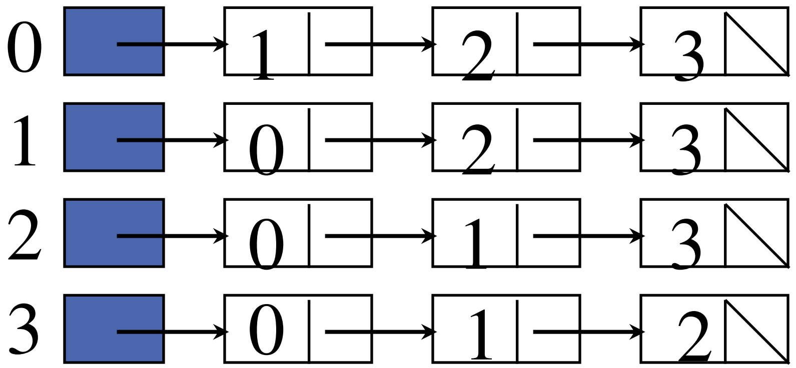
$$ind(vi) = \sum_{j=0}^{n-1} A[j,i] \quad outd(vi) = \sum_{j=0}^{n-1} A[i,j]$$

Adjacency Lists

```
class Gnode
{   int vertex;
    node *next;
   friend class Graph;
};
class Graph
{
private:
        Gnode *Head[20];
        int n;
public:
        Graph()
        {       create head nodes for n vertices
        }
};
```



Graph G_1



Adjacency List for Graph G_1

Adjacency List: Interesting Operations

- degree of a vertex in an undirected graph

  No. of nodes in adjacency list
- No. of edges in a graph

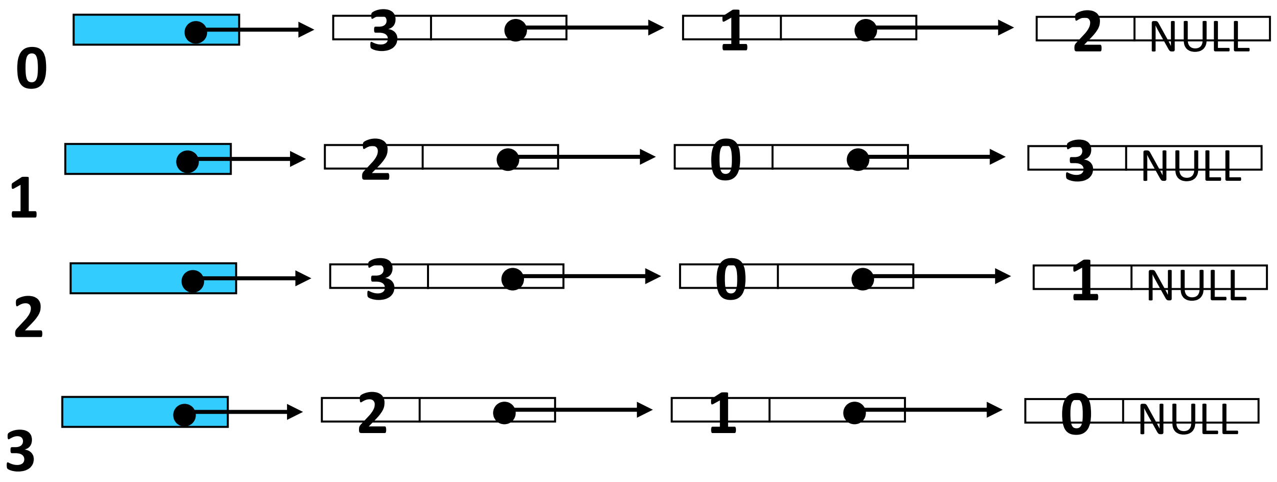
  determined in O(n+e)
- out-degree of a vertex in a directed graph

  No. of nodes in its adjacency list
- in-degree of a vertex in a directed graph

  traverse the whole data structure

```
graph()
{
  Accept no of vertices;
  for i=0 to n-1
     {Allocate a memory for head[i] node (array)
     head[i]->vertex=i; }
  }

create()
{
  for i=0 to n-1
  {
    temp=head[i];
    do
    {
      Accept adjacent vertex v;
      if(v==i)
          Print Self loop are not allowed;
      else
      {
       Allocate  memory for curr node;
         curr->vertex=v;
         temp->next=curr;
         temp=temp->next;
       }
       accept the choice ;
    }while(ans=='y'||ans=='Y');
  }
}
```



Graph Traversal

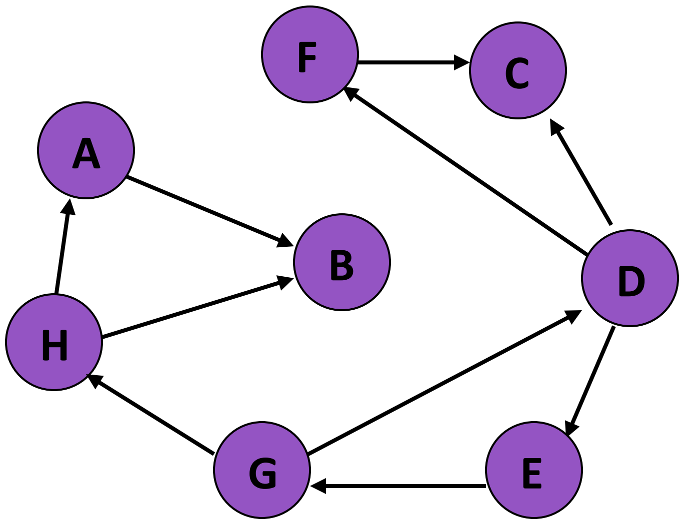
- Depth First Traversal
- Breadth First Traversal

Depth First Traversal (Recursive)

```
Algorithm DFS()
{
  //initially no vertex will be visited
        for(int i=0;i<n;i++)
                visited[i]=0;
//start search at vertex v
      accept starting vertex
        DFS(v);
}
```

```
Algorithm DFS(int v)
{       print v;
        visited[v]=1;
        for(each vertex w adjacent to v)
                if(!visited[w])
                        DFS(w);
}
```

Depth First Search Traversal

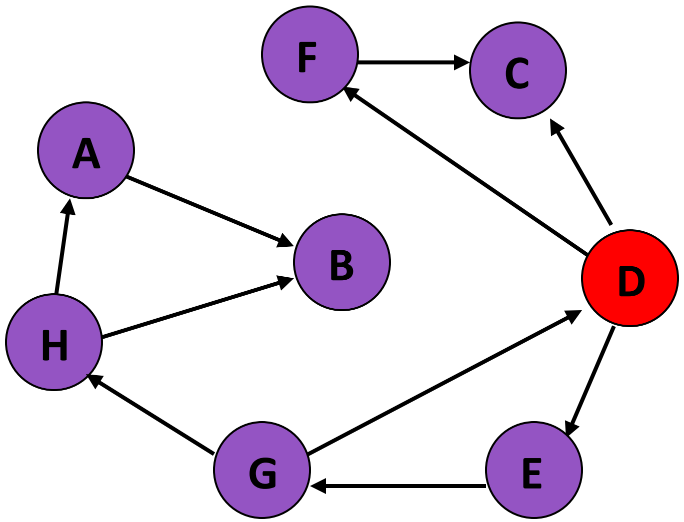


Visited Array

A	
B	
C	
D	
E	
F	
G	
H	

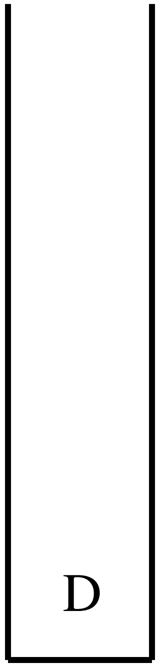
Task: Conduct a depth-first search of the graph starting with node D

Depth First SearchTraversal



Visited Array

A	
B	
C	
D	1
E	
F	
G	
H	

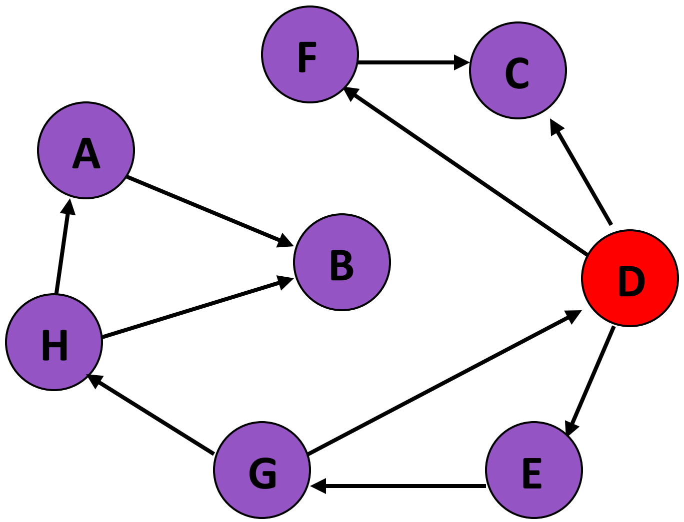


The DFT of nodes in graph :

D

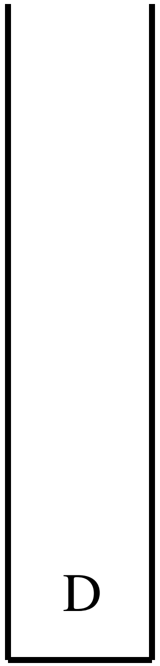
Visit D

Depth First Traversal



Visited Array

A	
B	
C	
D	1
E	
F	
G	
H	

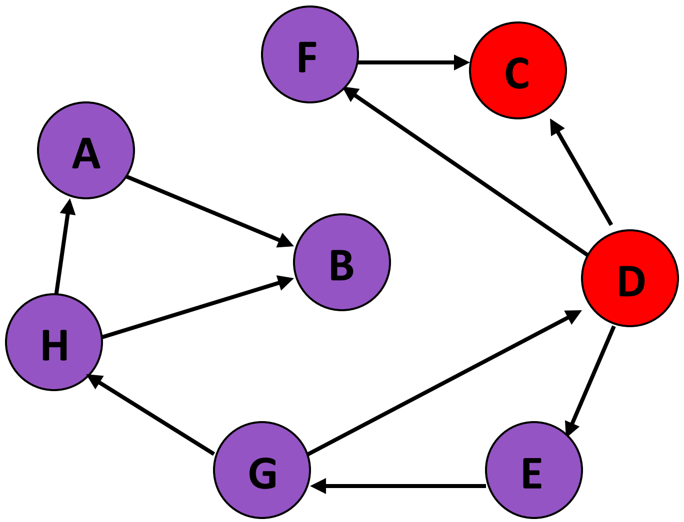


The DFT of nodes in graph :

D

Consider nodes adjacent to D, decide to visit C first (Rule: visit adjacent nodes in alphabetical order or in order of the adjacency list)

Depth First Traversal



Visited Array

A	
B	
C	1
D	1
E	
F	
G	
H	

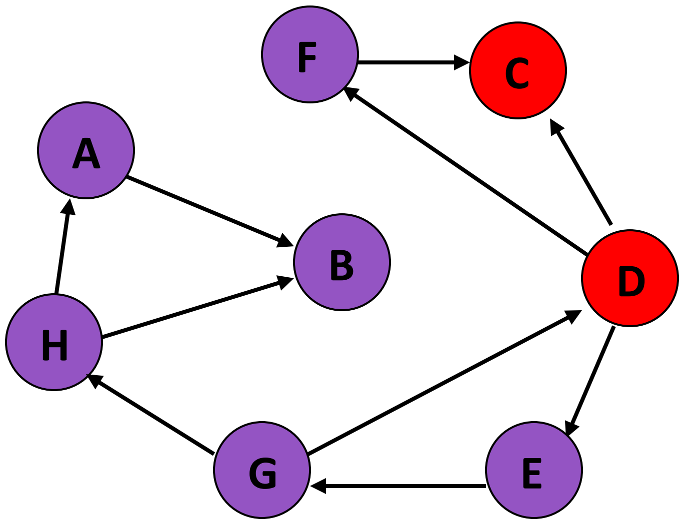
C
D

Visit C

The DFT of nodes in graph :

D, C

Depth First Traversal



Visited Array

A	
B	
C	1
D	1
E	
F	
G	
H	

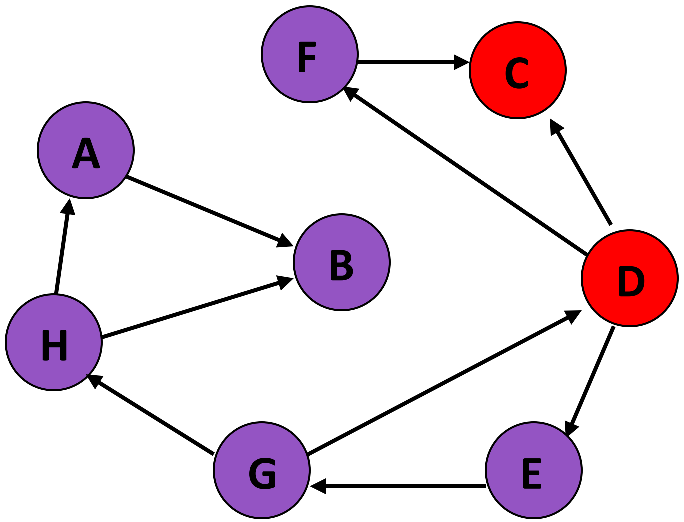
C
D

The DFT of nodes in graph :

D, C

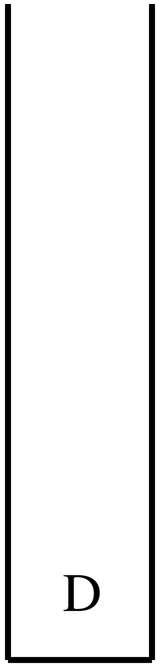
No nodes adjacent to C; cannot continue *backtrack*, i.e., pop stack and restore previous state

Depth First Traversal



Visited Array

A	
B	
C	1
D	1
E	
F	
G	
H	

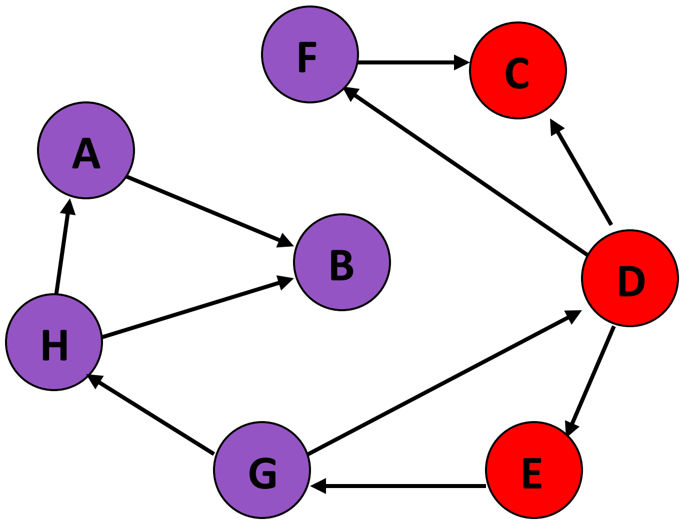


The DFT of nodes in graph :

D, C

Back to D – C has been visited, decide to visit E next

Depth First Traversal



Visited Array

A	
B	
C	1
D	1
E	1
F	
G	
H	

E

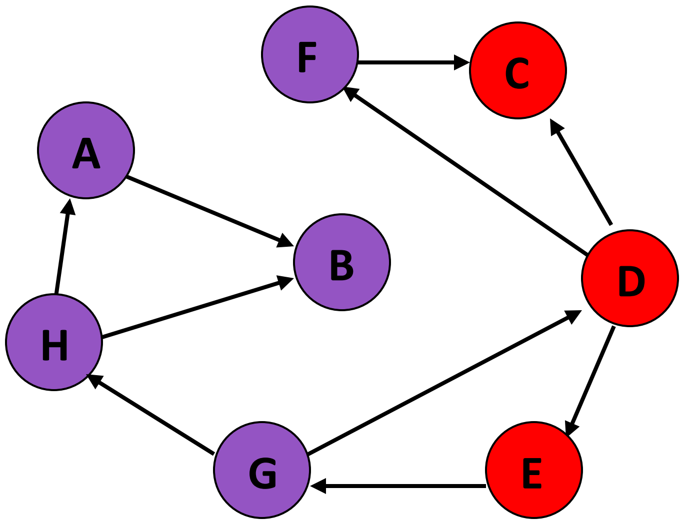
D

The DFT of nodes in graph :

D, C, E

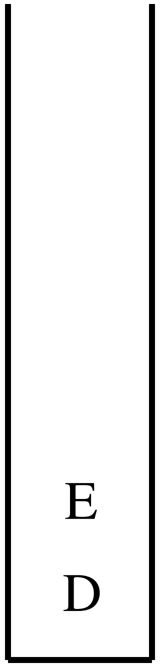
Back to D – C has been visited, decide to visit E next

Depth First Traversal



Visited Array

A	
B	
C	1
D	1
E	1
F	
G	
H	

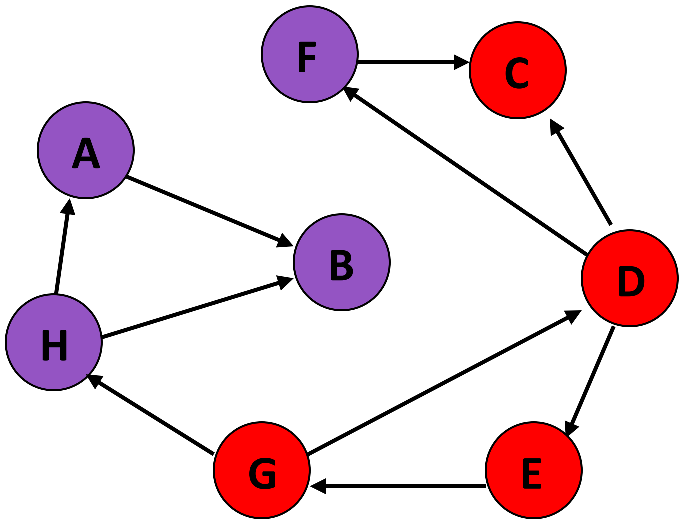


The DFT of nodes in graph :

D, C, E

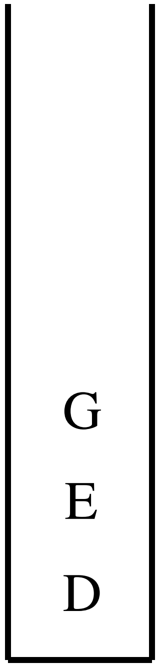
Only G is adjacent to E

Depth First Traversal



Visited Array

A	
B	
C	1
D	1
E	1
F	
G	1
H	

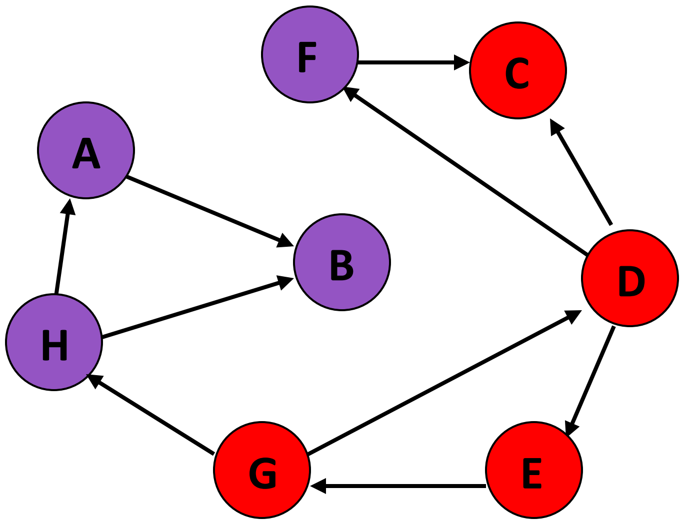


Visit G

The DFT of nodes in graph :

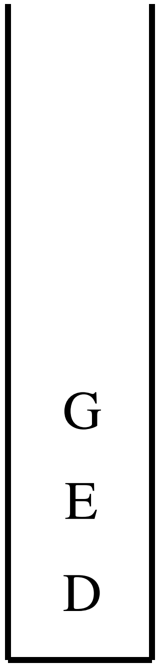
D, C, E, G

Depth First Traversal



Visited Array

A	
B	
C	**1**
D	**1**
E	**1**
F	
G	1
H	

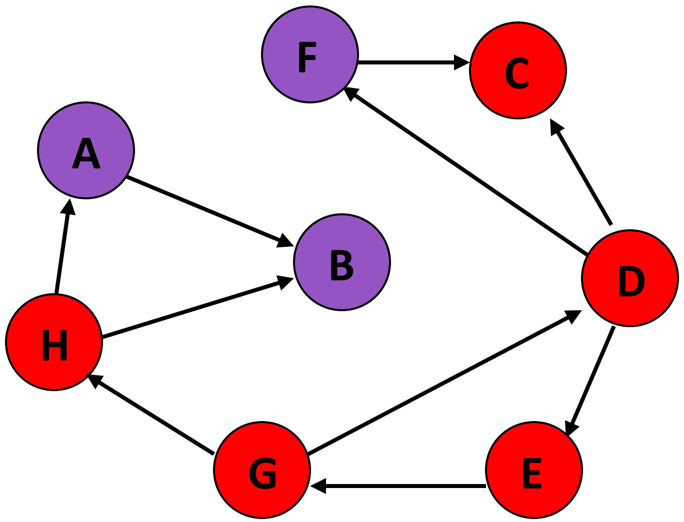


The DFT of nodes in graph :

D, C, E, G

Nodes D and H are adjacent to G. D has already been visited. Decide to visit H.

Depth First Traversal



Visited Array

A	
B	
C	1
D	1
E	1
F	
G	1
H	1

H

G

E

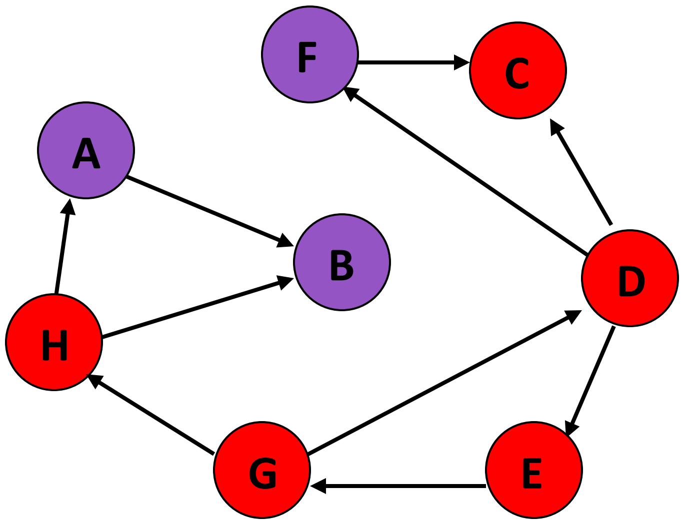
D

Visit H

The DFT of nodes in graph :

D, C, E, G, H

Depth First Traversal



Visited Array

A	
B	
C	1
D	1
E	1
F	
G	1
H	1

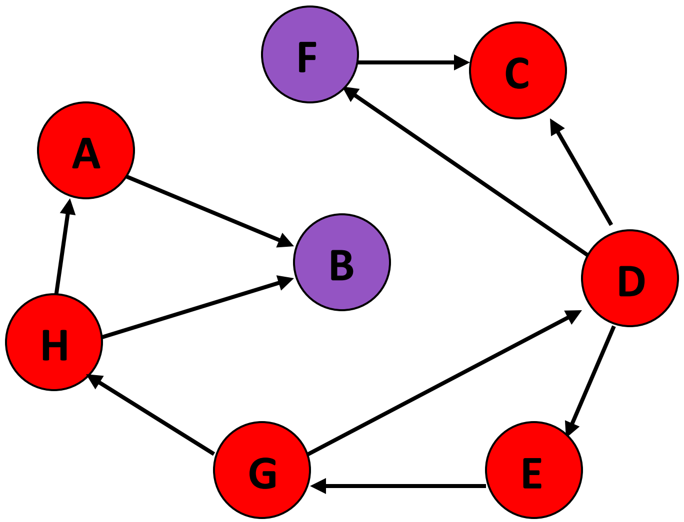
H
G
E
D

The DFT of nodes in graph :

D, C, E, G, H

Nodes A and B are adjacent to F. Decide to visit A next.

Depth First Traversal



Visited Array

A	1
B	
C	1
D	1
E	1
F	
G	1
H	1

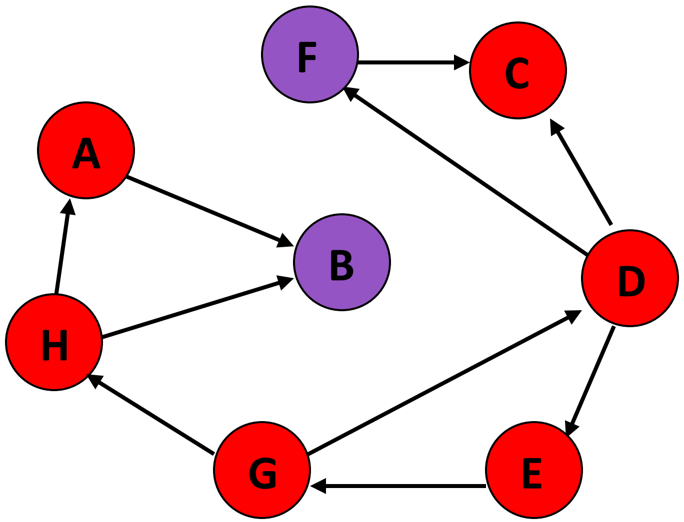
A
H
G
E
D

Visit A

The DFT of nodes in graph :

D, C, E, G, H, A

Depth First Traversal



The DFT of nodes in graph :

D, C, E, G, H, A

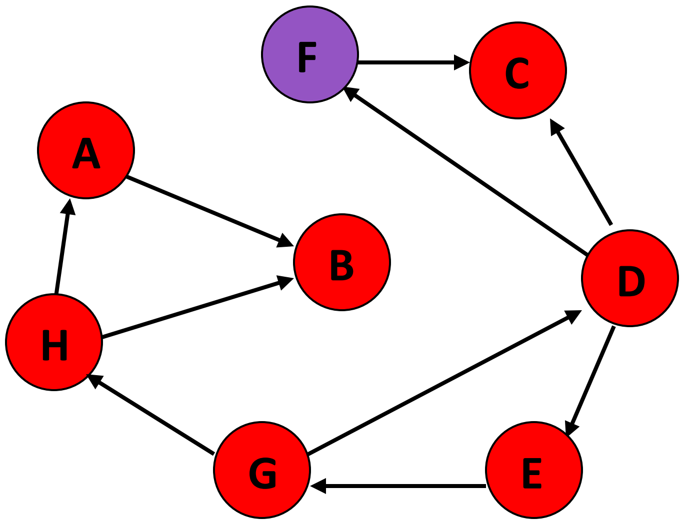
Visited Array

A	1
B	
C	1
D	1
E	1
F	
G	1
H	1

A
H
G
E
D

Only Node B is adjacent to A. Decide to visit B next.

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1

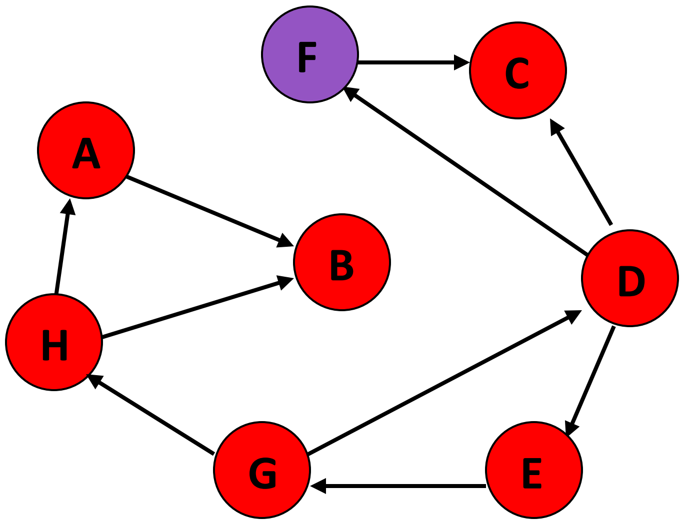
B
A
H
G
E
D

Visit B

The DFT of nodes in graph :

D, C, E, G, H, A, B

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1

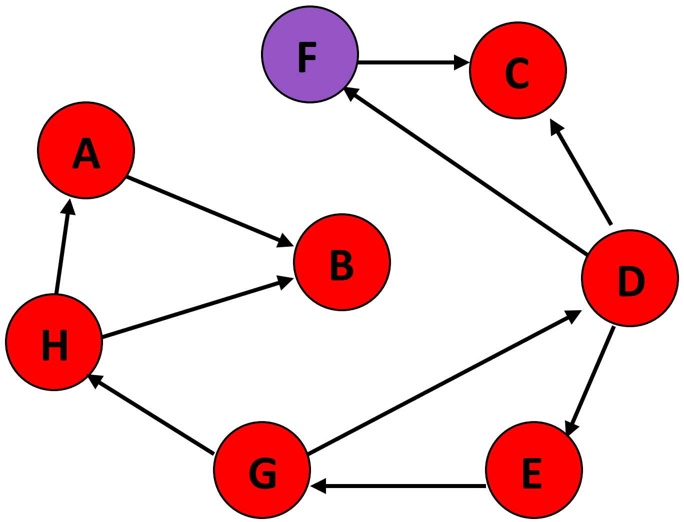
A
H
G
E
D

The DFT of nodes in graph :

D, C, E, G, H, A, B

No unvisited nodes adjacent to B. Backtrack (pop the stack).

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1

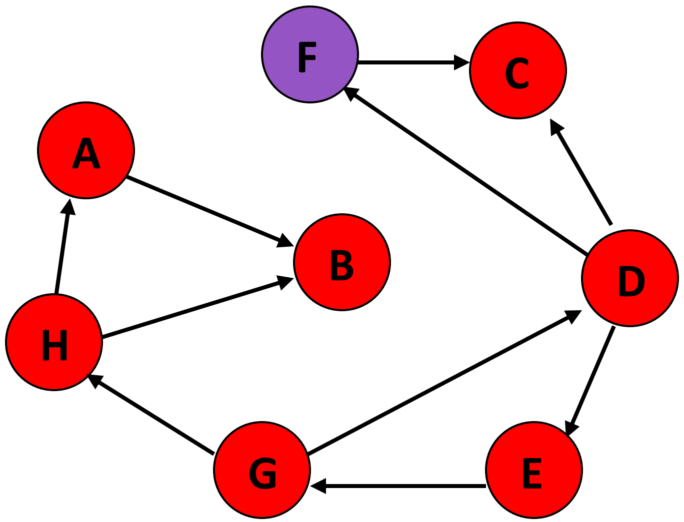
H
G
E
D

The DFT of nodes in graph :

D, C, E, G, H, A, B

No unvisited nodes adjacent to A. Backtrack (pop the stack).

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1

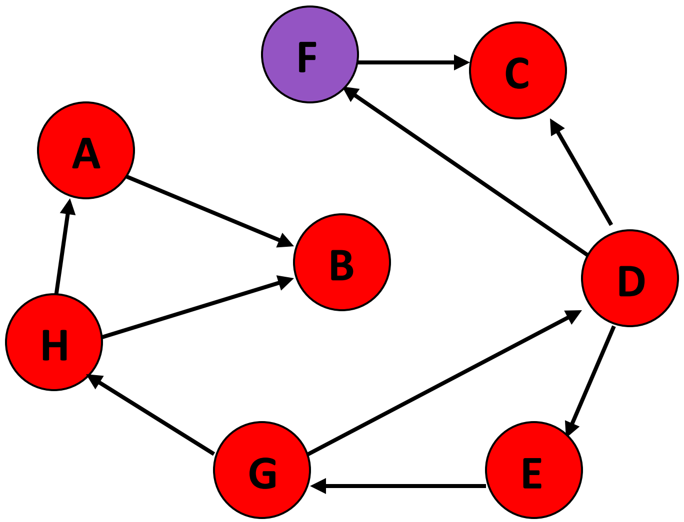
G
E
D

The DFT of nodes in graph :

D, C, E, G, H, A, B

No unvisited nodes adjacent to H. Backtrack (pop the stack).

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1

E

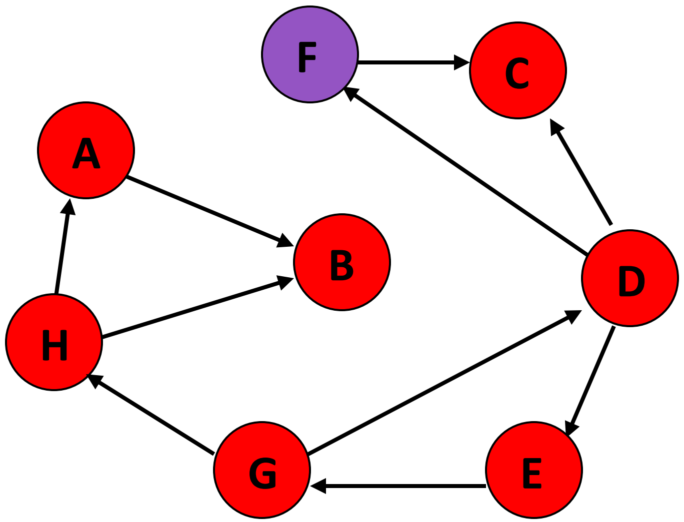
D

The DFT of nodes in graph :

D, C, E, G, H, A, B

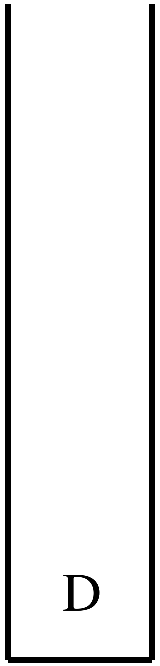
No unvisited nodes adjacent to G. Backtrack (pop the stack).

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1

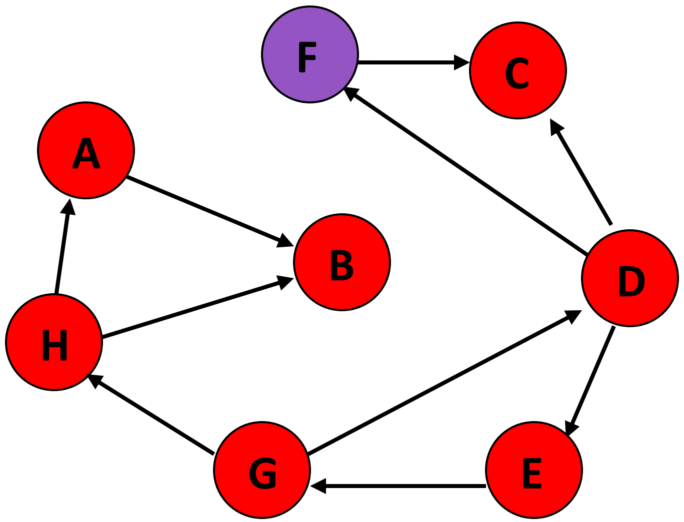


The DFT of nodes in graph :

D, C, E, G, H, A, B

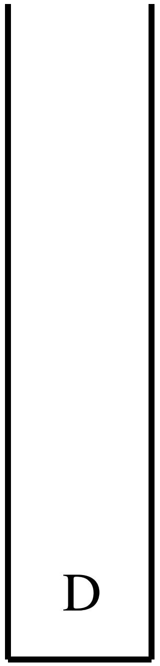
No unvisited nodes adjacent to E. Backtrack (pop the stack).

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1

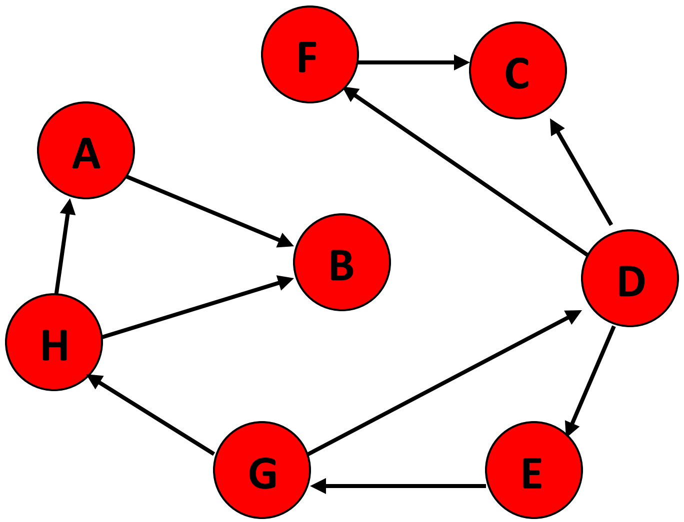


The DFT of nodes in graph :

D, C, E, G, H, A, B

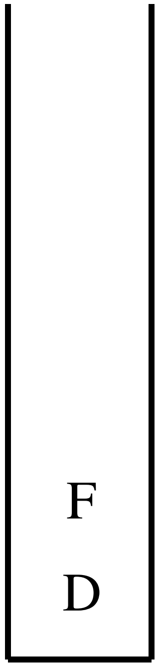
F is unvisited and is adjacent to D. Decide to visit F next.

Depth First Traversal



Visited Array

A	**1**
B	**1**
C	**1**
D	**1**
E	**1**
F	1
G	1
H	**1**

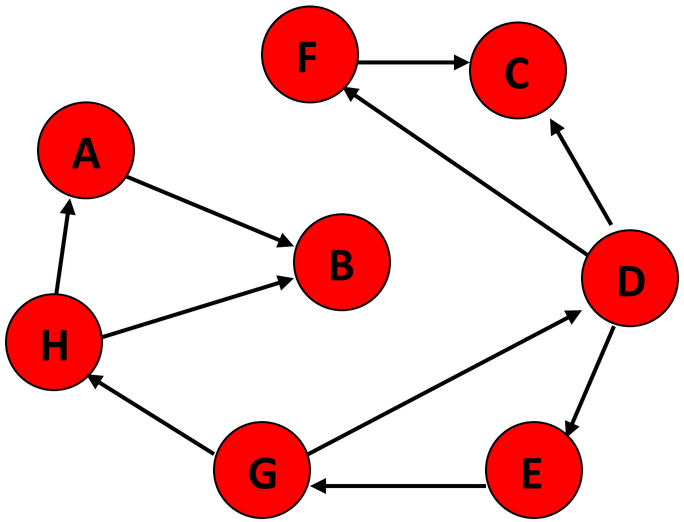


Visit F

The DFT of nodes in graph :

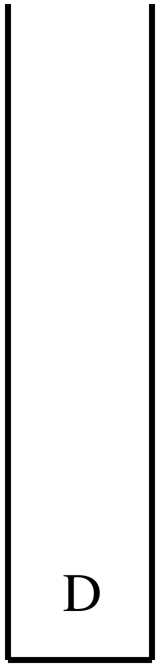
D, C, E, G, H, A, B, F

Depth First Traversal



Visited Array

A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

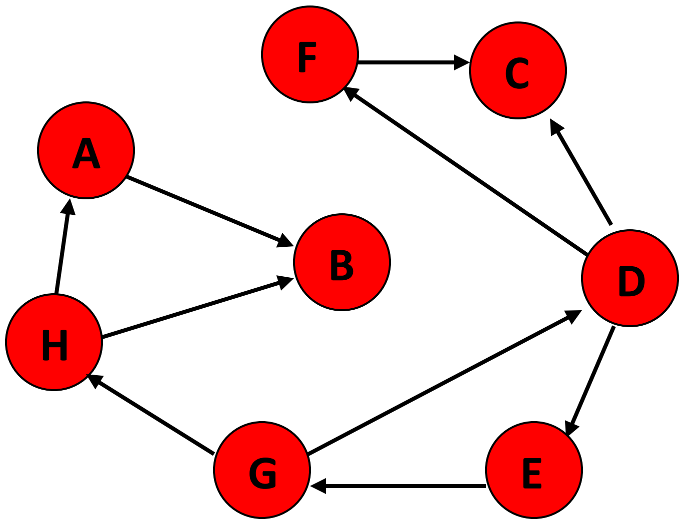


The DFT of nodes in graph :

D, C, E, G, H, A, B, F

No unvisited nodes adjacent to F. Backtrack.

Depth First Traversal



Visited Array

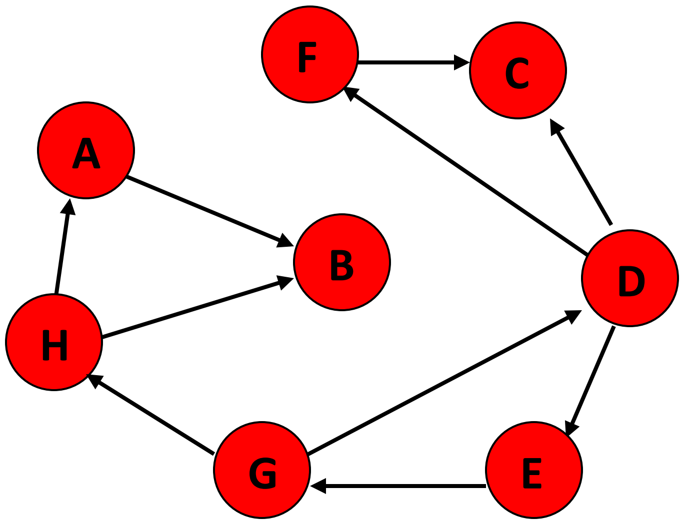
A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

The order nodes are visited:

D, C, E, G, H, A, B, F

No unvisited nodes adjacent to D. Backtrack.

Depth First Traversal



Visited Array

A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

The DFT of nodes in graph :

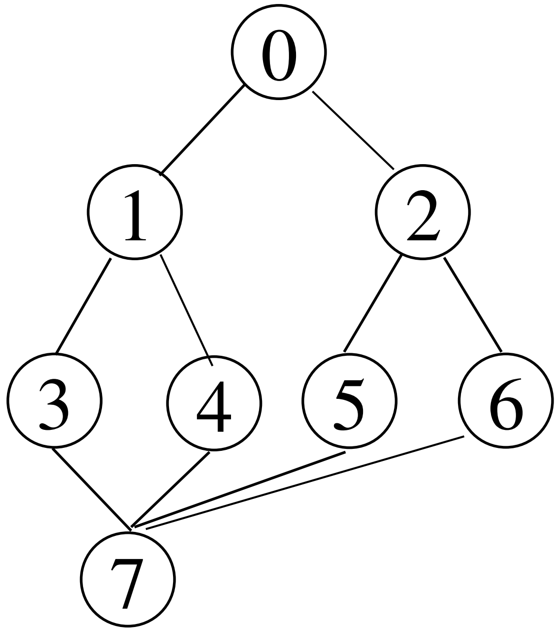
D, C, E, G, H, A, B, F

Stack is empty. Depth-first traversal is done.

Depth First Traversal (Non-recursive)

```
Algorithm DFS(int v)
{
  for all vertices of graph
        visited[i]=0;
   push(v);
   visited[v]=1;
   do
    {
      v=pop();
     print(v);
      for(each vertex w adjacent to v)
 {
           if(!visited[w])
              {  push(w); visited[w]=1;}
 } //end for
 } while(stack not empty)
 } //end dfs
```

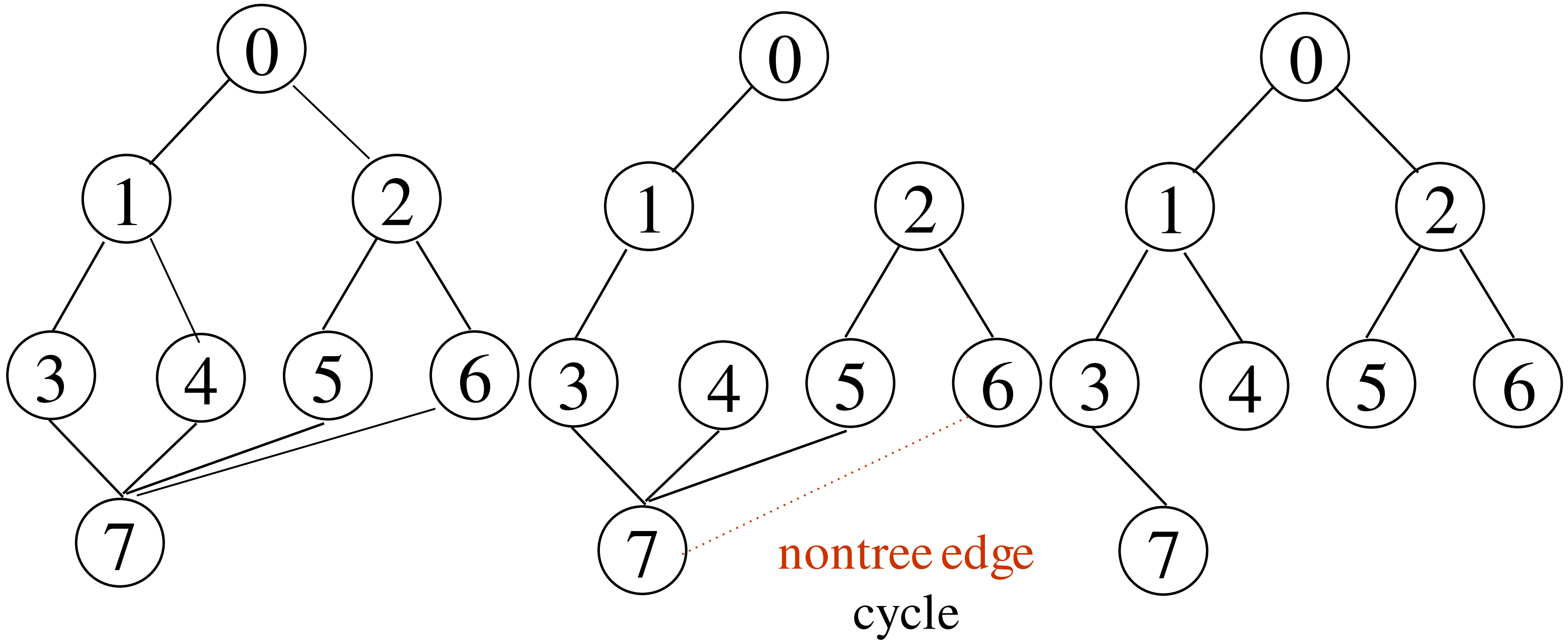
Depth First Traversal



Graph G1

Find DFT for given graph G1 starting at vertex 0

Home Assignment



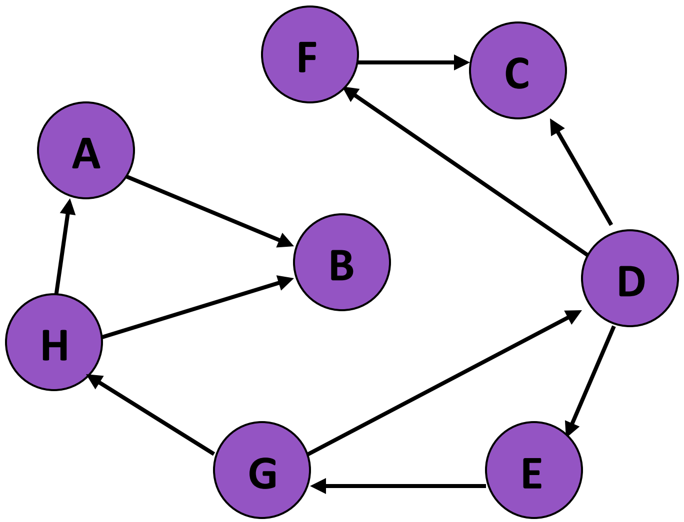
DFS Spanning

BFS Spanning

Breadth First Traversal

```
Algorithm BFS(int v) {
 for(int i=0;i<n;i++)
       visited[i]=0;
         Queue  q;
         q.insert(v);
         while(!q.IsEmpty())
         {
                   v=q.Delete(v);
                   for(all vertices w adjacent to v)
                   if(!vsited[w])
                   {
                             q.insert(w);
                             visited[w]=true;
                   }
         }
}
```

Breadth First Traversal



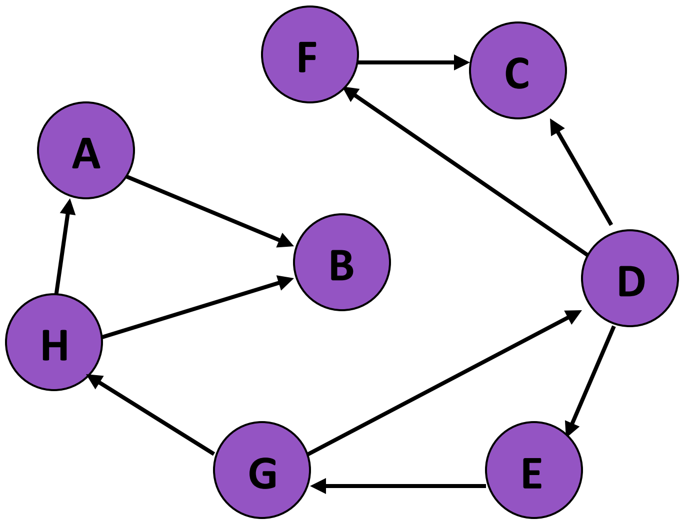
Enqueued Array

A	
B	
C	
D	
E	
F	
G	
H	

Q □

How is this accomplished? Simply replace the stack with a queue! Rules: (1) Maintain an *enqueued* array. (2) Visit node when *dequeued*.

Breadth First Traversal



Enqueued Array

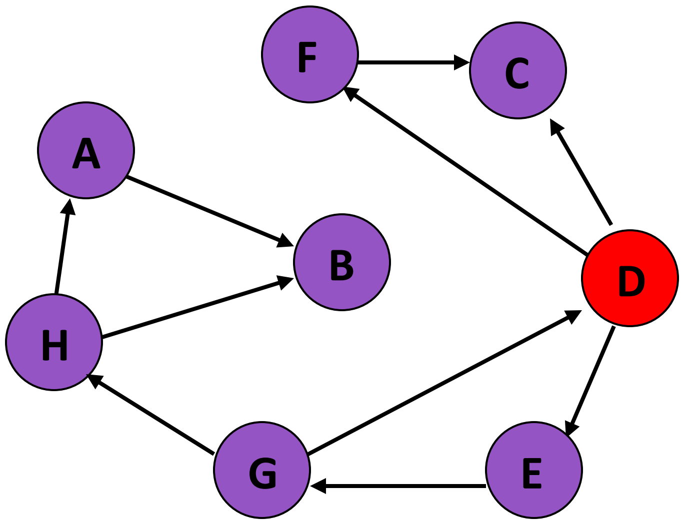
A	
B	
C	
D	√
E	
F	
G	
H	

Q D

Nodes visited:

Enqueue D. Notice, D not yet visited.

Breadth First Traversal



Enqueued Array

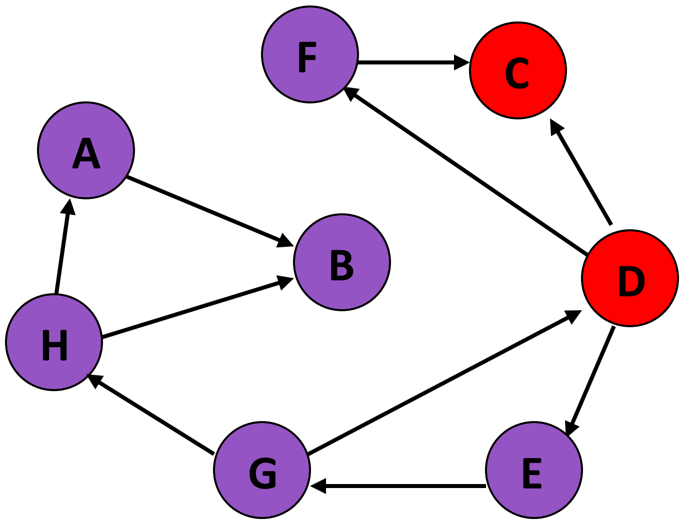
A	
B	
C	√
D	√
E	√
F	√
G	
H	

Q □ C □ E □ F

Nodes visited: D

Dequeue D. Visit D. Enqueue unenqueued nodes adjacent to D.

Breadth First Traversal



Enqueued Array

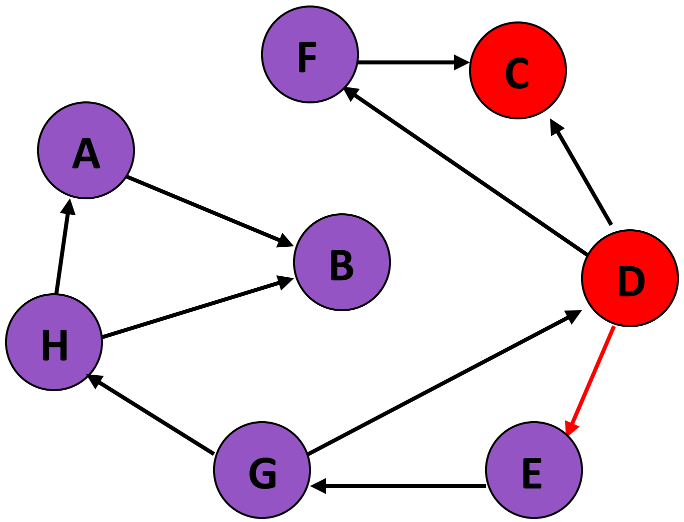
A	
B	
C	√
D	√
E	√
F	√
G	
H	

Q □ E □ F

Nodes visited: D, C

Dequeue C. Visit C. Enqueue unenqueued nodes adjacent to C.

Breadth First Traversal



Enqueued Array

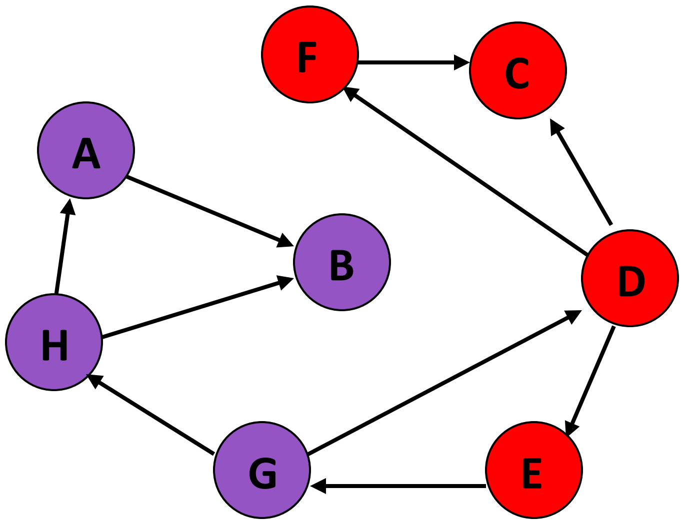
A	
B	
C	√
D	√
E	√
F	√
G	
H	

Q □ F □ G

Nodes visited: D, C, E

Dequeue E. Visit E. Enqueue unenqueued nodes adjacent to E.

Breadth First Traversal



Enqueued Array

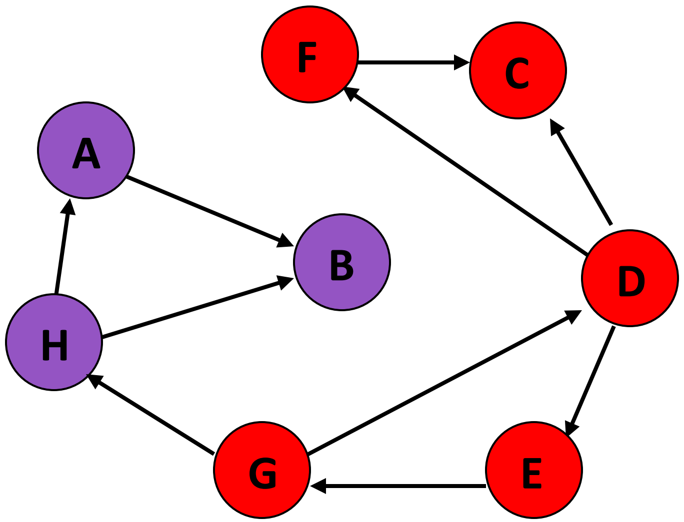
A	
B	
C	√
D	√
E	√
F	√
G	√
H	

Q □ G

Nodes visited: D, C, E, F

Dequeue F. Visit F. Enqueue unenqueued nodes adjacent to F.

Breadth First Traversal



Enqueued Array

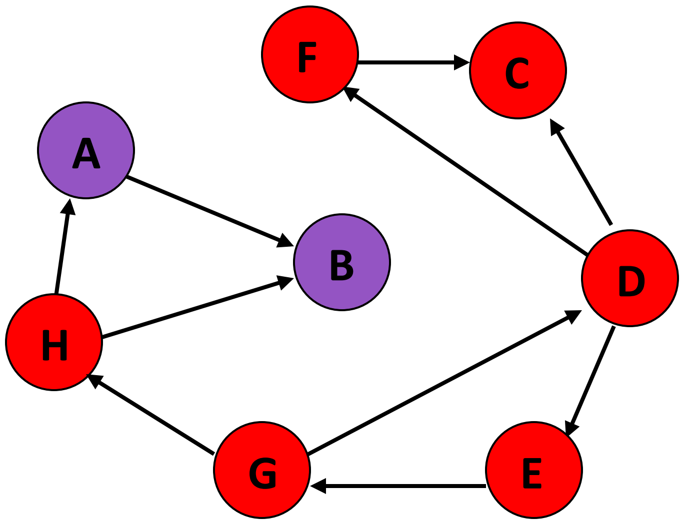
A	
B	
C	√
D	√
E	√
F	√
G	√
H	√

Q □ H

Nodes visited: D, C, E, F, G

Dequeue G. Visit G. Enqueue unenqueued nodes adjacent to G.

Breadth First Traversal



Enqueued Array

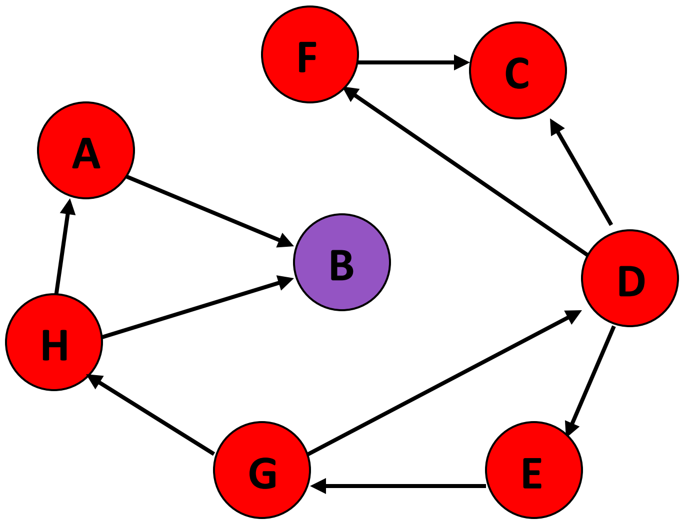
A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

Q □ A □ B

Nodes visited: D, C, E, F, G, H

Dequeue H. Visit H. Enqueue unenqueued nodes adjacent to H.

Breadth First Traversal



Enqueued Array

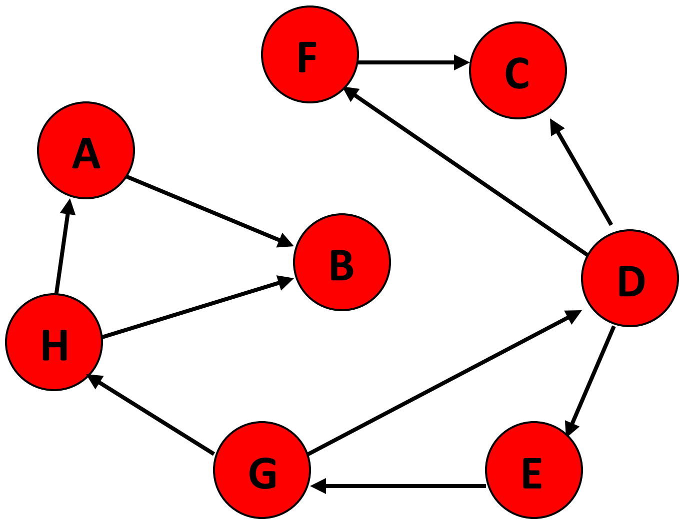
A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

Q □ B

Nodes visited: D, C, E, F, G, H, A

Dequeue A. Visit A. Enqueue unenqueued nodes adjacent to A.

Breadth First Traversal



Enqueued Array

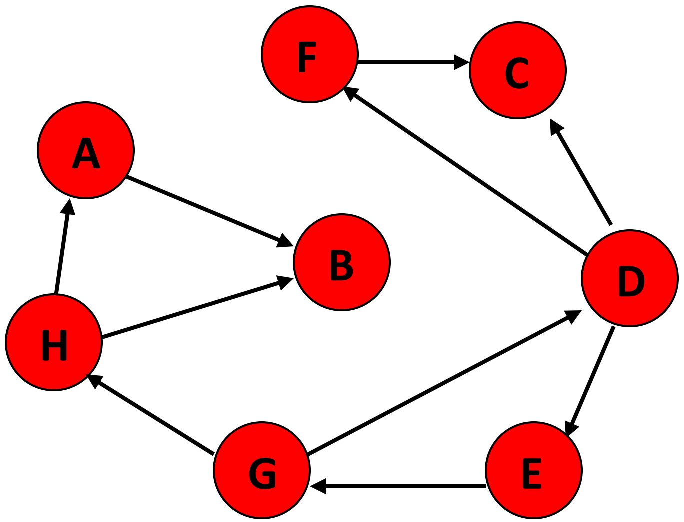
A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

Q empty

Nodes visited: D, C, E, F, G, H, A, B

Dequeue B. Visit B. Enqueue unenqueued nodes adjacent to B.

Breadth First Traversal



Enqueued Array

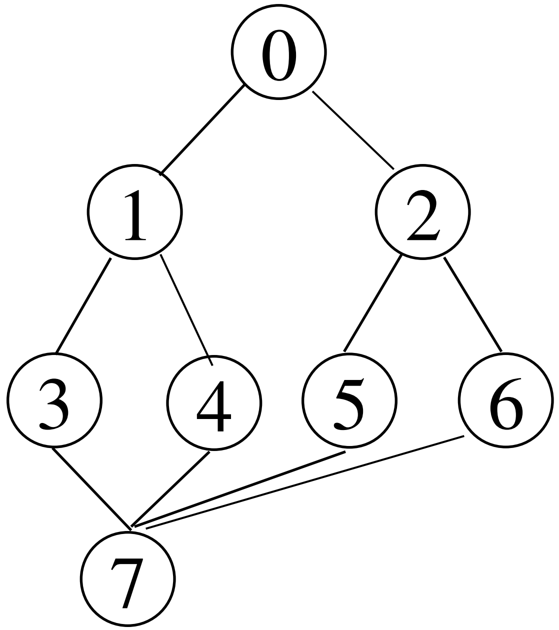
A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

Q empty

Nodes visited: D, C, E, F, G, H, A, B

Q empty. Algorithm done.

Breadth First Traversal



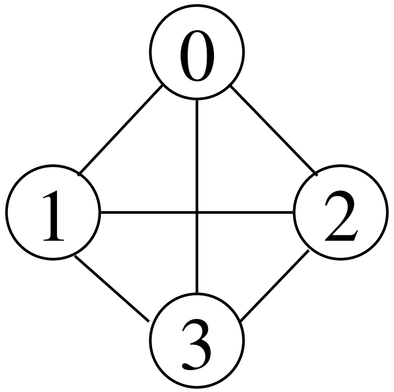
Graph G1

Find BFT for given graph G1 starting at vertex 0

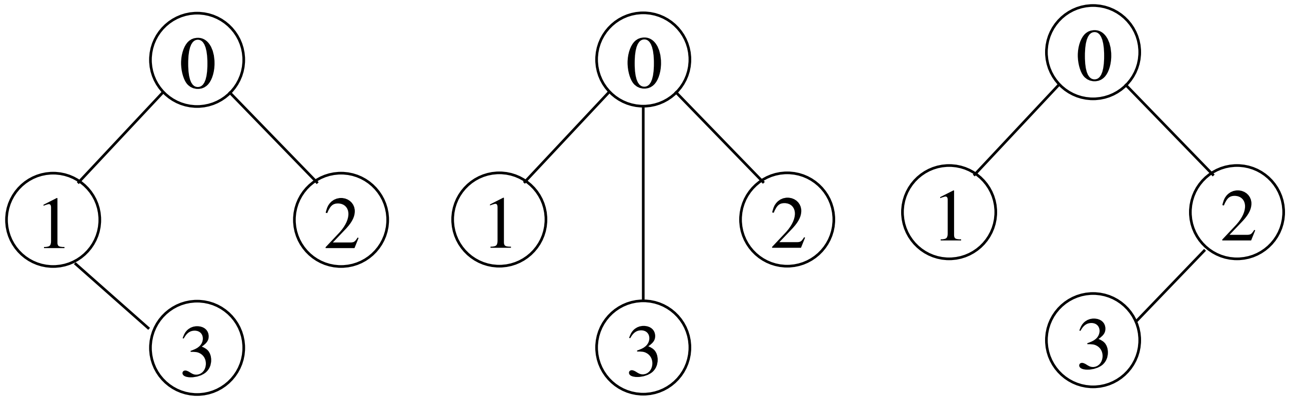
Spanning Trees

- A spanning tree is any tree that consists solely of edges in G and that includes all the vertices
- A spanning tree is a minimal subgraph, G', of G such that V(G')=V(G) and G' is connected.
- E(G): T (tree edges) + N (nontree edges)
  where T: set of edges used during search
  N: set of remaining edges
- Either dfs or bfs can be used to create a spanning tree
  - When dfs is used, the resulting spanning tree is known as a depth first spanning tree
  - When bfs is used, the resulting spanning tree is known as a breadth first spanning tree

Examples of Spanning Trees

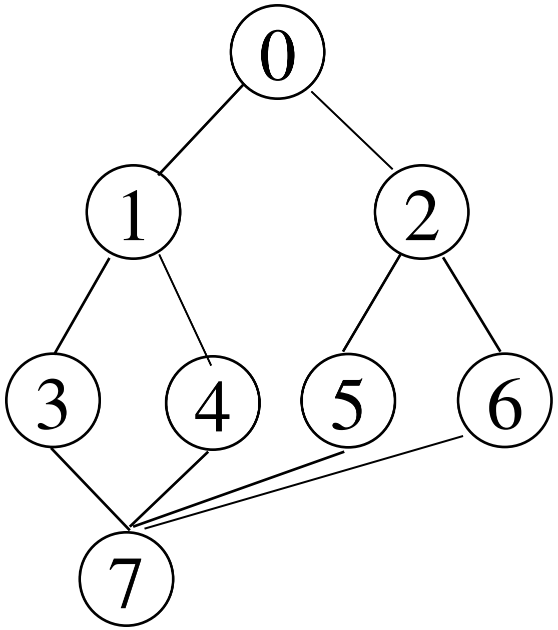


Graph G_1

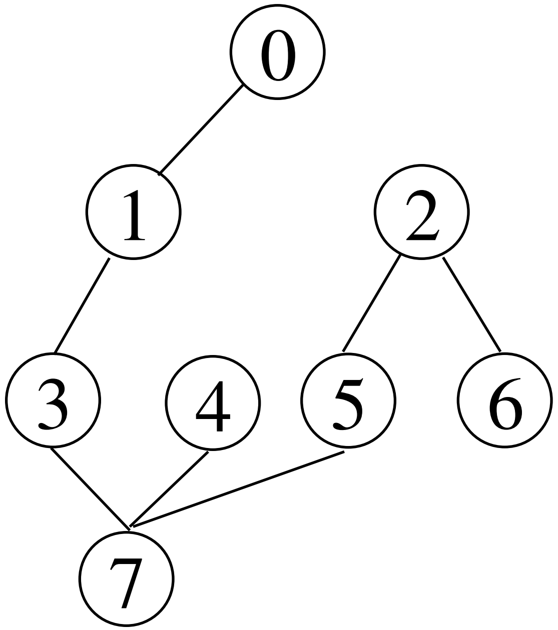


Possible spanning trees

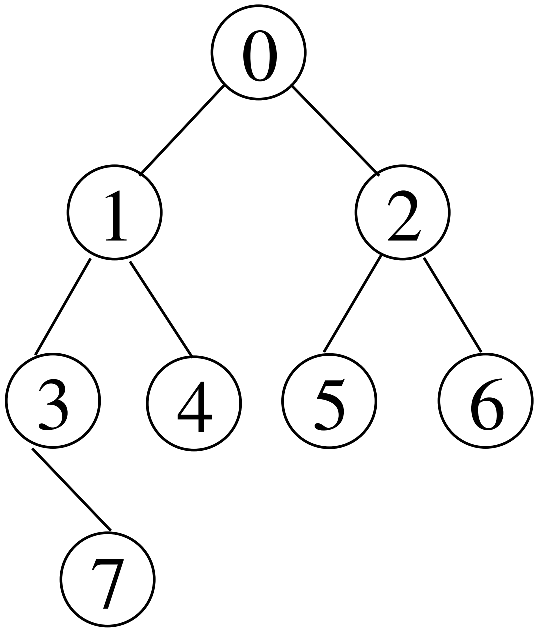
DFS VS BFS Spanning Trees



Graph



DFS Spanning Tree



BFS Spanning Tree

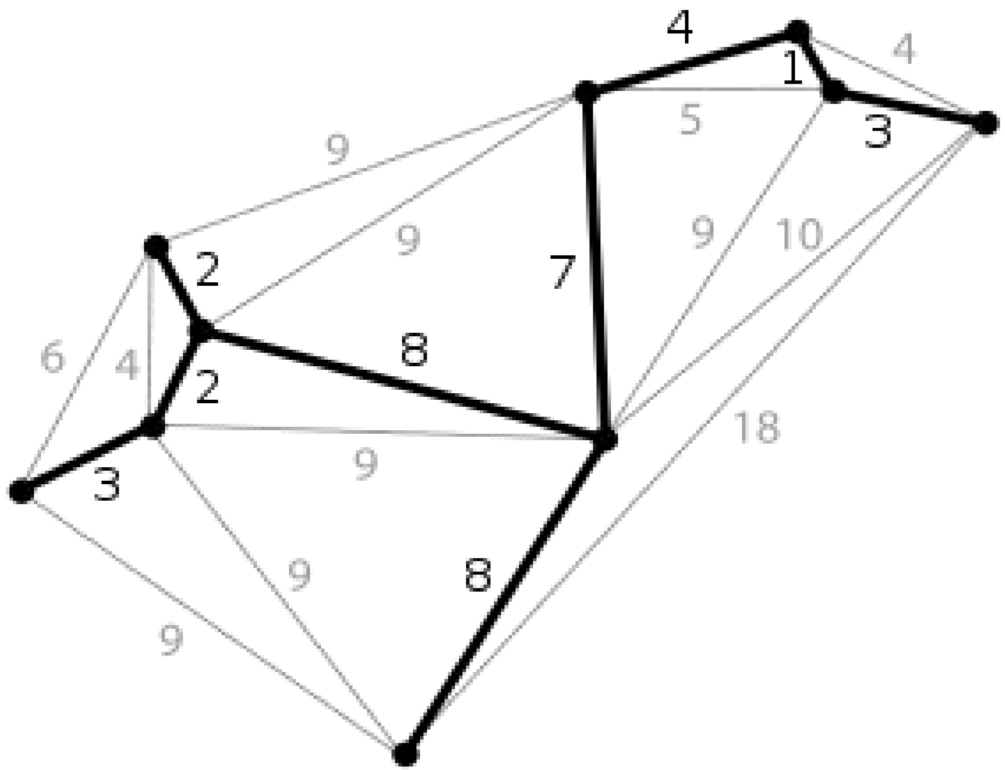
Minimum Spanning Tree

- The cost of a spanning tree of a weighted undirected graph is the sum of the costs of the edges in the spanning tree
- A minimum cost spanning tree is a spanning tree of least cost
- Two different algorithms can be used
  - Kruskal
  - Prim

Select n-1 edges from a weighted graph of n vertices with minimum cost.

Minimum Spanning Tree

- Applications of MST in Network design
  - *Telephone*
  - *Electrical*
  - *TV cable*
  - *Computer*
  - *road*



Greedy Strategy

- An optimal solution is constructed in stages
- At each stage, the best decision is made at this time
- Since this decision cannot be changed later, we make sure that the decision will result in a feasible solution
- Typically, the selection of an item at each stage is based on a least cost or a highest profit criterion

Kruskal's Algorithm

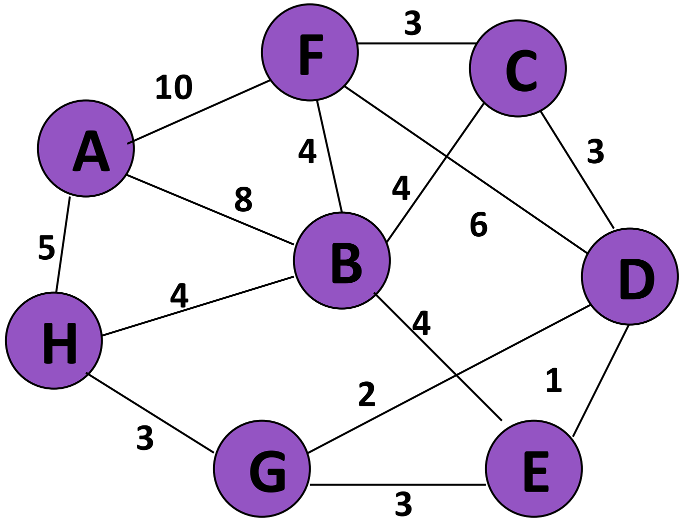
- Build a minimum cost spanning tree T by adding edges to T one at a time
- Select the edges for inclusion in T in nondecreasing order of the cost
- An edge is added to T if it does not form a cycle
- Since G is connected and has $n > 0$ vertices, exactly $n-1$ edges will be selected

Kruskal's Algorithm

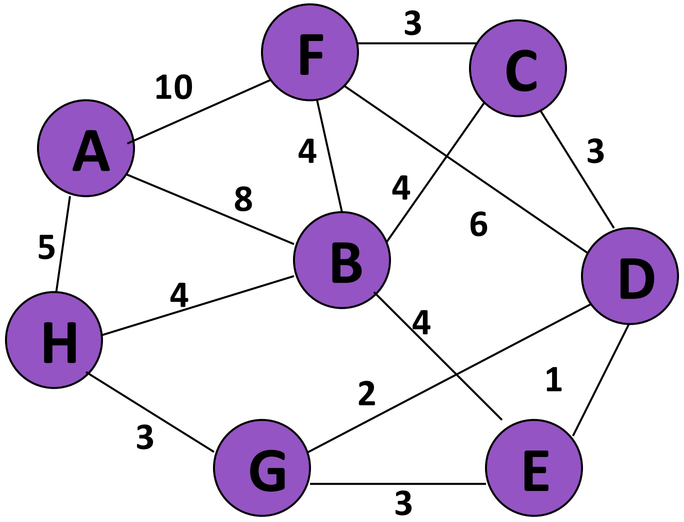
```
T= {};
while (T contains less than n-1 edges
       && E is not empty)
{
  choose a least cost edge (v,w) from E;
  delete (v,w) from E;
  if ((v,w) does not create a cycle in T)
    add (v,w) to T
 else
     discard (v,w);
}
if (T contains fewer than n-1 edges)
  printf("No spanning tree\n");
```

Kruskal's Algorithm

Consider an undirected, weight graph



Kruskal's Algorithm

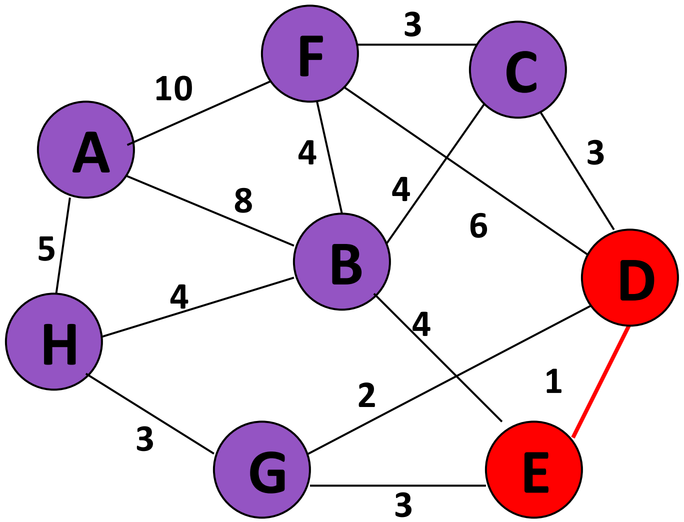


Sort the edges by increasing edge weight

edge	d_v	
(D,E)	1	
(D,G)	2	
(E,G)	3	
(C,D)	3	
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's Algorithm

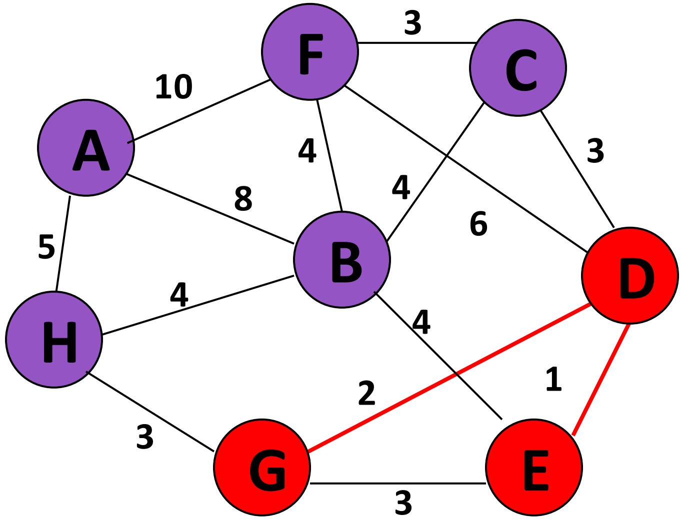


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	
(E,G)	3	
(C,D)	3	
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

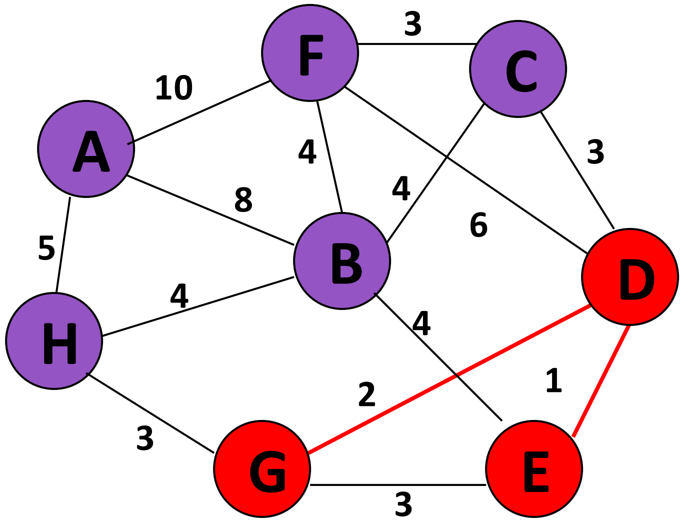


Select first |V|−1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	
(C,D)	3	
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's Algorithm



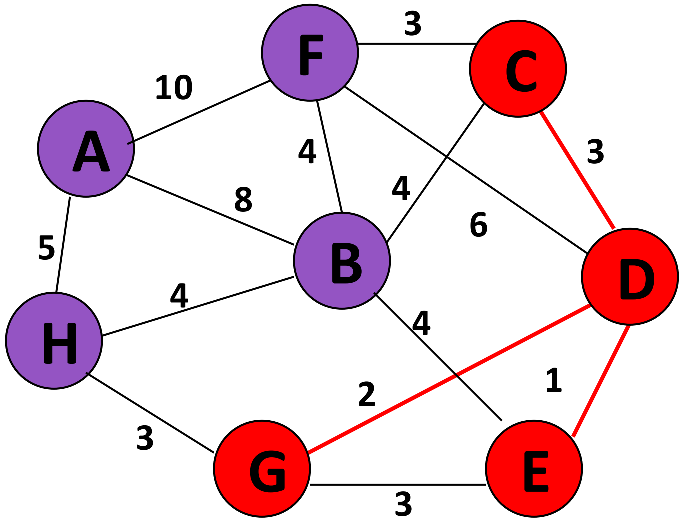
Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Accepting edge (E,G) would create a cycle

Kruskal's Algorithm

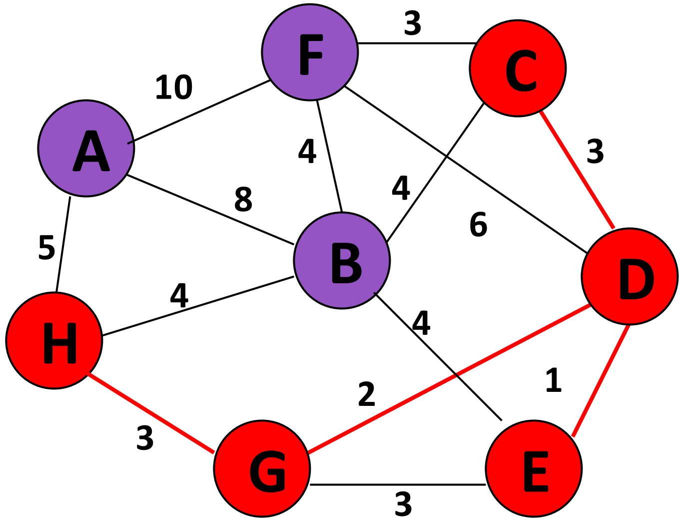


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's Algorithm

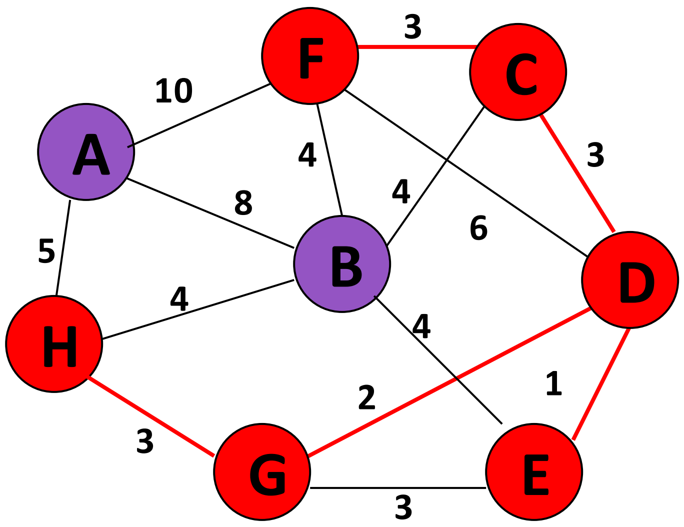


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

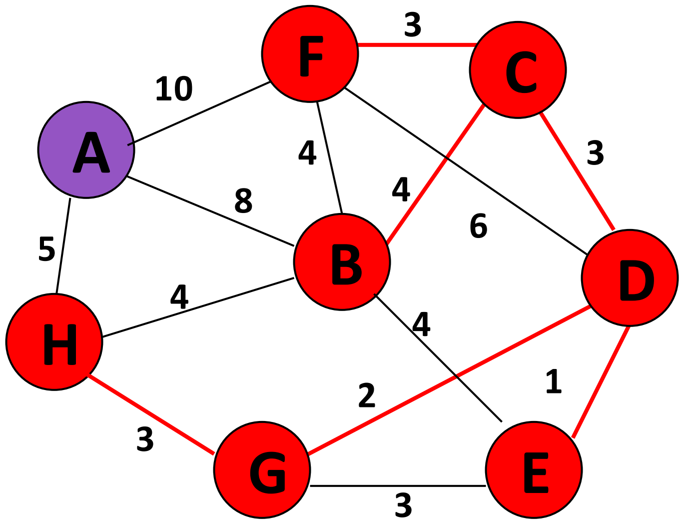


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's Algorithm

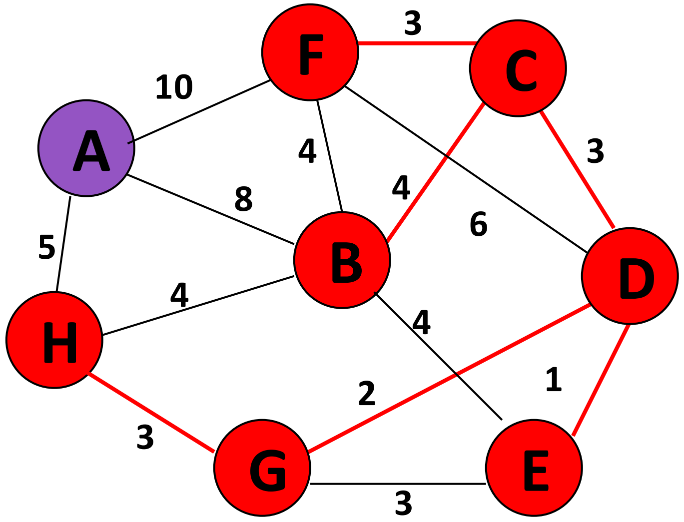


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's Algorithm

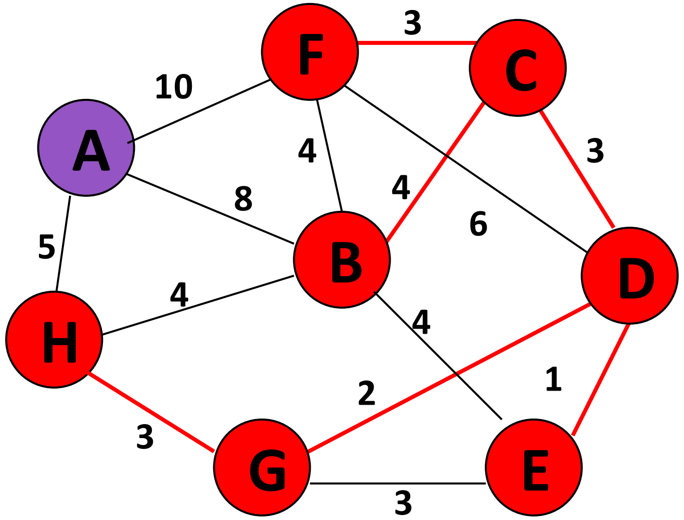


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

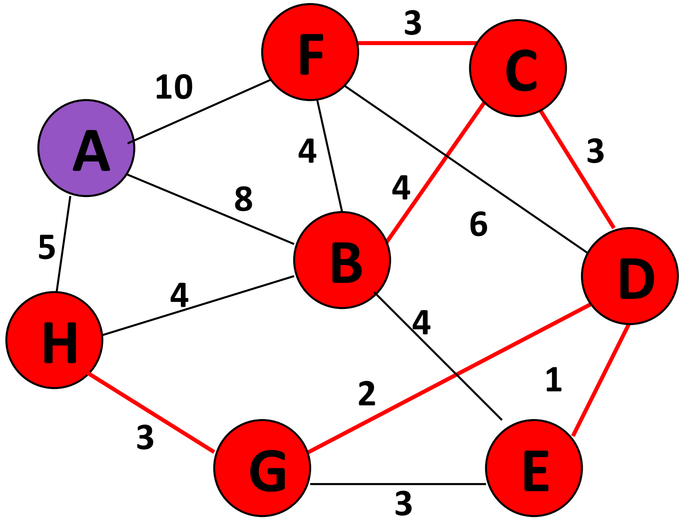


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	χ
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's Algorithm

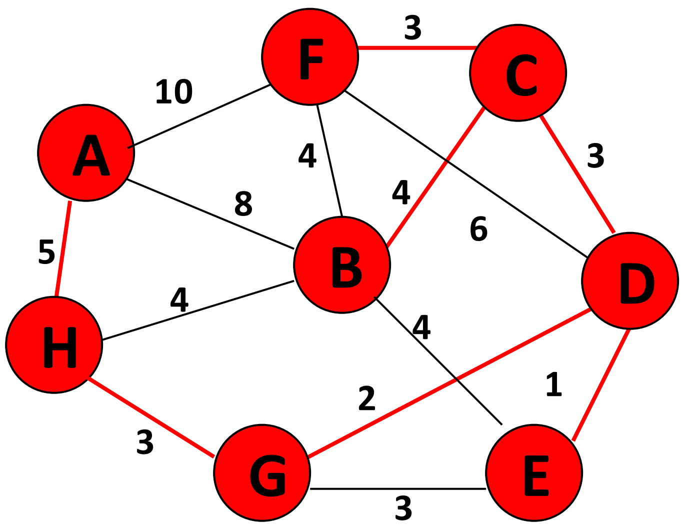


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	χ
(B,H)	4	χ
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's Algorithm



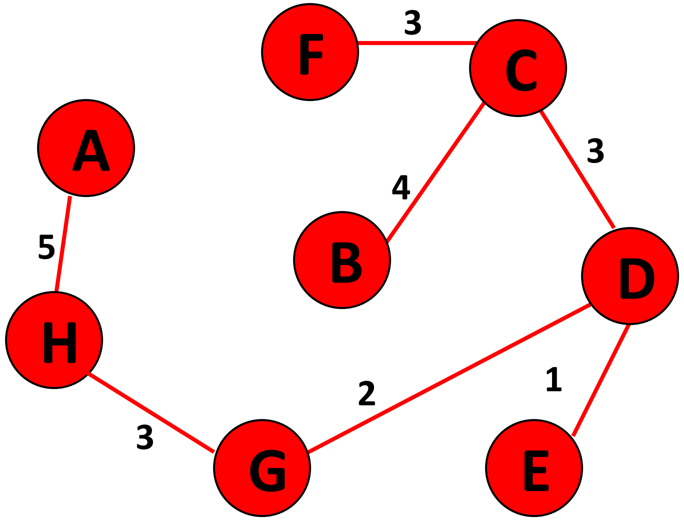
Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	χ
(B,H)	4	χ
(A,H)	5	√
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal's Algorithm

Select first |V|–1 edges which do not generate a cycle



edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	χ
(B,H)	4	χ
(A,H)	5	√
(D,F)	6	
(A,B)	8	
(A,F)	10	

} not considered

Done

Total Cost = $\Sigma\ d_v = 21$

Analysis of Kruskal’s Algorithm

Running Time = O(m log n) (m = edges, n = nodes)

Testing if an edge creates a cycle can be slow unless a complicated data structure called a “union-find” structure is used.

It usually only has to check a small fraction of the edges, but in some cases (like if there was a vertex connected to the graph by only one edge and it was the longest edge) it would have to check all the edges.

This algorithm works best, of course, if the number of edges is kept to a minimum.

∉

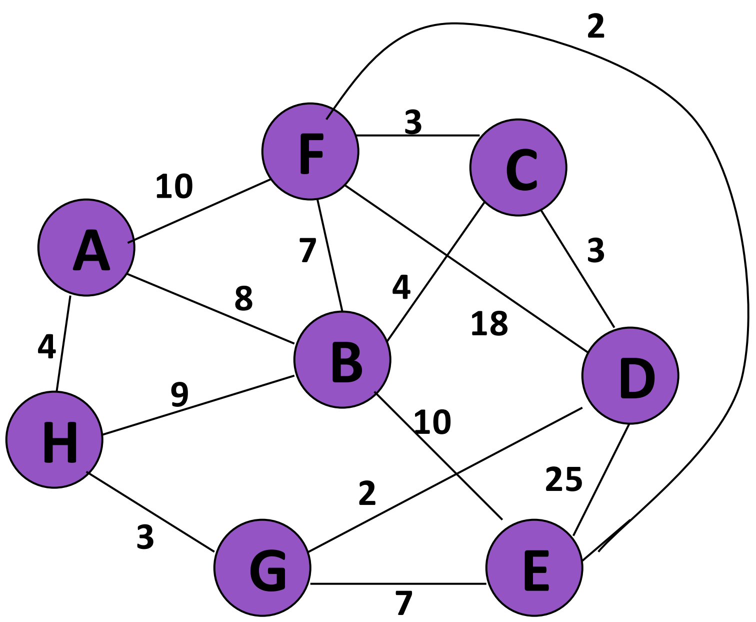
Prim’s Algorithm

```
//Assume G has at least one vertex
TV={0}; //start with vertex 0 and no edges

for (T=Ø; T contains less than n-1 edges; add(u,v) to T)
{
  let (u,v) be a least cost edge such that u ∈ TV and v ∉ TV;
  if (there is no such edge ) break;
  add v to TV;
}

if (T contains fewer than n-1 edges)
cout<<“No spanning tree\n”;
```

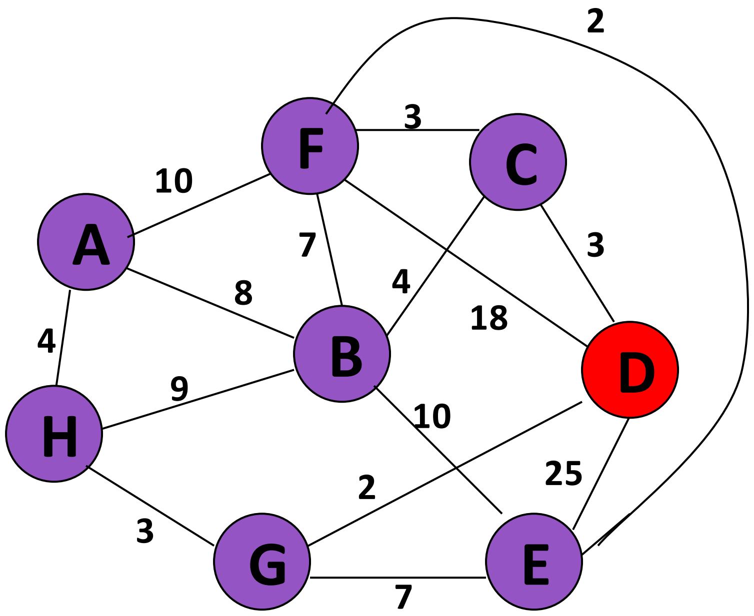
Prim's Algorithm



Initialize array

	K	d_v	p_v
A	F	∞	–
B	F	∞	–
C	F	∞	–
D	F	∞	–
E	F	∞	–
F	F	∞	–
G	F	∞	–
H	F	∞	–

Prim’s Algorithm

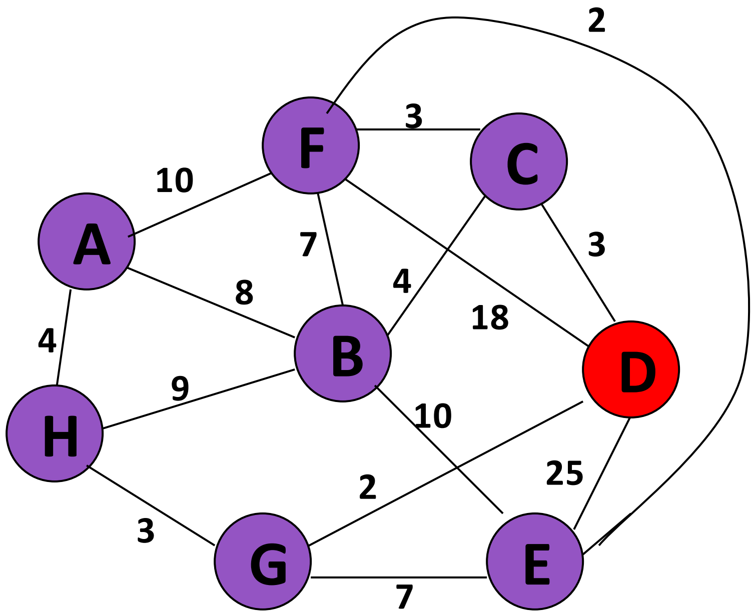


Start with any node, say D

	K	d_v	p_v
A			
B			
C			
D	T	0	–
E			
F			
G			
H			

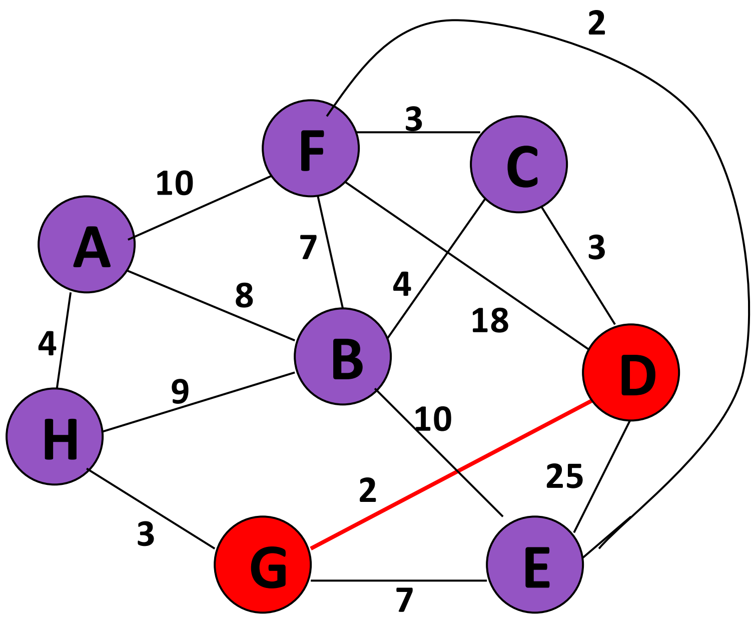
Prim's Algorithm

Update distances of adjacent, unselected nodes



	K	d_v	p_v
A			
B			
C		3	D
D	T	0	–
E		25	D
F		18	D
G		2	D
H			

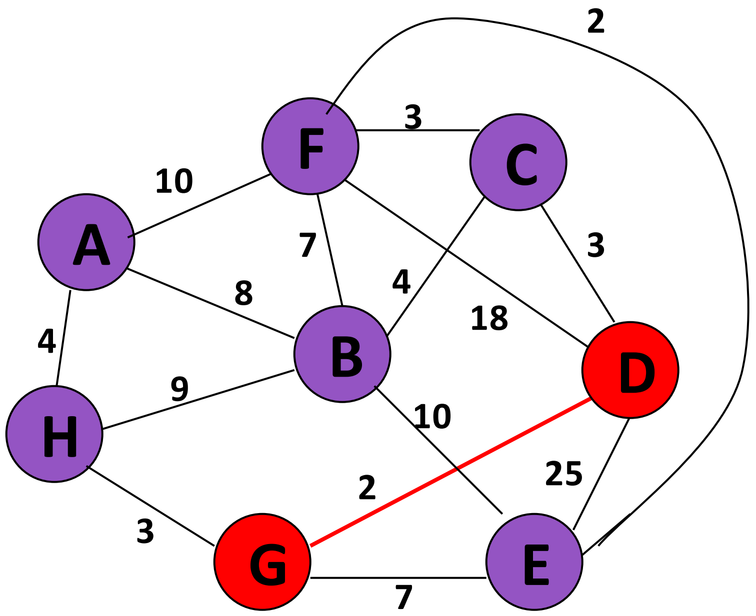
Prim’s Algorithm



Select node with minimum distance

	K	d_v	p_v
A			
B			
C		3	D
D	T	0	–
E		25	D
F		18	D
G	T	2	D
H			

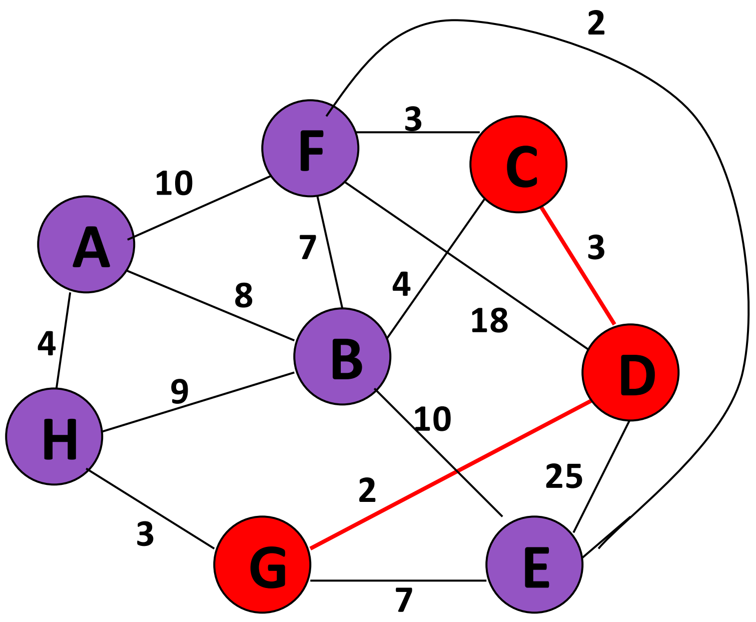
Prim’s Algorithm



Update distances of adjacent, unselected nodes

	K	d_v	p_v
A			
B			
C		3	D
D	T	0	–
E		7	G
F		18	D
G	T	2	D
H		3	G

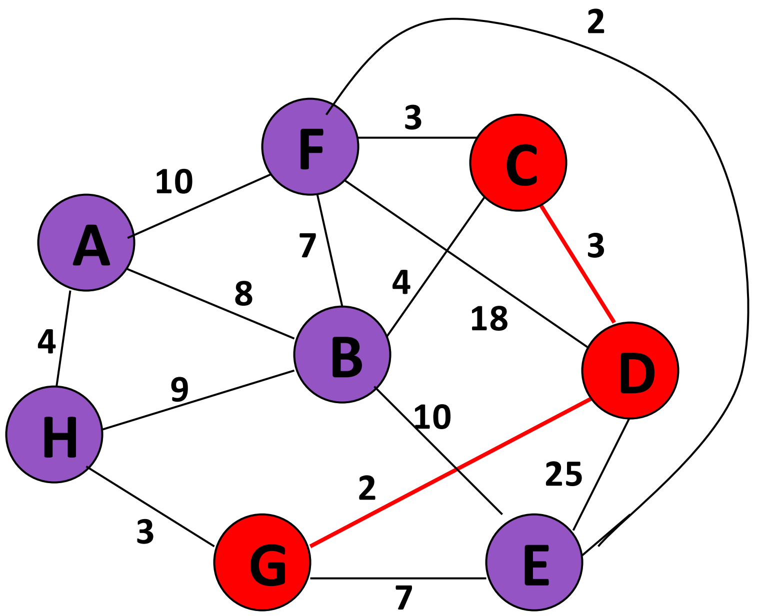
Prim's Algorithm



Select node with minimum distance

	K	d_v	p_v
A			
B			
C	T	3	D
D	T	0	–
E		7	G
F		18	D
G	T	2	D
H		3	G

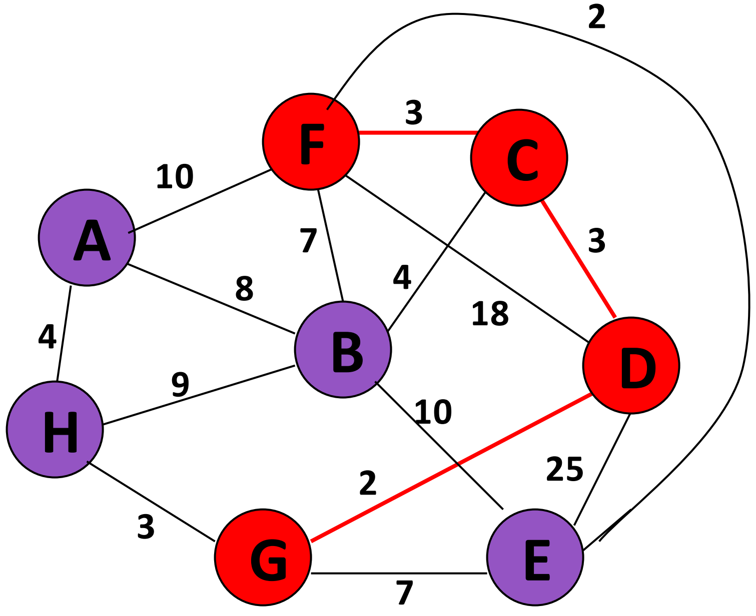
Prim’s Algorithm



Update distances of adjacent, unselected nodes

	K	d_v	p_v
A			
B		4	C
C	T	3	D
D	T	0	–
E		7	G
F		3	C
G	T	2	D
H		3	G

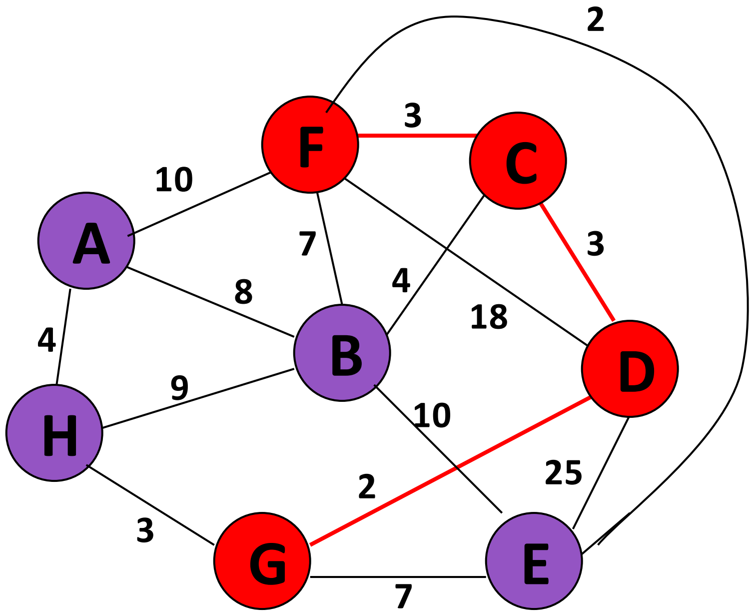
Prim’s Algorithm



Select node with minimum distance

	K	d_v	p_v
A			
B		4	C
C	T	3	D
D	T	0	–
E		7	G
F	T	3	C
G	T	2	D
H		3	G

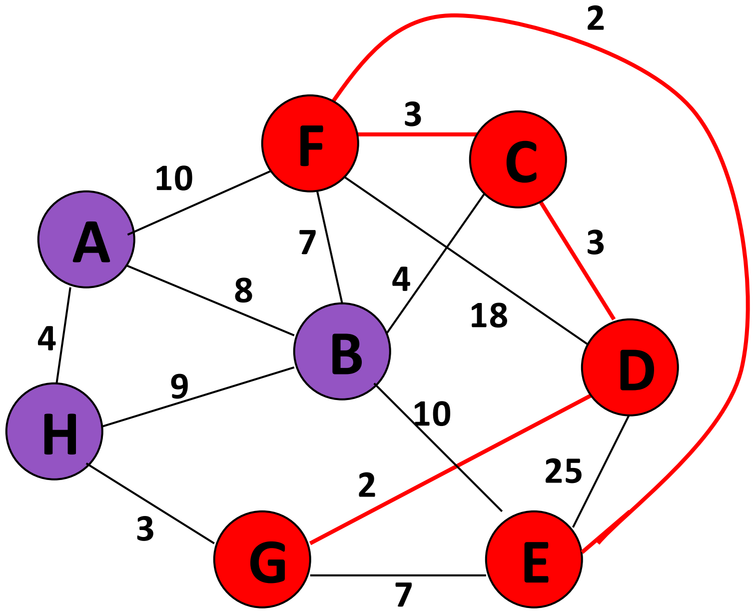
Prim’s Algorithm



Update distances of adjacent, unselected nodes

	K	d_v	p_v
A		10	F
B		4	C
C	T	3	D
D	T	0	–
E		2	F
F	T	3	C
G	T	2	D
H		3	G

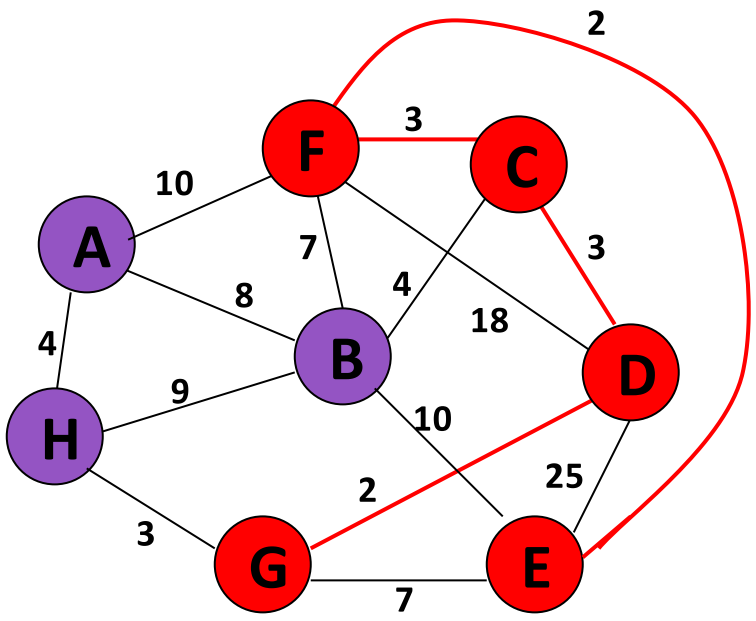
Prim's Algorithm



Select node with minimum distance

	K	d_v	p_v
A		10	F
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H		3	G

Prim's Algorithm

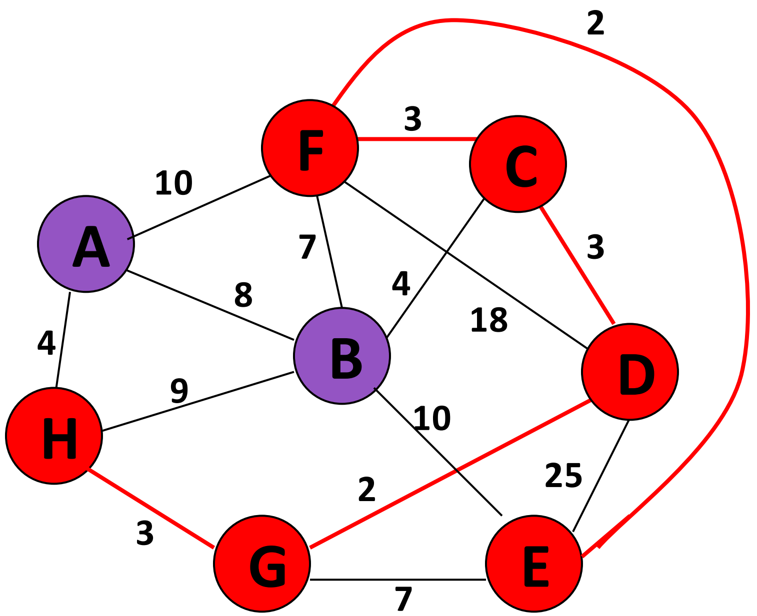


Update distances of
adjacent, unselected nodes

	K	d_v	p_v
A		10	F
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H		3	G

Table entries unchanged

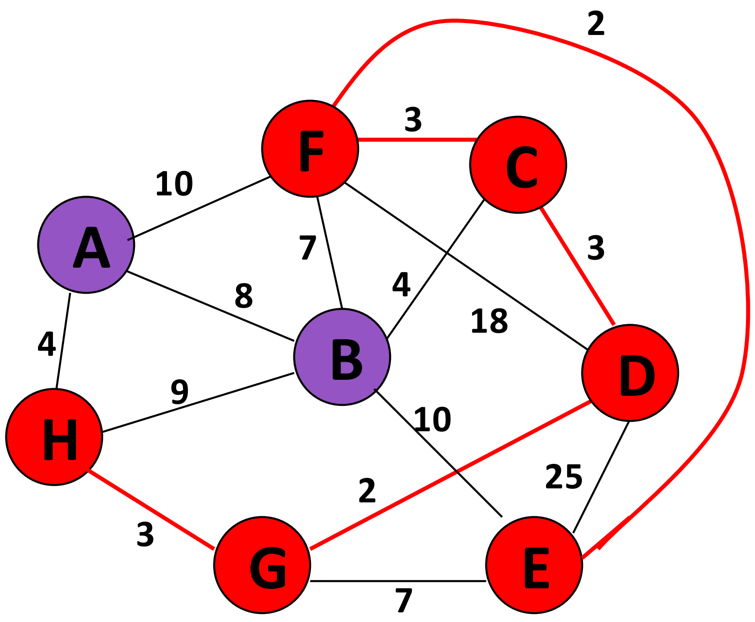
Prim's Algorithm



Select node with minimum distance

	K	d_v	p_v
A		10	F
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

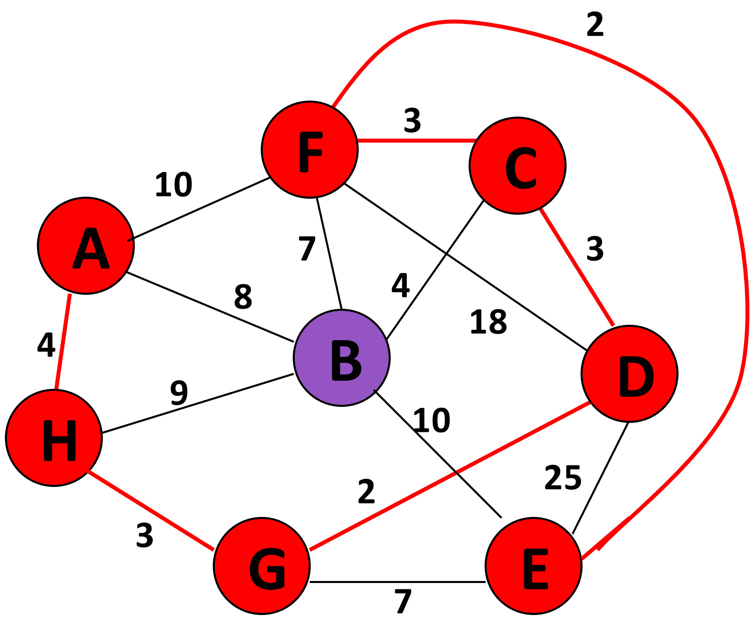
Prim's Algorithm



Update distances of adjacent, unselected nodes

	K	d_v	p_v
A		4	H
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

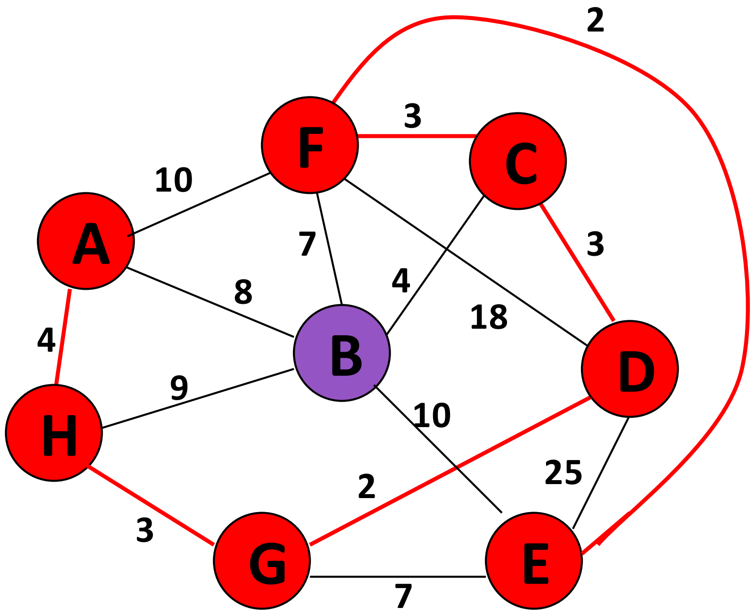
Prim's Algorithm



Select node with minimum distance

	K	d_v	p_v
A	T	4	H
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

Prim's Algorithm

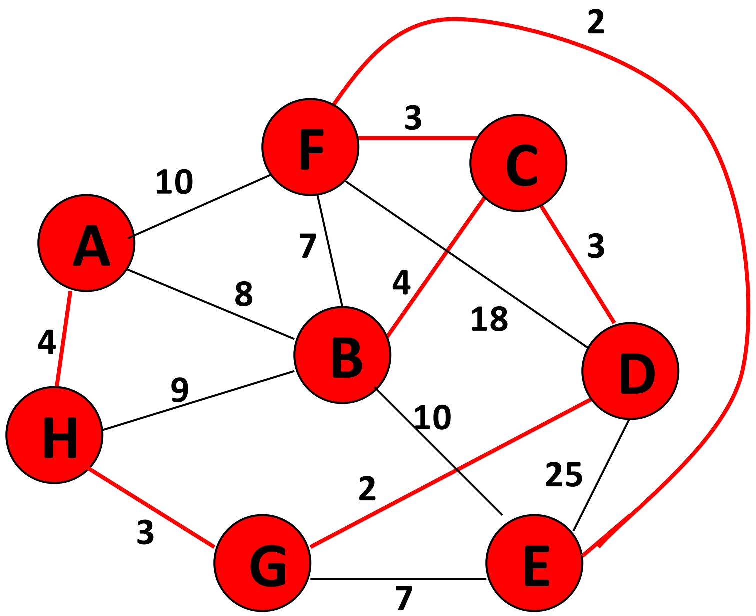


Update distances of adjacent, unselected nodes

	K	d_v	p_v
A	T	4	H
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

Table entries unchanged

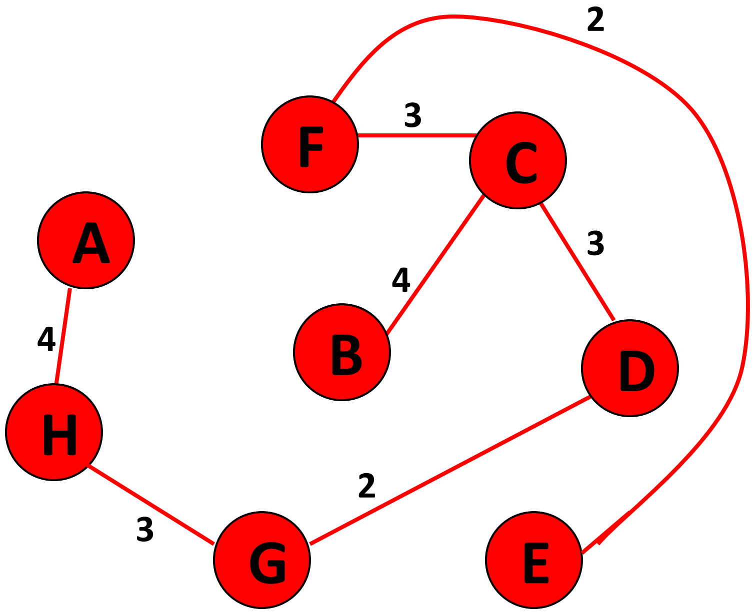
Prim's Algorithm



Select node with minimum distance

	K	d_v	p_v
A	T	4	H
B	T	4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

Prim's Algorithm



Cost of Minimum Spanning Tree = $\Sigma\, d_v$ = 21

	K	d_v	p_v
A	T	4	H
B	T	4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

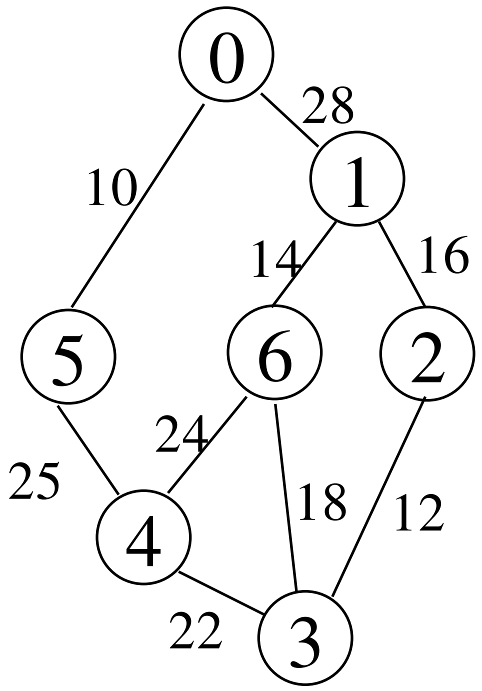
Done

Analysis of Prim’s Algorithm

- Run time will be O(n^2) .
- Unlike Kruskal’s, it doesn’t need to see all of the graph at once. It can deal with it one piece at a time. It also doesn’t need to worry if adding an edge will create a cycle since this algorithm deals primarily with the nodes, and not the edges.
- For this algorithm the number of nodes needs to be kept to a minimum in addition to the number of edges. For small graphs, the edges matter more, while for large graphs the number of nodes matters more.

Home Assignment

Find MST for given Graph G1 using Prim’s and Kruskal Algorithm



Comparison Prim's and Kruskal's Algorithm

- Kruskal's has better running times if the number of edges is low, while Prim's has a better running time if both the number of edges and the number of nodes are low.

- So, of course, the best algorithm depends on the graph and if you want to bear the cost of complex data structures.

Shortest Path Problems

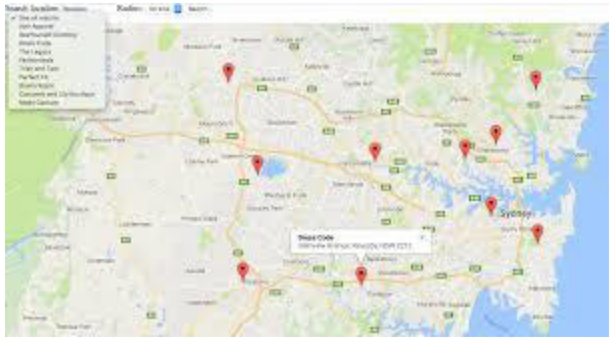
- Directed weighted graph.
- Path length is sum of weights of edges on path.
- The vertex at which the path begins is the source vertex.
- The vertex at which the path ends is the destination vertex.

Shortest Path Problems

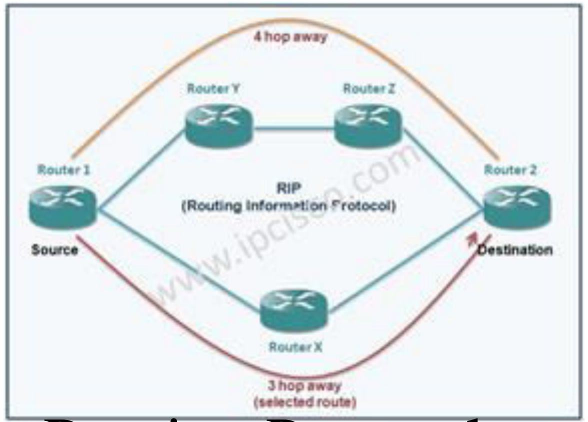
- Single source single destination.
- Single source all destinations.
- All pairs (every vertex is a source and destination).

Dijkstra Algorithm

- Finds Single source all destination shortest paths
- Uses Greedy Method
- No negative weights are allowed
- Application
  - Routing protocols in computer networks
  - Google Maps and many more..



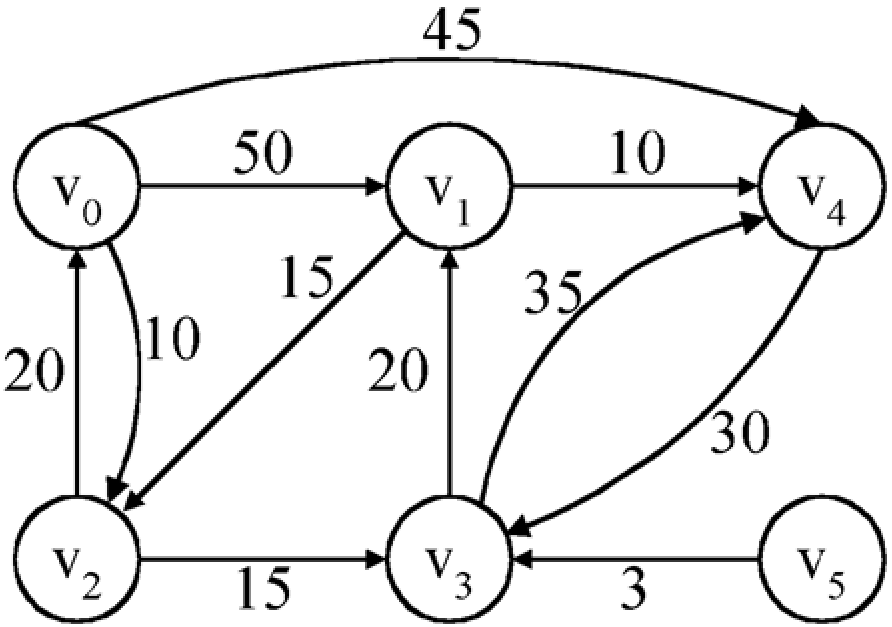
Google Maps



Routing Protocols

Dijkstra Algorithm

shortest paths from v_0(Single source) to all destinations



(a)

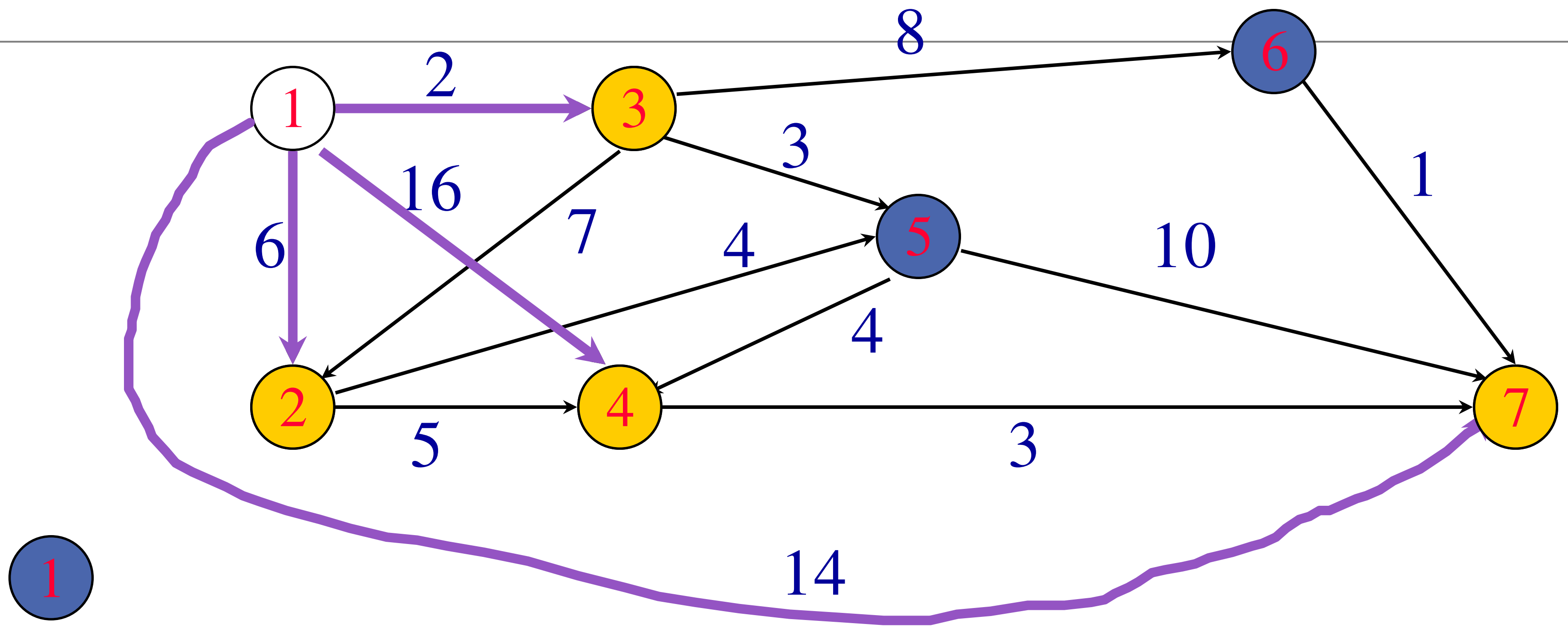
	Path	Length
1)	v_0v_2	10
2)	$v_0v_2v_3$	25
3)	$v_0v_2v_3v_1$	45
4)	v_0v_4	45

(b)

Dijkstra Algorithm

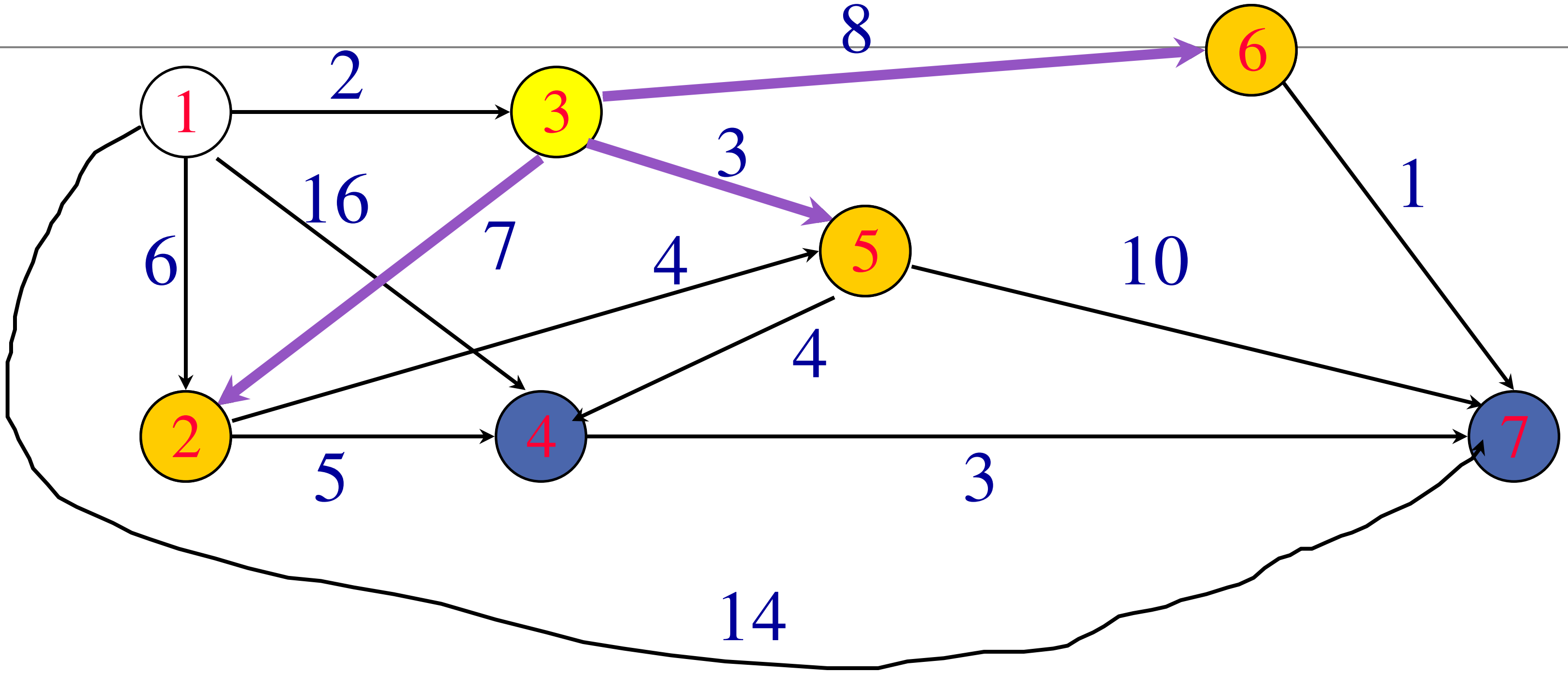
```
void shortestPath(int v,int g[][],int
    n,int dist[])
{
      for (i=0;i<n;i++)
      {
            s[i] = 0;
            dist[i]=g[v][i];
      }
      //put v into s
       s[v] = 1;
      dist[v] =0;
      for(j=2;j<n;  j++)
      {
            //choose u from among those vertices not in s
              such that dist[u] is minimum;
            s[u] = 1;
            for each(w adjacent to u with s[w] = 0)
            {
              if(dist[w] > dist[u]+g[u][w] )
                        dist[w] = dist[u]+cost[u][w];
            }
      }
}
```

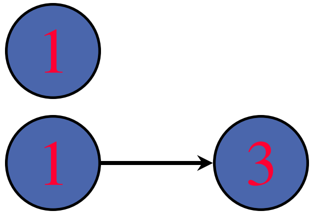
Dijkstra Algorithm



	[1]	[2]	[3]	[4]	[5]	[6]	[7]
d	0	6	$\boxed{2}$	16	-	-	14
p	-	1	1	1	-	-	1

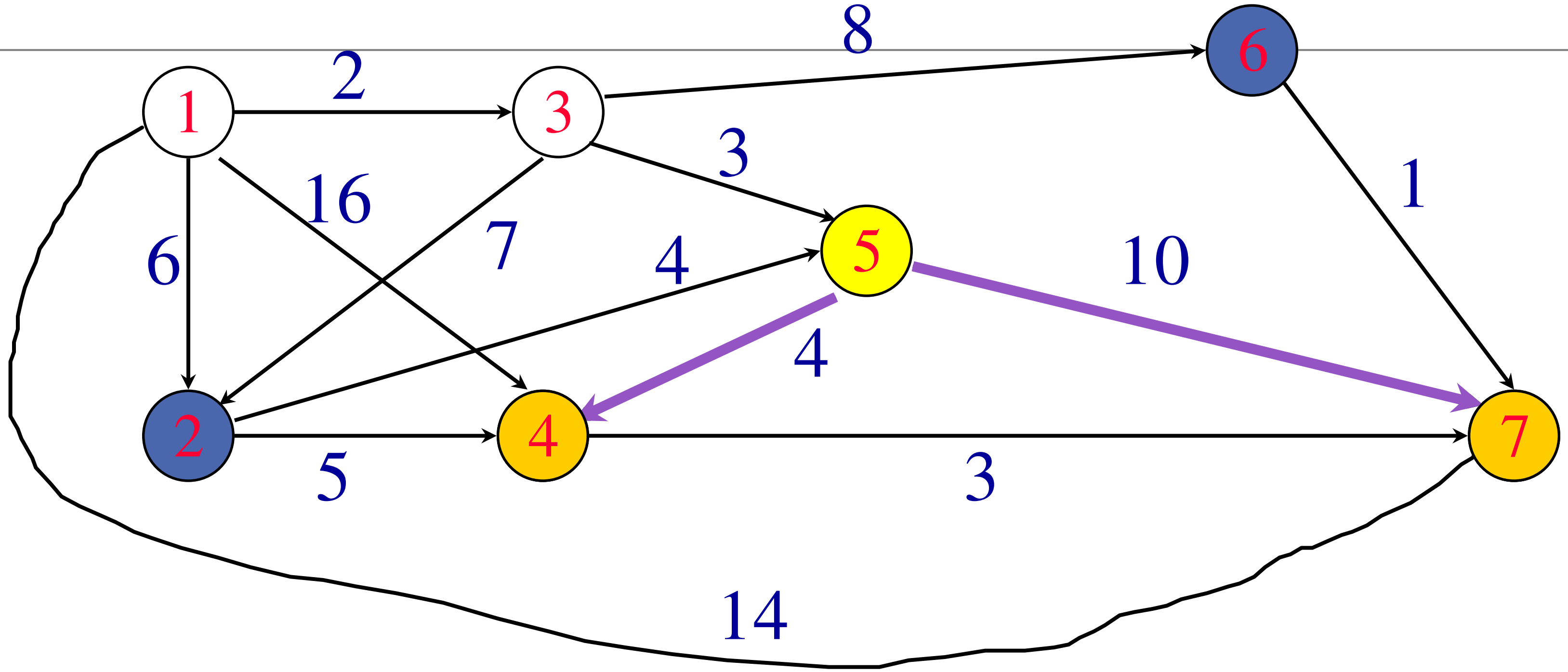
Dijkstra Algorithm

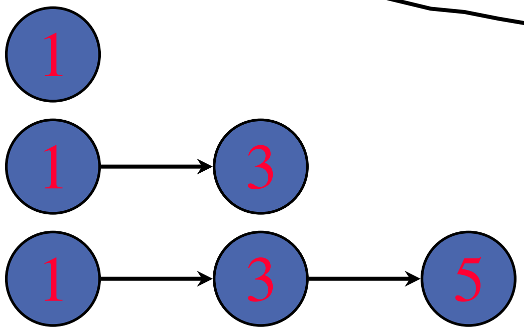




	[1]	[2]	[3]	[4]	[5]	[6]	[7]
d	0	6	2	16	5	10	14
p	-	1	1	1	3	3	1

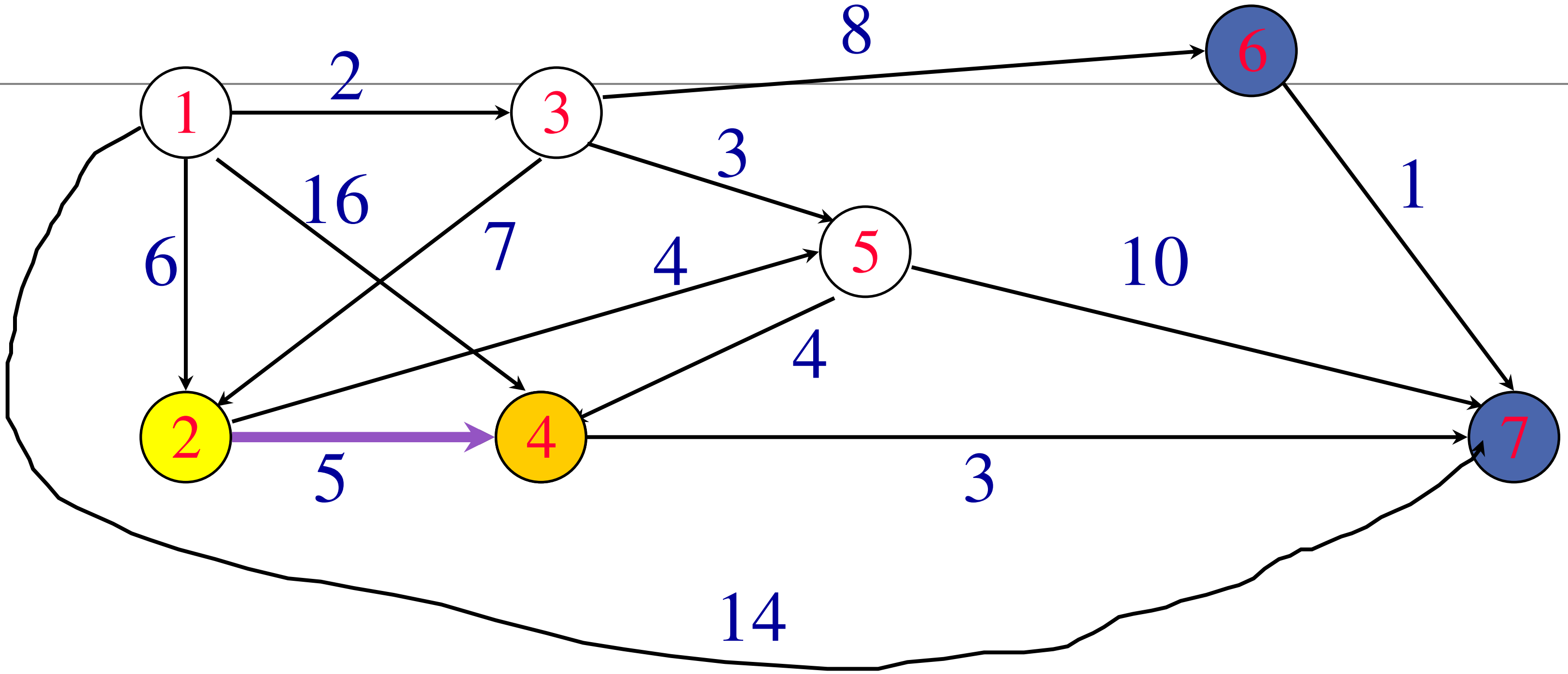
Dijkstra Algorithm

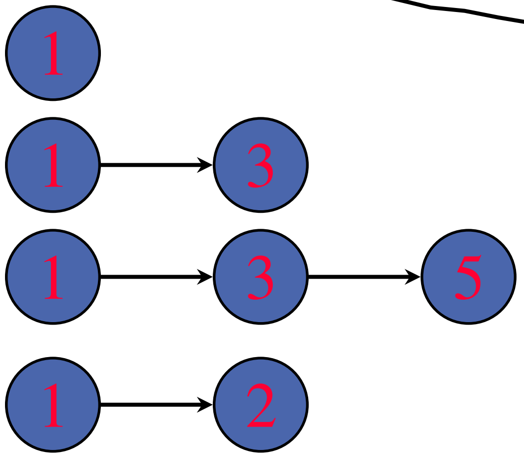




	[1]	[2]	[3]	[4]	[5]	[6]	[7]
d	0	6	2	9	5	10	14
p	-	1	1	5	3	3	1

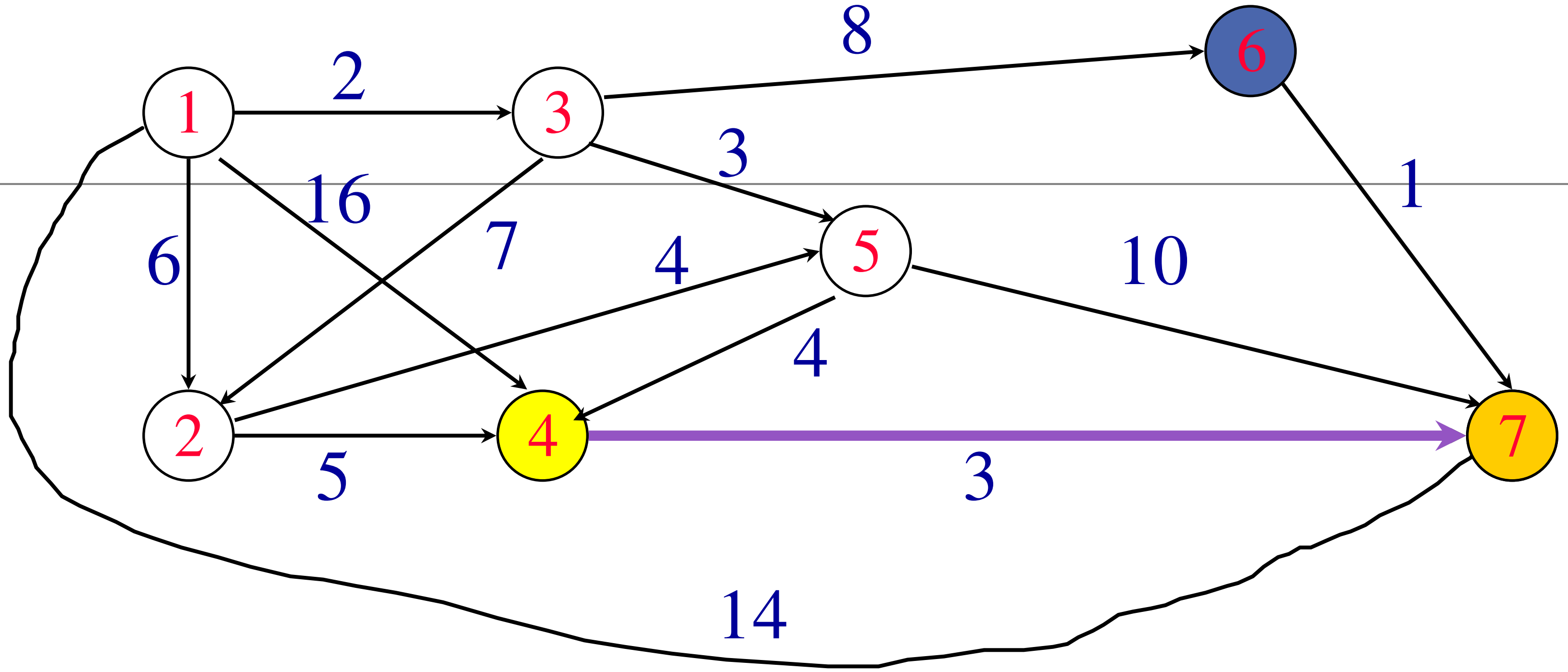
Dijkstra Algorithm

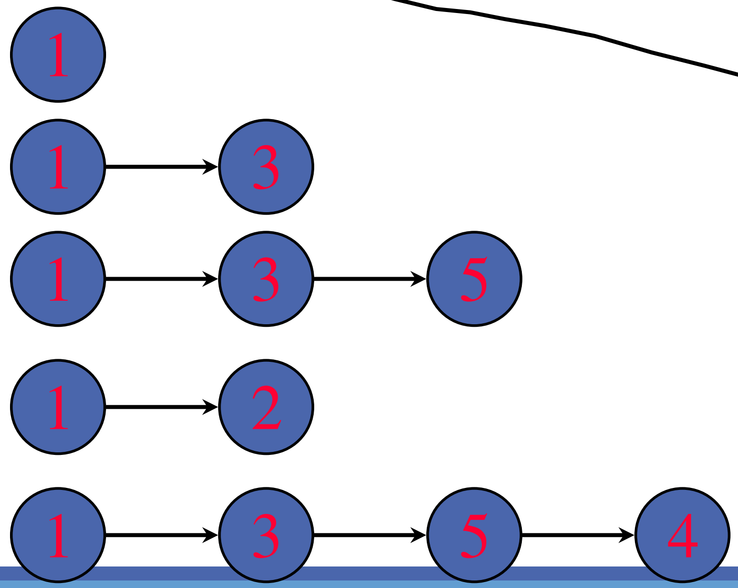




	[1]	[2]	[3]	[4]	[5]	[6]	[7]
d	0	6	2	9	5	10	14
p	-	1	1	5	3	3	1

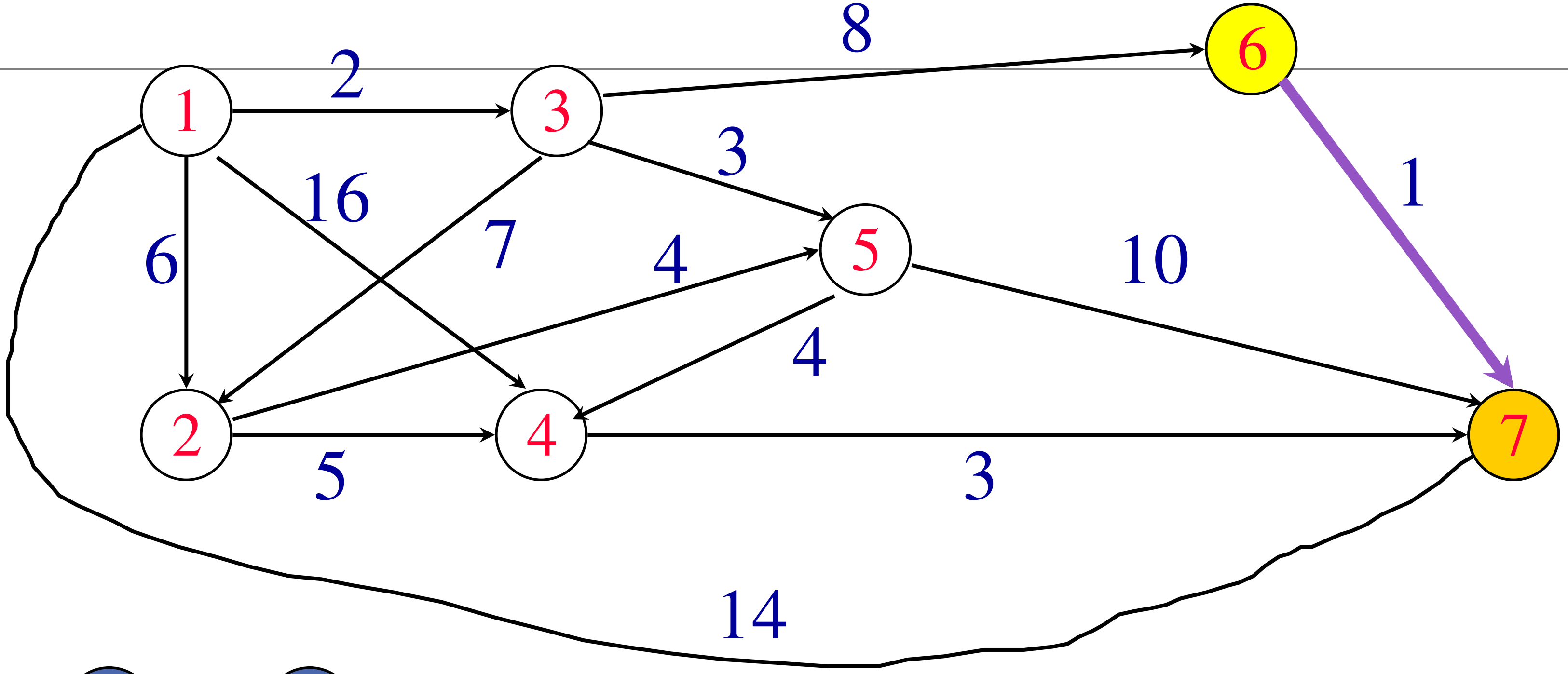
Dijkstra Algorithm

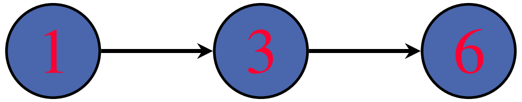




	[1]	[2]	[3]	[4]	[5]	[6]	[7]
d	0	6	2	9	5	10	12
p	-	1	1	5	3	3	4

Dijkstra Algorithm





	[1]	[2]	[3]	[4]	[5]	[6]	[7]
d	0	6	2	9	5	10	11
p	-	1	1	5	3	3	6

Algorithm

Path	Length
1	0
1 → 3	2
1 → 3 → 5	5
1 → 2	6
1 → 3 → 5 → 4	9
1 → 3 → 6	10
1 → 3 → 6 → 7	11

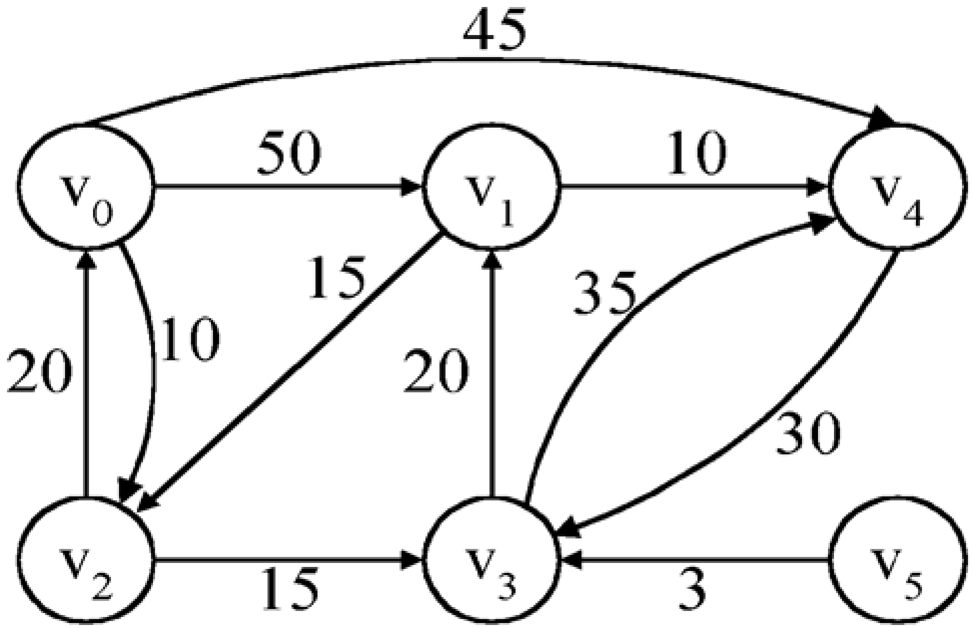
[1]	[2]	[3]	[4]	[5]	[6]	[7]
0	6	2	9	5	10	11
-	1	1	5	3	3	6

Analysis of Dijkstra Algorithm

- O(n) to select next destination vertex.
- O(out-degree) to update d() and p() values when adjacency lists are used.
- O(n) to update d() and p() values when adjacency matrix is used.
- Selection and update done once for each vertex to which a shortest path is found.
- Total time is $O(n^2 + e) = O(n^2)$.

Home Assignment

Find shortest paths from v_0 to all destinations

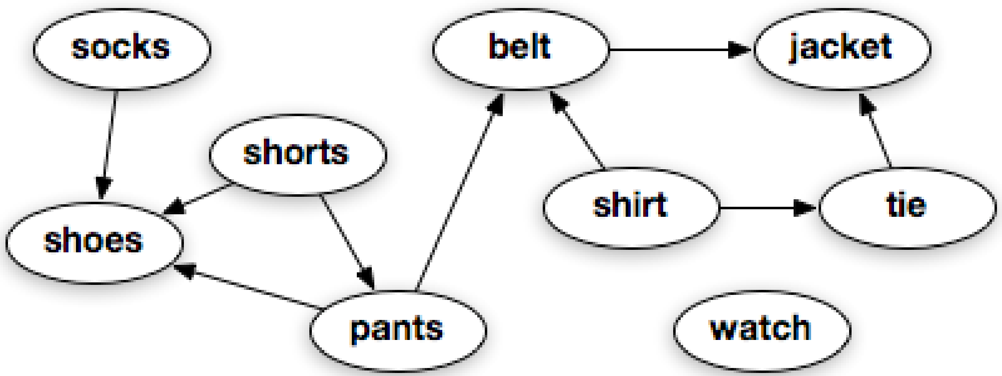


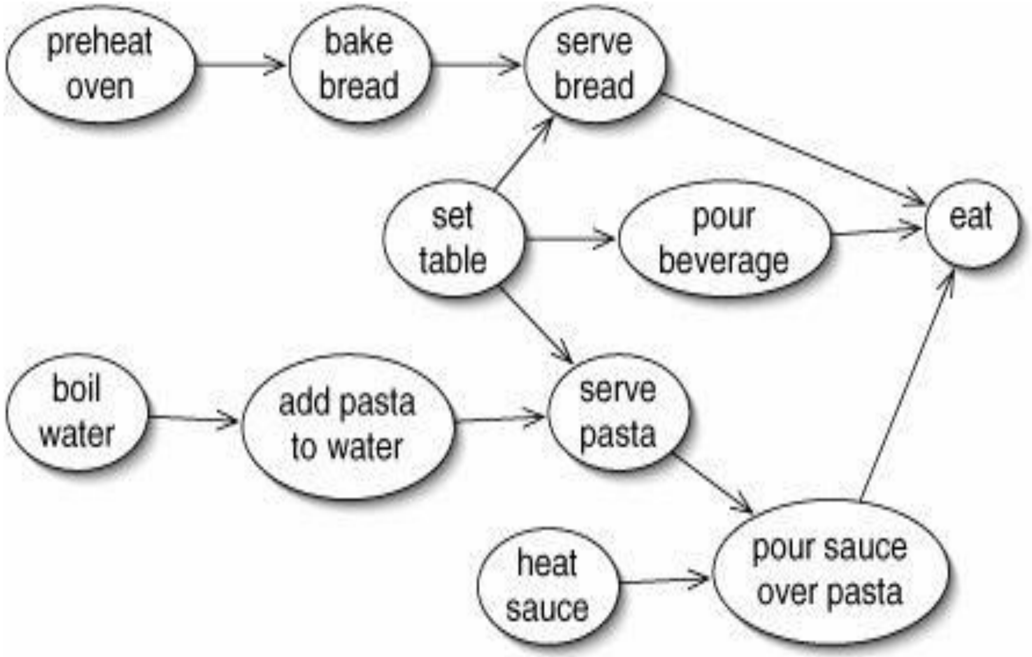
Topological sort

- We have a set of tasks and a set of dependencies (precedence constraints) of form *“task A must be done before task B”*
- Topological sort: An ordering of the tasks that conforms with the given dependencies
- Goal: Find a topological sort of the tasks or decide that there is no such ordering

Topological sort

- Applications
  - Assembly lines in industries
  - Courses arrangement in schools
  - Life related applications: Dressing order





Topological sort

```
void topological_sort()
{
for (i = 0; i <n; i++) {
      if every vertex has a predecessor {
         fprintf(stderr, "Network has a cycle. \n " );
         exit(1);
       }
       pick a vertex v that has no predecessors;
       output v;
       delete v and all edges leading out of v    from the network;
}
```

Topology sort Algorithm

A job consists of 10 tasks with the following precedence rules:

Must start with 7, 5, 4 or 9.

Task 1 must follow 7.

Tasks 3 & 6 must follow both 7 & 5.

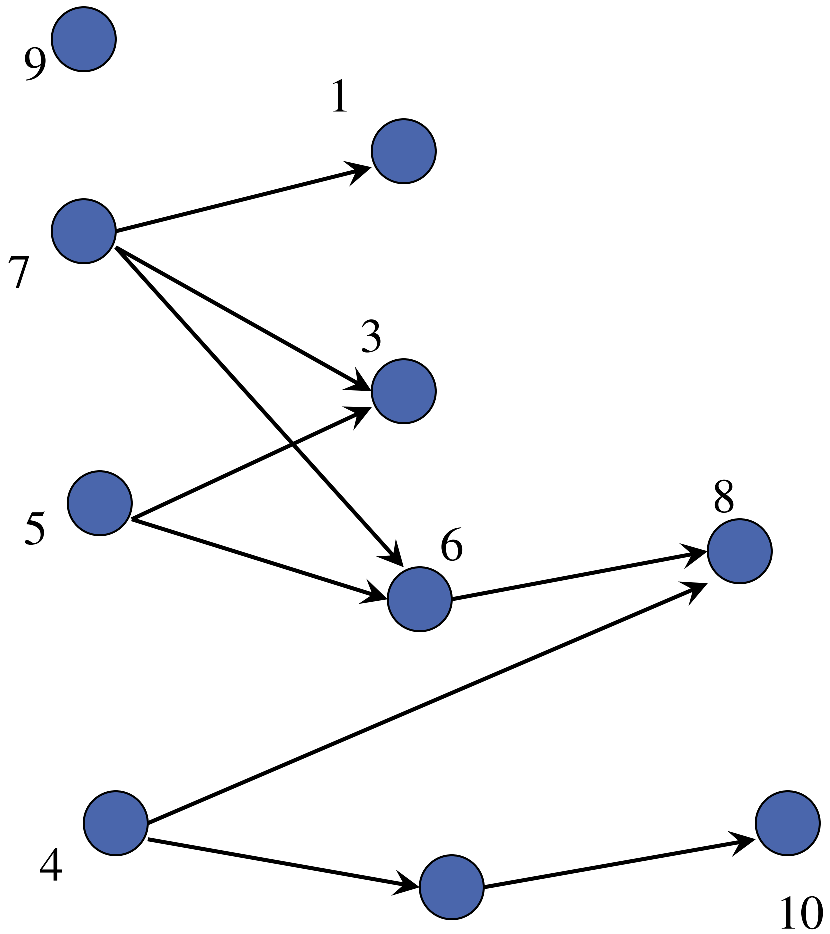
8 must follow 6 & 4.

2 must follow 4.

10 must follow 2.

Make a directed graph and then do DFS.

Topology sort Algorithm



Tasks shown as a directed graph.

Topological Sort using DFS

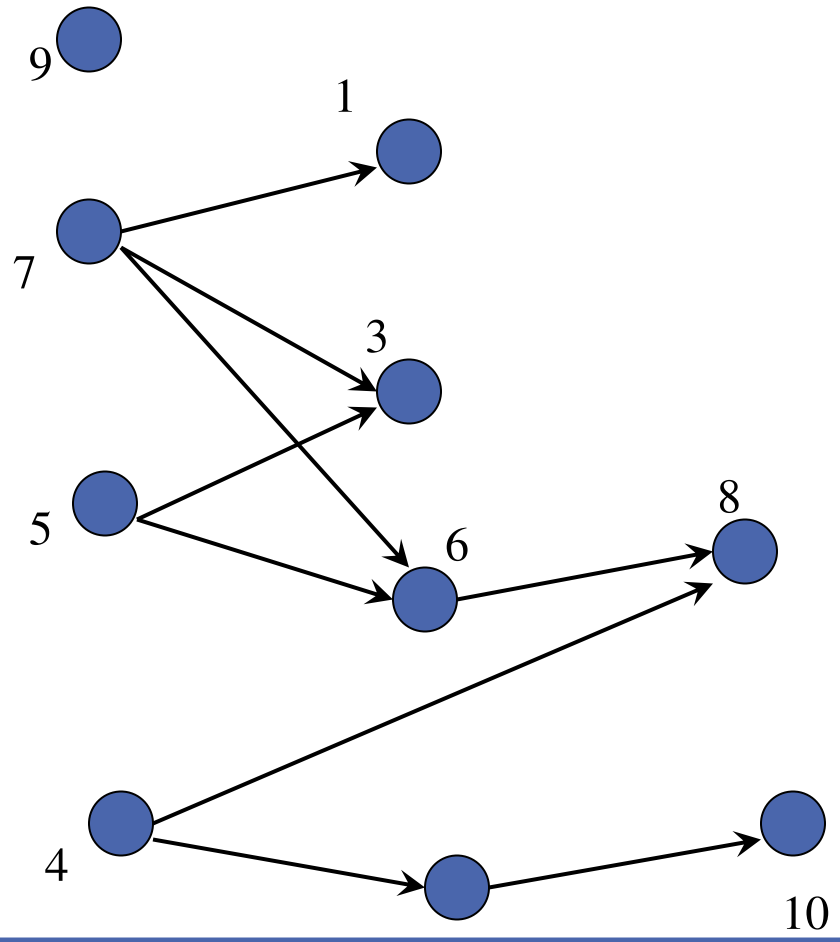
To create a topological sort from a DAG

1- Final linked list is empty

2- Run DFS

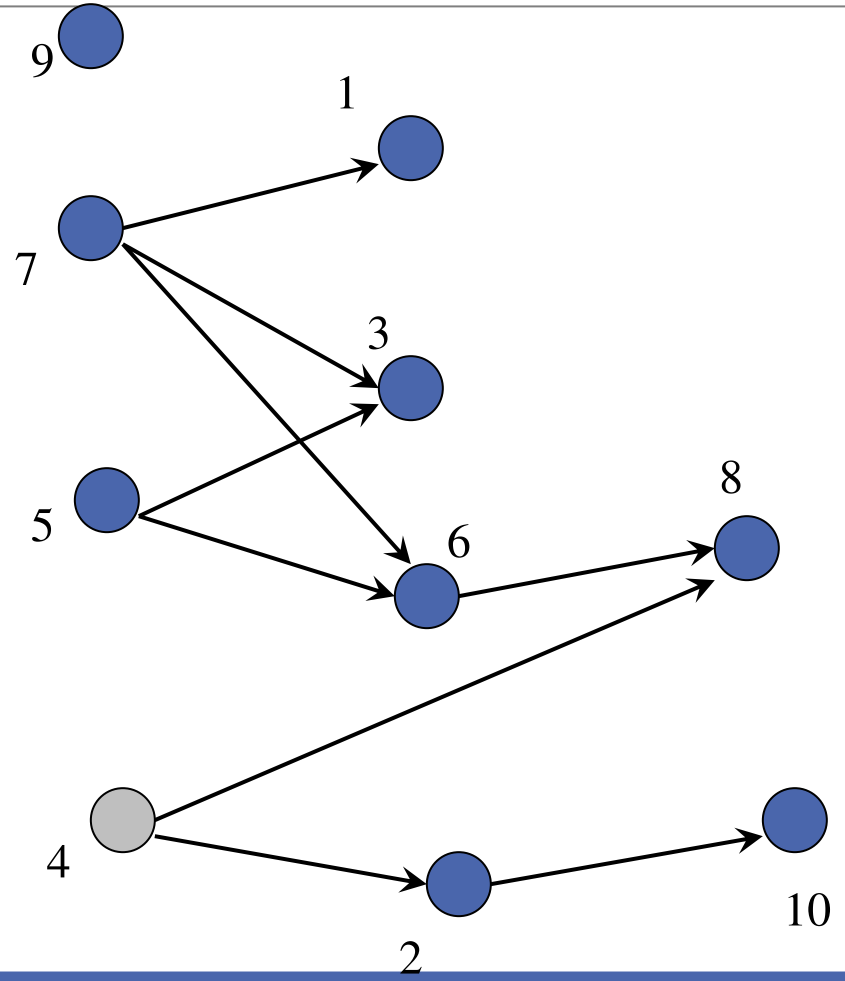
3- When a node becomes black (finishes) insert it to the top of a linked list

Topology sort Algorithm

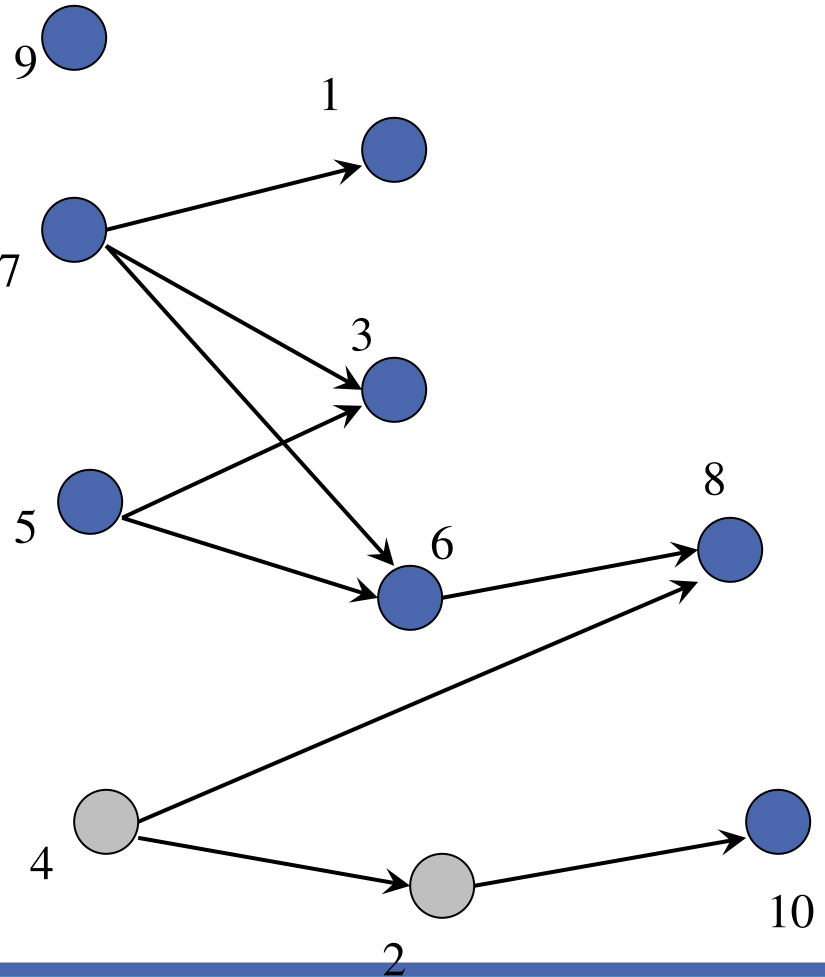


Tasks shown as a directed graph.

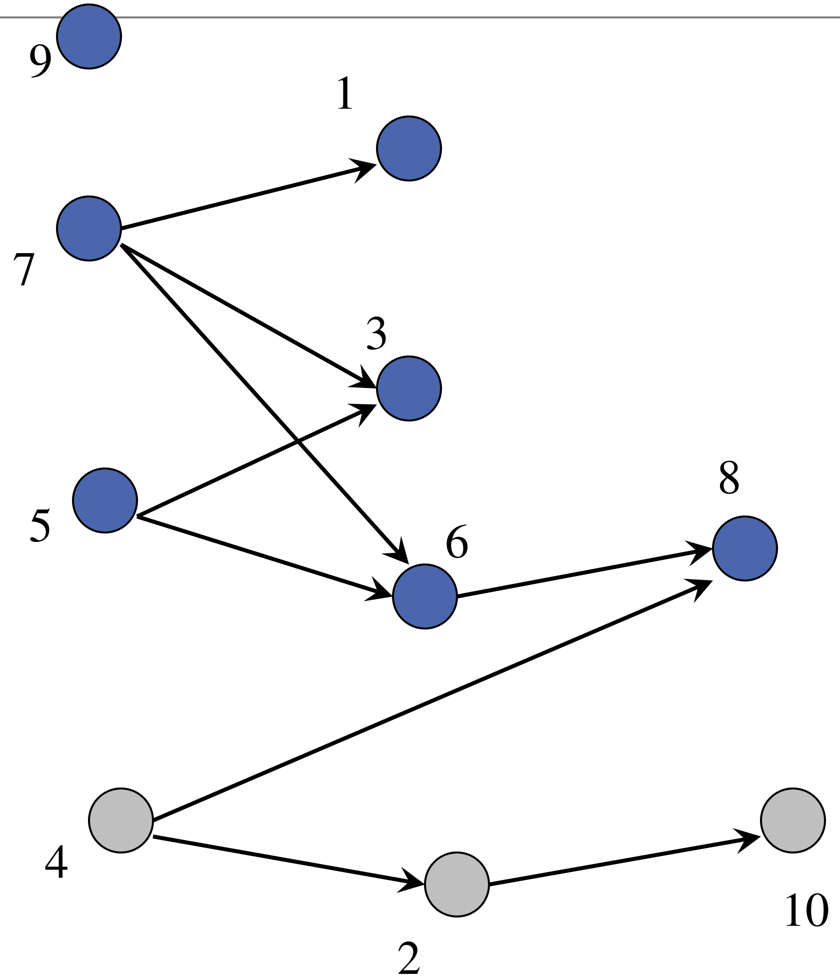
Topology sort Algorithm



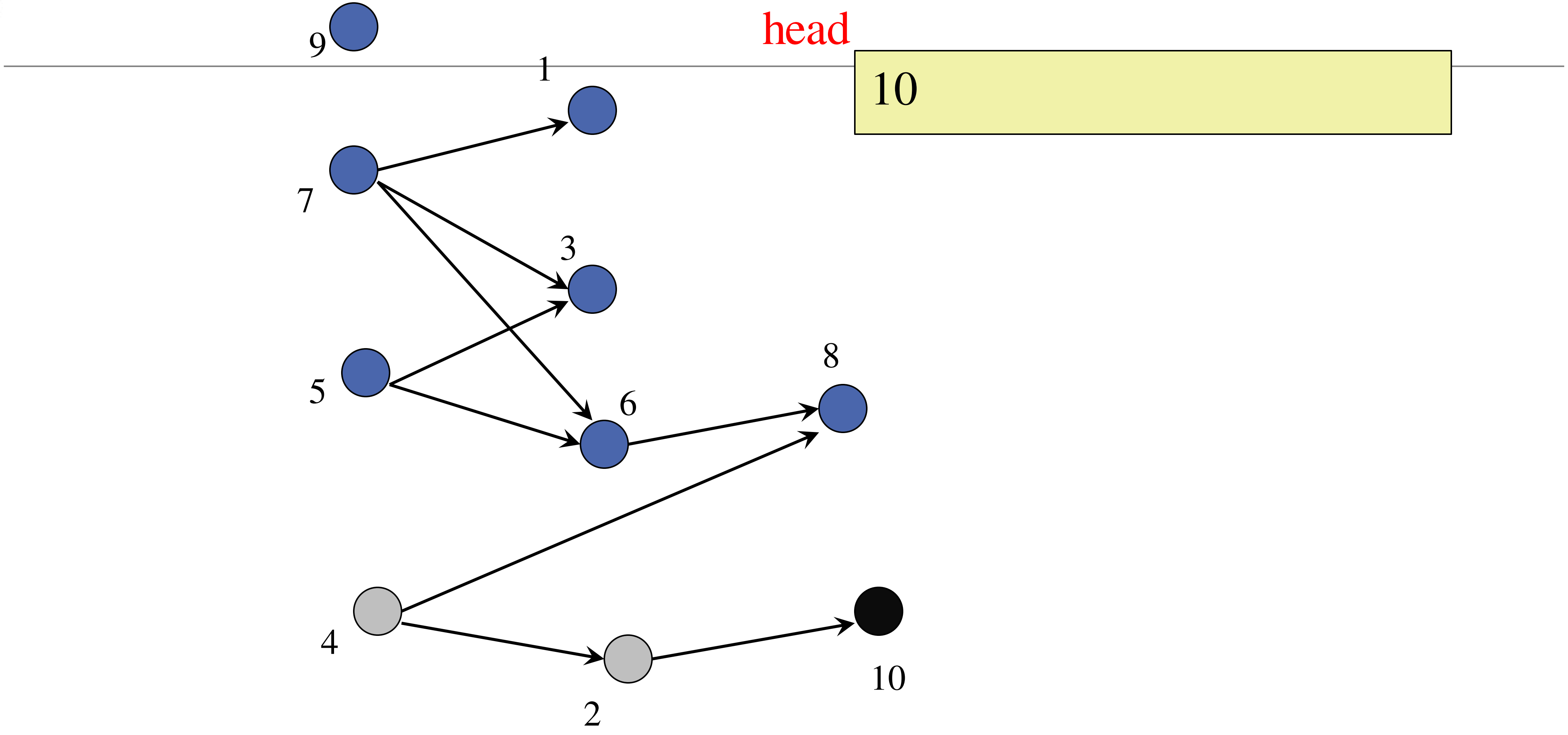
Topology sort Algorithm



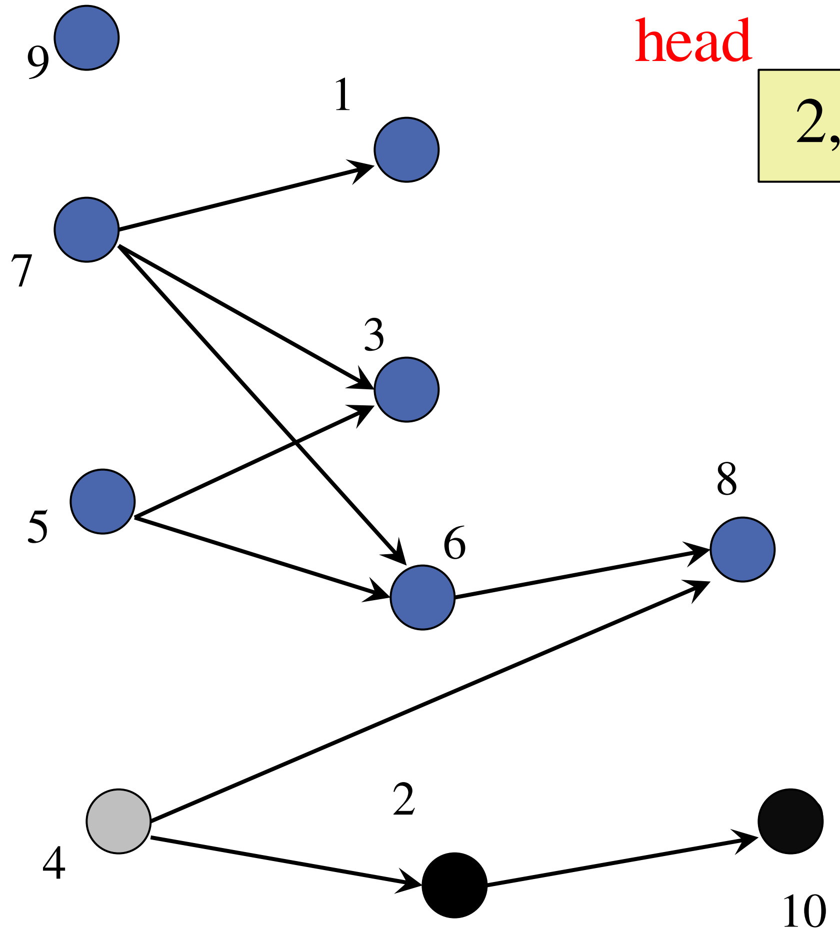
Topology sort Algorithm



Topology sort Algorithm



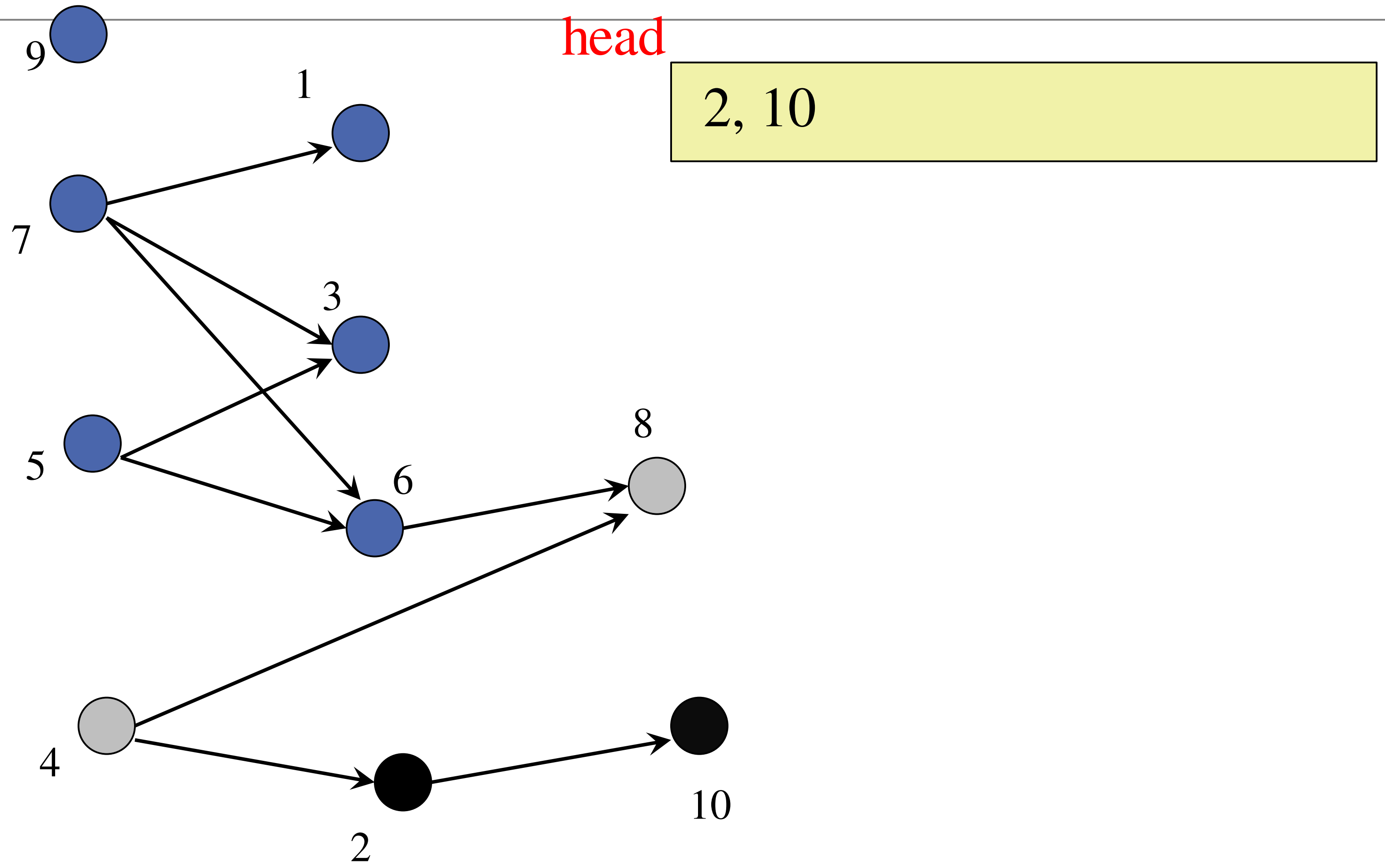
Topology sort Algorithm



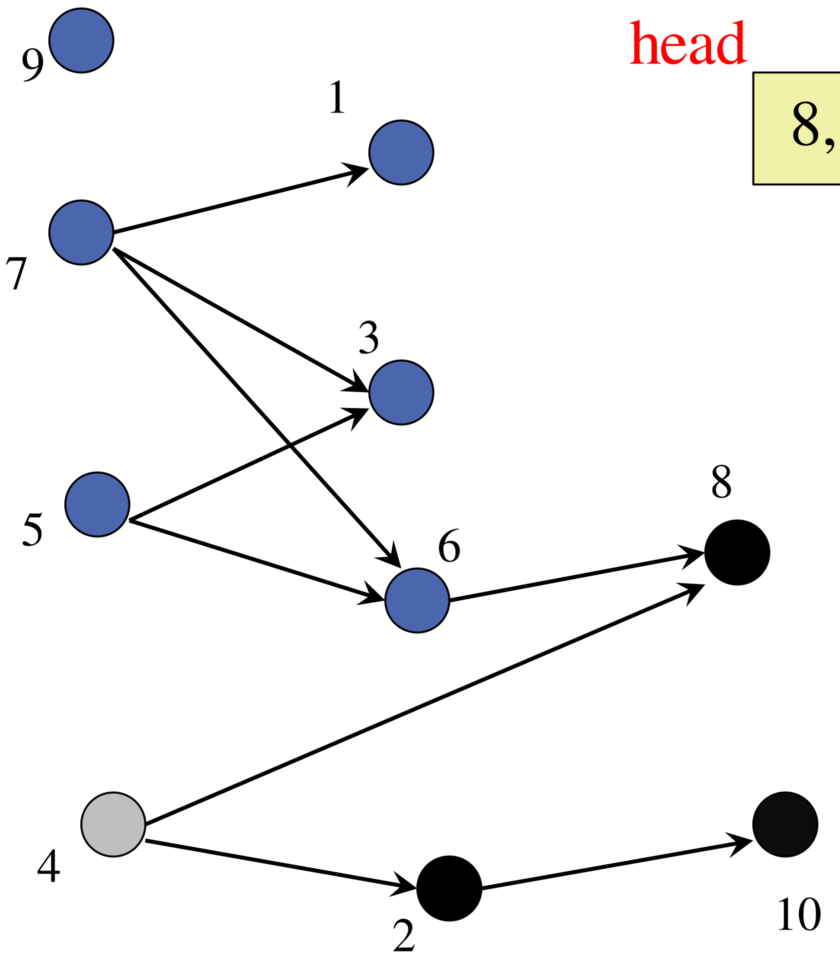
head

2, 10

Topology sort Algorithm



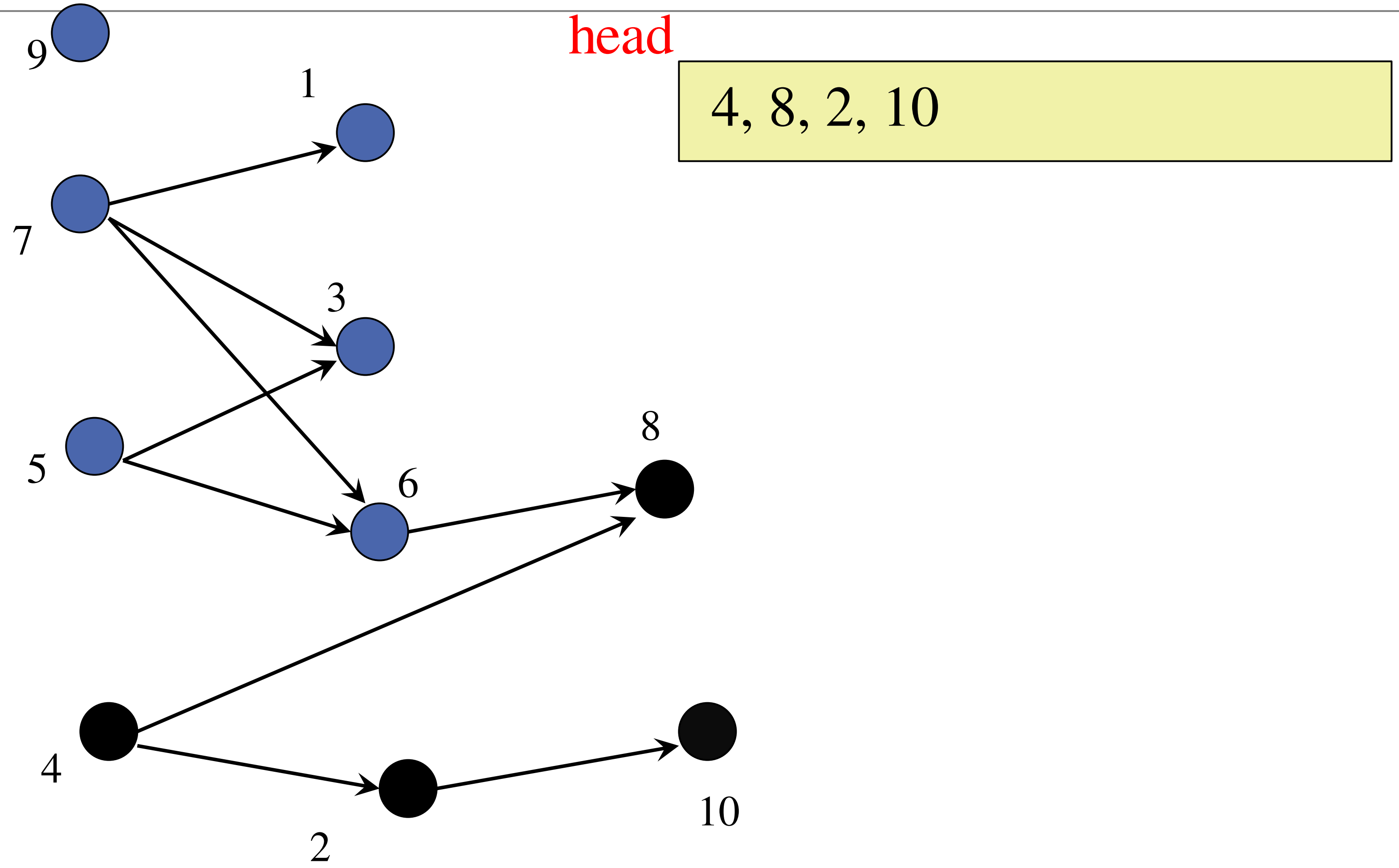
Topology sort Algorithm



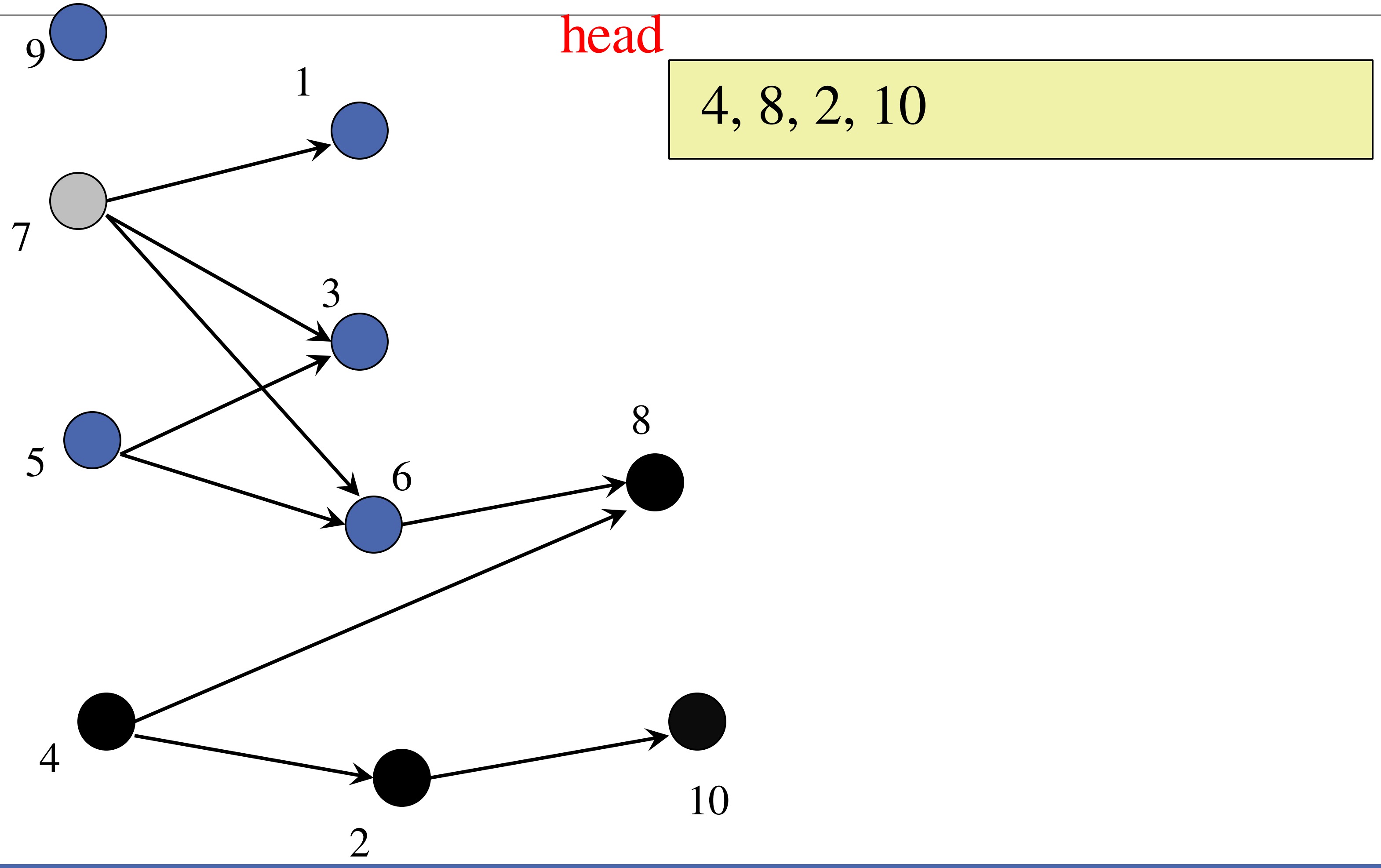
head

8, 2, 10

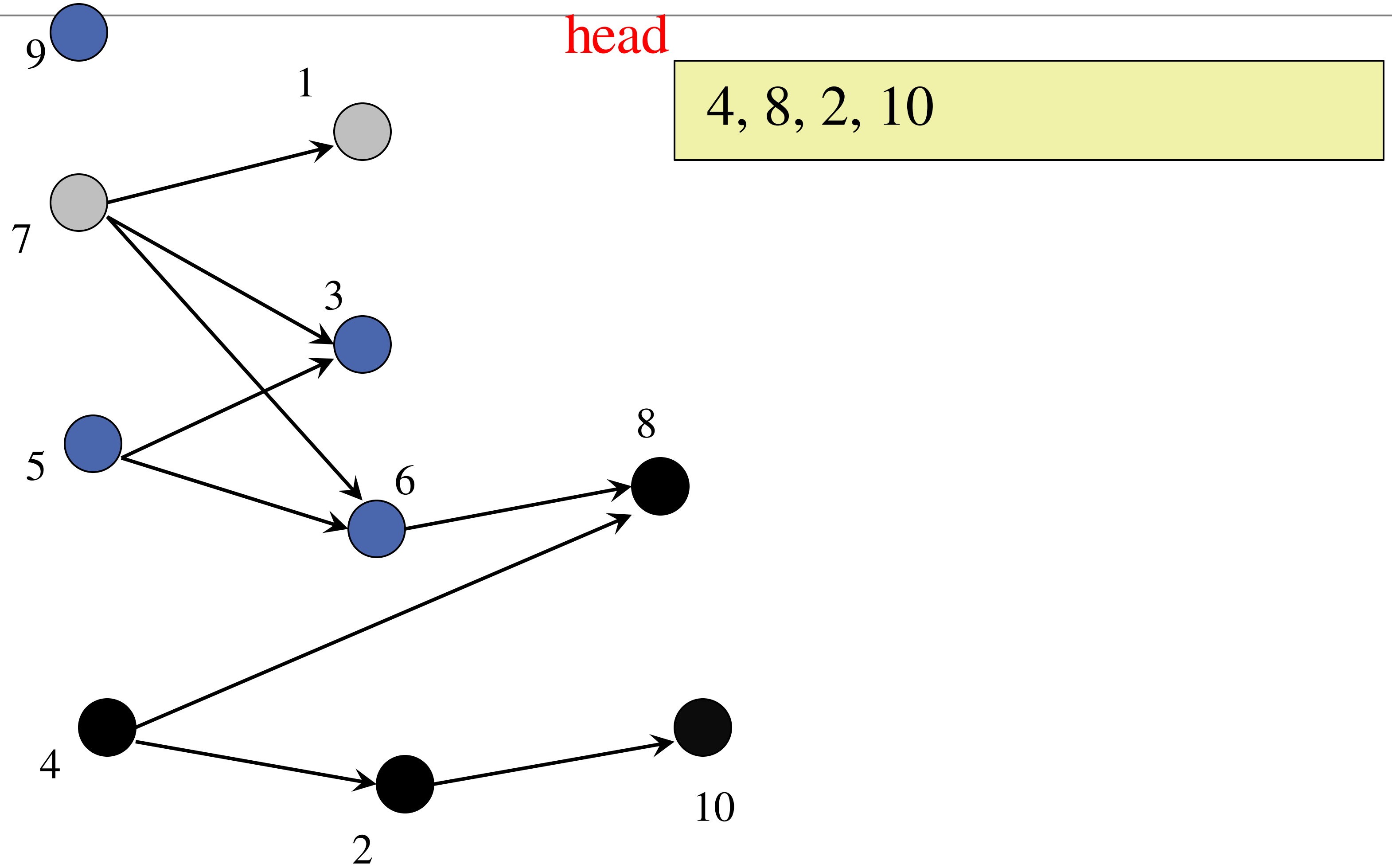
Topology sort Algorithm



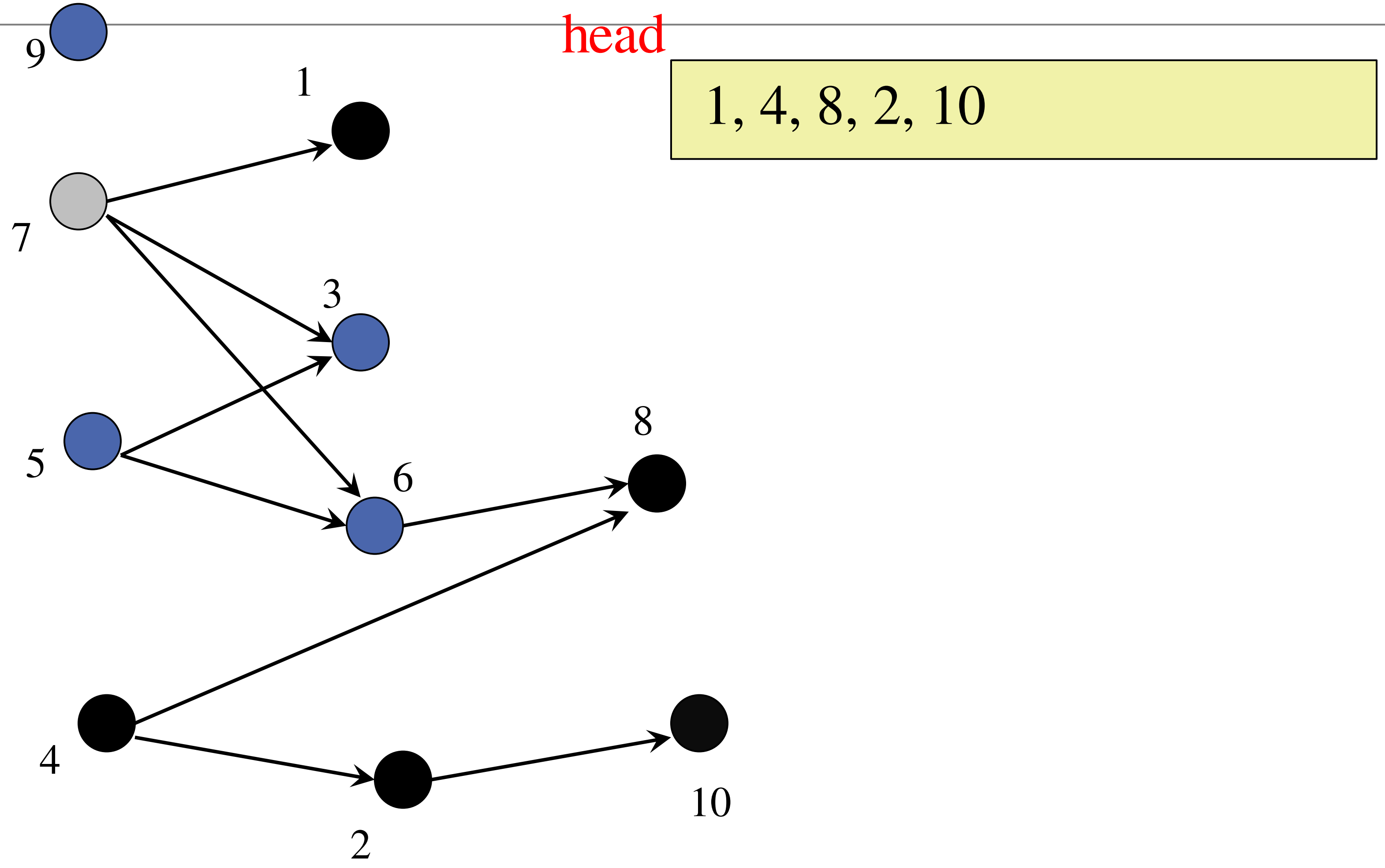
Topology sort Algorithm



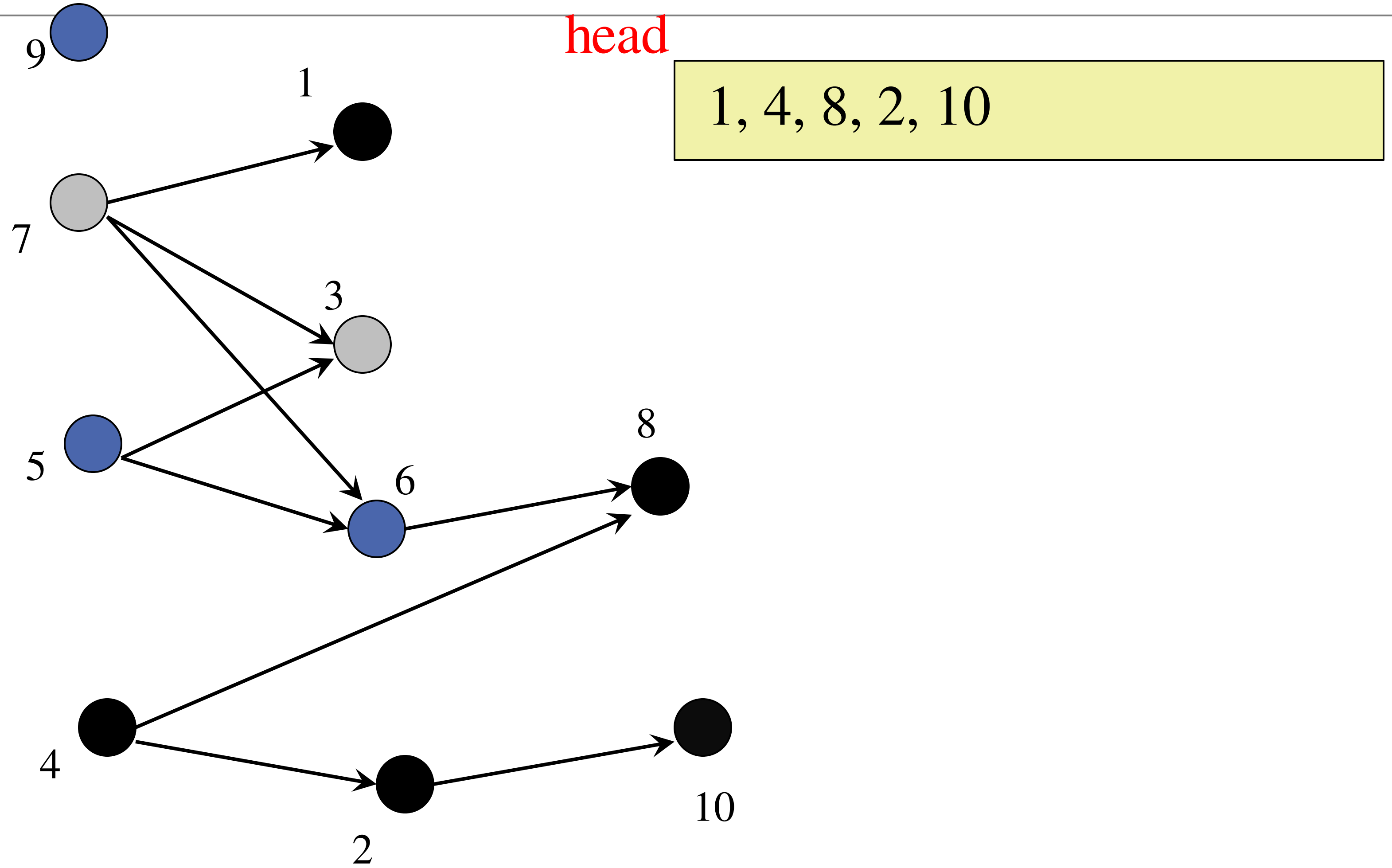
Topology sort Algorithm



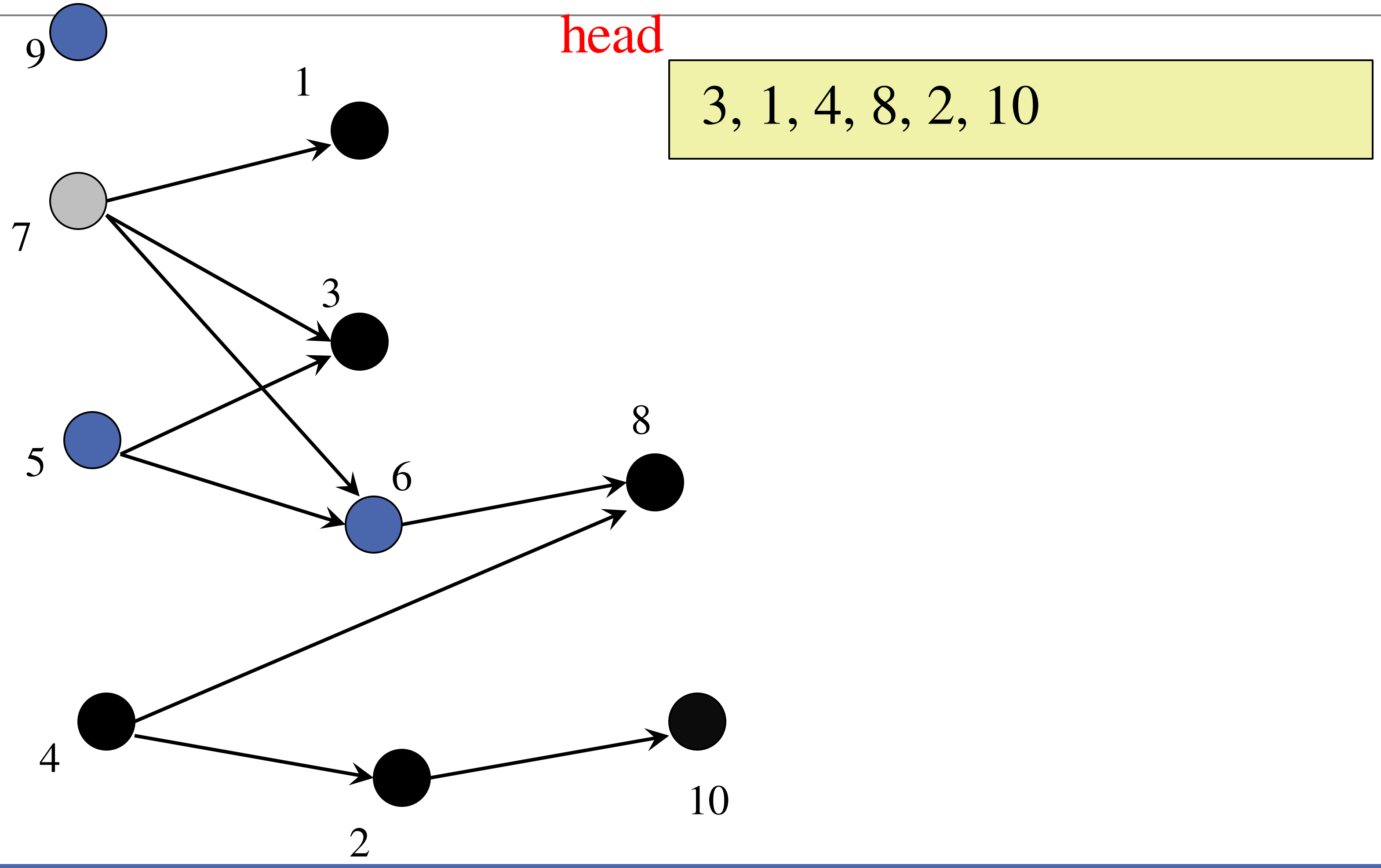
Topology sort Algorithm



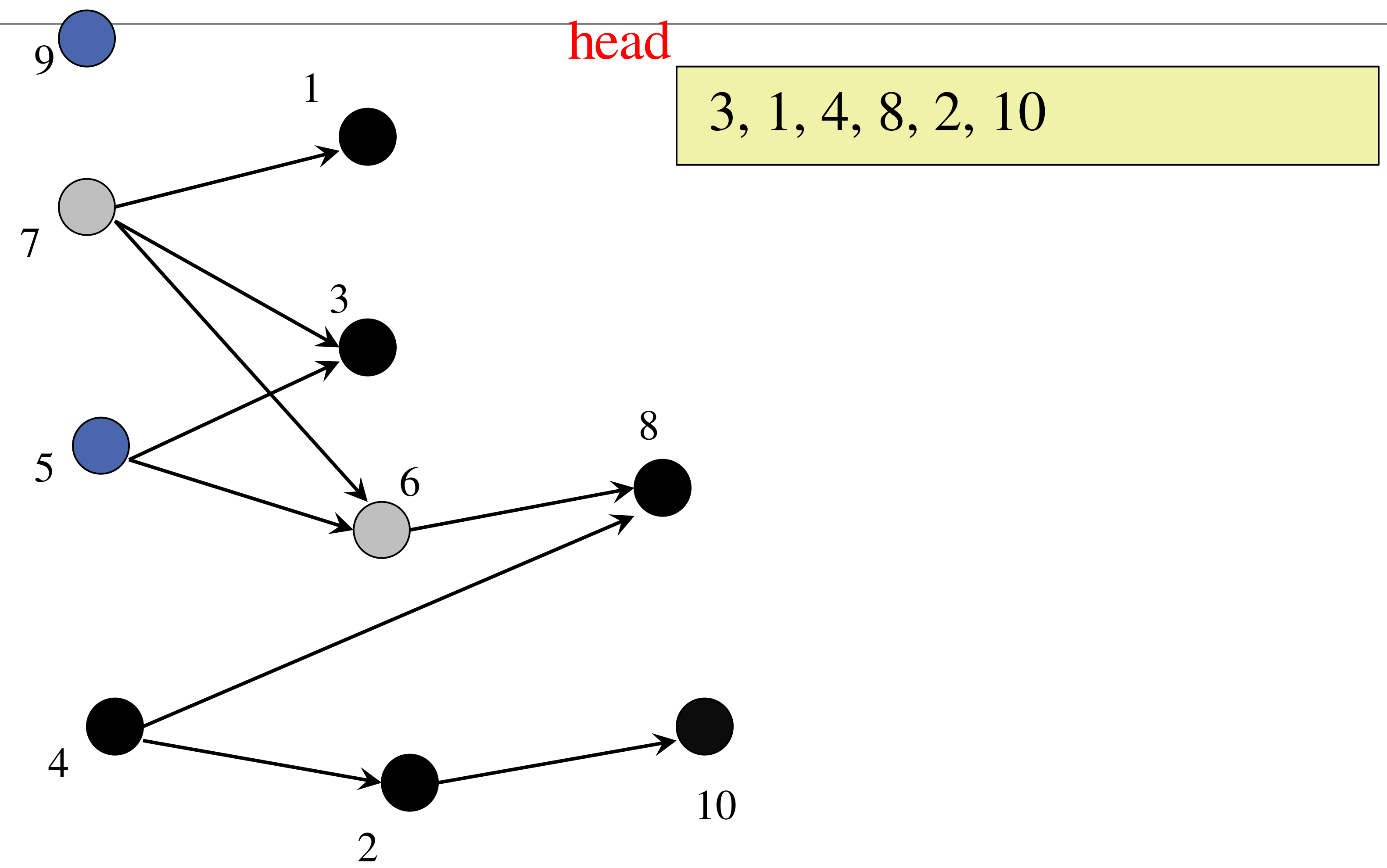
Topology sort Algorithm



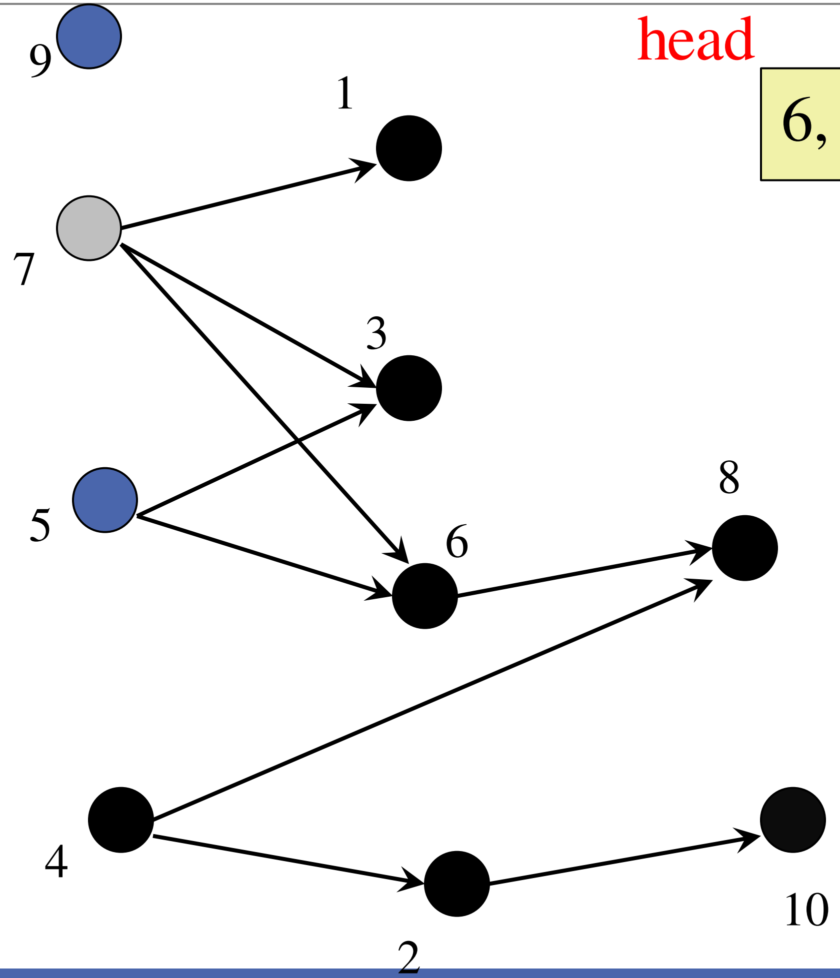
Topology sort Algorithm



Topology sort Algorithm



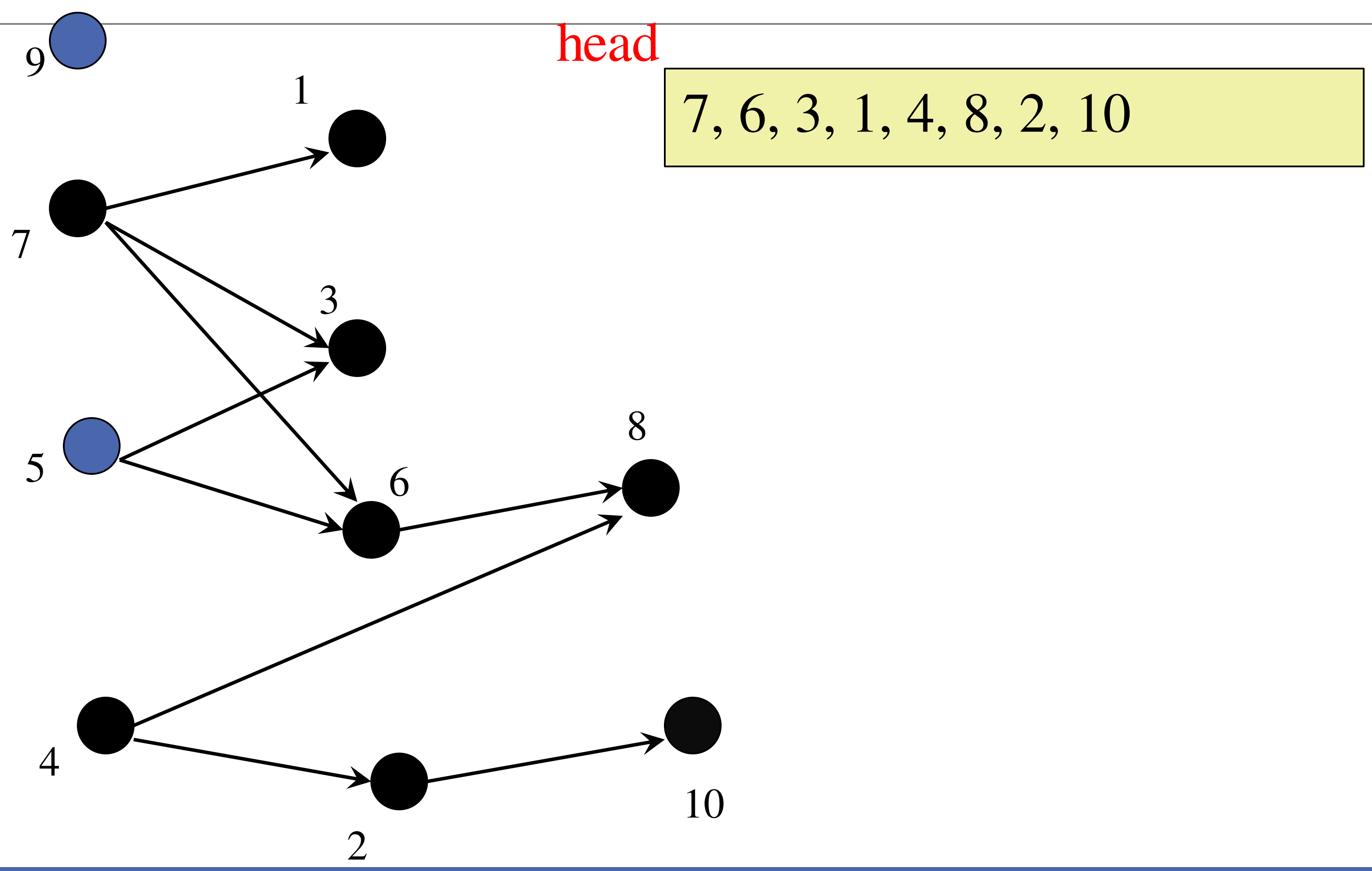
Topology sort Algorithm



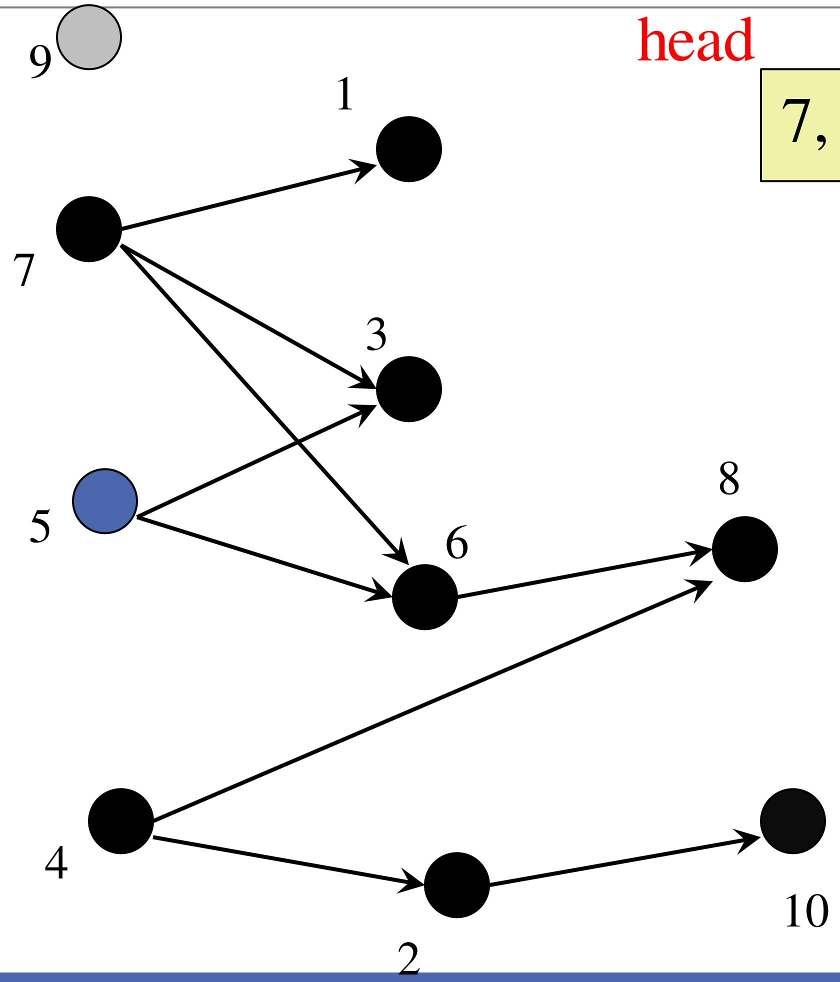
head

6, 3, 1, 4, 8, 2, 10

Topology sort Algorithm



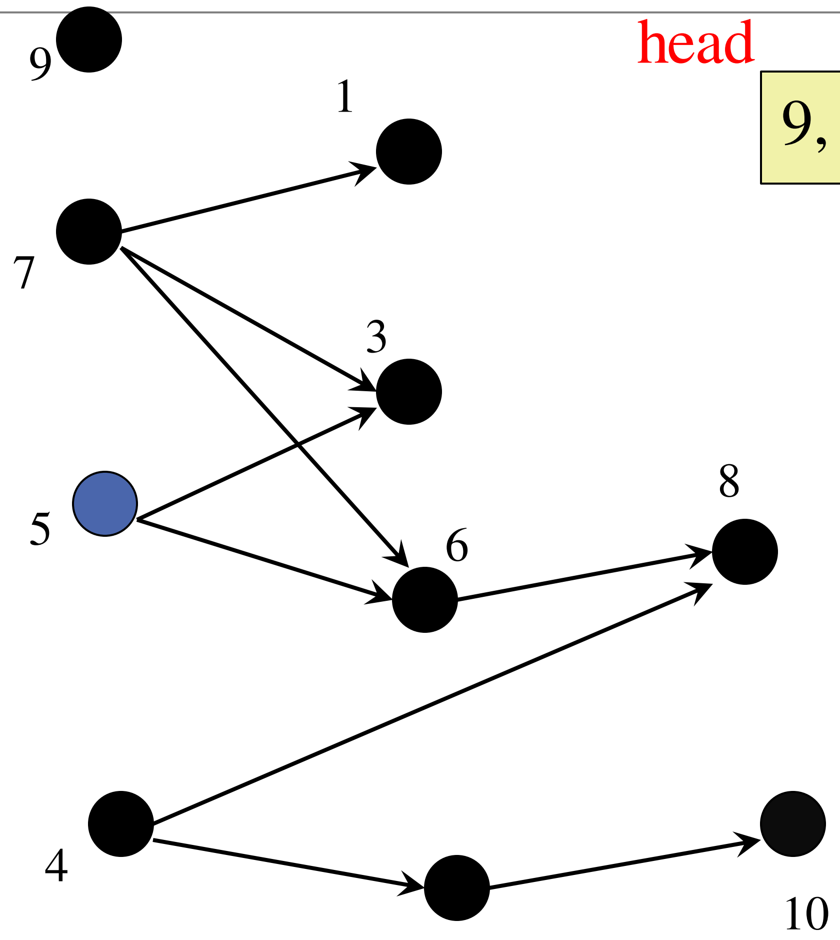
Topology sort Algorithm



head

7, 6, 3, 1, 4, 8, 2, 10

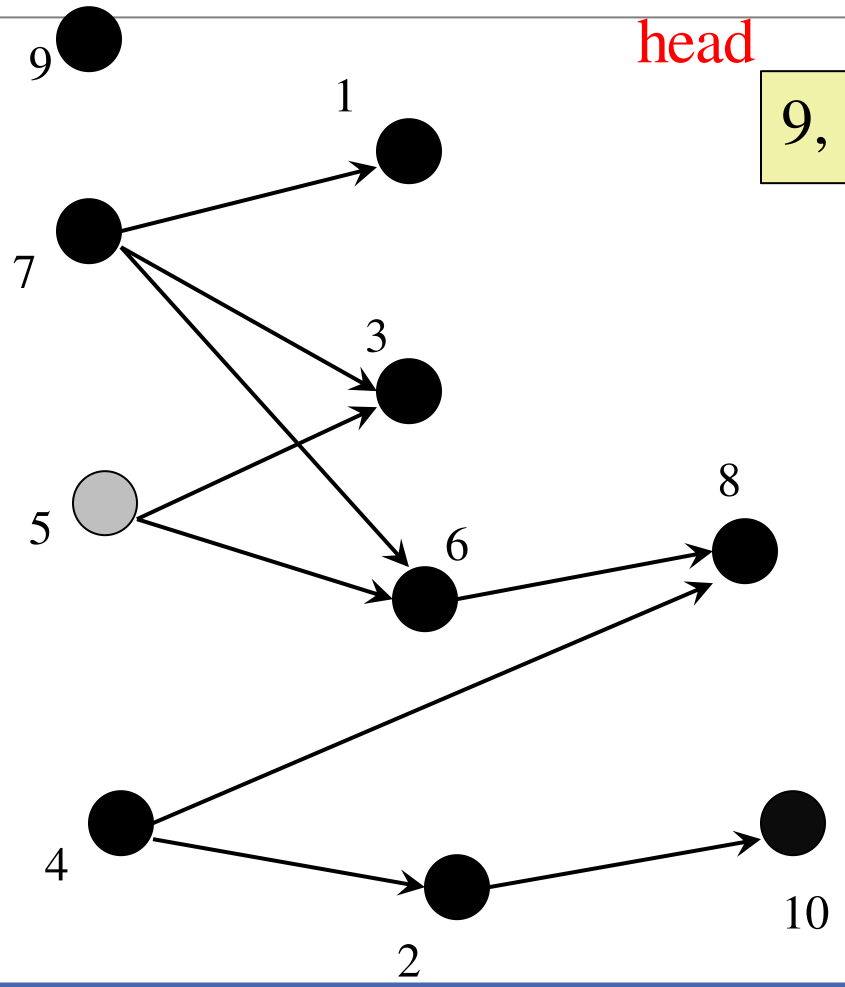
Topology sort Algorithm



head

9, 7, 6, 3, 1, 4, 8, 2, 10

Topology sort Algorithm



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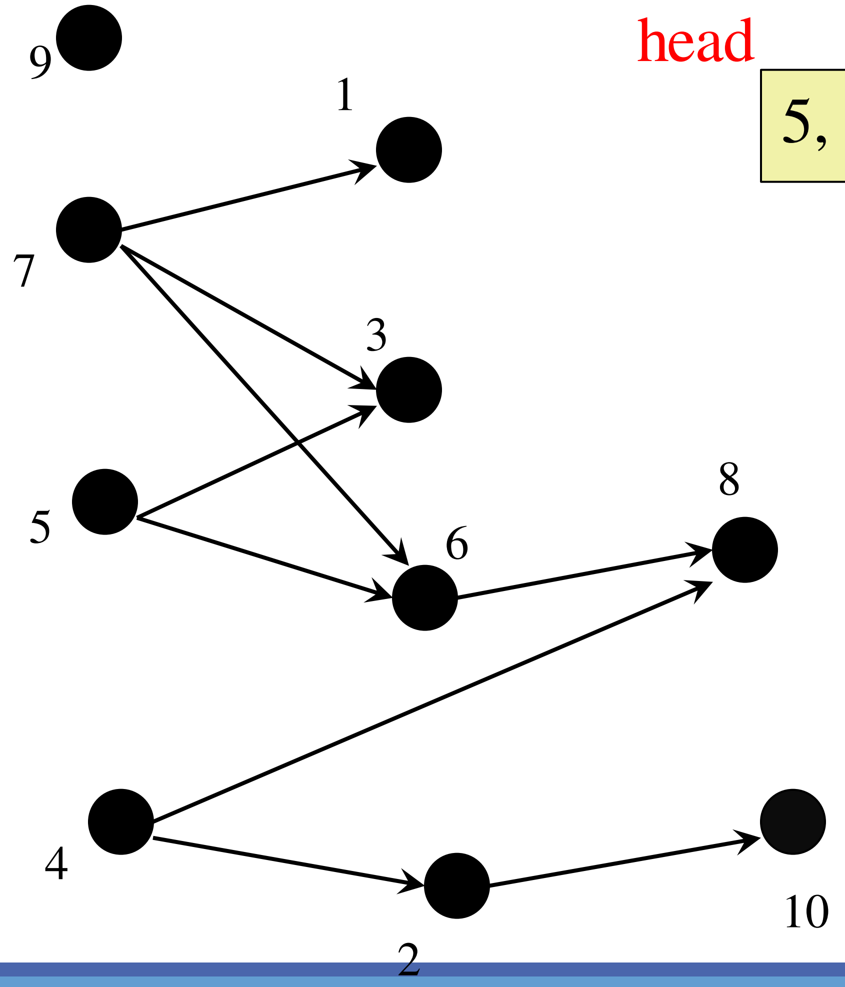
9, 7, 6, 3, 1, 4, 8, 2, 10

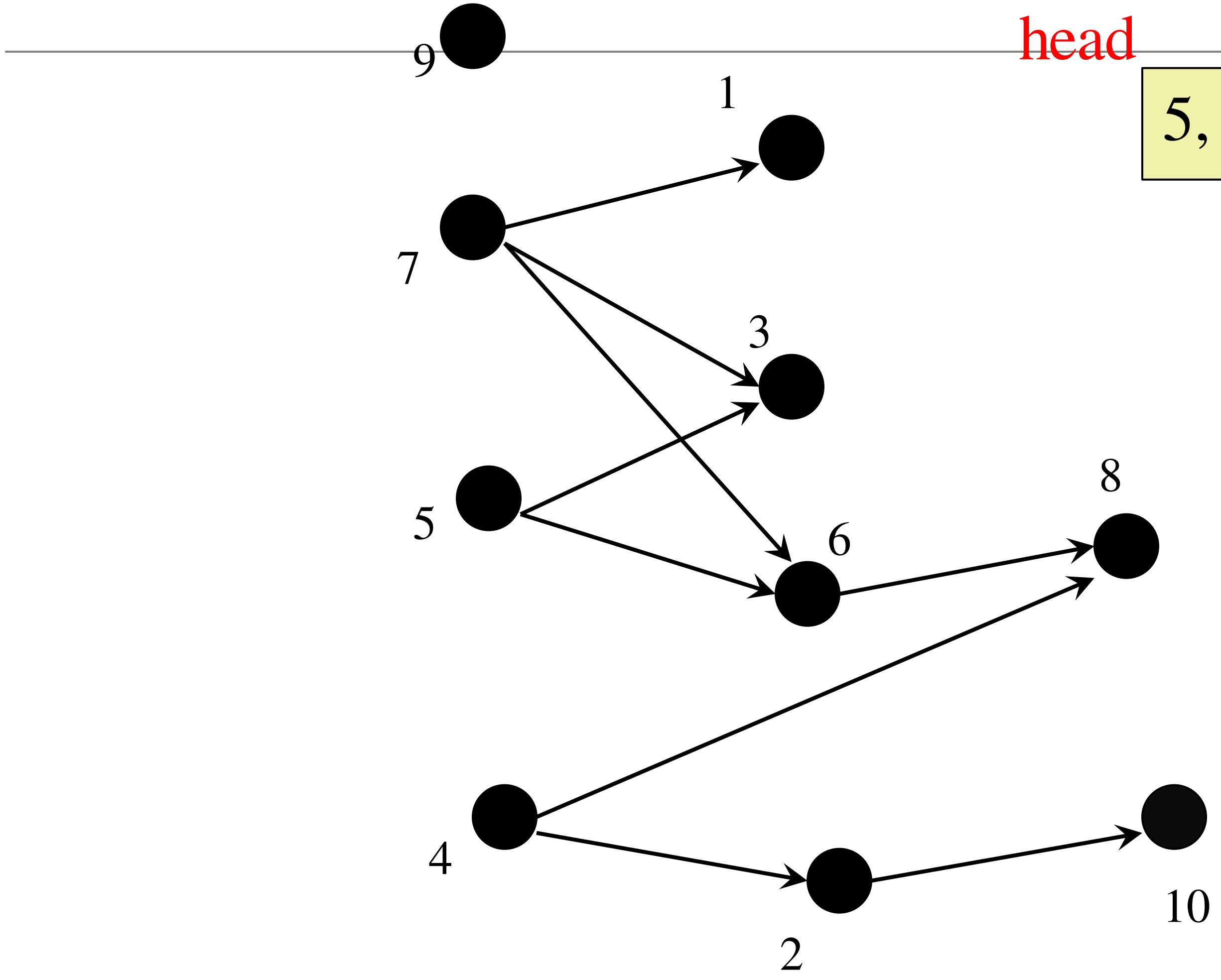
Topology sort Algorithm



head

5, 9, 7, 6, 3, 1, 4, 8, 2, 10

Topology sort Algorithm



5, 9, 7, 6, 3, 1, 4, 8, 2, 10

The final order or jobs

Time complexity = DFS complexity O(V + E)

There can be many orders that meet the requirements

References

www.csie.ntu.edu.tw/~ds/ppt/ch6/chapter6.PPT

https://people.cs.clemson.edu/~pargas/courses/cs212/.../ppt/21DepthFirstSearch.ppt

https://people.cs.clemson.edu/~pargas/courses/cs212/.../ppt/21BreadthFirstSearch.ppt

http://www.serc.iisc.ernet.in/~viren/Courses/2009/SE286/Graph_Dijkstra_Prim_Kruskal.ppt