homework02

P90. 习题 8.4　(A) 3-2，3-4，3-8；6；8；12-2，12-4；13

3. 求下列方程所确定的隐函数的导数或偏导数.

(2) $z^3-3xyz=a^3$，求$\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$；　(4) $e^z=xyz$，求$\frac{\partial z}{\partial x},\frac{\partial z}{\partial y},\frac{\partial^2 z}{\partial x^2}$；

(8) $e^{x+y}\sin(x+z)=0$，求$\frac{\partial z}{\partial x},\frac{\partial z}{\partial y},\frac{\partial x}{\partial y}$.

6. 设函数 $u=u(x,y)$，$v=v(x,y)$ 由

$$\begin{cases} u+v=x+y, \\ xu+yv=1 \end{cases}$$

确定，求 du，dv.

8. 设

$$\begin{cases} x=-u^2+v+z, \\ y=u+vz, \end{cases}$$

求$\frac{\partial u}{\partial x},\frac{\partial v}{\partial x},\frac{\partial u}{\partial z}$.

12. 求由下列方程组所确定的函数的导数或偏导数

(2) $\begin{cases} u=f(ux,v+y), \\ v=g(u-x,v^2y), \end{cases}$ 其中 f,g 具有一阶连续偏导数，求$\frac{\partial u}{\partial x},\frac{\partial v}{\partial x}$；　(4) $\begin{cases} lx+my+nz=a, \\ x^2+y^2+z^2=b, \end{cases}$ 求$\frac{dy}{dx},\frac{dz}{dx}$.

13. 求方程组

$$\begin{cases} x+y+z=0, \\ x^3+y^3-z^3=10 \end{cases}$$

确定的隐函数组 $y=y(x)$，$z=z(x)$ 在点 $P(1,1,-2)$ 的导数 y',z' 与 y'',z''.

P90 习题 8.4

3. 求下列方程所确定的隐函数的导数或偏导数.

(2) $z^3-3xyz=a^3$，求$\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$；　(4) $e^z=xyz$，求$\frac{\partial z}{\partial x},\frac{\partial z}{\partial y},\frac{\partial^2 z}{\partial x^2}$；

A.3-2. 解. 依题意

$$3z^2\,dz-3xy\,dz-3yz\,dx-3zx\,dy=0\Rightarrow z_x=\frac{yz}{z^2-xy}\quad z_y=\frac{zx}{z^2-xy}$$

A.3-4. 解. 依题意

$$e^z\,dz=xy\,dz+yz\,dx+zx\,dy\Rightarrow z_y=\frac{zx}{e^z-xy}=\frac{zx}{xyz-xy}=\frac{z}{yz-y}$$

$$z_x=\frac{yz}{e^z-xy}=\frac{yz}{xyz-xy}=\frac{z}{xz-x}=\frac{1}{x}\left(\frac{1}{z-1}+1\right)$$

$$\Rightarrow z_{xx}=\frac{-1}{x^2}\frac{z}{z-1}-\frac{1}{x}\frac{z_x}{(z-1)^2}=\frac{-z}{x^2(z-1)}+\frac{-z}{x^2(z-1)^3}$$

P90 习题 8.4

(8) $e^{x+y}\sin(x+z)=0$，求$\frac{\partial z}{\partial x},\frac{\partial z}{\partial y},\frac{\partial x}{\partial y}$.

6. 设函数 $u=u(x,y)$，$v=v(x,y)$ 由
$$\begin{cases} u+v=x+y, \\ xu+yv=1 \end{cases}$$
确定，求 du,dv.

A.3-8. **解. 依题意**

$$e^{x+y}\sin(x+z)(dx+dy)+e^{x+y}\cos(x+z)(dx+dz)=0$$
$$(\sin(x+z)+\cos(x+z))\,dx+\sin(x+z)\,dy+\cos(x+z)\,dz=0$$
$$\Rightarrow z_x=-1-\tan(x+z),\quad z_y=-\tan(x+z),\quad x_y=\frac{-\sin(x+z)}{\sin(x+z)+\cos(x+z)}$$

A.6. **解. 依题意**

$$\begin{cases} du+dv=dx+dy \\ x\,du+y\,dv=-u\,dx-v\,dy \end{cases} \Rightarrow \begin{cases} (y-x)\,du=(y+u)\,dx+(y+v)\,dy \\ (x-y)\,dv=(x+u)\,dx+(x+v)\,dy \end{cases}$$

$$\Rightarrow du=\frac{y+u}{y-x}dx+\frac{y+v}{y-x}dy,\quad dv=\frac{x+u}{x-y}dx+\frac{x+v}{x-y}dy$$

P90 习题 8.4

8. 设

$$\begin{cases} x = -u^2 + v + z, \\ y = u + vz, \end{cases}$$

求 $\dfrac{\partial u}{\partial x}, \dfrac{\partial v}{\partial x}, \dfrac{\partial u}{\partial z}$.

A.8. **解. 依题意**

$$\begin{cases} 2u\,du - dv = -dx + dz \\ du + z\,dv = dy - v\,dz \end{cases} \Rightarrow \begin{cases} (2uz+1)\,du = -z\,dx + dy + (z-v)\,dz \\ (2uz+1)\,dv = dx + 2u\,dy - (2uv+1)\,dz \end{cases}$$

$$u_x = \frac{-z}{1+2uz}, \quad v_x = \frac{1}{1+2uz} \quad \text{and} \quad u_z = \frac{z-v}{1+2uz}$$

P90 习题 8.4

12. 求由下列方程组所确定的函数的导数或偏导数

(2) $\begin{cases} u=f(ux,v+y), \\ v=g(u-x,v^2y), \end{cases}$ 其中 f,g 具有一阶连续偏导数，求 $\frac{\partial u}{\partial x}$，$\frac{\partial v}{\partial x}$；

A.12-2. 解. 依题意，方程组两边对 x 求偏导

$$\begin{cases} u_x = f_1(u_x x + u) + f_2 v_x \\ v_x = g_1(u_x - 1) + 2vyg_2 v_x \end{cases} \Rightarrow \begin{cases} \big(1 - xf_1\big) u_x - f_2 v_x = f_1 u \\ g_1 u_x + \big(2vyg_2 - 1\big) v_x = g_1 \end{cases}$$

$$\Rightarrow \begin{cases} \big(g_1 f_2 + (1 - xf_1)(2vyg_2 - 1)\big) u_x = (2vyg_2 - 1)uf_1 + f_2 g_1 \\ \big(g_1 f_2 + (1 - xf_1)(2vyg_2 - 1)\big) v_x = g_1(1 - xf_1) - f_1 g_1 u \end{cases}$$

$$\Rightarrow u_x = \frac{(2vyg_2 - 1)uf_1 + f_2 g_1}{g_1 f_2 + (1 - xf_1)(2vyg_2 - 1)} \qquad v_x = \frac{g_1(1 - xf_1 - uf_1)}{g_1 f_2 + (1 - xf_1)(2vyg_2 - 1)}$$

12．求由下列方程组所确定的函数的导数或偏导数

(4) $\begin{cases} lx+my+nz=a, \\ x^2+y^2+z^2=b, \end{cases}$ 求$\dfrac{dy}{dx}$，$\dfrac{dz}{dx}$.

A.12-4. 解. 依题意，方程组两边对 x 求偏导

$$\begin{cases} l+my_x+nz_x=0 \\ 2x+2yy_x+2zz_x=0 \end{cases} \Rightarrow \begin{cases} (zl-nx)+(mz-ny)y_x=0 \\ (ly-mx)+(ny-zm)z_x=0 \end{cases}$$

$$\Rightarrow y_x=\frac{nx-zl}{mz-ny} \qquad z_x=\frac{ly-mx}{mz-ny}$$

13. 求方程组

$$\begin{cases} x+y+z=0, \\ x^3+y^3-z^3=10 \end{cases}$$

确定的隐函数组 $y=y(x)$, $z=z(x)$ 在点 $P(1,1,-2)$ 的导数 y', z' 与 y'', z''.

A.13. 解. 依题意，方程组两边对 x 求偏导

$$\begin{cases} 1+y'+z'=0 \\ x^2+y^2y'-z^2z'=0 \end{cases}$$

$$\Rightarrow \text{在 } (1,1,-1) \quad \begin{cases} 1+y'+z'=0 \\ 1+y'-4z'=0 \end{cases} \Rightarrow y'=-1, \quad z'=0$$

$$\begin{cases} y''+z''=0 \\ 2x+2y(y')^2+y^2y''-2z(z')^2-z^2z''=0 \end{cases}$$

$$\Rightarrow \text{在 } (1,1,-1) \quad \begin{cases} y''+z''=0 \\ 2+2+y''-4z''=0 \end{cases} \Rightarrow y''=\frac{-4}{5}, \quad z''=\frac{4}{5}$$