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# Active deep learning on entity resolution by risk sampling



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#### ABSTRACT

While the state-of-the-art performance on entity resolution (ER) has been achieved by deep learning, its effectiveness depends on large quantities of accurately labeled training data. To alleviate the data labeling burden, Active Learning (AL) presents itself as a feasible solution that focuses on data deemed useful for model training.

Building upon the recent advances in risk analysis for ER, which can provide a more refined estimate on label misprediction risk than the simpler classifier outputs, we propose a novel AL approach of risk sampling for ER. Risk sampling leverages misprediction risk estimation for active instance selection. Based on the core-set characterization for AL, we theoretically derive an optimization model which aims to minimize core-set loss with non-uniform Lipschitz continuity. Since the defined weighted K-medoids problem is NP-hard, we then present an efficient heuristic algorithm. Finally, we empirically verify the efficacy of the proposed approach on real data by a comparative study. Our extensive experiments have shown that it outperforms the existing alternatives by considerable margins.

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#### 1. Introduction

The purpose of entity resolution (ER) is to identify the equivalent records that refer to the same real-world entity. Considering the running example shown in Fig. 1, ER needs to match the paper records between two tables,  $R_1$  and  $R_2$ . A pair  $\langle r_{1i}, r_{2j} \rangle$ , in which  $r_{1i}$  and  $r_{2j}$  denote a record in  $R_1$  and  $R_2$  respectively, is called an *equivalent* pair if and only if  $r_{1i}$  and  $r_{2j}$  refer to the same paper; otherwise, it is called an *inequivalent* pair. In this example,  $r_{11}$  and  $r_{21}$  are *equivalent* while  $r_{11}$  and  $r_{22}$  are *inequivalent*. ER can be treated as a binary classification problem tasked with labeling record pairs as *equivalent* or *inequivalent*. Therefore, various learning models have been proposed for ER [1]. As many other classification tasks (e.g. image and speech recognition), the state-of-the-art performance on ER has been achieved by deep learning [2–7].

Unfortunately, the efficacy of Deep Neural Network (DNN) models depends on large quantities of accurately labeled training data, which may not be readily available in practical scenarios. One possible way to overcome this issue is by active learning,

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in which data are actively sampled to be labeled by human oracles with the goal of maximizing model performance while minimizing labeling costs. Various sampling strategies have been proposed for active learning over the years coming from different perspectives, e.g. uncertainty [8], representativeness [9] and expected model change [10]. There have also been different combinations of Uncertainty with Representativeness [11,12] or with Expected Model Change [13] in an attempt to get the best of both worlds. In the traditional setting, AL algorithms typically choose a single point at each iteration; however, this is not feasible for DNN models since (1) a single point is likely to have no statistically significant impact on the accuracy due to the locality of optimization methods, and (2) each iteration requires a full training until convergence which makes it intractable to query labels one-by-one. Hence, most proposed AL algorithms for DNNs [9,11,14-17], take the strategy of batch selection that queries labels for a large subset at each iteration.

Uncertainty, considered the cheapest to obtain, is the mostly used sampling strategy due to its robustness across architectures and domains [18]. Empirical studies [19] have also revealed that it is usually highly competitive with the existing but more complicated alternatives. We note that risk analysis for ER has been recently studied [20–22] with the latter representing the most recent interpretable and learnable solution, henceforth denoted LearnRisk. Risk analysis estimates the misprediction risk of a classifier when applied to a certain workload. It has been empirically shown [22] that LearnRisk can identify mislabeled instances

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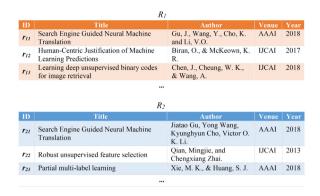


Fig. 1. An entity resolution running example.

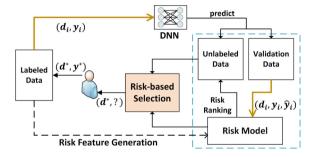


Fig. 2. The framework of risk sampling for active learning.

with considerably higher accuracy than the existing uncertainty measures, which are directly estimated upon classifier outputs. Traditionally, the motivation behind using uncertainty sampling in AL is to make the model more familiar with examples that come from uncertain areas. Risk analysis goes a step further by detecting mispredictions on unseen data regardless of the classifier's self-reported uncertainty. This enables access to more informative examples that could have a positive impact on classifier training. Hence, risk analysis is naturally fit as an AL sampling strategy.

Therefore, in this paper, we propose a novel AL approach of risk sampling for ER. Fig. 2 illustrates the risk sampling framework, which leverages the results of risk analysis in the sampling step of active learning. Based on the existing core-set characterization [9] for AL, we theoretically derive an optimization model which aims to minimize the core-set loss with non-uniform Lipschitz continuity. Since the defined weighted K-medoids problem is NP-hard, we then present an efficient heuristic algorithm. It is noteworthy that the strategy of risk sampling tends to sample challenging examples for the classifier, and spur it into learning better representations by looking farther than the low confidence regions in the current representation space. The main contributions of this paper can be summarized as follows:

- We propose a novel approach of risk sampling for ER active learning.
- We define a theoretically sound optimization model for risk sampling, and due to its NP-hardness, present an efficient weighted K-medoids algorithm.
- We validate the efficacy of the proposed solution on real datasets. Our extensive experiments have shown that it can effectively accelerate classifier training compared with the existing alternatives. Furthermore, its performance is very robust w.r.t the size of validation data.

#### 2. Related work

We review related work from the orthogonal perspectives of entity resolution and active learning.

**Entity Resolution.** Also known as Entity Matching or Record Linkage, ER plays a key role in data integration and has been extensively studied in the literature [23,24]. Other than the rule-based and probabilistic solutions [25–27], several machine learning models have been proposed, including Support Vector Machines [1], end-to-end deep neural network architectures [2–5], and pretrained models [6,7].

ER remains very challenging in real scenarios due to the prevalence of dirty data. Therefore, there is a need for risk analysis, alternatively called trust scoring or confidence ranking in the literature. It encompasses a multitude of methods that all intend to detect situations where a deployed DNN model is prone to misprediction. The proposed solutions range from those simply based on the model's output probabilities to more sophisticated, interpretable, and learnable ones [22,28-31]. Among them, Learn-Risk [22] is an interpretable and learnable framework for ER that is able to construct a dynamic risk model tuned towards a specific workload. It measures the risk using the VaR (Valueat-Risk) [32] metric from financial investment modeling. It is noteworthy that the concept of risk has also been leveraged in other works for different purposes as demonstrated in [33-35]. For instance, in [33], RecRisk considers two risk facets, Sense Drop and Blue lov, which affect recommendation quality from the user perspective, and presents an approach to minimize the value of risk facets in personalized recommendations.

Active Learning. Active learning has been extensively researched in the context of machine learning. The most prominent approaches that proved to perform well include margin-based, maximum entropy, Query by committee and Expected variance reduction to name a few [36]. However, many of the above methods pose challenges when applied to deep neural networks. The margin-based approaches are hindered by the fact that neural networks have an intractable decision boundary. Query by committee requires maintaining multiple classifiers and retraining them in each iteration which is not very practical. Similarly, the variance reduction methods require classifier retraining for each unlabeled point; this process is prohibitive even for shallow models. Active learning for ER has also received great attention [37-41]. In the low-resource setting, ER was also tackled using deep transfer active learning [16]. The proposed sampling method relies on Entropy to select a batch combined of uncertain and high-confidence pairs representing both class labels. The automatically labeled high-confidence pairs are added to prevent overfitting to uncertain examples.

More recently, active learning for DNNs has also been studied. mostly focusing on image classification. It has been shown in [42. 43] that applying dropout at test time can approximate Bayesian inference enabling the application of Bayesian methods to deep learning. It has also been shown in [44] that ensemble-based uncertainty performs better and is more calibrated than a single model or dropout-based uncertainty. The framework proposed in [45] automatically selected and pseudo-annotated unlabeled samples in addition to uncertain samples. The work in [46] approximated the distance to decision boundary by the distance to the nearest adversarial example. The works in [10,13] used an expected model change measure which chooses examples that maximize the impact on the learned model weights when labeled. Other recent works include generative data augmentation for AL [15], adversarial network-based discrimination of informative points [47] and detrimental point processes-based batch selection [14] to name a few. There also exist proposals combining

uncertainty with representativeness using data representation and entropy such as [11,12], or relying on gradient-based representation and gradient amplitude as a proxy to uncertainty [17]. It is worthy to point out that the proposed approach of risk sampling can be easily generalized to image classification when provided with effective risk analysis techniques.

#### 3. Preliminaries

In this section, we formally state the AL task, and then introduce the risk analysis technique for ER, LearnRisk.

### 3.1. Task statement

Suppose that we have a set of record pairs  $D = \{d_i, y_i\}$ , where a pair  $d_i$  can be labeled as *equivalent*  $(y_i = 1)$  or *inequivalent*  $(y_i = 0)$ . We follow the standard pool-based setting in which the set of training data, D, is partitioned into a small initial labeled set  $L = \{d_j, y_j\}$  and an unlabeled set U. We also assume the existence of two other sets: a validation set V that is commonly used for hyperparameter tuning as well as early stopping for DNN classifiers, and an independent test set T used to evaluate the classifier's generalization performance on unseen data.

The task of ER active learning is formally defined as follows:

**Definition 1.** Provided with the test and validation sets T and V, the labeled set L and the unlabeled set (the pool) U, active learning iteratively selects a batch of data  $Q \subseteq U$  that minimizes a specified criterion given a classifier  $h_L$  trained on L. At each iteration, once Q is labeled, it is removed from U and added to the labeled set L, i.e.  $U \leftarrow U \setminus Q$ ,  $L \leftarrow L \cup Q$ ; finally, a classifier is retrained on the updated set L.

#### 3.2. Risk analysis for ER: LearnRisk

Originally proposed in [22], the risk analysis pipeline operates in three main steps: *Risk feature generation* followed by *Risk model construction* and finally *Risk model training*.

## 3.2.1. Risk feature generation

This step automatically generates risk features in the form of interpretable rules based on one-sided decision trees. The algorithm ensures that the resulting rule-set is both discriminative, i.e, each rule is highly indicative of one class label over the other; and has a high data coverage, i.e, its validity spans over a subpopulation of the workload. As opposed to classical settings where a rule is used to label pairs to be equivalent or inequivalent, a risk feature focuses exclusively on one single class. Consequently, risk features act as indicators of the cases where a classifier's prediction goes against the knowledge embedded in them. An example of such rules is:

$$r_i[Year] \neq r_i[Year] \rightarrow inequivalent(r_i, r_i),$$

where  $r_i$  denotes a record and  $r_i[Year]$  denotes  $r_i$ 's Year attribute value. With this knowledge, a pair predicted as *equivalent* whose two records have different publication years is assumed to have a high risk of being mislabeled.

## 3.2.2. Risk model construction

Once high-quality features have been generated, the latter are readily available for the risk model to make use of, allowing it to be able to judge a classifier's outputs backing up its decisions with human-friendly explanations. To achieve this goal, Learn-Risk, drawing inspiration from investment theory, models each pair's equivalence probability distribution (portfolio reward) as

the aggregation of the distributions of its compositional features (stock rewards).

Practically, the equivalence probability of a pair  $d_i$  is modeled by a random variable  $p_i$  that follows a normal distribution  $\mathcal{N}(\mu_i, \sigma_i^2)$ , where  $\mu_i$  and  $\sigma_i^2$  denote its expectation and variance respectively. Given a set of m risk features  $f_1, f_2, \ldots, f_m$ , let  $\mathbf{w} = [w_1, w_2, \ldots, w_m]$  denote their corresponding weight vector. Suppose that  $\boldsymbol{\mu}_F = [\mu_{f_1}, \mu_{f_2}, \ldots, \mu_{f_m}]^T$  and  $\boldsymbol{\sigma}_F^2 = [\sigma_{f_1}^2, \sigma_{f_2}^2, \ldots, \sigma_{f_m}^2]^T$  represent their corresponding expectation and variance vectors respectively, such that  $\mathcal{N}(\mu_{f_j}, \sigma_{f_j}^2)$  denotes the equivalence probability distribution of the feature  $f_j$ . Accordingly,  $d_i$ 's distribution parameters are estimated by:

$$\mu_i = \mathbf{b}_i(\mathbf{w} \circ \boldsymbol{\mu}_F); \ \sigma_i^2 = \mathbf{b}_i(\mathbf{w}^2 \circ \boldsymbol{\sigma}_F^2)$$

Where  $\circ$  represents the element-wise product and  $\mathbf{b}_i$  is a one-hot feature vector.

Besides one-sided decision rules, LearnRisk also incorporates classifier's output probability as one of the risk features. Provided with the equivalence distribution  $p_i$  for  $d_i$ , its risk is estimated by the metric of Value-at-Risk (VaR) [32]. Note that compared with previous simpler alternatives using a single value to represent equivalence probability, LearnRisk can more accurately capture the uncertainty of the label status by a distribution.

#### 3.2.3. Risk model training

Finally, the risk model is trained on a classifier's validation data to optimize a learn-to-rank objective [48] by tuning the risk feature weight parameters  $(w_i)$  as well as their variances  $(\sigma_i^2)$ . As for their expectations  $(\mu_i)$ , they are considered as prior knowledge, and are estimated from labeled training data. Once trained, the risk model can be used to assess the misclassification risk on an unseen workload labeled by the classifier.

## 4. Risk sampling

In AL, each individual iteration can be seen as a standard supervised learning procedure in which a model is fit to labeled data, then the best configuration is selected based on the performance on a disjoint validation set. As shown in Fig. 2, the incorporation of risk analysis as an extra step into the process is therefore fairly straightforward. In this section, we illustrate the approach of risk sampling on the classical DeepMatcher model [2] for ER, which was built upon recurrent neural networks (RNN). The proposed approach is however similarly applicable to other DNN solutions, provided they can be shown to be Lipschitz-continuous.

In the rest of this section, we first theoretically derive the optimization model for risk sampling based on the core-set characterization, and then due to its NP-hardness, present a heuristic algorithm for its efficient solution. The notation used throughout this section as well as in Appendix is given in Table 1.

#### 4.1. Optimization model: Theoretical derivation

Suppose that the ER workload, D, is drawn from a distribution  $p_Z$ . Based on the core-set characterization for AL presented in [9], we consider the upper-bound of active learning loss in batch setting defined as

$$|E_{d,y \sim p_{Z}}[l(d,y)]| \leq \left| E_{d,y \sim p_{Z}}[l(d,y)] - \frac{1}{n} \sum_{(d_{i},y_{i}) \in D} l(d_{i},y_{i}) \right| + \frac{1}{|Q|} \sum_{(d_{j},y_{j}) \in Q} l(d_{j},y_{j})$$

Table 1

Notation.	
Symbol	Description
$d_i$	a pair of left and right records $\langle \overrightarrow{r_i}, \overrightarrow{r_i} \rangle$
$y_i$	pair's label
$n_A$	number of attributes per record
$T_{d_i} \atop \overleftarrow{a_k} (\overrightarrow{a_k}) \\ w_t^k$	total number of tokens in pair $d_i$
$\overleftarrow{a_k} \ (\overrightarrow{a_k})$	k'th attribute of the left (resp. right) record
$w_t^k$	$t$ 'th token for attribute $a_k$
$T_k$	number of tokens in attribute $a_k$
m	word embedding dimension
$X_{d_i} \in \mathbb{R}^{T_{d_i} \times m}$ $X^k \in \mathbb{R}^{T_k \times m}$	pair $d_i$ 's matrix representation
$X^{k} \in \mathbb{R}^{T_k \times m}$	representation of $a_k$
$X_t^k \in \mathbb{R}^m$	$t$ 'th token's vector representation in attribute $a_k$

+ 
$$\left| \frac{1}{n} \sum_{(d_i, y_i) \in D} l(d_i, y_i) - \frac{1}{|Q|} \sum_{(d_j, y_j) \in Q} l(d_j, y_j) \right|$$

in which the loss is controlled by the training error of the model on the labeled subset, the generalization error over the full dataset and a term referred to as the core-set loss. Core-set loss is simply the difference between average empirical loss over the set of points which have labels and the average empirical loss over the entire dataset including unlabeled points. Empirically, it is widely observed that DNNs are highly expressive leading to very low training error and they typically generalize well for various classification problems. Hence, the critical part for active learning is the core-set loss. Following this observation, we start off with the core-set loss defined as

$$\left| \frac{1}{n} \sum_{(d_i, y_i) \in D} l(d_i, y_i) - \frac{1}{|L \cup Q|} \sum_{(d_i, y_i) \in L \cup Q} l(d_j, y_j) \right|$$
 (1)

Where l is the loss of the model trained on  $L \cup Q$  ( $A_{L \cup Q}$ ). Informally, given an initial labeled set (L) and a budget (b), we are trying to find a set of points to query (Q), such that the learned model's performance on the labeled subset ( $L \cup Q$ ) and that on the whole dataset (D) will be as close as possible. In [9], it has been shown that provided with a  $\lambda$ -Lipschitz continuous convolutional neural network, if a set of balls, denoted by s, with radius  $\delta_s$  centered at each member of s can cover the entire set D, the core-set loss can be bound with the covering radius  $\delta_s$  and a term that goes to zero with rate depending only on n.

The existing core-set characterization applies the global Lipschitz value for all unlabeled points. However, it can be observed that, provided a Lipschitz continuous DNN, the local Lipschitz continuities of unlabeled points are usually not uniform, or their local Lipschitz values may be vastly different. Since the DeepMatcher model was built upon RNN, in what follows, we first theoretically establish the Lipschitz continuity of RNN and the DNN model of DeepMatcher, and then derive the optimization model for risk sampling based on non-uniform Lipschitz continuity.

**Lipschitz Continuity of RNN.** For a generic RNN, we have Lemma 1 on its Lipschitz continuity. We have provided the proofs of the lemmas and theorems in the appendix.

**Lemma 1.** The loss function defined as the 2-norm between one-hot class labels and the Softmax outputs of a stable RNN with T time steps and input dimension m, followed by  $n_{fc}$  fully connected layers defined over C classes is  $\frac{\sqrt{(C-1)T\,m}}{C}\,\alpha^{n_{fc}+1}$ -Lipschitz.

Note that  $\alpha$  in Lemma 1 is a bound over the operator norms of all trainable matrices in the RNN and fully connected layers. Although  $\alpha$  is in general unbounded, it can be made arbitrarily

small without changing the loss function's behavior. Moreover, an RNN is said to be *stable* when the gradients cannot explode, which is only valid when  $\alpha < 1$  [49]. In order to extend the result in Lemma 1 to the DeepMatcher solution for ER, we define a corresponding neural network model, then show that it is Lipschitz continuous in Theorem 1.

#### **Definition 2.**

**DNN Model for ER.** Suppose that each pair, denoted by  $d_i = \langle \overrightarrow{r_i}, \overrightarrow{r_i} \rangle$ , in an ER workload, consists of  $n_A$  attributes per record  $r_i$ , where each attribute  $a_k$  is a sequence of  $T_k$  tokens  $w_t^k$ . The model first embeds each attribute  $a_k$  as a sequence of vectors using an embedding matrix  $E(X_t^k = E[w_t^k])$ . Then, each attribute is encoded by a stable RNN into a representation  $\mathbf{s_k} \in \mathbb{R}^m$  as

$$s_k = RNN(X^k).$$

Let the attribute similarity layer be defined by a distance function  $F_D: \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^m$ . The k'th attribute pair similarity  $\tilde{s_k}$  between  $\overline{a_k}$  and  $\overline{a_k}$  is then defined as

$$\tilde{\mathbf{s}}_{\mathbf{k}} = F_D(\overleftarrow{\mathbf{s}_{\mathbf{k}}}, \overrightarrow{\mathbf{s}_{\mathbf{k}}}).$$

Finally, the classification layer  $F_C$  is defined by a fully-connected neural network followed by a Softmax function. The model takes the aggregated pair similarities as input and returns the match probability p by

$$p = F_{\mathcal{C}}(\{\tilde{\mathbf{s}}_{1}, \ldots, \tilde{\mathbf{s}}_{\mathbf{n}_{A}}\}).$$

The model defined in Definition 2 is consistent with the network structure defined in the RNN variant of DeepMatcher [2]. On its Lipschitz continuity, we have Theorem 1.

**Theorem 1.** The loss function defined as the 2-norm between one-hot class labels and the Softmax outputs of an RNN-based ER model as defined in Definition 2 with input representation dimension m and maximal number of tokens per pair  $\hat{T}$  is  $\frac{\alpha^{n_{fc}+1}}{2}\sqrt{\hat{T}}$  m-Lipschitz.

**Optimization Model.** Based on the Lipschitz continuity of the DNN model for ER, we establish an upper-bound on the core-set loss of active learning in Theorem 2.

**Theorem 2.** Given a dataset D of size n containing a labeled subset L and a Lipschitz continuous classifier, the core-set loss of active learning satisfies the following upper-bound:

$$\left| \frac{1}{n} \sum_{(d_i, y_i) \in D} l(d_i, y_i) - \frac{1}{|L \cup Q|} \sum_{(d_j, y_j) \in L \cup Q} l(d_j, y_j) \right|$$

$$\leq \frac{1}{n} \sum_{(d_i, y_i) \in L \cup Q} \sum_{(d_i, y_i) \in C_i} \mathbb{L}_i \|X_{d_i} - X_{d_j}\|_2$$
(2)

in which  $\mathbb{L}_i$  represents its Lipschitz constant for the loss of the model trained on  $L \cup Q$ ,  $C_j$  is D's j'th cluster with  $(d_j, y_j) \in L \cup Q$  representing its center and  $\|.\|_2$  is the  $L_2$  norm.

According to Theorem 2, we define the optimization objective for AL as:

$$\min_{Q} \sum_{(d_{i}, y_{j}) \in L \cup Q} \sum_{(d_{i}, y_{i}) \in C_{j}} \mathbb{L}_{i} \| X_{d_{i}} - X_{d_{j}} \|_{2}.$$
(3)

Unfortunately, in Eq. (3),  $\mathbb{L}_i$  is not available prior to the selection of Q and the training of  $A_{L\cup Q}$ . However, it can be observed that given an unlabeled point, its Lipschitz value is closely correlated with its misprediction risk. Indeed, if we consider an unlabeled

## Algorithm 1: Weighted fastPAM

```
Input: D
                  : Full data
        L
                 : Initial labeled data
        b > 0: Query budget
Output: Query Q.
 1: Let Q \leftarrow \text{top } b \text{ points ranked by } \mathbb{L}_i
 2: Calculate the total deviation TD for the initial solution L \cup Q
    by Eq. (6)
 3: repeat
        for all x_i \in D \setminus (L \cup Q) do
 4:
           d_i \leftarrow \mathbb{L}_i \cdot d_{nearest}(x_i)
 5:
           \Delta TD \leftarrow (0, ..., 0, -d_j, ..., -d_j)
 6:
          for all x_o \neq x_j do d_{oj} \leftarrow d(x_o, x_j)
7:
8:
              if n \in Q then
9:
                 Update \Delta TD_n
10:
              if d_{oi} \leq d_n then
11:
                 Update \Delta TD_i for m_i \in Q \setminus \{m_n\}
12:
           Save best swap (\Delta TD^*, m^*, x^*)
13:
        if \Delta TD^* < 0 then
14:
           Swap(m^*.x^*)
15:
           TD \leftarrow TD + \Delta TD^*
16:
17: until \Delta TD^* > 0
```

point  $d_i$ 's misprediction risk  $R_L(d_i)$  as its expected loss, i.e.  $R_L(d_i) = E[l(d_i, y_i)]$ , its Lipschitz value can be empirically estimated by

$$\mathbb{L}'_{i} = \frac{R_{L}(d_{i})}{\min_{(d_{j}, y_{j}) \in L} \|X_{d_{i}} - X_{d_{j}}\|_{2}},$$
(4)

in which  $d_i$  and  $d_j$  denote an unlabeled point and a labeled point, respectively.  $R_L(d_i)$  denotes the misprediction risk of  $d_i$ . This follows straightforwardly from the Lipschitz constant definition for the DNN loss function  $(|l(d_i, y_i) - l(d_j, y_j)| \leq \mathbb{L} \|X_{d_i} - X_{d_j}\|_2)$ . Since the loss of the labeled pair is assumed to be zero, the loss of the unlabeled pair is estimated via its misprediction risk  $R_L(d_i)$ . Therefore, we approximate  $\mathbb{L}_i$  with its empirical estimation based on the latest classifier, which is conveniently available as shown in Eq. (4). The optimization objective of risk sampling is finally defined as

$$\min_{Q} \sum_{(d_{j}, y_{j}) \in L \cup Q} \sum_{(d_{i}, y_{i}) \in C_{j}} \mathbb{L}'_{i} \|X_{d_{i}} - X_{d_{j}}\|_{2}.$$
(5)

#### 4.2. Algorithm

Clearly, the optimization problem defined in Eq. (5) is a sample-weighted version of the classical k-medoids clustering problem [50] with the addition of the weight  $\mathbb{L}_i$  for each non-medoid  $\mathbf{x_i}$ . Given a specified number of clusters k, k-medoids aims at finding k clusters where each cluster is centered around a point in the data. Due to its NP-hardness [51], the classic way to solve the k-medoids problem is via the heuristic Partitioning Around Medoids (PAM) algorithm [50], or its more recent optimized version, namely, fastPAM [52]. Hence, we adapt the fastPAM algorithm to risk sampling.

In the scenario of risk sampling, the number of clusters is the size of the labeled data in addition to the data to be queried, i.e.  $k = |L \cup Q|$ . The *total deviation* (TD) objective to be minimized as shown in Eq. (5) is measured by the sum of dissimilarities of each point to the medoid of its cluster weighted by its corresponding sample-weight with Euclidean distance as its dissimilarity measure, i.e.

$$TD = \sum_{(d_j, y_j) \in L \cup Q} \sum_{(d_i, y_i) \in C_j} \mathbb{L}'_i \| X_{d_i} - X_{d_j} \|_2$$
 (6)

For risk sampling, we need to only optimize Q while keeping L fixed. As fastPAM, the proposed algorithm similarly consists of two phases, BUILD and SWAP. To keep L fixed, we force the initial solution to contain L in BUILD, and then only allow the points in Q to be swapped out of the solution in the SWAP phase.

The algorithm is sketched in Algorithm 1. The first phase generates an initial solution  $L \cup Q$  in line 1. After that, the main search loop for phase two is started at line 3. In each iteration, the algorithm will go through candidate points in line 4, calculating the reduction in the total deviation  $(\Delta TD)$  for each candidate when swapped in place of any non-labeled medoid  $(m \notin L)$ . Lines 7–12 perform the actual calculation w.r.t each medoid and accumulate the values in the  $\Delta TD$  vector. The best swap across candidates and medoids is maintained in  $(\Delta TD^*, m^*, x^*)$  on line 13. The iteration ends by performing the swap between  $m^*$  and  $x^*$  as long as it provides a decrease in TD. Otherwise, the algorithm has converged and Q is returned as the selected query.

The asymptotic complexity of each iteration in Algorithm 1 is in the order of  $O(b(n-k)^2)$  in the worst case. Usually, the number of iterations is less than k as observed in [52] as well as in our experiments. As a result, the total worst-case complexity can be represented by  $O(kb(n-k)^2)$ . With the right cashing of the pairwise distances and the values returned by n=nearest(),  $d_n=d_{nearest}()$ , and  $d_s=d_{second}()$ ; the execution time is monopolized by the nested loops. In our implementation, we opted for a GPU-friendly version of the algorithm by transforming the internal loops into matrix operations and processing the candidates in a batch-wise manner. The execution time can be orders of magnitude faster than the CPU implementation. On the other hand, any future algorithm for the k-medoids clustering problem can be easily adapted to risk sampling.

### 5. Experiments

In this section, we empirically evaluate the performance of risk sampling on real benchmark datasets. It is organized as follows: Section 5.1 describes the experimental setting. Section 5.2 presents the comparative evaluation results. Section 5.3 evaluates the robustness of risk sampling w.r.t the size of validation data.

## 5.1. Experimental setting

Our testbed consists of four datasets from three domains: (1) **Publications.** From the literature domain, we used Citeseer-DBLP¹ and DBLP-Scholar² datasets; (2) **Products.** We selected a dataset containing the record pairs from Abt.com and Buy.com online shopping websites²; (3) **Music.** We manually created the Songs dataset from the 1-Million Songs corpus,¹ blocked to generate a dataset of size 30 k. The statistics of the test datasets are detailed in Table 2.

We compare risk sampling, denoted by **Risk**, with the following alternatives:

- Random sampling. The commonly used baseline method which selects points uniformly from the unlabeled set;
- Maximum Entropy and BALD [53]. Both are based on uncertainty measurement. Maximum Entropy samples points with the highest entropy value, while BALD chooses points that maximize the mutual information with the model parameters.
- 3. **ENS**. An ensemble-based uncertainty method that uses an ensemble of *N* classifiers and averages softmax vectors of each ensemble member as output. Uncertainty is measured using maximum entropy.

<sup>1</sup> https://sites.google.com/site/anhaidgroup/useful-stuff/data.

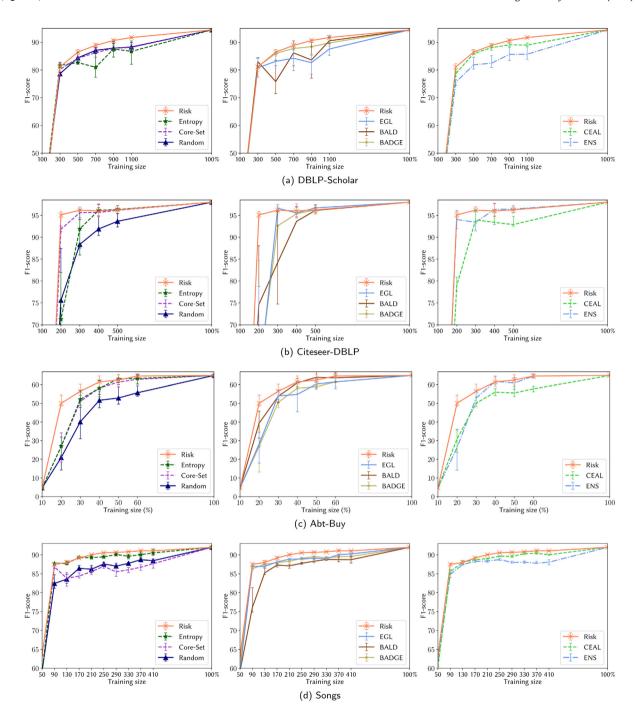


Fig. 3. Comparative evaluation: the comparison on each dataset is split in 3 method groups. Performance is evaluated by test F1-score per training data size. Error bars indicate the upper and lower quintiles among 10 runs.

**Table 2**Dataset statistics.

Dataset	Train			Validation			Test		
	Match	Unmatch	Total	Match	Unmatch	Total	Match	Unmatch	Total
Citeseer-DBLP	118	882	1000	222	1778	2000	827	6173	7000
DBLP-Scholar	3207	14016	17223	1070	4672	5742	1070	4672	5742
Abt-Buy	616	5127	5743	206	1710	1916	206	1710	1916
Songs	3655	8124	11779	1217	2710	3927	1236	2691	3927

- CEAL. Complements uncertain examples selected according to maximum entropy with a set of high-confidence examples which are softly labeled.
- 5. **Core-Set** [9]. It is the state-of-the-art *Representativeness*-based approach for DNNs;
- 6. **EGL** [13]. The state-of-the-art approach based on *Expected Model Change*, it chooses points that cause the biggest change to the embedding layer parameters;
- 7. **BADGE** [17]. A recently proposed approach which trades off between *diversity and uncertainty* by sampling points with diverse gradient embeddings.

These techniques can provide a good coverage of the existing effective AL approaches for deep models. We have implemented the AL solutions upon the hybrid variant of the classical DNN model for ER, DeepMatcher<sup>2</sup>. For the BALD method that requires test-time dropout, we use a dropout rate of 0.2 in the inputs to the RNN module in the embedding contextualization and word aggregation layers. The number of McDropout iterations is set to 100. The implementation of ENS uses 5 snapshot ensembles [54] as in the original work [44]. Their speed, compared with traditional ensembles, proves very useful in active learning's many iterations. In the implementation of CEAL, besides the b samples selected for query, a set of b high-confidence samples is automatically labeled using the DNN model's output probability as a soft label. Because EGL requires two backward passes for each example (each pass assumes a different class label), its application to the full unlabeled set can be very time-consuming. Thus, we randomly sample an unlabeled subset on which EGL-based selection is performed. For Core-Set, BADGE, and Risk, we use the representations of the classifier's penultimate representation layer, prior to the classification layer, for both representations and

As per Definition 1, we use a labeled seed set for initial model training. We provide 100 labeled examples for publications datasets, 50 examples for Songs, and 575 examples (10% of the unlabeled pool) for Abt-Buy. Similarly, the budget *b* was chosen to be in a reasonable range w.r.t each specific domain. *b* cannot be too small that it does not provide enough data for the DNN model, nor can it be too large that more data is labeled than needed. For example, Songs dataset can converge faster with only a few dozens of pairs while Abt-Buy needs a larger budget to show considerable improvements. This is true regardless of the AL method applied. We use a budget of 100 examples for publications datasets, 20 for Songs and 10% for Abt-Buy.

AL starts with an instance of the deepmatcher model trained on the initial seed set. Every AL iteration consists of running the sampling strategy of choice, retrieving the labels for the query from the dataset (simulating a human oracle), appending the newly labeled data to the training set and finally retraining a new model from scratch on the full labeled data so far. In each iteration, the risk features are re-extracted from the labeled data and their distributions are re-estimated. Then, the risk model is re-trained on the validation data. Finally, the risk scores for unlabeled data are estimated by the risk model and they are actively sampled by Algorithms 1.

The default hyper-parameters and loss functions for the deep-matcher model training were used as presented in [2]. The deepmatcher model is trained for 20 epochs with a batch size of 32 pairs using the Adam optimizer with a learning rate of  $10^{-3}$ . The risk model is similarly optimized using the Adam optimizer with a learning rate of  $10^{-3}$ , and VaR confidence is set to 0.9. It is trained for 100 epochs with a batch size of 100 pairs. To

overcome the randomness caused by different model initializations and training data shuffling, we perform 10 training sessions and report the mean test F1-score. For fair comparison, we make sure that all the methods use the same set of model initializations. For the approaches that require access to the classifier (all except *Random*), we use the model with the best validation performance.

### 5.2. Comparative evaluation

The evaluation results have been presented in Fig. 3. Due to the large number of compared methods, we report their performance on each test dataset in three separate sub-figures.

It can be observed that random sampling has the overall lowest performance. This confirms the need for active selection. The simple uncertainty method of maximum entropy achieves highly competitive performance on most of the test datasets, e.g. Abt-Buy, Citeseer-DBLP and Songs. While the other uncertainty method of BALD shows slightly higher performance than deterministic maximum entropy on some datasets. However, the improvement is not sufficiently consistent, possibly due to the quality of the MCDropout approximation. On the other hand, ensemble-based uncertainty (ENS) manages to outperform BALD on most datasets and is more stable. It can also be observed that the Core-Set approach can be highly competitive while only considering instance representativeness on most of the test datasets, e.g. Abt-Buy and Citeseer-DBLP. However, purely built upon instance representation, it is not very stable; on Songs, its performance fluctuates wildly. By maximizing the impact on the classifier. EGL is also able to positively impact its performance. Similarly, making use of gradient information, BADGE was mostly on par with EGL except on DBLP-Scholar, where the gradient-based diversification gave a better and more stable performance. Although CEAL labels more data given the same budget as other methods, thanks to soft-labels, it outperforms uncertainty methods only slightly.

It is clear that risk sampling is able to consistently increase the classifier's performance across the test datasets. It can be observed that the performance margins between risk sampling and alternative methods are considerable in most cases, especially in earlier iterations (low training sizes). This result clearly demonstrates that exposing the classifier to high-risk examples in an early stage can effectively accelerate training. Coupled with the representativeness achieved by core-set clustering, it is able to maintain an advantage over alternative methods. Finally, as shown in Fig. 3, the error bar plots for risk sampling are relatively short, even for the Abt-Buy products dataset which seems to show high variance overall. This means that the data selected via risk sampling yields less variance in the classifiers across random initializations.

**An Illustrative Example.** The major difference of risk sampling from previous alternatives is the criterion of misprediction risk. Therefore, we illustrate the efficacy of risk sampling by examining the number of mispredictions in the selected batches on the Abt-Buy dataset, which is the most challenging one. The results are reported in Fig. 4. It can be seen that risk sampling ends up selecting batches dominated by mispredictions. For reference, maximum entropy, which is likely to select mispredictions (since many uncertain points might turn out to be mispredicted), does not pick up as many as risk sampling. The same can be said about the core-set approach which only considers instance representation. The decreasing number of mispredictions throughout iterations is due to the reduction of such cases in the unlabeled pool that we are sampling from. Combined with the observation on their comparative performance in the first two iterations, these results clearly indicate that misprediction risk is an informative measure for AL.

<sup>&</sup>lt;sup>2</sup> https://github.com/anhaidgroup/deepmatcher/.

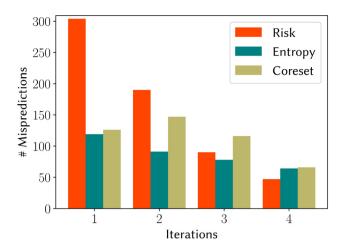


Fig. 4. Misprediction selection rate on Abt-Buy.

#### 5.3. Robustness w.r.t validation data size

Since risk sampling leverages validation data, we further investigate its performance robustness w.r.t the size of validation data. To this end, we re-run the AL experiment by varying the validation data ratio used for risk training among 0.25, 0.50 and 1. The results on all datasets are presented in Fig. 5a–d. For performance reference, we also plot the result of the core-set approach in the figure. It can be observed that the performance of risk sampling is overall very robust across ratios, and it consistently outperforms the core-set approach. It is noteworthy that our evaluation results are consistent with those reported in [22], which showed that the performance of LearnRisk is very robust w.r.t the size of validation data. These experimental results bode well for the application of risk sampling in real scenarios.

#### 5.4. Risk sampling efficiency

In this section we evaluate the efficiency of the risk sampling algorithm presented in Section 4.2. To this end, we evaluate its scalability w.r.t the total data size (n) both in terms of total runtime and number of swaps till convergence. We fix the number of clusters k = 200 (|L| = 100, |Q| = 100) and variate the data size on the large dataset of DBLP-Scholar using the risk scores and data representations from the first iteration of active learning. The runtimes for the different data sizes are presented in Fig. 5(e). Knowing that the algorithm's time complexity of  $O(kb(n-k)^2)$  is dependent on n - k, it is clear that the combination of a small k(200) and a large n (10000), which produces an extremely high value for n-k, still converges in a reasonable time. Moreover, the number of iterations does not exceed k as assumed in Section 4.2. Note that the AL iteration runtime is monopolized by the DNN model training, during which the sampling step takes way less time.

Moreover, the plot presenting the number of swaps needed until convergence as a function of data size is given in Fig. 5(f). It clearly shows that the number of swaps increases at a slow rate with larger dataset size (n). Meaning that the execution time is greatly due to the time needed for the search for each swap.

## 6. Conclusion

In this work, we propose a novel strategy of risk sampling for active learning that selects representative points with high misclassification risk for labeling. Built upon the core-set characterization for AL, we theoretically derive an optimization model based on an upper-bound of the core-set loss with non-uniform Lipschitz continuity. Due to the NP-hardness of the defined problem, we then present an efficient algorithm for its solution. Finally, our empirical study has validated the efficacy of the proposed approach. For future work, it is worthy to point out that risk sampling is generally applicable to other classification tasks; their technical solutions however need further investigations.

#### **CRediT** authorship contribution statement

**Youcef Nafa:** Conceptualization, Formal analysis, Writing – original draft. **Qun Chen:** Conceptualization, Supervision, Writing – review & editing. **Zhaoqiang Chen:** Methodology. **Xingyu Lu:** Software, Investigation. **Haiyang He:** Software, Investigation. **Tianyi Duan:** Software. **Zhanhuai Li:** Project administration, Funding acquisition.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Appendix A. Proof of Lemma 1

We use the following definition of RNN:

$$\mathbf{h_t} = \sigma(W \cdot \mathbf{h_{t-1}} + U \cdot \mathbf{x_t})$$

s.t.  $\mathbf{h_0} = \Phi$ ;  $U \in \mathbb{R}^{m \times m'}$ ,  $W \in \mathbb{R}^{m' \times m'}$  and  $\sigma$  is an  $L_{\sigma}$ -Lipschitz activation function. Note that, the commonly used activation functions for RNNs (ex. tanh) are 1-Lipschitz (i.e.  $L_{\sigma} = 1$ ).

**Proof of Lemma 1.** Let  $X \in \mathbb{R}^{T \times m}$  be an input sequence of size T (i.e.  $X = \{\mathbf{x_1}, \dots, \mathbf{x_T}\}$ ). For two distinct inputs X, X' generating hidden states  $\mathbf{h_t}, \mathbf{h_t'} \in \mathbb{R}^{m'}$  respectively, we have:

$$\|\mathbf{h}_{\mathbf{T}} - \mathbf{h}_{\mathbf{T}}'\|_{p} \le L_{\sigma} \|W\|_{p} \|\mathbf{h}_{\mathbf{T}-\mathbf{1}} - \mathbf{h}_{\mathbf{T}-\mathbf{1}}'\|_{p} + L_{\sigma} \|U\|_{p} \|\mathbf{x}_{\mathbf{T}} - \mathbf{x}_{\mathbf{T}}'\|_{p}$$

By unfolding the right-hand side in the above inequality,

$$\|\mathbf{h}_{\mathbf{T}} - \mathbf{h}_{\mathbf{T}}'\|_{p} \leq L_{\sigma}^{T} \|W\|_{p}^{T} \|\mathbf{h}_{\mathbf{0}} - \mathbf{h}_{\mathbf{0}}'\|_{p}$$

$$+ L_{\sigma}^{T} \|W\|_{p}^{T-1} \|U\|_{p} \|\mathbf{x}_{\mathbf{1}} - \mathbf{x}_{\mathbf{1}}'\|_{p}$$

$$+ \ldots + L_{\sigma} \|U\|_{p} \|\mathbf{x}_{\mathbf{T}} - \mathbf{x}_{\mathbf{T}}'\|_{p}$$

For  $||U||_p$ ,  $||W||_p \le \alpha$ ,

$$\|\mathbf{h}_{\mathbf{T}} - \mathbf{h}_{\mathbf{T}}'\|_{p} \leq \sum_{t=1}^{T} \alpha^{T-t+1} L_{\sigma}^{T-t+1} \|\mathbf{x}_{\mathbf{t}} - \mathbf{x}_{\mathbf{t}}'\|_{p}$$

When p=2, for an  $L_2$ -regularized and stable RNN [49] ( $\alpha \leq 1$ ) we have  $\max_{t \in [1,T]} \alpha^t = \alpha$ ,

$$\|\mathbf{h}_{\mathbf{T}} - \mathbf{h}_{\mathbf{T}}'\|_{2} \leq \alpha \sum_{t=1}^{T} \|\mathbf{x}_{t} - \mathbf{x}_{t}'\|_{2}$$

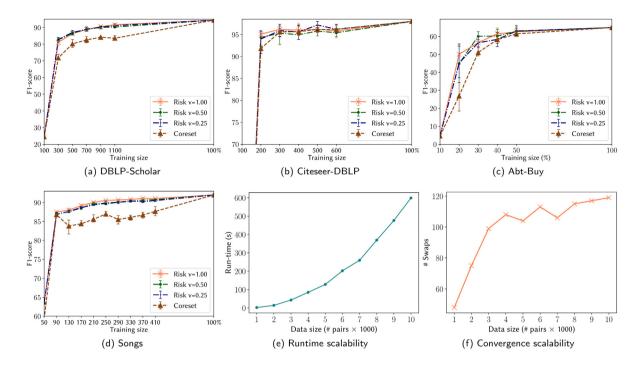


Fig. 5. a-d: Robustness evaluation of risk sampling. Evaluates risk sampling with different validation data ratios (0.5, 0.75, and 1). Core-set is plotted for performance reference. e-f: Risk sampling scalability. Evaluates the scalability in terms of runtime and number of swaps.

Then by applying Cauchy-Schwartz inequality,

$$\|\mathbf{h}_{\mathbf{T}} - \mathbf{h}_{\mathbf{T}}'\|_{2} \leq \alpha \sqrt{T} \|X - X'\|_{F}$$

For a fully-connected network module  $F_C$  with  $n_{fc}$  layers, trainable parameters  $w_{fc}$  and  $\sigma$  activations, followed by a C-class Softmax function [9],

$$||F_{C}(\mathbf{h}_{T}, w_{fc}) - F_{C}(\mathbf{h}'_{T}, w_{fc})||_{2}$$

$$\leq \frac{\sqrt{C-1}}{C} \alpha^{n_{fc}+1} \sqrt{T} ||X - X'||_{F}$$

For  $w = \{w_{fc}, W, U\}$  and knowing that for a matrix  $X \in \mathbb{R}^{n \times m}$ :  $\|X\|_F \leq \sqrt{m} \|X\|_2$ ,

$$||RNN(X, w) - RNN(X', w)||_{2} = ||F_{C}(\mathbf{h}_{T}, w_{fc}) - F_{C}(\mathbf{h}_{T}', w_{fc})||_{2}$$

$$\leq \frac{\sqrt{(C-1)Tm}}{C} \alpha^{n_{fc}+1} ||X-X'||_{2}$$

Knowing that  $l(X, y, w) = ||RNN(X, w) - y||_2$ . For any fixed y, using the reverse triangle inequality we get,

$$|l(X, y, w) - l(X', y, w)| \le ||RNN(X, w) - RNN(X', w)||_2$$
  
  $\le \frac{\sqrt{(C-1)Tm}}{C} \alpha^{n_{fc}+1} ||X - X'||_2$ 

## Appendix B. Proof of Theorem 1

Here, we study the Lipschitz continuity for the DNN model defined in Definition 2. We suppose a distance function  $F_D(\vec{s_k}, \vec{s_k}) = |\vec{s_k} - \vec{s_k}|$  as used by the DeepMatcher model.

**Proof of Theorem 1.** We start with the expression

$$\|\mathbf{s_k} - \mathbf{s'_k}\|_2 \le \alpha \sqrt{T_k} \|X^k - X^{k'}\|_F$$

Let 
$$\tilde{X}^{k} = \{\overrightarrow{X^{k}}, \overrightarrow{X^{k}}\} \in \mathbb{R}^{(\overrightarrow{T_{k}} + \overrightarrow{T_{k}}) \times m},$$

$$\|\tilde{\mathbf{s}_{k}} - \tilde{\mathbf{s}'_{k}}\|_{2} \leq \|\dot{\overline{\mathbf{s}_{k}}} - \dot{\overline{\mathbf{s}'_{k}}}\|_{2} + \|\dot{\overline{\mathbf{s}_{k}}} - \dot{\overline{\mathbf{s}'_{k}}}\|_{2}$$

$$\leq \alpha(\sum_{t=1}^{\overleftarrow{T_{k}}} \|\dot{\overline{\mathbf{x}_{t}}} - \dot{\overline{\mathbf{x}'_{t}}}\|_{2} + \sum_{t=1}^{\overrightarrow{T_{k}}} \|\ddot{\overline{\mathbf{x}_{t}}} - \dot{\overline{\mathbf{x}'_{t}}}\|_{2})$$

$$\leq \alpha\sqrt{\overleftarrow{T_{k}} + \overrightarrow{T_{k}}} \|\tilde{X^{k}} - \tilde{X^{k'}}\|_{F}$$

Finally, the classifier module  $F_C$  takes in the concatenated similarities  $S = \{\tilde{\mathbf{s_k}}\}_{k=1}^{n_A}$ . Let  $X_d = \{X^k\}_{k=1}^{n_A} \in \mathbb{R}^{T_d \times m}$  be the representation for pair d, s.t  $T_d = \sum_{k=1}^{n_A} (\overrightarrow{T_k} + \overrightarrow{T_k})$ . And let  $\hat{T} = \max_{d_i} T_{d_i}$  be the maximal pair length in D. Then, the resulting similarity matrix satisfies,

$$||S - S'||_F \le \alpha \sqrt{\hat{T}} ||X_d - X_d'||_F$$

The final expression for the loss function following the same steps as in the proof of Lemma 1 and setting C=2:

$$|l(d, y, w) - l(d', y, w)| \le \frac{\alpha^{n_{fc}+1}}{2} \sqrt{\hat{T} m} \|X_d - X_d'\|_2$$

#### Appendix C. Proof of Theorem 2

**Proof of Theorem 2.** Let  $(d_i, y_i) \in U$ ,  $(d_j, y_j) \in L$  be an unlabeled and a labeled pair respectively. Let l(d, y) be an  $\mathbb{L}$ -Lipschitz continuous loss function for any pair d with ground-truth label y w.r.t the model  $A_{L \cup Q}$  trained on  $L \cup Q$ . We have:

$$|l(d_i, y_i) - l(d_j, y_j)| \le \mathbb{L}_i ||X_{d_i} - X_{d_i}||_2$$

where  $\mathbb{L}_i$  represents the Lipschitz bound over the slope of the loss landscape between  $d_i$  and  $d_j$  ( $\mathbb{L}_i \leq \mathbb{L}$ ). Let  $\{C_1, C_2, \ldots, C_{|L \cup Q|}\}$  represent a clustering of D ( $D = \bigcup_j C_j$ ) where each cluster  $C_j$  is centered around  $(d_i, y_i) \in C_i$ . Using triangle inequality and

summing over  $(d_i, y_i) \in C_i$ ,

$$\left| \sum_{(d_i, y_i) \in C_j} l(d_i, y_i) - |C_j| \cdot l(d_j, y_j) \right| \le \sum_{(d_i, y_i) \in C_j} \mathbb{L}_i \|X_{d_i} - X_{d_j}\|_2$$

By summing over all clusters  $C_j$  and applying triangle inequality, then multiplying both sides by  $\frac{1}{n}$ ,

$$\begin{split} &\left| \frac{1}{n} \sum_{(d_i, y_i) \in D} l(d_i, y_i) - \frac{1}{n} \sum_{(d_j, y_j) \in L \cup Q} |C_j| \, l(d_j, y_j) \right| \\ &\leq \frac{1}{n} \sum_{(d_j, y_j) \in L \cup Q} \sum_{(d_i, y_i) \in C_j} \mathbb{L}_i \, \|X_{d_i} - X_{d_j}\|_2 \end{split}$$

Assuming zero loss for labeled data, i.e.  $\forall (d_j, y_j) \in L \cup Q$ :  $l(d_j, y_j) = 0$ , the cluster-weighted loss average and the simple loss average are equal, yielding:

$$\left| \frac{1}{n} \sum_{(d_i, y_i) \in D} l(d_i, y_i) - \frac{1}{|L \cup Q|} \sum_{(d_j, y_j) \in L \cup Q} l(d_j, y_j) \right| \\ \leq \frac{1}{n} \sum_{(d_i, y_j) \in L \cup Q} \sum_{(d_i, y_i) \in C_j} \mathbb{L}_i \|X_{d_i} - X_{d_j}\|_2$$

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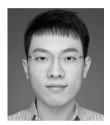
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