

2008-2009 二学期高等数学期中(A 卷)解答

一 1.  $1, 2\sqrt{7}$  (2 分, 2 分)

2.  $\sqrt{\frac{26}{65}}$

3.  $\{4,4,2\}, 6$  (2 分, 2 分)

4. (柱面)螺线,  $\{-2,0,5\}$  (1 分, 3 分)

5.  $y + \frac{1}{2}(2xy - y^2) + o(\rho^2)$  (一次项 1 分, 二次项 2 分, 余项 1 分)

6.  $\frac{x^2}{2}\arctan y + \frac{2}{3}(1+x)^{\frac{3}{2}} + y - \frac{2}{3}$

7.  $2xf_1' + e^{x+y}f_2', 4xyf_{11}'' + 2(x+y)e^{x+y}f_{12}'' + e^{2(x+y)}f_{22}'' + e^{x+y}f_2'$  (2 分, 2 分)

8.  $-3, -5$  (2 分, 2 分)

二.  $2x + 2z \frac{\partial z}{\partial x} = f_1' \cdot y + f_2' \cdot (\frac{\partial z}{\partial x} - 2)$  .....(4 分)

$\frac{\partial z}{\partial x} = \frac{2x - yf_1' + 2f_2'}{f_2' - 2z}$  .....(5 分)

$2y + 2z \frac{\partial z}{\partial y} = f_1' \cdot x + f_2' \cdot \frac{\partial z}{\partial y}$  .....(9 分)

$\frac{\partial z}{\partial y} = \frac{2y - xf_1'}{f_2' - 2z}$  .....(10 分)

三.  $L_1$  的方向向量为  $\vec{s}_1 = \{3, -2, 2\}$ ,  $P_1(2, -1, 3) \in L_1$

$L_2$  的方向向量为  $\vec{s}_2 = \{1, 2, 0\} \times \{0, 1, 1\} = \{2, -1, 1\}$ ,  $P_2(1, 0, 2) \in L_2$  .....(3 分)

$\vec{P_1P_2} = \{-1, 1, -1\}$

$$(\vec{s}_1, \vec{s}_2, \vec{P_1P_2}) = \begin{vmatrix} 3 & -2 & 2 \\ 2 & -1 & 1 \\ -1 & 1 & -1 \end{vmatrix} = 0$$

故  $L_1, L_2$  共面; .....(8 分)

所求平面法向量为  $\vec{n} = \vec{s}_1 \times \vec{s}_2 = \{0, 1, 1\}$  .....(10 分)

所求平面方程为  $1 \times (y - 0) + 1 \times (z - 2) = 0$

即  $y + z = 2$  .....(12 分)

四.  $I = \iint_{D_1} \frac{x-y}{x^2+y^2} dx dy + \iint_{D_2} \frac{y-x}{x^2+y^2} dx dy$  .....(2 分)

$$= \int_0^{\frac{\pi}{4}} d\theta \int_0^{2\sin\theta} (\cos\theta - \sin\theta) d\rho + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta \int_{\frac{2}{2\sin\theta}}^{\frac{2}{\sin\theta}} (\sin\theta - \cos\theta) d\rho$$
 .....(6 分)

$$= 2 \int_0^{\frac{\pi}{4}} (\sin\theta \cos\theta - \sin^2\theta) d\theta + 2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1 - \sin^2\theta - \frac{\cos\theta}{\sin\theta} + \sin\theta \cos\theta) d\theta$$
 .....(10 分)

$$= 1 - \ln 2$$
 .....(12 分)

五. 设切点  $P(x_0, y_0, z_0)$ , 则  $3x_0^2 + y_0^2 + z_0^2 = 16$  .....(2 分)

切平面法向量为  $\vec{n} = \{6x_0, 2y_0, 2z_0\}$  .....(4 分)

$L_1, L_2$  的方向向量分别为  $\vec{s}_1 = \{4, 5, 8\}, \vec{s}_2 = \{1, 1, 1\}$

$$\vec{s} = \vec{s}_1 \times \vec{s}_2 = \{-3, 4, -1\}$$
 .....(6 分)

由题意, 有  $\vec{n} // \vec{s}$ , 故  $\frac{3x_0}{-3} = \frac{y_0}{4} = \frac{z_0}{-1}$  .....(8 分)

解得  $x_0 = \pm \frac{2}{\sqrt{5}} \quad y_0 = \mp \frac{8}{\sqrt{5}} \quad z_0 = \pm \frac{2}{\sqrt{5}}$

所求点为  $(-\frac{2}{\sqrt{5}}, \frac{8}{\sqrt{5}}, -\frac{2}{\sqrt{5}})$  或  $(\frac{2}{\sqrt{5}}, -\frac{8}{\sqrt{5}}, \frac{2}{\sqrt{5}})$  .....(11 分)

六.  $I = \int_0^1 dx \int_0^{1-x} dz \int_0^{1-x-z} e^{\frac{y}{1-x-z}} dy \dots\dots\dots(4 \text{ 分})$

$$= (e-1) \int_0^1 dx \int_0^{1-x} (1-x-z) dz \dots\dots\dots(7 \text{ 分})$$

$$= \frac{1}{2} (e-1) \int_0^1 (1-x)^2 dx \dots\dots\dots(9 \text{ 分})$$

$$= \frac{1}{6} (e-1) \dots\dots\dots(11 \text{ 分})$$

七.  $\vec{e} = \{\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\} \dots\dots\dots(1 \text{ 分})$

$$f'_x = 2x \quad f'_y = 2y \quad f'_z = 2z$$

$$\frac{\partial f}{\partial \vec{e}} = \frac{2}{\sqrt{3}} (x-y+z) \dots\dots\dots(3 \text{ 分})$$

令  $g(x, y, z) = x - y + z$

$$F(x, y, z) = x - y + z + \lambda(2x^2 - y^2 + z^2 - 5) + \mu(x + y) \dots\dots\dots(6 \text{ 分})$$

$$\begin{cases} F'_x = 1 + 4\lambda x + \mu = 0 \\ F'_y = -1 - 2\lambda y + \mu = 0 \\ F'_z = 1 + 2\lambda z = 0 \\ 2x^2 - y^2 + z^2 = 5 \\ x + y = 0 \end{cases} \dots\dots\dots(8 \text{ 分})$$

解得  $x = \mp 2 \quad y = \pm 2 \quad z = \mp 1$

得两点  $M_1(-2, 2, -1) \quad M_2(2, -2, 1) \dots\dots\dots(10 \text{ 分})$

$$\left. \frac{\partial f}{\partial \vec{e}} \right|_{M_1} = -\frac{10}{\sqrt{3}} \quad \left. \frac{\partial f}{\partial \vec{e}} \right|_{M_2} = \frac{10}{\sqrt{3}}$$

由于  $\frac{\partial f}{\partial \vec{e}}$  在曲线上确有最大值和最小值, 故  $M_1, M_2$  为所求, 且

$$\max_M \left\{ \frac{\partial f}{\partial \vec{e}} \right\} = \frac{10}{\sqrt{3}} \quad \min_M \left\{ \frac{\partial f}{\partial \vec{e}} \right\} = -\frac{10}{\sqrt{3}} \dots\dots\dots(12 \text{ 分})$$