

2007-2008 学年第二学期期中试题(B 卷)参考解答及评分标准

2008 年 4 月 18 日

一、填空题 (每小题 4 分, 共 24 分)

1. $-\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{k}$ or $\{-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\};$

2. $2x^2 + y^2 + 2z^2 = 8,$ $-\vec{i} + \vec{j} + \vec{k}$ or $\{-1, 1, 1\}$ or $\{-4, 4, 4\};$

3. $\frac{1}{\sqrt{17}}(\vec{i} - 4\vec{j})$ or $\{\frac{1}{\sqrt{17}}, -\frac{4}{\sqrt{17}}\};$ 4. $\frac{3}{5}, (\frac{418}{25}, \frac{524}{25}, \frac{131}{5})$

5. $dx + 2dy$ 6. $\int_{-1}^2 dy \int_{y^2}^{y+2} f(x, y) dx$

二、(10 分)

$\frac{\partial z}{\partial x} = yf'_1 + e^{x-y}f'_2$ 3 分

$\frac{\partial z}{\partial y} = xf'_1 - e^{x-y}f'_2$ 6 分

$\frac{\partial^2 z}{\partial x \partial y} = f'_1 - e^{x-y}f'_2 + xyf''_{11} + (x-y)e^{x-y}f''_{12} - e^{2(x-y)}f''_{22}$ 10 分

三、(12 分)

$$\begin{cases} 2x - \frac{dz}{dx} = 0 \\ 3 + 2\frac{dy}{dx} = 0 \end{cases}$$
 将点 M 代入得 $\frac{dy}{dx} = -\frac{3}{2}$ $\frac{dz}{dx} = 2$

$\vec{T} = \{1, -\frac{3}{2}, 2\}$ 4 分

$\pi: (x-1) - \frac{3}{2}(y+2) + 2(z-1) = 0,$

即 $2x - 3y + 4z = 12$ 5 分

$\vec{s} = \{2, 1, -1\} \times \{1, -1, 1\} = \{0, -3, -3\}$ 9 分

$(\frac{1}{3}, -\frac{2}{3}, 0)$ 是 L 上一点, L 的标准方程为

$\frac{x - \frac{1}{3}}{0} = \frac{y + \frac{2}{3}}{1} = \frac{z}{1}$ 10 分

$\sin \varphi = \frac{|\vec{T} \cdot \vec{s}|}{|\vec{T}| |\vec{s}|} = \frac{1}{\sqrt{58}}$ $\varphi = \arcsin \frac{1}{\sqrt{58}}$ 12 分

四、(10 分) $\frac{\partial z}{\partial x} + \frac{1}{z} \frac{\partial z}{\partial x} - e^{-x^2} = 0$, $\frac{\partial z}{\partial x} = \frac{ze^{-x^2}}{z+1}$ 3 分

$$\frac{\partial z}{\partial y} + \frac{1}{z} \frac{\partial z}{\partial y} + e^{-y^2} = 0 \quad , \quad \frac{\partial z}{\partial y} = \frac{-ze^{-y^2}}{z+1} \quad \dots\dots\dots 6 \text{ 分}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\frac{\partial z}{\partial y} e^{-x^2} (z+1) - ze^{-x^2} \frac{\partial z}{\partial y}}{(z+1)^2} = \frac{-ze^{-x^2-y^2}}{(z+1)^3} \quad \dots\dots\dots 10 \text{ 分}$$

五、(10 分) $f'_x = y - 2x = 0$ $f'_y = 3y^2 - 2y + x = 0$

解得 $x = 0, y = 0$ 或 $x = \frac{1}{4}, y = \frac{1}{2}$

$$f''_{x^2} = -2 \quad f''_{xy} = 1 \quad f''_{y^2} = 6y - 2 \quad \dots\dots\dots 4 \text{ 分}$$

在 $x = 0, y = 0$, $AC - B^2 = 3 > 0$, $A = -2 < 0$

故 $(0,0)$ 是极大值点, 极大值为 $f(0,0) = 0$ 7 分

在 $x = \frac{1}{4}, y = \frac{1}{2}$, $AC - B^2 = -3 < 0$, $(\frac{1}{4}, \frac{1}{2})$ 不是极值点.10 分

六、(12 分) $y = \sqrt{8-x^2}$ 与 $2y = x^2$ 交点为 $(2,2)$

$$I = \int_0^{\frac{\pi}{4}} d\theta \int_0^{\frac{2\sin\theta}{\cos^2\theta}} f(\rho \cos\theta, \sin\theta) \rho d\rho + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta \int_0^{2\sqrt{2}} f(\rho \cos\theta, \sin\theta) \rho d\rho \quad \dots\dots\dots 7 \text{ 分}$$

$$\iint_D \sqrt{x^2 + y^2} dx dy = \int_0^{\frac{\pi}{4}} d\theta \int_0^{\frac{2\sin\theta}{\cos^2\theta}} \rho^2 d\rho + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta \int_0^{2\sqrt{2}} \rho^2 d\rho$$

$$= \int_0^{\frac{\pi}{4}} \frac{8}{3} \frac{\sin^3\theta}{\cos^6\theta} d\theta + \frac{4\sqrt{2}}{3} \pi = \frac{16}{45}(\sqrt{2}+1) + \frac{4\sqrt{2}}{3} \pi \quad \dots\dots\dots 12 \text{ 分}$$

七、(10 分) $V = \iint_D (x-y+2) dx dy$ 3 分

$$= \int_{-1}^2 dx \int_{x^2}^{x+2} (x-y+2) dy \quad \dots\dots\dots 7 \text{ 分}$$

$$= \frac{81}{20} \quad \dots\dots\dots 10 \text{ 分}$$

八、(12 分) 设 (x,y,z) 是椭球面上任一点, (x,y,z) 到平面 π 的距离为

$$d = \frac{|x-y+2z-6|}{\sqrt{6}} \quad \dots\dots\dots 2 \text{ 分}$$

考虑函数 $f(x, y, z) = (x - y + 2z - 6)^2$ 在限制条件 $x^2 + y^2 + 2z^2 = 1$ 下的极值问题.

令 $F(x, y, z) = (x - y + 2z - 6)^2 + \lambda(x^2 + y^2 + 2z^2 - 1)$

由
$$\begin{cases} F'_x = 2(x - y + 2z - 6) + 2\lambda x = 0 \\ F'_y = -2(x - y + 2z - 6) + 2\lambda y = 0 \\ F'_z = 4(x - y + 2z - 6) + 4\lambda z = 0 \\ x^2 + y^2 + 2z^2 = 1 \end{cases}$$

解得 $x = \pm \frac{1}{2} \quad y = \mp \frac{1}{2} \quad z = \pm \frac{1}{2} \quad \dots\dots\dots 6 \text{ 分}$

记 $M(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}), \quad N(-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}),$

$d|_M = \frac{4}{\sqrt{6}} \quad d|_N = \frac{8}{\sqrt{6}}$

由问题实际意义可判别出 M 为最近点, N 为最远点 $\dots\dots\dots 8 \text{ 分}$

$\vec{n} = \{2x, 2y, 4z\}$

$\vec{n}|_M = \{1, -1, 2\}, \quad \vec{n}|_N = \{-1, 1, -2\}$

$\pi_M: \quad x - y + 2z - 2 = 0$

$\pi_N: \quad x - y + 2z + 2 = 0 \quad \dots\dots\dots 12 \text{ 分}$