

高等数学期中试题(B 卷)解答 (2010.5)

一. 1. 2, $\frac{2}{3}$ (2 分, 2 分)

2. $\frac{7}{\sqrt{14}}$

3. $e^{\frac{1}{2}}$, 不存在 (2 分, 2 分)

4. $\{-\frac{3}{5}, \frac{1}{2}, -\frac{1}{5}\}, -\frac{1}{2\sqrt{6}}$ (2 分, 2 分)

5. $\{3, -4, -1\}, \arccos \frac{5}{\sqrt{39}}$ (2 分, 2 分)

6. $xyf'_1 + \frac{1}{y}f'_2 + 2x\varphi', f'_1 - \frac{1}{y^2}f'_2 + xyf''_{11} - \frac{x}{y^3}f''_{22} + 2x\varphi''$ (2 分, 2 分)

7. $\frac{1}{2}(1 - e^{-4})$

二. 设 $L: \frac{x+1}{l} = \frac{y}{m} = \frac{z-4}{n}$ (3 分)

有 $\begin{vmatrix} l & m & n \\ 1 & 2 & 3 \\ -1 & 0 & 4 \end{vmatrix} = 8l - 7m + 2n = 0$ (5 分)

$\{l, m, n\} \cdot \{2, 1, 4\} = 2l + m + 4n = 0$ (7 分)

解得 $l = \frac{15}{14}m, n = -\frac{11}{14}m$

故 $L: \frac{x+1}{15} = \frac{y}{14} = \frac{z-4}{-11}$ (9 分)

三. $\begin{cases} \frac{dz}{dx} = 2x + 2y\frac{dy}{dx} \\ 2x + 4y\frac{dy}{dx} + 6z\frac{dz}{dx} = 0 \end{cases}$ (4 分)

将点 P 代入得 $\begin{cases} \frac{dz}{dx} = 2 - 2\frac{dy}{dx} \\ 1 - 2\frac{dy}{dx} + 6\frac{dz}{dx} = 0 \end{cases}$

解得 $\frac{dy}{dx} = \frac{13}{14}, \frac{dz}{dx} = \frac{1}{7}$ (6 分)

故切向量 $\vec{s} = \{1, \frac{13}{14}, \frac{1}{7}\}$ (7 分)

法平面 $\pi: 14(x-1) + 13(y+1) + 2(z-2) = 0$

即 $14x + 13y + 2z - 5 = 0$ (9 分)

四. 令 $\frac{\partial z}{\partial x} = -(1+e^y)\sin x = 0$

$$\frac{\partial z}{\partial y} = e^y(\cos x - 1 - y) = 0 \quad \dots\dots\dots(2 \text{ 分})$$

得驻点 $P_1(\pi, -2) \quad P_2(2\pi, 0) \quad \dots\dots\dots(4 \text{ 分})$

$$\frac{\partial^2 z}{\partial x^2} = -(1+e^y)\cos x \quad \frac{\partial^2 z}{\partial x \partial y} = -e^y \sin x \quad \frac{\partial^2 z}{\partial y^2} = e^y(\cos x - 2 - y) \quad \dots\dots\dots(7 \text{ 分})$$

在点 $P_1 \quad A = 1 + e^{-2} \quad B = 0 \quad C = -e^{-2}$

$$AC - B^2 = -(1 + e^{-2})e^{-2} < 0$$

故 P_1 不是极值点 $\dots\dots\dots(9 \text{ 分})$

在点 $P_2 \quad A = -2 \quad B = 0 \quad C = -1$

$$AC - B^2 = 2 > 0 \quad \text{且} \quad A < 0$$

故 P_2 是极大值点, 极大值 $z = 2 \quad \dots\dots\dots(11 \text{ 分})$

五. $I = \int_0^{\frac{\pi}{2}} dx \int_0^{\sqrt{x}} dy \int_0^{\frac{\pi}{2}-x} \frac{y \sin x}{x} dz \quad \dots\dots\dots(4 \text{ 分})$

$$= \int_0^{\frac{\pi}{2}} dx \int_0^{\sqrt{x}} \frac{y \sin x}{x} (\frac{\pi}{2} - x) dy \quad \dots\dots\dots(6 \text{ 分})$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} (\frac{\pi}{2} - x) \sin x dx \quad \dots\dots\dots(8 \text{ 分})$$

$$= \frac{\pi}{4} - \frac{1}{2} \quad \dots\dots\dots(9 \text{ 分})$$

六. (1) $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} + 2x \frac{\partial z}{\partial v} \quad \frac{\partial z}{\partial y} = -2y \frac{\partial z}{\partial v} \quad \dots\dots\dots(6 \text{ 分})$

代入方程得 $\frac{\partial z}{\partial u} = \frac{1}{\sqrt{1-u^2}} \quad \dots\dots\dots(7 \text{ 分})$

(2) $z = \arcsin u + f(v) \quad (f \text{ 是任意可导函数})$

$$= \arcsin x + f(x^2 - y^2) \quad \dots\dots\dots(9 \text{ 分})$$

七. (1) $S: x^2 + y^2 + z^2 = 2z$ (1 分)

(2) $C: \begin{cases} x^2 + y^2 = 1 \\ z = 0 \end{cases}$ (3 分)

(3) $I = \int_0^{2\pi} d\theta \int_0^{\frac{\pi}{4}} d\varphi \int_0^{2\cos\varphi} r^7 \sin\varphi dr$ (6 分)

$$= 2^6 \pi \int_0^{\frac{\pi}{4}} \sin\varphi \cos^8\varphi d\varphi$$
(9 分)

$$= \frac{64}{9} (1 - \frac{\sqrt{2}}{32}) \pi$$
(11 分)

八. 设切点为 $M(x, y, z)$

则切平面 $\frac{x}{a^2}(X-x) + \frac{y}{b^2}(Y-y) + \frac{z}{c^2}(Z-z) = 0$

即 $\frac{x}{a^2}X + \frac{y}{b^2}Y + \frac{z}{c^2}Z = 1$ (3 分)

三截距为 $\frac{a^2}{x}, \frac{b^2}{y}, \frac{c^2}{z}$

$$V = \frac{1}{6} \frac{a^2 b^2 c^2}{xyz}$$
(6 分)

令 $f(x, y, z) = xyz$ 其中 $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$$F(x, y, z) = xyz + \lambda(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1)$$
(8 分)

$$\begin{cases} F'_x = yz + \frac{2\lambda}{a^2}x = 0 \\ F'_y = xz + \frac{2\lambda}{b^2}y = 0 \\ F'_z = xy + \frac{2\lambda}{c^2}z = 0 \\ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \end{cases}$$
(10 分)

解得 $x = \frac{a}{\sqrt{3}}, y = \frac{b}{\sqrt{3}}, z = \frac{c}{\sqrt{3}}$ (12 分)

由问题的实际意义, ..., 故 $(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}})$ 为所求点

$$V_{\min} = \frac{\sqrt{3}}{2} abc$$
(14 分)