

# 2008-2009 第二学期期中试题(B 卷)解答

一. 1. (4,4,5)

2.  $\sqrt{3}, 5\sqrt{3}$  (2 分, 2 分)

3.  $\{-3,3\}, 3\sqrt{2}$  (2 分, 2 分)

4.  $1+x+\frac{1}{2}(x^2+2y^2)+o(\rho^2)$  (一次项 1 分, 二次项 2 分, 余项 1 分)

5.  $yf'_1+e^x f'_2, f'_1+xyf''_{11}+(y+xe^x)f''_{12}+e^x f''_{22}$  (2 分, 2 分)

6.  $\{1, -\frac{3}{5}, -\frac{2}{5}\}$

7.  $\frac{y^2}{2}\arcsin x - \ln|\cos x| + \frac{y^2}{2}$

8. 25, 9 (2 分, 2 分)

二.  $2xyz + x^2 y \frac{\partial z}{\partial x} = f' \cdot (-\frac{\partial z}{\partial x})$  .....(4 分)

$\frac{\partial z}{\partial x} = \frac{-2xyz}{f' + x^2 y}$  .....(5 分)

$x^2 z + x^2 y \frac{\partial z}{\partial y} = f' \cdot (1 - \frac{\partial z}{\partial y})$  .....(9 分)

$\frac{\partial z}{\partial x} = \frac{f' - x^2 z}{f' + x^2 y}$  .....(10 分)

三.  $I = 2 \int_0^{\frac{\pi}{2}} d\theta \int_0^{2R \cos \theta} \rho \sqrt{4R^2 - \rho^2} d\rho$  .....(4 分)

$= -\frac{16}{3} R^3 \int_0^{\frac{\pi}{2}} (\sin^3 \theta - 1) d\theta$  .....(8 分)

$= -\frac{8(4-3\pi)}{9} R^3$  .....(10 分)

四. 设  $L: \frac{x-2}{l} = \frac{y+1}{m} = \frac{z-3}{n}$  .....(2 分)

$\vec{s} = \{l, m, n\}$        $M(2, -1, 3) \in L$        $N(1, 0, -2) \in L_1$

$L_1$  的方向向量  $\vec{s}_1 = \{2, -1, 1\}$

由于  $L, L_1$  相交, 有

$$(\vec{s}, \vec{s}_1, \vec{MN}) = \begin{vmatrix} l & m & n \\ 2 & -1 & 1 \\ -1 & 1 & -5 \end{vmatrix} = 4l + 9m + n = 0 \quad \text{.....(7 分)}$$

由  $L // \pi$ , 得  $3l - 2m + n = 0$  .....(9 分)

解得  $l = -11m$        $n = 35m$  .....(11 分)

$$L: \frac{x-2}{-11} = \frac{y+1}{1} = \frac{z-3}{35} \quad \text{.....(12 分)}$$

五. 设切点  $P(x_0, y_0, z_0)$ , 则  $x_0^2 + y_0^2 + z_0^2 = 4$  .....(2 分)

平面  $\pi$  法向量为  $\vec{n} = \{2x_0, 2y_0, 2z_0\} = 2\{x_0, y_0, z_0\}$  .....(4 分)

$L$  的方向向量为  $\vec{s} = \{1, -1, 1\} \times \{2, -1, 3\} = \{-2, -1, 1\}$  .....(6 分)

由于  $\pi \perp L$ , 有  $\frac{x_0}{-2} = \frac{y_0}{-1} = \frac{z_0}{1}$  .....(8 分)

解得  $x_0 = \pm 2\sqrt{\frac{2}{3}}$        $y_0 = \pm \sqrt{\frac{2}{3}}$        $z_0 = \mp \sqrt{\frac{2}{3}}$  .....(10 分)

$\pi$  的方程为  $2x + y - z = \pm 2\sqrt{6}$  .....(12 分)

六. 曲面  $y = x^2$  将  $V$  分成  $V_1, V_2$ ,

$$I = \iiint_{V_1} xz(y - x^2) dV + \iiint_{V_2} xz(x^2 - y) dV \quad \text{.....(2 分)}$$

$$= \int_0^1 dx \int_{x^2}^1 dy \int_0^1 xz(y - x^2) dy + \int_0^1 dx \int_0^{x^2} dy \int_0^1 xz(x^2 - y) dy \quad \text{.....(6 分)}$$

$$= \frac{1}{2} \int_0^1 x dx \int_{x^2}^1 (y - x^2) dy + \frac{1}{2} \int_0^1 x dx \int_0^{x^2} (x^2 - y) dy \quad \text{.....}$$

.....(8 分)

$$= \frac{1}{2} \int_0^1 x \left( \frac{1}{2} - x^2 + \frac{1}{2} x^4 \right) dx + \frac{1}{4} \int_0^1 x^5 dx \quad \dots\dots\dots(10 \text{ 分})$$

$$= \frac{1}{12} \quad \dots\dots\dots(12 \text{ 分})$$

七.  $\vec{e} = \left\{ \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\} \quad \dots\dots\dots(1 \text{ 分})$

$$f'_x = 2x \quad f'_y = 2y \quad f'_z = 2z$$

$$\frac{\partial f}{\partial \vec{e}} = \frac{2}{\sqrt{3}}(x - y + z) \quad \dots\dots\dots(3 \text{ 分})$$

令  $g(x, y, z) = x - y + z$

$$F(x, y, z) = x - y + z + \lambda(2x^2 - y^2 + z^2 - 5) + \mu(x + y) \quad \dots\dots\dots(6 \text{ 分})$$

$$\begin{cases} F'_x = 1 + 4\lambda x + \mu = 0 \\ F'_y = -1 - 2\lambda y + \mu = 0 \\ F'_z = 1 + 2\lambda z = 0 \\ 2x^2 - y^2 + z^2 = 5 \\ x + y = 0 \end{cases} \quad \dots\dots\dots(8 \text{ 分})$$

解得  $x = \mp 2 \quad y = \pm 2 \quad z = \mp 1$

得两点  $M_1(-2, 2, -1) \quad M_2(2, -2, 1) \quad \dots\dots\dots(10 \text{ 分})$

$$\left. \frac{\partial f}{\partial \vec{e}} \right|_{M_1} = -\frac{10}{\sqrt{3}} \quad \left. \frac{\partial f}{\partial \vec{e}} \right|_{M_2} = \frac{10}{\sqrt{3}}$$

由于  $\frac{\partial f}{\partial \vec{e}}$  在曲线上确有最大值和最小值, 故  $M_1, M_2$  为所求, 且

$$\max_M \left\{ \frac{\partial f}{\partial \vec{e}} \right\} = \frac{10}{\sqrt{3}} \quad \min_M \left\{ \frac{\partial f}{\partial \vec{e}} \right\} = -\frac{10}{\sqrt{3}} \quad \dots\dots\dots(12 \text{ 分})$$