



On the sensitivity of extracting the astrophysical cross section factor of the $^{12}\text{C}(\alpha, \gamma)$ reaction from existing data.

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Abstract

We address a conflicting report on the value and uncertainty of the astrophysical cross section factor of the $^{12}\text{C}(\alpha, \gamma)$ reaction extracted from existing data. In sharp contrast to previously reported ambiguities (by up to a factor 8), Schuermann *et al.* suggest an accuracy of 12%. We demonstrate that the so claimed “rigorous data selection criteria” used by Schuermann *et al.* relies on the s-factors extracted by Assuncao *et al.* But these results were shown in a later analysis (by this author) to have large error bars (considerably larger than claimed by Assuncao *et al.*) which render these data not appropriate for a rigorous analysis. When their “rigorous data selection” is adjusted to remove the results of Assuncao *et al.* the astrophysical cross section factor cannot be extracted with 12% accuracy, or even close to it. Such data on the S_{E2} values at low energies deviate by up to a factor two from their fit and exhibit a sharper slope rising toward low energies, leading to strong doubt on their extrapolated $S_{E2}(300)$ value and the quoted small error bar. Contrary to their claim the small value of $S_{E1}(300) \approx 10 \text{ keVb}$ cannot be ruled out by current data including the most modern gamma-ray data. As previously observed by several authors current data reveal ambiguities in the value of $S_{E1}(300) = \text{approximately } 10 \text{ keVb}$ or $\text{approximately } 80 \text{ keVb}$, and the new ambiguity that was recently revealed (by this author) of $S_{E2}(300) = \text{approximately } 60 \text{ keVb}$ or $\text{approximately } 154 \text{ keVb}$, appear to be a more reasonable evaluation the status of current data.

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During stellar helium burning oxygen is formed by the fusion of helium with carbon and is denoted by the $^{12}\text{C}(\alpha, \gamma)$ reaction. It has been announced three decades ago [1] as the most important and uncertain nuclear input of stellar evolution and it remains so today. In sharp contrast Schuermann *et al.* [2] claimed that the astrophysical cross section factor of the $^{12}\text{C}(\alpha, \gamma)$ reaction (as defined for example in [1]) can be deduced with high accuracy of approximately 12%, using a global R-matrix analysis of existing data. We demonstrate that existing data do not permit the claimed 12% accuracy, or even close to it, notwithstanding sophisticated R-matrix analyses. We also raise strong doubts on the values of the astrophysical cross section factors quoted by Schuermann *et al.* [2].

Two comments are in order from the outset. First, the “rigorous data selection criteria” of Schuermann *et al.* [2] include ten data points shown with the measured 100% (or more) error bars. Such data points should be considered as upper limits and as such they cannot be included in a rigorous chi-square analysis, since the contribution of an upper limit to chi-square cannot be rigorously evaluated. Most bothersome are the five such data points shown at

low energies below 2 MeV (2 data points for S_{E1} and 3 for S_{E2}). More significantly as we discuss below, other data included by Schuermann *et al.* [2] were shown in a later analysis (by this author [3]) to have similar large error bars (close to 100%) which also render these data not useful for a rigorous chi-sqaure analysis.

Second, the data points below 2 MeV are essential for having high sensitivity to the contributions of the bound 1^- and 2^+ states at 7.1169 and 6.9171 MeV in ^{16}O , respectively, that govern the value of the astrophysical cross section factors at stellar energies: $S_{E1}(300)$ and $S_{E2}(300)$, respectively. But the contribution to χ^2 of these low energy data points is overwhelmed by the large number of high energy data points included by Schuermann *et al.*, leading to a reduced sensitivity to the low energy data. In addition the inclusion of higher energy data points raises additional question(s): for example concerning the sign of the interference of the 2^+ at $E_{cm} = 2.68$ MeV that was shown to be positive and leading to an extrapolated $S_{E2}(300) = 62^{+9}_{-6}$ keVb [4, 5]. This S-factor is 16% lower than quoted by Schuermann *et al.* with a difference that is considerably larger than the quoted uncertainty of $S_{E2}(300)$. We note that the exclusion of high energy data points and consideration of data points only below 1.7 MeV was shown by this author [3] to lead to very different conclusions as we discuss below.

More troubling is the inclusion and in fact their reliance on the published data of Assuncao *et al.* [6]. We note from the outset that Brune *et al.* [7] concluded several systematical problem with the data of Assuncao *et al.* [6]. These data [6] were re-analyzed by this author [3] and I revealed very large error bars of the extracted s-factors [3], considerably larger than stated by Assuncao *et al.* [6].

We recapitulate here a few observation made in my chi-square analysis [3] of the data of Assuncao *et al.* [6]. For example even though no discernible peaks can be established in the gamma-ray spectra measured at $90^\circ - 130^\circ$ shown in Fig. 6 of Assuncao *et al.* [6], an angular distribution is claimed to have been measured at these backward angles at the indicated energy of $E_{\alpha,lab} = 1.850$ MeV. I showed that the angular distribution labeled as $E_{\alpha,lab} = 1.900$ MeV ($E = 1.340$ MeV) [6] can be fitted with $\frac{S_{E2}}{S_{E1}}$ values that vary by a factor of six with similar reduced χ^2 values; see Fig. 1 of Ref. [3]. My chi-square analysis of all data points measured by Assuncao *et al.* [6] below 1.7 MeV lead to $\frac{S_{E2}}{S_{E1}}$ values that vary by a large factor without significant variation in chi-square. The resulting uncertainties are shown in Fig. 2 of my paper [3] and they are close to 100%. As discussed above such data points must be considered as upper limits and the data of Assuncao *et al.* [6] cannot be included in a rigorous chi-square analysis. We emphasize that excluding the results of Assuncao *et al.* is not a matter of choice for “data selection”, rather it is dictate by standard considerations of a rigorous chi-square analysis.

In the same paper I also demonstrated that the disagreement of the E1-E2 measured phase angle (ϕ_{12}) with (theoretical) predictions, shown in Fig. 11 of Assuncao *et al.* [6], is a violation of unitarity [3] and not just a mere disagreement with the prediction of R-matrix theory as suggested by them [6]. A violation of unitarity is a clear indication of serious problems with the data or the data analysis.

We conclude that the “rigorous data selection” employed by Schuermann *et al.* [2] should not include the data of Assuncao *et al.* [6] and we remove the data of Assuncao *et al.* from the data sample analyzed by Schuermann *et al.* But since they already removed the data of Redder *et al.* [8] and Ouellet *et al.* [9] from their choice for the current precise data, the adjusted choice of data of S_{E2} measured below 2 MeV includes only the data of Kunz *et al.* [10]. This is clearly an unacceptable situation that must be alleviated by new data measured at low energies and it cannot be remedied by sophisticated global R-matrix analyses. In the same time it is important to comment here that my analysis of the data of Kunz *et al.* [10] agrees with Kunz *et al.* as stated in my paper [3].

Nevertheless the comparison of the low energy R-matrix curve shown in Fig. 1, taken from their Fig. 4 [2], reveals a disagreement by up to a factor of 2 with the adjusted choice of data sample used by Schuermann *et al.* (which does not include the data of Assuncao *et al.* and only includes the data of Kunz *et al.*, excluding the data points measured with 100% error bars). More troubling is the fact that in comparison to the data their R-matrix fit has the wrong slope at low energies as shown in Fig. 1. Clearly these data points below 2 MeV are crucial for delineating the contribution of the bound 2^+ state of ^{16}O and the slope is very important for an accurate extrapolation of $S_{E2}(300)$.

We conclude that when considering the adjusted choice of the data sample used by Schuermann *et al.*, the value they quote for S_{E2} cannot be substantiated and certainly we cannot support their claimed small uncertainty of the extracted $S_{E2}(300)$. We refer the reader to Ref. [3] for a complete chi-square analysis of all currently available data below 1.7 MeV that bifurcate and lead to ambiguity in the extracted value of $S_{E2}(300) = 60 \pm 12$ or 154 ± 31 keVb.

Concerning the value of $S_{E1}(300)$: Schuermann *et al.* [2] consider the possibility of destructive interference of the 1^- sub-threshold state at 7.1169 MeV and the 2.42 MeV 1^- resonance in ^{16}O and they claim “the constructive

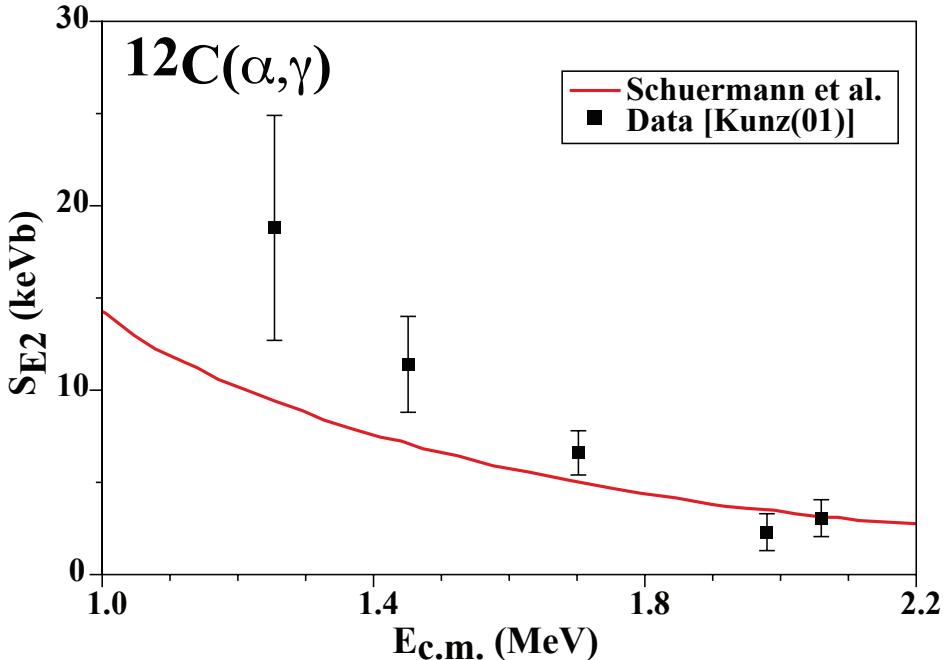


Figure 1. (Color online) Comparison of the low energy portion of the R-matrix fit of Schuermann *et al.* [2] with the S_{E2} data measured by Kunz *et al.* [10]. As discussed in the text the data of Assuncao *et al.* [6] have been removed from their “rigorously selected data” leading to a major discrepancy with the remainder of the data (i.e. the Kunz *et al.* data) and specifically with the wrong predicted slope.

solution is strongly favored and the destructive interference pattern has been rejected”. This claim is based on the obtained $\chi^2_{\text{cap}} = 265$ and 233 for $S_{E1}(300) = 7.9$ and 83.4 keVb, respectively. When considering the large number of capture data points included by Schuermann *et al.* (243) we conclude that the obtained χ^2 difference of 32 is hardly significant to warrant the rejection of the destructive interference pattern [yielding the small $S_{E1}(300) = 7.9$ keVb].

Furthermore, since they state that a one sigma variation of the total s-factor leads to $\Delta\chi^2 \approx 21$ [2], the obtained χ^2 difference of 32 is not significant enough and certainly far from the usual five sigma used to substantiate a claim. Clearly both fits have reduced χ^2 values that are close to unity, and the fit that leads to the small $S_{E1}(300) = 7.9$ keVb cannot be considered as ruled out by the data since the reduced χ^2 is close to unity. As such the destructive interference cannot be ruled out with the certainty claimed by Schuermann *et al.* It is important to note that it is not sufficient to demonstrate the good fit for the large $S_{E1}(300)$ solution, but one must also rule out the small $S_{E1}(300)$ solution. This has not been achieved by Schuermann *et al.* and current data do not allow us to rule out the small $S_{E1}(300)$ solution, leading to the ambiguous value of $S_{E1}(300)$ [3].

The rejection of the small S_{E1} solution by Schuermann *et al.* based on χ^2 consideration is made more doubtful since they included and relied on the data of Assuncao *et al.* which were found to have unrealistic small error bars [3]. This makes the minuscule difference in the reduced χ^2 even more troublesome for rejecting the destructive interference pattern.

Similarly the fit to the modern data that was published in a peer reviewed conference proceedings [11] (and was neglected by Schuermann *et al.*) states the numerical values: $S_{E1}(300) = 77.9$ and 4.3 keVb with $\chi^2 = 9.0$ and 9.6, respectively, see Fig. 5 of [11]. Such a small difference in χ^2 in of itself does not allow rejecting the small value solution and the need to re-evaluate the error bars quoted in Ref. [6, 11] weakens the possibility of rejecting the small s-factor solution using these modern data alone. We thus conclude that this modern gamma-ray data analysis [11] just the same as the analysis of Schuermann *et al.* [2] and previous data analyses [12, 13] do not support ruling out the small $S_{E1} \approx 10$ keVb solution.

We conclude that a realistic evaluation of current data does not permit the determination of the astrophysical cross section factors with the 12% accuracy suggested by Schuermann *et al.*, nor can we support their claimed values of

$S_{E1}(300)$ and $S_{E2}(300)$. A more suitable conclusion is that both extrapolated $S_{E1}(300)$ and $S_{E2}(300)$ cross section factors are ambiguous with the values listed in my previous publication [3].

Unlike the strong claim of 12% accuracy suggested by Schuermann *et al.* the observed large ambiguities justify and promote a new and different research effort to determine the astrophysical cross section factors with the required uncertainty of 10% or better. Indeed proposals for determining the cross section at very low energy have been developed for the HI γ S gamma-ray facility in the USA [14], and the newly constructed ELI-NP facility in Bucharest as shown in [15]. Measurements with gamma-beams are favored by the detailed balance factors of 50 - 100 and are made possible due to the anticipated high intensity ($10^9 \gamma/s$) that will allow a measurement at $E_{c.m.} = 1.0$ MeV within two weeks of beam time. The design goal sensitivity of these measurements which is shown in Figs. 2 and 3 of [15], promises to resolve the observed ambiguities in $S_{E1}(300)$ and $S_{E2}(300)$.

In closing we note that for example as shown in Fig. 1 of [16] the suggested value of the total astrophysical cross section factor quoted in [2] of $161 \pm 19 + 8 - 2$ keVb (i.e. a multiplicative factor of 1.61 as defined in [16]) is exactly at the boundary (170 keVb) where a 25 solar masses star is predicted to be oxygen rich ($C/O < 1$) and thus skip the carbon burning stage and collapse to a black hole. Thus a resolution of the ambiguities in $S_{E1}(300)$ (approximately 10 or 80 keVb) and $S_{E2}(300)$ (approximately 60 or 154 keVb) noted in Ref [3] is essential for progress in stellar evolution theory.

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