1.18)

a) $A=\begin{pmatrix}1&1&-3\\2&1&m\\1&m&3\end{pmatrix}\xrightarrow[(3)\to(3)-(1)]{(2)\to(2)-2(1)}\begin{pmatrix}1&1&-3\\0&-1&m+6\\0&m-1&6\end{pmatrix}$

$\xrightarrow{(3)\to(3)+(m-1)(2)}\begin{pmatrix}1&1&-3\\0&-1&m+6\\0&0&m^2+5m\end{pmatrix}$

Với $m^2+5m=0\Leftrightarrow\left[\begin{array}{l}m=0\\m=-5\end{array}\right.\Rightarrow r(A)=2$

Với $m^2+5m\neq 0\Rightarrow\left[\begin{array}{l}m\neq 0\\m\neq -5\end{array}\right.\Rightarrow r(A)=3$

b) $A=\begin{pmatrix}m&1&1\\1&m&1\\1&1&m\end{pmatrix}\xrightarrow[(3)\to(3)-m(1)]{(3)\to(3)\leftrightarrow(1)\\(2)\to(2)-(1)}\begin{pmatrix}1&1&m\\0&m-1&1-m\\0&1-m&1-m^2\end{pmatrix}$

$\xrightarrow{(3)\to(3)+(2)}\begin{pmatrix}1&1&m\\0&m-1&1-m\\0&0&2-m-m^2\end{pmatrix}$

* Nếu $m-1=0$ hay $m=1$ thì

$A\sim\begin{pmatrix}1&1&1\\0&0&0\\0&0&0\end{pmatrix}\Rightarrow r(A)=1$

* Nếu $m-1\neq 0$ hay $m\neq 1$. Xét

. Nếu $m\neq -2, m\neq 1$ thì $2-m-m^2\neq 0$, $m-1\neq 0$

nên $r(A)=3$

. Nếu $m\neq -2$ thì $m-1\neq 0$, $2-m-m^2=0$

nên $r(A)=2$

c) $A=\begin{pmatrix}1&1&3&3\\1&m&3&1\\1&2&-1&2\\2&3&2&4\end{pmatrix}\begin{matrix}(2)\to(2)\leftrightarrow(1)\\(3)\to(3)-(1)\\(4)\to(4)-2(1)\end{matrix}\begin{pmatrix}1&1&3&3\\0&m-1&0&-2\\0&1&-4&-1\\0&1&-4&-2\end{pmatrix}$

$\begin{matrix}(3)\to(3)\leftrightarrow(2)\\(3)\to(3)-(m-1)(2)\\(4)\to(4)-(2)\end{matrix}\begin{pmatrix}1&1&3&3\\0&1&-4&-1\\0&0&4m-4&m-3\\0&0&0&-1\end{pmatrix}$

Nếu $4m-4=0\Leftrightarrow m=1$ thì

$A\sim\begin{pmatrix}1&1&3&3\\0&1&-4&-1\\0&0&0&-4\\0&0&0&-1\end{pmatrix}\to r(A)=3$

Nếu $4m-4\neq0\Leftrightarrow m\neq1$ thì $r(A)=4$

d)

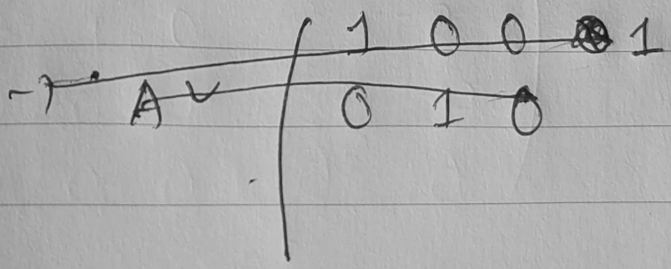
$\begin{pmatrix}1&0&0&1\\1&m&0&1\\0&1&m&0\end{pmatrix}\longrightarrow$

d7

$$A = \begin{pmatrix} m & 0 & 0 & 1 \\ 1 & m & 0 & 0 \\ 0 & 1 & m & 0 \\ 0 & 0 & 1 & m \end{pmatrix} \xrightarrow[(2)\to(2)-m(4)]{(2)\to(2)\leftrightarrow(4)} \begin{pmatrix} 1 & m & 0 & 0 \\ 0 & -m^2 & 0 & 1 \\ 0 & 1 & m & 0 \\ 0 & 0 & 1 & m \end{pmatrix}$$

$$\xrightarrow[(3)\to(3)+m^2(2)]{(2)\to(2)\leftrightarrow(3)} \begin{pmatrix} 1 & 0 & -m^2 & 0 \\ 0 & 1 & m & 0 \\ 0 & 0 & m^3 & 1 \\ 0 & 0 & 1 & m \end{pmatrix} \xrightarrow[(4)\to(4)-m^3(3)]{(3)\to(3)\leftrightarrow(4)} \begin{pmatrix} 1 & 0 & 0 & m^3 \\ 0 & 1 & 0 & -m^2 \\ 0 & 0 & 1 & m \\ 0 & 0 & 0 & 1-m^4 \end{pmatrix}$$

Với ~~$m \neq 1$~~ $1 - m^4 = 0 \Leftrightarrow m^4 = 1 \Leftrightarrow m = \pm 1$



Với $1 - m^4 = 0 \Leftrightarrow m = \pm 1 \Rightarrow r(A) = 3$

Với $1 - m^4 \neq 0 \Leftrightarrow m \neq \pm 1 \Rightarrow r(A) = 4$

1.19)

a) $\begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 4 \\ 6 & 0 & 0 \end{pmatrix} \xrightarrow{(1)\to(1)-(2)} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 5 \end{pmatrix} \begin{matrix} (2)\to\frac{1}{2}(2) \\ (1)\to(1)-2(2) \\ (3)\to(3)\cdot\frac{1}{5} \end{matrix} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$\xrightarrow[(2)\to(2)-3(1)]{(1)\to(1)+2(3)} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

c)

$\begin{pmatrix} 1 & 3 & -2 & -1 \\ 2 & 5 & -2 & 1 \\ 1 & 1 & 6 & 13 \\ -2 & -6 & 8 & 10 \end{pmatrix} \begin{matrix} (2)\to(2)-2(1) \\ (3)\to(3)-(1) \\ (4)\to(4)+2(1) \end{matrix} \begin{pmatrix} 1 & 3 & -2 & -1 \\ 0 & -1 & 2 & 3 \\ 0 & -2 & 8 & 14 \\ 0 & 0 & 4 & 8 \end{pmatrix}$

$\xrightarrow[(1)\to(1)-3(2)\\(3)\to(3)+2(2)]{(2)\to-(2)} \begin{pmatrix} 1 & 0 & 4 & 8 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 4 & 8 \end{pmatrix} \begin{matrix} (3)\to\frac{1}{4}(3) \\ (4)\to(1)-4(3) \\ (2)\to(2)+2(3) \\ (4)\to(4)-4(3) \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

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d)

$$\begin{pmatrix} 1 & 2 & 3 & 1 & -1 \\ 1 & 2 & 1 & -5 & 3 \\ 1 & 3 & 2 & 1 & -1 \\ 2 & 1 & 6 & -7 & 4 \end{pmatrix} \xrightarrow[(3)\to(3)-(4)]{(2)\to(2)-(4)} \begin{pmatrix} 1 & 2 & 3 & 1 & -1 \\ 0 & 0 & -2 & -6 & 4 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & -3 & 0 & -9 & 6 \end{pmatrix}$$

$$\xrightarrow[(4)\to(4)+3(2)]{(2)\to(2)\leftrightarrow(3)} \begin{pmatrix} 1 & 0 & 5 & 1 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -2 & -6 & 4 \\ 0 & 0 & -3 & -9 & 6 \end{pmatrix} \xrightarrow[(2)\to(2)+(3)]{(1)\to(1)-5(3)} \begin{pmatrix} 1 & 0 & 0 & -14 & 9 \\ 0 & 1 & 0 & 3 & -2 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$