Họ Tên: Nguyễn Nam Khánh ; MSSV: 22120157

Lớp: 22-2 Toán UDTK

Bài 1: $x=(1,2,3)$, $y=(y_1,y_2,y_3)$, $z=(4,2,1)$

① $\langle x,z\rangle = 1.4+2.2+3.1 = 11$

② $\langle x, 2y+z\rangle$

có: $2y+z = 2(y_1,y_2,y_3)+(4,2,1)$

$= (2y_1+4, 2y_2+2; 2y_3+1)$

$\Rightarrow \langle x,2y+z\rangle = 1.(2y_1+4)+2(2y_2+2)+3(2y_3+1)$

$= 2y_1+4y_2+6y_3+11$

③ $||x|| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

④ $x-y = (1,2,3)-(y_1,y_2,y_3)$

$= (1-y_1, 2-y_2, 3-y_3)$

$\Rightarrow ||x-y|| = \sqrt{(1-y_1)^2+(2-y_2)^2+(3-y_3)^2}$

Bài 2: Gọi e_i là vector trực chuẩn của các vector u_i

a) $u_1=(1,1,1)$, $u_2=(0,1,1)$, $u_3=(0,0,1)$

Có: • $f_1=u_1=(1,1,1) \Rightarrow e_1 = \frac{f_1}{||f_1||} = \left(\frac{1}{\sqrt3}, \frac{1}{\sqrt3}, \frac{1}{\sqrt3}\right)$

• $f_2 = u_2 - \frac{\langle u_2,f_1\rangle}{\langle f_1,f_1\rangle} f_1 = (0,1,1) - \frac{2}{3}(1,1,1) = \left(\frac{-2}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$\Rightarrow e_2 = \frac{f_2}{||f_2||} = \left(-\frac{\sqrt6}{3}, \frac{\sqrt6}{6}, \frac{\sqrt6}{6}\right)$

• $f_3 = u_3 - \frac{\langle u_3,f_1\rangle}{\langle f_1,f_1\rangle} f_1 - \frac{\langle u_3,f_2\rangle}{\langle f_2,f_2\rangle} f_2$

$= (0,0,1) - \frac{1}{3}(1,1,1) - \frac{1/3}{2/3}\left(\frac{-2}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$= \left(0, \frac{-1}{2}, \frac{1}{2}\right)$

$\Rightarrow e_3 = \frac{f_3}{||f_3||} = \left(0, \frac{-\sqrt2}{2}, \frac{\sqrt2}{2}\right)$

Vậy $e_1 = \left(\frac{1}{\sqrt3}, \frac{1}{\sqrt3}, \frac{1}{\sqrt3}\right)$, $e_2 = \left(\frac{-\sqrt6}{3}, \frac{\sqrt6}{6}, \frac{\sqrt6}{6}\right)$, $e_3 = \left(0, \frac{-\sqrt2}{2}, \frac{\sqrt2}{2}\right)$

b) $u_1=(1,1,1),\ u_2=(-1,1,0),\ u_3=(1,2,1)$

Có: $f_1=u_1=(1,1,1) \Rightarrow e_1=\dfrac{f_1}{\|f_1\|}=\left(\dfrac{1}{\sqrt3},\dfrac{1}{\sqrt3},\dfrac{1}{\sqrt3}\right)$

. $f_2=u_2-\dfrac{\langle u_2,f_1\rangle}{\langle f_1,f_1\rangle}f_1=(-1,1,0)-\dfrac{0}{3}(1,1,1)=(-1,1,0)$

$\Rightarrow e_2=\dfrac{f_2}{\|f_2\|}=\left(\dfrac{-1}{\sqrt2},\dfrac{1}{\sqrt2},0\right)$

. $f_3=u_3-\dfrac{\langle u_3,f_1\rangle}{\langle f_1,f_1\rangle}f_1-\dfrac{\langle u_3,f_2\rangle}{\langle f_2,f_2\rangle}f_2=(1,2,1)-\dfrac{4}{3}(1,1,1)$
$-\dfrac{1}{2}(-1,1,0)=\left(\dfrac16,\dfrac16,\dfrac{-2}{3}\right)$

$\Rightarrow e_3=\dfrac{f_3}{\|f_3\|}=\left(\dfrac{1}{\sqrt6},\dfrac{1}{\sqrt6},\dfrac{-2}{\sqrt6}\right)$

c) $u_1=(1,0,0),\ u_2=(3,7,-2),\ u_3=(0,4,1)$

Có: $f_1=u_1=(1,0,0)\rightarrow e_1=\dfrac{f_1}{\|f_1\|}=(1,0,0)$

$f_2=u_2-\dfrac{\langle u_2,f_1\rangle}{\langle f_1,f_1\rangle}f_1=(3,7,-2)-\dfrac{3}{1}(1,0,0)=(0,7,-2)$

$\Rightarrow e_2=\dfrac{f_2}{\|f_2\|}=\left(0,\dfrac{7}{\sqrt{53}},\dfrac{-2}{\sqrt{53}}\right)$

$f_3=u_3-\dfrac{\langle u_3,f_1\rangle}{\langle f_1,f_2\rangle}f_1-\dfrac{\langle u_3,f_2\rangle}{\langle f_2,f_2\rangle}f_2=(0,4,1)-\dfrac{0}{1}(1,0,0)$

$-\dfrac{26}{53}\cdot\left(0,\dfrac{7}{\cancel{\sqrt{53}}},\dfrac{-2}{\cancel{\sqrt{53}}}\right)=\left(0,\dfrac{30}{53},\dfrac{105}{53}\right)$

$\Rightarrow e_3=\dfrac{f_3}{\|f_3\|}=\left(0,\dfrac{2}{\sqrt{53}},\dfrac{7}{\sqrt{53}}\right)$

d) $u_1=(0,2,1,0),\ u_2=(1,-1,0,0),\ u_3=(1,2,0,-1)$
$u_4=(1,0,0,1)$

Có: $f_1=u_1=(0,2,1,0)\Rightarrow e_1=\left(0,\dfrac{2}{\sqrt5},\dfrac{1}{\sqrt5},0\right)$

$f_2=u_2-\dfrac{\langle u_2,f_1\rangle}{\langle f_1,f_1\rangle}f_1=(1,-1,0,0)-\dfrac{-2}{5}(0,2,1,0)=\left(1,\dfrac{-1}{5},\dfrac{2}{5},0\right)$

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$\cancel{f_3} = 0 \Rightarrow e_2 = \left(\frac{\sqrt{30}}{6}, \frac{-\sqrt{30}}{30}, \frac{\sqrt{30}}{15}, 0\right)$

$\cdot\ f_3 = u_3 - \frac{\langle u_3, f_1\rangle}{\langle f_1, f_1\rangle} f_1 - \frac{\langle u_3, f_2\rangle}{\langle f_2, f_2\rangle} f_2 = (1,2,0,-1) - \frac{4}{5}(0,2,1,0)$

$- \frac{3/3}{6/5}\left(1, \frac{-1}{5}, \frac{2}{5}, 0\right) = \left(\frac{1}{2}, \frac{1}{2}, -1, -1\right)$

$\Rightarrow e_3 = \left(\frac{\sqrt{10}}{10}, \frac{\sqrt{10}}{10}, \frac{-\sqrt{10}}{5}, \frac{-\sqrt{10}}{5}\right)$

$f_4 = u_4 - \frac{\langle u_4, f_1\rangle}{\langle f_1, f_1\rangle} f_1 - \frac{\langle u_4, f_2\rangle}{\langle f_2, f_2\rangle} f_2 - \frac{\langle u_4, f_3\rangle}{\langle f_3, f_3\rangle} f_3$

$= (1,0,0,1) - \frac{0}{5}(0,2,1,0) - \frac{1}{6/5}\left(1, \frac{-1}{5}, \frac{2}{5}, 0\right) - \frac{-1/2}{5/2}\left(\frac{1}{2}, \frac{1}{2}, -1, -1\right) =$

$= \left(\frac{1}{15}, \frac{4}{15}, \frac{-8}{15}, \frac{4}{5}\right)$

$\rightarrow e_4 = \left(\frac{1}{15}, \frac{4}{15}, \frac{-8}{15}, \frac{4}{5}\right)$

Bài 3: Phân rã QR ma trận

a) $\begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 1 \\ -2 & 4 & 1 \end{bmatrix}$, $a_1 = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$, $a_2 = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$, $a_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

$\cdot\ f_1 = a_1 \Rightarrow e_1 = \frac{f_1}{\|e_1\|} = \begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix}$

$\cdot\ f_2 = a_2 - \langle a_2, f_1\rangle e_1 = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + 3\begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

$\Rightarrow e_2 = \begin{pmatrix} 2/3 \\ 1/3 \\ 2/3 \end{pmatrix}$

$\cdot\ f_3 = a_3 - \langle a_3, e_1\rangle e_1 - \langle a_3, e_2\rangle e_2$

$= \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \frac{2}{3}\begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix} - \frac{7}{3}\begin{pmatrix} 2/3 \\ 1/3 \\ 2/3 \end{pmatrix} = \begin{pmatrix} 2/9 \\ -2/9 \\ -1/9 \end{pmatrix} \rightarrow e_3 = \begin{pmatrix} 2/3 \\ -2/3 \\ -1/3 \end{pmatrix}$

Vậy $Q = [e_1 \; e_2 \; e_3] = \begin{pmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ -2/3 & 2/3 & -1/3 \end{pmatrix}$

$$R = \begin{pmatrix} \langle a_1,e_1\rangle & \langle a_2,e_1\rangle & \langle a_3,e_1\rangle \\ 0 & \|f_2\| & \langle a_3,e_2\rangle \\ 0 & 0 & \|f_3\| \end{pmatrix} = \begin{pmatrix} 3 & -3 & \frac{2}{3} \\ 0 & 3 & \frac{7}{3} \\ 0 & 0 & \frac{-1}{3} \end{pmatrix}$$

b)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix}, a_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, a_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, a_3 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

. $f_1 = a_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \to e_1 = \dfrac{f_1}{\|f_1\|} = \left(\dfrac{1}{\sqrt6}, \dfrac{2}{\sqrt6}, \dfrac{1}{\sqrt6}\right)$

. $f_2 = a_2 - \langle a_2,e_1\rangle e_1 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} - \left(\dfrac{-\sqrt6}{3}\right)\begin{pmatrix} 1/\sqrt6 \\ 2/\sqrt6 \\ 1/\sqrt6 \end{pmatrix}$

$= \begin{pmatrix} 4/3 \\ -4/3 \\ 4/3 \end{pmatrix} \to e_2 = \dfrac{f_2}{\|f_2\|} = \dfrac{f_2}{4/\sqrt3} = \left(\sqrt3/3, \dfrac{-\sqrt3}{3}, \dfrac{\sqrt3}{3}\right)$

. $f_3 = a_3 - \langle a_3,e_1\rangle e_1 - \langle a_3,e_2\rangle e_2$

$= (1,2,-1) - \dfrac{4}{\sqrt6}\left(\dfrac{1}{\sqrt6}, \dfrac{2}{\sqrt6}, \dfrac{1}{\sqrt6}\right) - \dfrac{2}{\sqrt3}\left(\sqrt3/3, \dfrac{-\sqrt3}{3}, \dfrac{\sqrt3}{3}\right)$

$= \left(\dfrac{-1}{3}, \dfrac{4}{3}, \dfrac{-7}{3}\right) \to e_3 = \dfrac{f_3}{\|f_3\|} = \dfrac{f_3}{\sqrt{66}/3} = \left(\dfrac{-\sqrt{66}}{66}, \dfrac{2\sqrt{66}}{33}, \dfrac{-7\sqrt{66}}{66}\right)$

Vậy $Q = [e_1 \; e_2 \; e_3] = \begin{pmatrix} 1/\sqrt6 & \sqrt3/3 & \frac{-\sqrt{66}}{66} \\ 2/\sqrt6 & -\sqrt3/3 & 2\sqrt{66}/33 \\ 1/\sqrt6 & \sqrt3/3 & -7\sqrt{66}/66 \end{pmatrix}$

$$R = \begin{pmatrix} \sqrt6 & \frac{-\sqrt6}{3} & 4/\sqrt6 \\ 0 & 4/\sqrt3 & 2/\sqrt3 \\ 0 & 0 & \frac{\sqrt{66}}{3} \end{pmatrix}$$

c) $\begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$, $a_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $a_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $a_3 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$

$f_1 = a_1 = (1, 0, 1) \Rightarrow e_1 = \dfrac{f_1}{\|f_1\|} = \left(\dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right)$

$f_2 = a_2 - \langle a_2, e_1\rangle e_1 = (1,1,1) - \sqrt{2}\left(\dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right) = (0,1,0)$

$\Rightarrow e_2 = \dfrac{f_2}{\|f_2\|} = (0,1,0)$

$f_3 = a_3 - \langle a_3, e_1\rangle e_1 - \langle a_3, e_2\rangle e_2 = (-1,2,1) - 0.\left(\dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right)$
$- 2(0,1,0) = (-1,0,1)$

$\Rightarrow e_3 = \dfrac{f_3}{\|f_3\|} = \dfrac{f_3}{\sqrt{2}} = \left(\dfrac{-1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right)$

Vậy $Q = (e_1\ e_2\ e_3) = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$

$R = \begin{pmatrix} \sqrt{2} & \sqrt{2} & 0 \\ 0 & 1 & 2 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$

d) $\begin{pmatrix} -1 & -1 & 1 \\ 1 & 3 & 3 \\ -1 & -1 & 5 \\ 1 & 3 & 7 \end{pmatrix}$ $a_1 = \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$, $a_2 = \begin{pmatrix} -1 \\ 3 \\ -1 \\ 3 \end{pmatrix}$, $a_3 = \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \end{pmatrix}$

$f_1 = a_1 = (-1, 1, -1, 1) \Rightarrow e_1 = \left(\dfrac{-1}{2}, \dfrac{1}{2}, \dfrac{-1}{2}, \dfrac{1}{2}\right)$

$f_2 = a_2 - \langle a_2, e_1\rangle e_1 = (-1, 3, -1, 3) - 4\left(\dfrac{-1}{2}, \dfrac{1}{2}, \dfrac{-1}{2}, \dfrac{1}{2}\right) = (1, 1, 1, 1)$

$\Rightarrow e_2 = \left(\dfrac{1}{2}, \dfrac{1}{2}, \dfrac{1}{2}, \dfrac{1}{2}\right)$

$f_3 = a_3 - \langle a_3, e_1\rangle e_1 - \langle a_3, e_2\rangle e_2 = (1,3,5,7) - 2\left(\dfrac{-1}{2}, \dfrac{1}{2}, \dfrac{-1}{2}, \dfrac{1}{2}\right)$
$- 8\left(\dfrac{1}{2}, \dfrac{1}{2}, \dfrac{1}{2}, \dfrac{1}{2}\right) = (-2, -2, 2, 2)$

$\Rightarrow e_3 = \left(\frac{-1}{2}, \frac{-1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

$$\text{Vậy } Q = (e_1, e_2, e_3) = \begin{pmatrix} -\frac{1}{2} & 1/2 & -1/2 \\ \frac{1}{2} & 1/2 & -1/2 \\ -\frac{1}{2} & 1/2 & 1/2 \\ \frac{1}{2} & 1/2 & 1/2 \end{pmatrix}$$

$$R = \begin{pmatrix} 2 & 4 & 2 \\ 0 & 2 & 8 \\ 0 & 0 & 4 \end{pmatrix}$$

e)
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad a_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}, a_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix}, a_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$f_1 = a_1 = (1, 2, 3, 0) \Rightarrow e_1 = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, 0\right)$

$f_2 = a_2 - \langle a_2, e_1\rangle e_1 = (1, 2, 0, 0) - \frac{5}{\sqrt{14}} \cdot \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, 0\right)$

$= \left(\frac{9}{14}, \frac{2}{7}, \frac{-15}{14}, 0\right) \Rightarrow e_2 = \left(\frac{3}{\sqrt{70}}, \frac{6}{\sqrt{70}}, \frac{-\sqrt{70}}{14}, 0\right)$

$f_3 = a_3 - \langle a_3, e_1\rangle e_1 - \langle a_3, e_2\rangle e_2$

$= (1, 0, 0, 1) - \frac{1}{\sqrt{14}} \cdot \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, 0\right) - \frac{3}{\sqrt{70}}\left(\frac{3}{\sqrt{70}}, \frac{6}{\sqrt{70}}, \frac{-\sqrt{70}}{14}, 0\right)$

$= \left(\frac{4}{5}, \frac{-2}{5}, 0, 1\right) \Rightarrow e_3 = \left(\frac{4\sqrt{5}}{15}, \frac{-2\sqrt{5}}{15}, 0, \frac{\sqrt{5}}{3}\right)$

$$\text{Vậy } Q = (e_1 \cdot e_2 \; e_3) = \begin{pmatrix} 1/\sqrt{14} & 3/\sqrt{70} & 4\sqrt{5}/15 \\ 2/\sqrt{14} & 6/\sqrt{70} & -2\sqrt{5}/15 \\ 3/\sqrt{14} & -\sqrt{70}/14 & 0 \\ 0 & 0 & \sqrt{5}/3 \end{pmatrix}$$

$$R = \begin{pmatrix} \sqrt{14} & 5/\sqrt{14} & 1/\sqrt{14} \\ 0 & 3\sqrt{70}/14 & 3/\sqrt{70} \\ 0 & 0 & 3/\sqrt{5} \end{pmatrix}$$

f)
$$A=\begin{pmatrix}-2&1&3\\1&0&0\\0&1&0\\0&0&1\end{pmatrix},\quad a_1=\begin{pmatrix}-2\\1\\0\\0\end{pmatrix},\quad a_2=\begin{pmatrix}1\\0\\1\\0\end{pmatrix},\quad a_3=\begin{pmatrix}3\\0\\0\\1\end{pmatrix}$$

$f_1=a_1=(-2,1,0,0) \Rightarrow e_1=\left(\frac{-2}{\sqrt5},\frac{1}{\sqrt5},0,0\right)$

$f_2=a_2-\langle a_2,e_1\rangle e_1=(1,0,1,0)-\left(\frac{-2}{\sqrt5}\right)\cdot\left(\frac{-2}{\sqrt5},\frac{1}{\sqrt5},0,0\right)$

$=\left(\frac15,\frac25,1,0\right)\Rightarrow e_2=\left(\frac{1}{\sqrt{30}},\frac{2}{\sqrt{30}},\frac{5}{\sqrt{30}},0\right)$

$f_3=a_3-\langle a_3,e_1\rangle e_1-\langle a_3,e_2\rangle e_2=(3,0,0,1)-\left(-\frac{6}{\sqrt5}\right)\left(\frac{-2}{\sqrt5},\frac{1}{\sqrt5},0,0\right)$

$-\frac{\sqrt{30}}{10}\left(\frac{1}{\sqrt{30}},\frac{2}{\sqrt{30}},\frac{5}{\sqrt{30}},0\right)=\left(\frac12,1,\frac{-1}{2},1\right)$

$\Rightarrow e_3=\left(\frac{1}{\sqrt{10}},\frac{2}{\sqrt{10}},\frac{-1}{\sqrt{10}},\frac{2}{\sqrt{10}}\right)$

Vậy $Q=\begin{pmatrix}e_1&e_2&e_3\end{pmatrix}=\begin{pmatrix}\frac{-2}{\sqrt5}&1/\sqrt{30}&1/\sqrt{10}\\1/\sqrt5&2/\sqrt{30}&2/\sqrt{10}\\0&5/\sqrt{30}&-1/\sqrt{10}\\0&0&2/\sqrt{10}\end{pmatrix}$

$R=\begin{pmatrix}\sqrt5&-2/\sqrt5&-6/\sqrt5\\0&\sqrt{30}/5&\sqrt{30}/10\\0&0&\sqrt{10}/2\end{pmatrix}$

g)
$$A=\begin{pmatrix}1&-1&2\\1&0&-1\\-1&1&2\\0&1&1\end{pmatrix},\quad a_1=\begin{pmatrix}1\\1\\-1\\0\end{pmatrix},\quad a_2=\begin{pmatrix}-1\\0\\1\\1\end{pmatrix},\quad a_3=\begin{pmatrix}2\\-1\\2\\1\end{pmatrix}$$

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$f_1 = a_1 = (1,1,-1,0) \Rightarrow e_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, 0\right)$

$f_2 = a_2 - \langle a_2, e_1\rangle e_1 = (-1,0,1,1) - \frac{2}{\sqrt{3}} \cdot \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, 0\right)$

$= \left(\frac{-5}{3}, \frac{-2}{3}, \frac{5}{3}, 1\right) \Rightarrow e_2 = \left(\frac{-5}{3\sqrt{7}}, \frac{-2}{3\sqrt{7}}, \frac{5}{3\sqrt{7}}, \frac{1}{\sqrt{7}}\right)$

$f_3 = a_3 - \langle a_3, e_1\rangle e_1 - \langle a_3, e_2\rangle e_2 = (2,-1,2,1) - \frac{\sqrt{3}}{3} \cdot \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, 0\right)$

$- \frac{5\sqrt{7}}{21} \cdot \left(\frac{-5}{3\sqrt{7}}, \frac{-2}{3\sqrt{7}}, \frac{5}{3\sqrt{7}}, \frac{1}{\sqrt{7}}\right) = \left(\frac{130}{63}, \frac{-74}{63}, \frac{122}{63}, \frac{16}{21}\right)$

$\Rightarrow e_3 = \left(\frac{130\sqrt{63}}{63\sqrt{628}}, \frac{-74\sqrt{63}}{63\sqrt{628}}, \frac{122\sqrt{63}}{63\sqrt{628}}, \frac{16\sqrt{63}}{21\sqrt{628}}\right)$

Vậy $Q = (e_1 \quad e_2 \quad e_3) = \begin{pmatrix} 1/\sqrt{3} & -5/3\sqrt{7} & \frac{130\sqrt{63}}{63\sqrt{628}} \\ 1/\sqrt{3} & -2/3\sqrt{7} & \frac{-74\sqrt{63}}{63\sqrt{628}} \\ -1/\sqrt{3} & 5/3\sqrt{7} & 122\sqrt{63}/63\sqrt{628} \\ 0 & 1/\sqrt{7} & \frac{16\sqrt{63}}{21\sqrt{628}} \end{pmatrix}$

$R = \begin{pmatrix} \sqrt{3} & 2/\sqrt{3} & \sqrt{3}/3 \\ 0 & \sqrt{7} & 5\sqrt{7}/21 \\ 0 & 0 & 2\sqrt{\frac{628}{63}} \end{pmatrix}$