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$$17) \begin{cases} x_1 + 2x_2 - x_3 = -1 \\ 2x_1 + 2x_2 + x_3 = 2 \\ 3x_1 + 5x_2 - 2x_3 = -1 \end{cases} \xrightarrow{\text{mở rộng}} \begin{pmatrix} 1 & 2 & -1 & -1 \\ 2 & 2 & 1 & 1 \\ 3 & 5 & -2 & -1 \end{pmatrix}$$

$$\xrightarrow{\begin{array}{l} d_1-d_2 \\ d_3-2d_1 \end{array}} \begin{pmatrix} 1 & 2 & -1 & -1 \\ 0 & -2 & 3 & 3 \\ 0 & -1 & 1 & 2 \end{pmatrix} \xrightarrow{\begin{array}{l} d_3-d_1-\frac{1}{2}d_2 \\ d_2 \end{array}} \begin{pmatrix} 1 & 2 & -1 & -1 \\ 0 & -2 & 3 & 3 \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\Rightarrow \begin{cases} x_1 = -2x_2 + x_3 - 1 \\ x_2 = \frac{3 - 3x_3}{-2} \\ -\frac{1}{2}x_3 = \frac{1}{2} \end{cases} \quad \begin{cases} x_2 = 4 \\ x_2 = -3 \\ x_3 = -1 \end{cases} \quad \text{vậy } \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 \\ 4 & -3 & -1 \end{pmatrix}$$

$$27) \begin{cases} x_1 - 2x_2 - x_3 = 1 \\ 2x_1 - 3x_2 + x_3 = 6 \\ 3x_1 - 5x_2 = 7 \\ x_1 + 5x_3 = 9 \end{cases} \xrightarrow{\begin{array}{l} \text{mở rộng} \\ \text{hết} \end{array}} \begin{pmatrix} 1 & -2 & -1 & 1 \\ 2 & -3 & 1 & 6 \\ 3 & -5 & 0 & 7 \\ 1 & 0 & 5 & 9 \end{pmatrix}$$

$$\xrightarrow{\begin{array}{l} d_2-d_1-2d_3 \\ d_3-d_1-3d_2 \\ d_4-d_1-d_3 \end{array}} \begin{pmatrix} 1 & -2 & -1 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 1 & 3 & 4 \\ 0 & 2 & 6 & 8 \end{pmatrix} \xrightarrow{\begin{array}{l} d_3-d_2-d_4 \\ d_4-d_2-2d_3 \end{array}} \begin{pmatrix} 1 & -2 & -1 & 1 \\ 0 & 1 & 3 & 9 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Có  $\text{rank}(A) = \text{rank}(A|I) = 2 \leq n = 3 = 1$  hệ C.S.S. không  
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$$\left\{ \begin{array}{l} x_1 = 2x_2 + x_3 + 1 = 2(4 - 3a) + a + 1 = 7 - 5a \\ x_2 = 4 - 3a \\ x_3 = a \end{array} \right. \quad (a \in \mathbb{R})$$

$\nabla_{\text{ag}} (x_1, x_2, x_3) = (-5a + 9, 4 - 3a, a) \quad \forall a \in \mathbb{R}$

$$\begin{aligned} & \left| \begin{array}{l} x_1 + 2x_2 + 2x_3 = 6 \\ 3x_1 + 5x_2 - x_3 + 6x_4 = 17 \\ 2x_1 + 4x_2 + x_3 + 2x_4 = 12 \\ 2x_1 - 7x_2 + 11x_4 = 7 \end{array} \right. \quad \xrightarrow{\substack{\text{消去法} \\ \text{Row}}} \quad \begin{pmatrix} 1 & 2 & 0 & 2 & 6 \\ 3 & 5 & -1 & 6 & 17 \\ 2 & 4 & 1 & 2 & 12 \\ 2 & 0 & -7 & 11 & 7 \end{pmatrix} \end{aligned}$$

$$\xrightarrow{\substack{(1)-(2)-3(3) \\ (3)-(1)-(2) \\ (4)-(1)-(2)}} \begin{pmatrix} 1 & 2 & 0 & 2 & 6 \\ 0 & -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & -7 & -7 & 7 & -5 \end{pmatrix} \xrightarrow{d_4 - 1d_1 - 4d_2} \begin{pmatrix} 1 & 2 & 0 & 2 & 6 \\ 0 & -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 7 & 7 \end{pmatrix}$$

$$\xrightarrow{d_1 \rightarrow d_1 + 3d_3} \begin{pmatrix} 1 & 2 & 0 & 2 & 6 \\ 0 & -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 2 & -1 \end{pmatrix} \Rightarrow \begin{cases} x_1 = 6(2x_2 - 2x_4 + 6) = 2 \\ x_2 = -(x_3 - 1) = 3 \\ x_3 = 2x_4 = -2 \\ x_4 = -1 \end{cases}$$

$\nabla_{\text{ag}} (x_1, x_2, x_3, x_4) = (2, 3, -2, -1)$

$$\left\{ \begin{array}{l} 2x_1 - 4x_2 - x_3 = 1 \\ x_1 - 3x_2 + x_3 = 1 \\ 3x_1 - 5x_2 - 3x_3 = 1 \end{array} \right. \xrightarrow{\text{mở rộng}} \left( \begin{array}{ccc|c} 2 & -4 & -1 & 1 \\ 1 & -3 & 1 & 1 \\ 3 & -5 & -3 & 1 \end{array} \right)$$

$$\xrightarrow{\begin{array}{l} d_1 \leftrightarrow d_2 \\ d_2 - d_1 - 2d_2 \\ d_3 - d_2 - 3d_2 \end{array}} \left( \begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 2 & -3 & -1 \\ 0 & 4 & -8 & -1 \end{array} \right) \xrightarrow{\begin{array}{l} d_3 - 2d_2 \\ d_2 \end{array}} \left( \begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 2 & -3 & -1 \\ 0 & 0 & 0 & 1 \end{array} \right)$$

G:  $\text{rank}(A) \neq \text{rank}(A|\vec{b})$  nên hệ pt vô nghiệm

3.5.

$$\left\{ \begin{array}{l} x_1 + 2x_2 - 2x_3 = 3 \\ 3x_1 - 2x_2 + x_3 = 2 \\ -x_1 + 5x_2 - 5x_3 = 5 \end{array} \right. \xrightarrow{\begin{array}{l} \text{mở rộng} \\ \text{học} \end{array}} \left( \begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 3 & -2 & 1 & 2 \\ -1 & 5 & -5 & 5 \end{array} \right)$$

$$\xrightarrow{\begin{array}{l} d_2 - d_1 - 3d_2 \\ d_3 - d_2 + d_1 \end{array}} \left( \begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & -7 & 7 & -8 \\ 0 & 7 & -7 & 8 \end{array} \right) \xrightarrow{d_3 + 7d_2} \left( \begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & -7 & 7 & -8 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

G:  $\text{rank}(A) = \text{rank}(A|\vec{b}) = 2 < n = 3 \Rightarrow$  Hệ pt có vô số sol

$$x_3 = -2x_2 + 2x_3 + 3 = -2(\alpha + \frac{8}{7}) + 2\alpha + \frac{3}{7} = \frac{5}{7}$$

$$\left\{ \begin{array}{l} x_2 = -\frac{7x_3 - 8}{7} = x_3 + \frac{8}{7} = \alpha + \frac{1}{7} \\ x_3 = \alpha \end{array} \right. \quad \forall \alpha \in \mathbb{R}$$

$$\sqrt[3]{x_1, x_2, x_3} = \left( \frac{5}{7} \mid \alpha + \frac{1}{7} \mid \alpha \right) \forall \alpha \in \mathbb{R}$$



HÒA BÌNH

$$\left\{ \begin{array}{l} 2x_1 - 4x_2 + 6x_3 = 8 \\ x_1 - 2x_2 + x_3 = -1 \end{array} \right.$$

matto  
nó là

$$x_1 - 3x_2 + 4x_3 = 0$$

$$2 - 4 6 8$$

$$1 - 1 1 - 1$$

$$1 - 3 4 0$$

$d_2 \leftarrow d_2$

$\overrightarrow{d_2 - d_2 - 2d_2}$

$d_5 - d_3 - d_1$

$$\left( \begin{array}{cccc} 1 & -1 & 1 & -1 \\ 0 & 2 & 4 & 10 \end{array} \right) \xrightarrow[d_5 - d_3 - d_1]{}$$

$$0 - 2 3 1$$

$$\left( \begin{array}{cccc} 1 & -1 & 1 & -1 \\ 0 & -2 & 4 & 10 \end{array} \right)$$

$$0 0 -1 -9$$

$$x_1 = x_2 - x_3 - 1 = 3$$

$$x_2 = \frac{-4x_3 - 10}{-2} = 13$$

$$x_3 = 9$$

$$\text{Vôj } (x_1, x_2, x_3) = (3, 13, 9)$$

$$\text{Bên } \Rightarrow \left\{ \begin{array}{l} 4x_2 - 2x_3 - 4x_4 + 2x_5 = 2 \\ 6x_2 - 3x_3 - 5x_4 = 3 \end{array} \right.$$

$$4 - 2 - 4 2 1$$

$$6x_2 - 3x_3 - 5x_4 = 3 \xrightarrow[nó]{}$$

$$6 - 3 8 5 3$$

$$8x_2 - 4x_3 + 28x_4 - 44x_5 = 22$$

$$8 - 9 28 - 44 22$$

$$-8x_2 + 9x_3 - 4x_4 + 12x_5 = -5$$

$$-8 9 -9 32 -5$$

$d_3 - d_2 - \frac{3}{2}d_1$

$d_5 - d_3 - 2d_2$

$d_4 - d_4 - 2d_2$

$$4 - 2 - 4 2 1$$

$$0 0 6 - 8 \frac{3}{2}$$

$$0 0 36 - 48 9$$

$$0 0 -12 16 -3$$

$$4 - 2 - 4 2 2$$

$$0 0 6 - 8 \frac{3}{2}$$

$$0 0 0 0 0$$

$$0 0 0 0 0$$



Ta có:  $\text{rank}(A) = \text{rank}(A|I) = 2 < n = 4$  nên bộ phâcô không

(b)  $A - I = 2$  làm mì do

$$x_2 = \frac{1}{4}(2x_2 + 4x_3 - 2x_4 + 1) = \frac{1}{2}a_1 + \frac{5}{6}b + \frac{1}{2}$$

$$x_3 = a_2 + b + \frac{1}{4}$$

$$x_4 = b$$

$a, b \in \mathbb{R}$

$$\text{Vg } (x_1, x_2, x_3, x_4) = \left( \frac{1}{2}a_1 + \frac{5}{6}b + \frac{1}{2}, a_2 + \frac{4}{3}b + \frac{1}{4}, b, b \right)^T \in \mathbb{R}^4$$

$$\begin{array}{l} \text{Bài 8: } \begin{cases} x_1 - 2x_2 + 3x_3 = -3 \\ 2x_1 + 2x_2 = 0 \\ -3x_2 + 4x_3 = 7 \\ x_1 + x_3 = -1 \end{cases} \quad \left( \begin{array}{cccc} 1 & -2 & 3 & -3 \\ 2 & 2 & 0 & 0 \\ 0 & -3 & 4 & 1 \\ 1 & 0 & 1 & -1 \end{array} \right) \\ \xrightarrow{\text{d}_1 - d_2 - d_3} \begin{cases} x_1 - 2x_2 + 3x_3 = -3 \\ 0 & 6 & -6 & 6 \\ 0 & -3 & 4 & 1 \\ 0 & 2 & -2 & 2 \end{cases} \quad \left( \begin{array}{cccc} 1 & -2 & 3 & -3 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{array} \right) \end{array}$$

$$\xrightarrow{\text{d}_2 - \frac{1}{6}\text{d}_1} \begin{cases} x_1 - 2x_2 + 3x_3 = -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{cases} \quad \left( \begin{array}{cccc} 1 & -2 & 3 & -3 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

G:  $\text{rank}(A) = \text{rank}(A|I) = n = 4 \Rightarrow$  hệ có nghiệm

$$\begin{cases} x_1 = x_2 - 3x_3 - 3 = -5 \\ x_2 = x_3 + 1 = 5 \end{cases}$$

$$x_3 = 9$$

$$\text{Vg } (x_1, x_2, x_3) = (-5, 5, 9)$$

$$\left. \begin{array}{l} 3x_1 - 3x_2 + 3x_3 = -3 \\ -x_1 - 5x_2 + 2x_3 = 4 \\ -4x_2 + 2x_3 = 2 \\ 8x_2 - x_2 + 2x_3 = -4 \end{array} \right\} \quad \left| \begin{array}{cccc} 3 & -3 & 3 & -3 \\ -1 & -5 & 2 & 4 \\ 0 & -4 & 2 & 2 \\ 3 & -1 & 2 & -4 \end{array} \right.$$

$$\left. \begin{array}{l} d_2 \rightarrow \frac{1}{3}d_2 \\ d_2 - d_1 + d_3 \\ d_4 - d_4 - 3d_3 \end{array} \right\} \quad \left| \begin{array}{cccc} 1 & -1 & 1 & -1 \\ 0 & -6 & 3 & 3 \\ 0 & -4 & 2 & 2 \\ 0 & 2 & -1 & 1 \end{array} \right. \quad \left. \begin{array}{l} d_2 \rightarrow \frac{1}{3}d_2 \\ d_3 - d_3 - 2d_2 \\ d_4 - 1d_4 + d_2 \end{array} \right\} \quad \left| \begin{array}{cccc} 1 & -1 & 1 & -1 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right.$$

6)  $\text{rank}(A) \geq \text{rank}(A|A^T) = 2 \quad \langle n=3 \rangle \Rightarrow \text{högl. Punkt g. ob so mögl.}$

$$\left. \begin{array}{l} x_1 = x_2 - x_3 - 1 = \frac{1}{2}a + \frac{1}{2} \\ x_2 = \frac{-x_3 + 1 - 1}{-2} = \frac{1}{2}a - \frac{1}{2} \\ x_3 = a \quad (a \in \mathbb{R}) \end{array} \right. \quad \text{Vektoren } (x_1, x_2, x_3) = \left( \frac{1}{2}a - \frac{3}{2}, \frac{1}{2}a - \frac{1}{2}, a \right) \quad \forall a \in \mathbb{R}$$

$$\left. \begin{array}{l} \text{Bsp. 10} \\ \left. \begin{array}{l} x_2 - x_2 + x_3 - 3x_3 = 0 \\ 2x_2 - x_2 + 4x_3 - 2x_3 = 0 \end{array} \right\} \end{array} \right| \xrightarrow{\text{normieren}} \left| \begin{array}{cccc} 1 & -1 & 1 & -3 \\ 2 & -1 & 4 & -2 \end{array} \right| \quad \langle n=2 \rangle$$

$$\xrightarrow{d_2 - 2d_1} \left| \begin{array}{cccc} 1 & -1 & 1 & -3 \\ 0 & 1 & 2 & 9 \end{array} \right| \quad \langle n=2 \rangle$$

6)  $\text{rank}(A) = \text{rank}(A^T) = 2 \quad \langle n=2 \rangle \text{ högl. Punkt g. ob so mögl.}$

$$\left. \begin{array}{l} x_2 - x_3 - 3x_4 = -2a - 4b + a(3b) = -3a - b \\ x_2 = -2x_3 - 4x_4 = -2a - 4b \end{array} \right\} \forall a, b \in \mathbb{R}$$

$$x_3 = a$$

$$x_4 = 5$$

$$v_{\mathcal{G}}(x_1, x_2, x_3, x_4) = (-3a - b, -2a - 4b, a, 5) \quad \forall a, b \in \mathbb{R}$$

$$\text{Béz 12. } \left. \begin{array}{l} 2x_2 - 3x_3 + 9x_4 - x_1 = 0 \\ 6x_2 + 3 - 8x_3 + 9x_4 = 0 \\ 2x_1 + 6x_2 + 2x_3 - x_4 = 0 \end{array} \right\} \begin{matrix} \xrightarrow{x_1 + x_2} \\ \xrightarrow{3x_1} \end{matrix} \left| \begin{array}{cccc|c} 2 & -3 & 4 & -1 & 0 \\ 0 & 10 & -20 & 12 & 0 \\ 0 & 3 & -3 & 0 & 0 \end{array} \right| \begin{matrix} \xrightarrow{x_2 - 5x_3} \\ \xrightarrow{5x_2} \end{matrix} \left| \begin{array}{cccc|c} 2 & -3 & 4 & -1 & 0 \\ 0 & 10 & -20 & 12 & 0 \\ 0 & 0 & -15 & -5 & 0 \end{array} \right|$$

$$\left. \begin{array}{l} \xrightarrow{C_1 - C_2 - C_3} \\ \xrightarrow{C_3 - 10C_2} \end{array} \right| \left| \begin{array}{cccc|c} 2 & -3 & 4 & -1 & 0 \\ 0 & 10 & -20 & 12 & 0 \\ 0 & 3 & -3 & 0 & 0 \end{array} \right| \xrightarrow{C_3 - 10C_2 - \frac{3}{10}C_1} \left| \begin{array}{cccc|c} 2 & -3 & 4 & -1 & 0 \\ 0 & 10 & -20 & 12 & 0 \\ 0 & 0 & -15 & -5 & 0 \end{array} \right|$$

$$\Rightarrow x_2 = \frac{1}{2}(3x_3 - 4x_4 + x_1) = \frac{-1}{5}a$$

$$x_2 = \frac{20x_3 + 12x_4}{10} = \frac{6}{25}a \quad \forall a \in \mathbb{R}$$

$$x_3 = \frac{18}{25}a$$

$$x_4 = a$$

$$v_{\mathcal{G}}(x_1, x_2, x_3, x_4) = \left( -\frac{29}{50}a, \frac{6}{25}a, \frac{18}{25}a, a \right) \quad \forall a \in \mathbb{R}$$

$$\text{Bsp 32: } \left\{ \begin{array}{l} x_1 + 6x_2 + 4x_3 = 0 \\ 2x_1 + 4x_2 - x_3 = 0 \\ -x_1 + 2x_2 + 5x_3 = 0 \end{array} \right. \xrightarrow{\substack{\text{norm} \\ \text{rc}}} \left( \begin{array}{ccc|c} 1 & 6 & 4 & 0 \\ 2 & 4 & -1 & 0 \\ -1 & 2 & 5 & 0 \end{array} \right)$$

$$\xrightarrow{\substack{d_1 - 2d_2 \\ d_3 - d_2}} \left( \begin{array}{ccc|c} 1 & 6 & 4 & 0 \\ 0 & -8 & -9 & 0 \\ 0 & 8 & 9 & 0 \end{array} \right) \xrightarrow{\substack{d_3 + d_2 \\ \text{div} 8}} \left( \begin{array}{ccc|c} 1 & 6 & 4 & 0 \\ 0 & -8 & -9 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

6.  $\text{rank}(A) = \text{rank}(A|I) = 2 < n = 3 \Rightarrow$  kein VSS

$$\left\{ \begin{array}{l} x_1 = -6x_2 - 4x_3 = \frac{11}{4}a \\ x_2 = \frac{9x_3}{-8} = -\frac{9}{8}a \\ x_3 = a \end{array} \right. \quad a \in \mathbb{R}$$

$$V_A(x_1, x_2, x_3) = \left( \frac{11}{4}a, -\frac{9}{8}a, a \right)^T \in \mathbb{R}^3$$